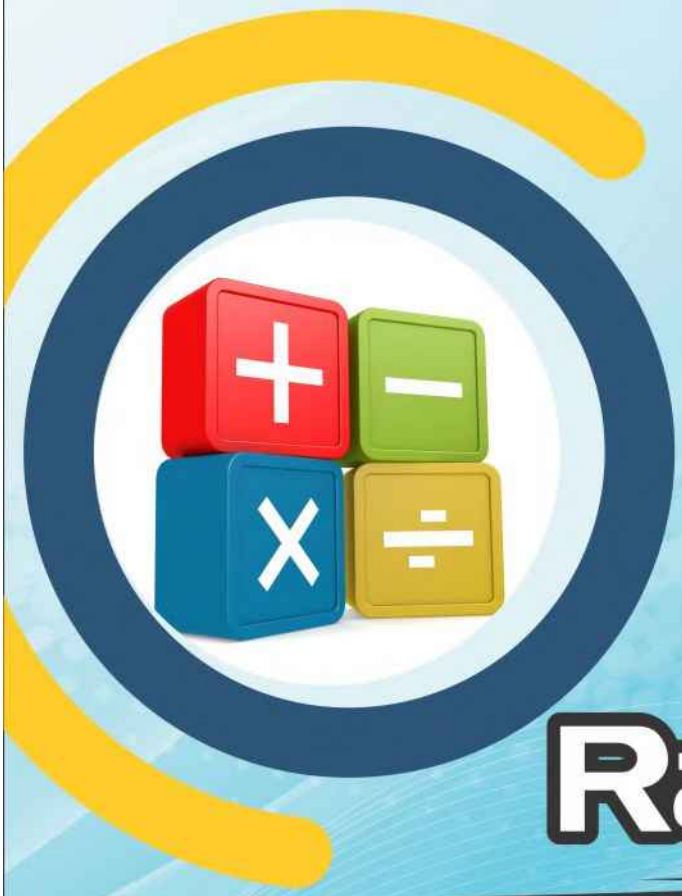


disha
Nurturing Ambitions

Rapid Quantitative Aptitude

with Shortcuts & Tricks for Competitive Exams

Er. Deepak Agarwal
Mr. D.P. Gupta

Includes Arithmetic & Advanced Mathematics

Must for SSC, Railways, Banking, Defence, UPSC & MBA Aspirants

Complete Coverage of all new Pattern Competitive Exams (Including Caselet, Data Analysis & Missing Data Table)

More At @Aj_ebooks

disha
Nurturing Ambitions

Rapid

Quantitative Aptitude

with Shortcuts & Tricks for

Competitive Exams

GP_3386

• **Corporate Office :** 45, 2nd Floor, Maharishi Dayanand Marg, Corner Market,
Malviya Nagar, New Delhi-110017
Tel. : 011-49842349 / 49842350

Typeset by Disha DTP Team

DISHA PUBLICATION

ALL RIGHTS RESERVED

© Copyright Publisher

No part of this publication may be reproduced in any form without prior permission of the publisher. The author and the publisher do not take any legal responsibility for any errors or misrepresentations that might have crept in. We have tried and made our best efforts to provide accurate up-to-date information in this book.

For further information about the books from DISHA,

Log on to **www.dishapublication.com** or email to **info@dishapublication.com**

INDEX

More At @Aj_ebooks

Chapter Name	Heading	Page No.
1. Number System (Including Divisibility)	• Numbers • Prime Numbers • Face Value and Place Value • Rational & Irrational Numbers • Real Numbers • Roman Numbers • Modulus & Its Properties • Divisibility Rules • Properties of Divisibility • Division Algorithm • Prime Factorization • SHORTCUTS.	**1-26**
2. HCF and LCM (Including Factors, Multiples and Prime Factorization)	• Factors and Multiples • Highest Common Factor (HCF) • Least Common Multiple (LCM) • Method to find the HCF & LCM • Rule for finding HCF And LCM of Fractions & Decimals • SHORTCUTS.	**27-46**
3. Fractions and Decimals	• Fractions & their Types • Equivalent Fractions • Multiplicative Inverse of a Non-Zero Fraction • Comparison of Fractions • Addition Subtraction, Multiplication and Division of Fractions • Decimal Fractions • Rounding Off (Approximation) of Decimal Numbers • Special Types of Decimal Numbers • Non-Terminating Recurring Or Non-Recurring Decimal Numbers • SHORTCUTS.	**47-66**
4. Square and square roots, Cube and Cube Roots, Indices and Surds	• Square • Properties Of Squares • Method Of Finding Square of a Number • Pythagorean Triplets • Square Roots • Properties Of Square Roots • Methods To Find Square Roots • Square Root Of Decimal Number & Fractions • Cube • Properties Of Cube • Cube Roots • Method To Find Cube Root Of A Perfect Cube Number • Properties Of Cube Root • Relation Between Unit Digit of a Perfect Cube Number and Unit Digit of its Cube Root • Indices And Surds, Comparison Of Surds • SHORTCUTS.	**67-94**
5. Fundamental Operations, VBODMAS Rule, Algebraic Identities, Approximation and Simplification	• Fundamental Operations • Brackets • VBODMAS Rule • Algebraic Identities • Standard Identities • Basic rules of approximation • SIMPLIFICATION • SHORTCUTS.	**95-118**

6. Polynomials and Rational Expressions (Including Factorization of Polynomial, Factor Theorem & Remainder Theorem)	• Constant • Variables • Algebraic Expressions • Terms of an Algebraic Expression • coefficient of a term • Like and Unlike terms • Types of Algebraic expressions on the basis of number of terms • Polynomials • Degree of a Polynomial • Types of Polynomials on the basis of their degree • Addition, Subtraction, Multiplication of Algebraic Expressions • Division of Polynomials • Remainder Theorem • Factor Theorem • Value of a Polynomial • Zero of a Polynomial • Relation between zero(s) and coefficient of a polynomial • Factorization of Polynomial • Methods of factorization • HCF (or GCD) & LCM of two polynomials • Relation between two polynomials and their HCF and LCM • Rational Expressions • Operations on Rational Expressions • Additive Inverse and Multiplicative Inverse of Rational Expression • SHORTCUTS.	**119-138**
7. Linear Equations, Quadratic equations & Inequalities (Including Simultaneous Linear equations)	• Equation, Linear Equations • Solution of a linear Equation of one variable • Rules for solving a Linear equation in one variable • To find the solutions or Roots of a Linear Equation in two variables a pair of linear equations in two variables • Quadratic Equations • Solutions or Roots of quadratic equation • Discriminate and Nature of the roots of Quadratic Equations, Sum and Product of roots, to find the roots (or solutions) of a quadratic equation • Inequality • Properties of Inequality • Important Results • Notation and Ranges • Sign of roots • SHORTCUTS.	**139-160**
8. Ratio, proportion, variation and partnership	• Ratio, Proportion & Variation • Properties Of Ratios • Comparison Of Ratios • Types Of Ratios • Proportion • Continued Proportion • Properties Of Proportion • Variations • Compound Variation (Repeated Use Of 'Rule Of Three) • SHORTCUTS.	**161-186**
9. Average (Including Mean, Median & Mode	• Average • Weighted Average • Ages and Averages • SHORTCUTS	**187-204**
10. Percentage	• Basic definition of percentage • Decimal to Percentage Conversion • Percentage to Decimal Conversion • Percentage Increase, Percentage Decrease and Percentage Change • Changing a Quantity by a Given Percent • Successive change in a quantity by different given per cent • SHORTCUTS	**205-226**

11. Profit, Loss and Discount	• Profit, Loss and Discount• Cost Price • Selling Price • Profit • Loss • Profit and Loss percentage • Overheads • Dishonest Dealing • Real Profit/Loss Percentage • Marked Price, List Price, Discount, and Successive Discounts • SHORTCUTS.	**227-248**
12. Simple Interest, Compound Interest and Instalments	• Interest • Simple Interest • Compound Interest (C.I.) • Instalments • SHORTCUTS.	**249-274**
13. Sequence and Series (AP, GP, Sum of Special Sequences and Number Series)	• Sequence • Series • Arithmetic Progressions (A.P.) • Arithmetic Mean of *n* Numbers • Geometric Progression (G.P.) • Geometric Mean of *n* Numbers, Relation between Arithmetic Mean and Geometric Mean • SHORTCUTS.	**275-296**
14. Allegation & Mixture	• Allegation, Solving the Problems of Allegation, Using Allegation Formula • Graphical Representation of Allegation– Cross Method • The Straight Line Approach to Solve the Problems Related to Allegations • Recognition of Different Situations Where Allegation can be Used, • Some Keys to Identify A_1, A_2 & A_w and Differentiate these from n_1 and n_2 • A Typical Problem • Mixture of more Than Two Ingredients • SHORTCUTS.	**297-316**
15. Time, Work and Wages (Including Pipes & Cistern)	• Concept of efficiency • Concept of negative work • Concept of man-days • Work Done • Work Done Equation • Extension of the Concept of Time and Work, Pipes and Cistern • Alternate Work • SHORTCUTS	**317-346**
16. Time, Speed and Distance (Including Trains, Boats and Stream, Circular Motion, Races and Games)	• Motion or Movement • Conversion of kmph to m/s and Vice-versa • Average Speed • Relative Speed • Concept Related to Motion of Trains • Boats and Streams • Circular Motion • Basic terminology Related to Races • SHORTCUTS	**347-374**
17. Clock and Calendar	• Clocks • Other Important Concepts • Concept of Incorrect Clocks • Calendars • Leap Year • Counting Odd Days • Concept of Total Odd Days in 100, 200, 300 and 400 Year • SHORTCUTS	**375-390**

18. Permutation & Combination and Probability	• Permutation & Combination • Fundamental Principle of Counting • Multiplication Principle • Addition Principle • Factorials • Problems of Permutations and Combinations • Counting Formula for linear Permutations (• Without Repetition & With Repetition) • Number of Linear Permutations under Certain Conditions • Circular Permutations • Counting Formula for Combination • Division and Distribution of objects • Finding the Rank of a Word • Probability • Concept of Probability • Mathematical Definition of Probability • Range of Probability of any Event • Odds Against and Odds in Favour of an Event • Mutually Exclusive Events • Independent Events • Geometrical Applications • Probability Regarding *n* letters and their Envelopes	**391-416**
19. Geometry-I Lines & Angles, Quadrilateral, Polygons, Circles (Including Chords, Secant and Tangent)	• Lines and Angles • Polygons • Interior and Exterior Angles of a Regular Polygon • Diagonals of a Regular Polygon • Properties of a Regular Polygons • Quadrilaterals • Types of Quadrilaterals • Diagonal Properties of all Parallelograms • Circles • Basic Properties of a Circle • Important Points	**417-464**
20. Geometry-II Triangles (Including Similarity and Congruence)	• Triangle • Basic Properties and Some Important Theorems of Triangles • Determination of Nature of Triangle • Important Terms Related to a Triangle • Congruency of Two Triangles • Conditions of Congruency • Similarity of Two Triangles • Conditions of Similarity • Important Points	**465-492**
21. Coordinate Geometry	• Rectangular Coordinate Axes • Coordinates • Quadrants of Coordinate Plane and Sign of *x* and *y*-Coordinate of a Point in Different Quadrants• Plotting Point whose Coordinates are Known • Distance Formula • Applications of Distance Formula• Section Formula • Coordinates of Some Particular Points • Area of a Triangle • Image of a Point • Equation of Straight Line Parallel to an Axis • Slope of a Straight Line • Equation of Straight Lines • Different Forms of the Equation of a Straight Line	**493-506**
22. Mensuration-Plane Figures`	• Basic Conversion of Units, Plane Figures, • Area of a Triangle • Area of an Equilateral TriangleArea. of a Quadrilateral • Area of a Parallelogram • Area of a Rectangle • Area of a Rhombus • Area of a Trapezium • Area of a Regular Hexagon • Area of Irregular Plane Figures • Paths • Area related to a circle • Circular Ring • Semi-circle • Sector of a Circle • Segment of a Circle	**507-538**

23. Mensuration-Solid Figures	• Cuboid • Cube • Cylinder • Hollow Cylinder • Cone • Frustum of a Cone • Sphere • Hemisphere • Hollow Sphere or Spherical Shell • Hemispherical Bowl • Prism • Pyramid	**539-558**
24. Set Theory (Including Venn Diagram)	• SETS • Representation of Sets • Standard Symbols of Some • Special Sets • Types of sets • Subsets • Universal set • Venn Diagrams • Disjoint sets • Number of elements in a set	**559-570**
25. Trigonometry (Including Height and Distances, Second, Minute, Digree, Radians, Relation $\theta = \frac{l}{r}$, Minimum and Maximum Value of Trigonometric Functions)	• Trigonometric Ratio • Trigonometric Ratios For Complementary Angles System of Measurement of • Angles • Relation Between Arc Radius And Central Angle Area of a Sector • Important Results Angle of Elevation and • Angle of Depression. (to Find the Height Distance of an Inaccessible tower Standing on a Horizontal Plane)	**571-602**
26. Logarithm	• Definition • Laws of logarithm • Some important properties	**603-614**
27. Data Analysis and Data Interpretation (Including Caselet, Table, Line Graph, Bar Graph, Mixed Bar)	• Data Analysis • Data Interpretation • Caselets • Tables • Graphs (Bar Graph, • Line Graph (LG), Pie Graph (PG)	**615-648**
28. Data sufficiency	• Introduction • Problem Format	**649-668**

CHAPTER

1

Number System

(Including Divisibility)

More At @Aj_ebooks

NUMBERS

Digits

The ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called digits.

Natural Numbers/Positive Numbers/Positive Integers

Counting numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, etc. are called natural numbers/ positive integers. There are infinite natural numbers and smallest natural number is 1.

Negative Integers

Natural numbers with negative (–) sign i.e. –1, –2, –3, –4, etc. are called negative numbers.

Whole Numbers

Zero (0) and natural numbers are called whole numbers i.e. 0, 1, 2, 3, 4, etc.

Digits

Natural numbers, their negative and zero (0) are called integers i.e. .....–4, –3, –2, –1, 0, 1, 2, 3, 4, ...

Even Natural Numbers

Those natural numbers which are divisible by 2 are called even natural numbers i.e. 2, 4, 6, 8, ...
Each even natural number is formed by $2n$ where n is number of terms from starting.

Odd Natural Numbers

Those natural numbers which are not divisible by 2 are called odd natural numbers i.e. 1, 3, 5, 7, ...
Each odd number is formed by $(2n - 1)$ where n is no. of given terms from starting.

Prime Numbers

A number other than 1 is called a **prime number** if it is divisible by only 1 and itself.
All prime numbers less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
Note that 2 is the smallest prime number. 2 is the only even prime number.
Smallest odd prime number is 3.

Twin Primes
A pair of prime numbers are said to be twin prime when they differ by 2. For example 3 and 5 are twin primes.

Co-primes or Relative Primes
A pair of numbers are said to be co-primes or relative primes to each other if they do not have any common factor other than 1. **For example:** 13 and 21.

Some Properties which Help in Finding Two Co-prime Numbers

(i) Two consecutive natural numbers are always co-prime.
For example: 8 and 9 are co-prime.
Also 12 and 13 are co-prime.

(ii) Two consecutive odd integers are always co-prime.
For example: 7, 9; 15, 17; 21, 23; etc.

(iii) Two prime numbers are always co-prime.
For example: 19 and 23 are co-prime.
Also 29 and 41 are co-prime.

(iv) A prime number and a composite number such that the composite number is not a multiple of the prime number are always co-prime.
For example: 7 and 15 are co-prime.

(v) Square of two co-prime numbers are always co-prime numbers.

GP_3386

Face Value and Place Value

Face value is absolute value of a digit.
Place value (or Local value) is value of a digit in relation to its position in the number.
For example : Face value and Place value of 9 in 14921 are 9 and 900 respectively.

RATIONAL NUMBERS

Any number that/which can be put in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a **rational number** i.e. $\frac{4}{7}, \frac{3}{5}, \frac{5}{1}, \frac{0}{2}$ etc.

Note that every integer is a rational number. Average of any two rational numbers is also a rational number. There are infinite number of rational number between any two rational numbers.
Decimal representation of any rational number is either terminating decimal number or non-terminating recurring/ repeating decimal number. So, all terminating decimal numbers (2.348, 0.07315, etc.) and all non-terminating repeating/recurring decimal numbers ($54.3\overline{42}, 0.06\bar{3}, 5.\bar{2}$, etc.) are also rational numbers.

IRRATIONAL NUMBERS

Any number which is not rational i.e. which can not be put in the form $\frac{p}{q}$; where p and q are integers and $q \neq 0$, is called **irrational number.**

For example: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \quad 2+\sqrt{3}, \sqrt{3}-5, 4\sqrt{7}$ etc. are irrational numbers.
Decimal representation of any irrational number is always non-terminating and non-terminating recurring/ repeating decimal number. So, all non-terminating and non-recurring/ repeating decimal numbers (68.3010010001....., 0.233223332223333..... etc.)

Conversion of Rational Number of the Form Non-terminating Recurring Decimal into the Rational Number of the Form $\frac{p}{q}$

First, write the non-terminating repeating decimal number in recurring form i.e., write

64.20132132132..... as $64.20\overline{132}$

Then, using formula given below we find the required $\frac{p}{q}$ form of the given number.

Rational number in the form $\frac{p}{q}$

$$= \frac{\left[\begin{array}{l}\text{Complete number neglecting}\\ \text{the decimal and bar over}\\ \text{repeating digit(s)}\end{array}\right] - \left[\begin{array}{l}\text{Non-recurring part of}\\ \text{the number neglecting}\\ \text{the decimal}\end{array}\right]}{m \text{ times 9 followed by } n \text{ times 0}}$$

where m = number of recurring digits in decimal part and n = number of non-recurring digits in decimals part

Thus, $\frac{p}{q}$ form of $64.20\overline{132} = \frac{6420132 - 6420}{99900}$

$$\frac{6413712}{99900} = \frac{534476}{8325}$$

In short ; $0.\bar{a} = \frac{a}{9}, 0.\overline{ab} = \frac{ab}{99}, 0.\overline{abc} = \frac{abc}{999}$, etc. and

$0.a\bar{b} = \frac{ab-a}{90}, 0.a\overline{bc} = \frac{abc-a}{990}, 0.ab\bar{c} = \frac{abc-ab}{900}$,

$0.ab\overline{cd} = \frac{abcd-ab}{9900}, ab \cdot cd\bar{e} = \frac{abcde-abc}{990}$, etc.

Illustration – 1

Convert $2.\overline{46102}$ in the $\frac{p}{q}$ form of rational number.

Solution : Required $\frac{p}{q}$ form $= \frac{246102-2}{99999} = \frac{246100}{99999}$

Illustration – 2

Convert $0.167\overline{3206}$ in the $\frac{p}{q}$ form of rational number.

Solution :

Required $\frac{p}{q}$ form $= \frac{1673206-167}{9999000} = \frac{1673039}{9999000}$

Illustration – 3

Convert 31.026415555... into $\frac{p}{q}$ form of rational number.

Solution : First write 31.026415555... as $31.02641\bar{5}$

Now required $\frac{p}{q}$ form

$= \frac{31026415-3102641}{900000} = \frac{27923774}{900000} = \frac{13961887}{450000}$

REAL NUMBERS

All the existing numbers are real numbers. They are collectively represented by 'R'.

For example: 5, –2, 0, $\frac{4}{7}$, $\sqrt{3}$, $\frac{\sqrt{4}}{3}$, $2+\sqrt{2}$, 5.4, 2.7070070007....., $3.41\overline{25}$ etc. are real numbers.

Sign of the Product of Two Numbers

(a) Product of two positive numbers is always a positive number.
$(+a)\times(+b)=+(a\times b)$
For example: $(+4)\times(+3)=(+)\,12=12$

(b) Product of two negative numbers is always a positive number.
$(-a)\times(-b)=+(a\times b)$
For example: $(-4)\times(-3)=(+)\,12=12$

(c) Product of a negative number and a positive number is always a negative number.
$(-a)\times(+b)=(+a)\times(-b)=-(a\times b)$
For example: $(4)\times(-3)=(-)12=-12$

GREATEST INTEGRAL VALUE

If x be a real number, then $[x]$ indicates greatest integer equal or just less than x.
If the given number is an integer, then the greatest integer gives the number itself, otherwise it gives an integer just less than the given number i.e.
the first integer towards the left of the number x on the number line.
For example: $[4]=4$, $[3,4]=3$, $[6,8]=6$, $[-2.3]=-3$, $[-5,6]=-6$ and so on.
Note that –3 is less than –2.3 and –6 is less than –5.6, etc.

Illustration – 4

What is the value of

$$\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+...+\left[\sqrt{49}\right]+\left[\sqrt{50}\right]$$

when [x] denotes greatest integer function?

Solution : $1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, $8^2=64$

Therefore, from $\left[\sqrt{1}\right]$ to $\left[\sqrt{3}\right]$, the value will be 1, from $\left[\sqrt{4}\right]$ to $\left[\sqrt{8}\right]$ the value will be 2, from $\left[\sqrt{9}\right]$ to $\left[\sqrt{15}\right]$ the value will be 3 and so on.
Therefore, the total value
$=1\times3+2\times5+3\times7+4\times9+5\times11+6\times13+7\times2$
$=3+10+21+36+55+75+14=217$

Illustration – 5

What is the value of x for which $x[x]=32$?

Solution : If the value of x is 5, $x[x]=25$, and if the value of x is 6, then $x[x]=36$
Therefore, the value of x lies between 5 and 6.
If x lies between 5 and 6, then $[x]=5$.

$$\Rightarrow x=\frac{28}{[x]}=\frac{32}{5}=6.4.$$

ROMAN NUMBERS

In this system there are basically seven symbols used to represent the whole Roman number system. The symbols and their respective values are given below.
I = 1, V = 5, X = 10, L = 50,
C = 100, D = 500 and M = 1000
In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur continuously more than three times; the value of the numeral is the sum of the values of the symbols.
For example: LXVII = 50 + 10 + 5 + 1 + 1 = 67.
An exception to the left to the right reading occurs when a symbol of smaller value is followed immediately by a symbol of greater value, then the smaller value is subtracted from the larger. For example.
CDXLVIII = (500 – 100) + (50 – 10) + 5 + 1 + 1 + 1 = 448.

Illustration – 6

The value of the numeral MCDLXIV is:
(a) 1666 (b) 664 (c) 1464 (d) 656

Solution : (c) MCDLXIV = 1000 + (500 – 100) + 50 + 10 + (5 – 1) = 1464

Illustration – 7

Which of the following represents the numeral for 2949?
(a) MMMIXL (b) MMXMIX
(c) MMCMIL (d) MMCMXLIX

Solution : (d) 2949 = 2000 + 900 + 40 + 9
= (1000 + 1000) + (1000 – 100) + (50 – 10) + (10 – 1)
= MMCMXLIX

IMPORTANT CONVERSION

1 trillion = 10^{12} = 1000000000000
1 billion = 10^9 = 1000000000
1 million = 10^6 = 1000000
1 crore = 10^7 = 100 lakh
10 lakh = 10^6 = 1 million
1 lakh = 10^5 = 100000 = 100 thousand
1 thousand = 10^3 = 1000

ABSOLUTE VALUE OR MODULUS OF A NUMBER

Absolute value of a number is its numerical value irrespective of its sign.

If x be a real number N then $|N|$ indicates the absolute value of N.

Thus $|6|=6, |-6|=6, |0|=0, |1|=1, |3.4|=3.4, |-6.8|=6.8$, etc.

$|-6|=6$ can also be written as $|-6|=-(-6)=6$. Thus, if x is a negative number, then $|x|=-x$ and if x is non-negative number, then $|x|=x$

$$\text{Hence } |x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

Properties of a Modulus

(i) $|a| = |-a|$ **(ii)** $|ab| = |a||b|$

(iii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ **(iv)** $|a+b| \leq |a|+|b|$

(The sign of equality holds only when the sign of a and b are same)

(v) If $|a| \leq k \Rightarrow -k \leq a \leq k$

(vi) If $|a-b| \leq k \Rightarrow -k \leq a-b \leq k \Rightarrow b-k \leq a \leq b+k$

Illustration – 8

Solution of the equation $|x-2|=5$ is

(a) 3, – 7 **(b)** – 3, 7

(c) 3, 6 **(d)** None of these

Solution : (b) $|x-2|=5 \Rightarrow x-2=5$ or $x-2=-5$

$\Rightarrow x=7$ or $x=-3$

Illustration – 9

The minimum value of the expression $|17x-8|-9$ is

(a) 0 **(b)** -9 **(c)** $\frac{8}{17}$ **(d)** None of these

Solution : (b) The value of expression $|17x-8|-9$ is minimum only when $|17x-8|$ is minimum. But the minimum value of $|k|$ is zero.

Hence minimum value of $|17x-8|-9=0-9=-9$

PROPERTIES OF OPERATIONS

The following properties of addition, subtraction and multiplication are valid for real numbers a , b and c.

(a) Commutative property of addition :
$a+b=b+a$

(b) Associative property of addition :
$(a+b)+c=a+(b+c)$

(c) Commutative property of multiplication:
$a \times b = b \times a$

(d) Associative property of multiplication :
$(a \times b) \times c = a \times (b \times c)$

(e) Distributive property of multiplication with respect to addition :
$(a+b) \times c = a \times c + b \times c$

DIVISIBILITY RULES

Divisibility by 2

A number is divisible by 2 if its unit's digit is even or 0.

Divisibility by 2

A number is divisible by 3 if the sum of its digits is divisible by 3.

Divisibility by 2

A number is divisible by 4 if the last 2 digits are divisible by 4, or if the last two digits are 0's.

Divisibility by 2

A number is divisible by 5 if its unit's digit is 5 or 0.

Divisibility by 2

A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

Divisibility by 7

A number is divisible by 7 if unit's place digit is multiplied by 2 and subtracted from the remaining digits and the number obtained is divisible by 7.

For example:

$1680\boxed{7} = 1680 - 7 \times 2 = 1666$

It is difficult to decide whether 1666 is divisible by 7 or not. In such cases, we continue the process again and again till it become easy to decide whether the number is divisible by 7 or not.

$166\boxed{6} \longrightarrow 166 - 6 \times 2 = 154$

Again $15\boxed{4} \longrightarrow 15 - 4 \times 2 = 7$, divisible by 7

Hence 16807 is divisible by 7.

Divisibility by 2

A number is divisible by 8 if the last 3 digits of the number are divisible by 8, or if the last three digits of a number are zeros.

Divisibility by 2

A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 2

A number is divisible by 10 if its unit's digit is 0.

Divisibility by 2

A number is divisible by 11 if the sum of digits at odd and even places are equal or differ by a number divisible by 11.

Divisibility by 2

A number is divisible by 12 if the number is divisible by both 4 and 3.

Divisibility by 2

A number is divisible by 13 if its unit's place digit is multiplied by 4 and added to the remaining digits and the number obtained is divisible by 13.

For example:

$219\boxed{7} \longrightarrow 219 + 7 \times 4 = 247$

Again $24\boxed{7} \longrightarrow 24 + 7 \times 4 = 52$, divisible by 13.

Hence 2197 is divisible by 13.

Divisibility by 2

A number is divisible by 14 if the number is divisible by both 2 and 7.

Divisibility by 2

A number is divisible by 15 if the number is divisible by both 3 and 5.

Divisibility by 2

A number is divisible by 16 if its last 4 digits is divisible by 16 or if the last four digits are zeros.

Divisibility by 2

A number is divisible by 17 if its unit's place digit is multiplied by 5 and subtracted from the remaining digits and the number obtained is divisible by 17.

For example:

$491\boxed{3} \longrightarrow 491 - 3 \times 5 = 476$

Again, $47\boxed{6} \longrightarrow 47 - 6 \times 5 = 17$, divisible by 17.

Hence 4913 is divisible by 17.

Divisibility by 2

A number is divisible by 18 if the number is divisible by both 2 and 9.

Divisibility by 2

A number is divisible by 19 if its unit's place digit is multiplied by 2 and added to the remaining digits and the number obtained is divisible by 19.

For example:

$4873\boxed{7} \longrightarrow 4873 + 7 \times 2 = 4887$

$488\boxed{7} \longrightarrow 488 + 7 \times 2 = 502$

$50\boxed{2} \longrightarrow 50 + 2 \times 2 = 54$ not divisible by 19.

Hence 48737 is not divisible by 19.

Properties of Divisibility

(i) The product of 3 consecutive natural numbers is divisible by 6.

(ii) The product of 3 consecutive natural numbers, the first of which is even, is divisible by 24.

(iii) Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

(iv) Any number written in the form $(10^n - 1)$ is divisible by 3 and 9.

(v) Any number with number of digits equal to multiple of 6, is divisible by each of 7, 11 and 13 if all of its digits are same.

For example: 666666, 888888, 333333333333 are all divisible by 7, 11 and 13.

Note that multiples of 6 are 6, 12, 18, 24, 30,.... etc.

(vi) Any number in the form abcabc (a, b, c are three different digits) is divisible by 1001.

(vii) (a) $(a^n - b^n)$ is divisible both by $(a + b)$ and $(a - b)$, when n is even.

(b) $(a^n - b^n)$ is divisible only by $(a - b)$, when n is odd.

DIVISION ALGORITHM

Dividend = (Divisor × Quotient) + Remainder

Divisor) Dividend (Quotient

Remainder

GP_3386

where, Dividend = The number which is being divided
Divisor = The number which performs the division process
Quotient = Greatest possible integer as a result of division
Remainder = Rest part of dividend which cannot be further divided by the divisor.

Illustration – 10

On dividing 397246 by a certain number, the quotient is 865 and the remainder is 211. Find the divisor.

(a) 549 (b) 546 (c) 548 (d) 459

Solution : (d) Applying are above formula, we get

$$\text{Divisor} = \frac{397246 - 211}{865} = 459$$

General or Expanded Form of 2 and 3 Digits Numbers

(i) In a two digits number AB, A is the digit of tenth place and B is the digit of unit place, therefore AB is written using place value in expanded form as

$AB = 10A + B$

For example: $35 = 10 \times 3 + 5$

(ii) In a three digits number ABC, A is the digit of hundred place, B is the digit of tenth place and C is the digit of unit place, therefore ABC is written using place value in expanded form as

$ABC = 100A + 10B + C$

For example: $247 = 100 \times 2 + 10 \times 4 + 7$

These expanded forms are used in forming equations related to 2- and 3- digits numbers.

Illustration – 11

In a two digit prime number, if 18 is added, we get another prime number with reversed digits. How many such numbers are possible ?

Solution : Let a two-digit number be pq.

$\therefore \quad 10p + q + 18 = 10q + p$

$\Rightarrow \quad -9p + 9q = 18 \Rightarrow q - p = 2$

Satisfying this condition and also the condition of being a prime number (pq and qp both), there are 2 numbers 13 and 79.

PRIME FACTORISATION

It is a process of representing a given number as a product of two or more prime numbers.

Each prime number which is present in the product is called a **prime factor** of the given number.

For example: 12 is expressed in the factorised form in terms of its prime factors as $12 = 2 \times 2 \times 3 = 2^2 \times 3$.

Number of Ways of Expressing a Composite Number as a Product of Two Factors

(i) Number of ways of expressing a composite number N which is not a perfect square as a product of two factors

$= \frac{1}{2} \times$ (Number of prime factors of the N)

(ii) Number of ways of expressing a perfect square number M as a product of two factors

$= \frac{1}{2}$ [(Number of prime factors of $M + 1$]

Illustration – 12

Find the number of ways of expressing 180 as a product of two factors.

Solution : $180 = 2^2 \times 3^2 \times 5^1$

Number of factors $= (2 + 1)(2 + 1)(1 + 1) = 18$

Since 180 is not a perfect square, hence there are total $\frac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

Illustration – 13

Find the number of ways expressing 36 as a product of two factors.

Solution : $36 = 2^2 \times 3^2$

Number of factors $= (2 + 1)(2 + 1) = 9$

Since 36 is a perfect square, hence the number of ways of expressing 36 as a product of two factors $= \frac{9+1}{2} = 5$, as $36 = 1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9$ and 6×6.

SHORTCUTS

Shortcut Approach - 1

When two numbers are divided by a third number, leave the same remainder, then the difference of these two numbers is always perfectly divisible by third number.

Illustration – 14

451 and 607 are divided by a number and we get the same remainder in both the cases. Find all the possible divisors (other than 1).

Shortcut Solution :

607 – 451 = 156 is perfectly divisible by those numbers (divisors).

Now, $156 = 2 \times 2 \times 3 \times 13$

Thus, 1-digit numbers = 2, 3, 2×2, 2×3 = 2, 3, 4, 6

2-digit numbers = 12, 13, 26, 39, 52, 78

3-digit number = 156

Shortcut Approach - 2

To find the greatest *n* digits number completely divisible by a divisor. Find the remainder by dividing greatest *n* digits number by the divisor.

Required number = (Greatest *n* digits number) – (Remainder)

Illustration – 15

Find the greatest 5 digits number completely divisible by 245.

(a) 99965 (b) 99958 (c) 99960 (d) 99975

Shortcut Solution : (c)

Greatest 5 digits number = 99999

Divisor = 245

Remainder = 39

Required number = 99999 – 39 = 99960

```
245 | 99999 ( 408
      980
      ----
       1999
       1960
       ----
         39
```

Shortcut Approach - 3

To find the least *n* digits number completely divisible by a divisor. Find the remainder by dividing least *n* digits number by the divisor.

Required number = (Least *n* digits number) + (Divisor – Remainder)

Illustration – 16

Find the least 4 digits number completely divisible by 65.

(a) 1040 (b) 1025

(c) 1020 (d) None of these

Shortcut Solution : (a)

Least 4 digits number = 1000

Divisor = 65

Remainder = 25

Required number = 1000 + (65 – 25) = 1040.

```
65 | 1000 ( 15
     65
     --
      350
      325
      ---
       25
```

Shortcut Approach - 4

If sum and difference of two numbers are given, then Product of the two numbers

$$= \frac{\text{(Sum + Difference)} \times \text{(Sum – Difference)}}{4}$$

Illustration – 17

The sum of two numbers is 33 and their difference is 21. Find the product of the two numbers.

(a) 162 (b) 126 (c) 102 (d) 216

Shortcut Solution : (a)

$$\text{Product} = \frac{(\text{Sum} + \text{Difference}) \times (\text{Sum} - \text{Difference})}{4}$$

$$= \frac{(33+21)\times(33-21)}{4} = \frac{54\times12}{4} = 162.$$

Shortcut Approach - 2

Sum of the digits of a given two digit number is *S*. When its digits are interchange their places, the number decreased by *D*. Then,

$$\text{Given number} = 5\left(S + \frac{D}{9}\right) + \frac{1}{2}\left(S - \frac{D}{9}\right)$$

Illustration – 18

Sum of the digits of a given two digit number is 13. When its digits interchange their places, the number decrease by 45. Find the number.

(a) 49 (b) 76 (c) 94 (d) 86

Shortcut Solution : (c)

$$\text{Given number} = 5\left(S + \frac{D}{9}\right) + \frac{1}{2}\left(S - \frac{D}{9}\right)$$

$$= 5\left(13 + \frac{45}{9}\right) + \frac{1}{2}\left(13 - \frac{45}{9}\right) = 5\times18 + \frac{1}{2}\times8 = 90 + 4 = 94.$$

Shortcut Approach - 2

Sum of the digits of a given two digit number is S. When its digits interchange their places, the number is increased by I. Then

$$\text{Given number} = 5\left(S - \frac{I}{9}\right) + \frac{1}{2}\left(S + \frac{I}{9}\right)$$

GP_3386

Illustration – 19

Sum of the digits of a given two digits number is 5. When its digits interchange their places, the number increased by 27. Find the given number.

(a) 14 (b) 32 (c) 41 (d) 23

Shortcut Solution : (a)

$$\text{Given number} = 5\left(S - \frac{I}{9}\right) + \frac{1}{2}\left(S + \frac{I}{9}\right)$$

$$= 5\left(5 - \frac{27}{9}\right) + \frac{1}{2}\left(5 + \frac{27}{9}\right) = 5 \times 2 + \frac{1}{2} \times 8 = 14.$$

Shortcut Approach - 7

If difference between a two-digit number and the number obtained by interchanging the digits is given, then difference of the two digits of the two digits number

$$= \frac{\text{Difference in original and new number}}{9}$$

Illustration – 20

If the difference between a two digit number and the number obtained by interchanging its digits is 45, then find the difference of two digits of the original number.

(a) 8 (b) 7 (c) 6 (d) 5

Shortcut Solution : (d)

$$\text{Required difference} = \frac{45}{9} = 5.$$

Shortcut Approach - 2

If ratio of the sum and the difference of two numbers is $a : b$, then

$$\text{Ratio of the two numbers} = \frac{a+b}{a-b}$$

Illustration – 21

Ratio of the sum and difference of two numbers is 2 : 1. Find the ratio of these two numbers.

(a) 3 : 4 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2

Shortcut Solution : (b)

$$\text{Required ratio} = \frac{2+1}{2-1} = \frac{3}{1} = 3:1$$

Shortcut Approach - 2

(i) $(a^n + b^n)$ is divisible by $(a + b)$ when n is odd

(ii) $(a^n - b^n)$ is divisible by both $(a + b)$ and $(a - b)$ when n is even

(iii) $(a^n - b^n)$ is divisible by only $(a - b)$ when n is odd

Illustration – 22

$a^8 - b^8$ is completely divisible by which number where $a = 5$ and $b = 3$?

(a) 16 (b) 31 (c) 25 (d) 33

Shortcut Solution : (a)

$$a^8 - b^8 = (a^4 - b^4)(a^4 + b^4)$$
$$= (a^2 - b^2)(a^2 + b^2)(a^4 + b^4)$$
$$= (5^2 - 3^2)(5^2 + 3^2)(5^4 + 3^4)$$
$$= 16 \times 34 \times 706$$

Hence, $a^8 - b^8$ is divisible by 16

Illustration – 23

$a^9 - b^9$ is completely divisible by

(a) $(a + b)$ (b) $(a^2 - b^2)$ (c) $(a - b)$ (d) $(a^6 - b^6)$

Shortcut Solution : (c)

$a^9 - b^9$. Here 9 is an odd number

Hence this is divisible by $a - b$

Also $(a^3)^3 - (b^3)^3 = (a^3 - b^3)(a^6 + a^3b^3 + b^6)$
$= (a - b)(a^2 + ab + b^2)(a^6 + a^3b^3 + b^6)$

Hence $a^9 - b^9$ is completely divisible by $a - b$

Illustration – 24

What is the HCF of $(a^6 - b^6)$ and $(a^8 - b^8)$?

(a) $a + b$ (b) $a - b$ (c) $a^2 - b^2$ (d) $a^4 - b^4$

Shortcut Solution : (c)

$a^6 - b^6 = (a^2)^3 - (b^2)^3 = (a^2 - b^2)(a^4 + a^2b^2 + b^4)$

$a^8 - b^8 = (a^4 - b^4)(a^4 + b^4) = (a^2 - b^2)(a^2 + b^2)(a^4 + b^4)$

Hence HCF of $(a^6 - b^6)$ and $a^8 - b^8 = a^2 - b^2$

Shortcut Approach - 10

When $(x^n + k)$ is divided by $(x - 1)$,

(a) Remainder $= 1 + k$; if $k < (x - 1)$

(b) Remainder = 1 + (Remainder obtained when k is divided by $x - 1$); if $k > x - 1$

Illustration – 25

Find the remainder when $(5^{23} + 3)$ is divisible by 4.

(a) 2 (b) 1 (c) 5 (d) 0

Shortcut Solution : (d)

Here $k = 3$ and $x - 1 = 4$

$\therefore \quad k < (x - 1)$

remainder $= 1 + 3 = 4$.

But 4 again divisible by 4, so remainder is zero.

Shortcut Approach - 15

A number when divided by d_1 and d_2 successively, leaves the remainder r_1 and r_2 respectively. If the number is divided by $d_1 \times d_2$, then
Remainder = $d_1 \times r_2 + r_1$

Illustration – 26

A number when divided by 8 and 9 successively, leaves the remainder 5 and 7 respectively. Find the remainder when the same number is divided by 72.
(a) 51 (b) 48 (c) 61 (d) 8

Shortcut Solution : (c)
Required remainder = $d_1 \times r_2 + r_1$
Here, $d_1 = 8, r_1 = 5, r_2 = 7$
Hence required remainder = $8 \times 7 + 5 = 61$.

Shortcut Approach - 16

To find the number of zeros at the end of a product, write the product as prime factorisation like (in the prime factorise bases are different prime numbers and powers are natural numbers)
Product = $2^m \times 5^n \times$ .....
Here m and n are natural numbers. Now you know that product of each pair of 2 and 5 gives a zero (0), therefore
The product has m zeros, if $m < n$ and
The product has n zeros, if $m > n$.
The product has m or n zeros, if $m = n$.

Illustration – 27

Find the number of zeros at the end of the product
$70 \times 80 \times 32 \times 2^4 \times 18$
(a) 4 (b) 2 (c) 8 (d) 7

Shortcut Solution : (b)
$70 \times 80 \times 32 \times (2)^4 \times 18$
$= (2 \times 5 \times 7) \times (2^4 \times 5) \times (2^5) \times (2^4) \times (2 \times 3^2)$
$= (2)^{1+4+5+2+1} \times (3)^2 \times (5)^{1+1} \times (7)^1$
$= (2)^{13} \times (3)^2 \times (5)^2 \times 7$
$= (2^{13} \times 5^2) \times (3)^2 \times 7$
Here, $m = 13, n = 2$.
$\therefore m > n$.
Hence, required number of zeros = $n = 2$.

Shortcut Approach - 17

(a) Unit digit of any power of the numbers with unit digit 0, 1, 5 and 6 are respectively 0, 1, 5 and 6 i.e.
$(\ldots\ldots 0)^n = (\ldots\ldots 0)$
$(\ldots\ldots 1)^n = (\ldots\ldots 1)$
$(\ldots\ldots 5)^n = (\ldots\ldots 5)$
$(\ldots\ldots 6)^n = (\ldots\ldots 6)$

(b) If unit digit of any number is other than 0, 1, 5 and 6; then multiply its unit digit by itself least number of times till you get the unit digit as 1, 5 or 6.
Count the number of times you multiply the unit digit by itself.
Let's see
$(2)^4 = 16$ $(3)^4 = 81$
$(4)^2 = 16$ $(7)^4 = 2401$
$(8)^4 = 4096$ $(9)^2 = 81$
Therefore, if n is a natural number, then
Unit digit of $(2)^{4n} = 6$ Unit digit of $(3)^{4n} = 1$
Unit digit of $(4)^{2n} = 6$ Unit digit of $(7)^{4n} = 1$
Unit digit of $(9)^{2n} = 1$

(c) Unit digit of a product $a \times b \times c \times$ ..... is the unit digit of the product of the unit digit of $a, b, c,$ .....
Using the facts given above in section (a), (b) and (c), you can find the unit digit of any power of a number.

Illustration – 28

Find the digit at unit place in $(4023)^{39}$
(a) 7 (b) 5 (c) 2 (d) 1

Shortcut Solution : (a)
Digit at unit place in 4023 = 3
Now, $(3)^{39} = (3)^{4 \times 9 + 3} = (3^{4 \times 9}) \times (3^3)$
Unit digit of $(3^{4 \times 9}) = 1$
Unit digit of $(3^3) = 7$
Hence unit digit of $(3^{4 \times 9}) \times (3^3) = 1 \times 7 = 7$.

Shortcut Approach - 18

If N is a composite number and $N = a^p b^q c^r$ .... where a, b, c are different prime numbers and p, q, r are natural numbers. Then
The number of divisors (factors) of N including 1 and the number N itself = $(p + 1)(q + 1)(r + 1)$ .....

Illustration – 29

Find the number of divisors of 2057, besides unity and itself.
(a) 18 (b) 21 (c) 23 (d) 4

Shortcut Solution : (d)
$2057 = 11 \times 187 = 11 \times 11 \times 17 = (11)^2 \times (17)^1$
Hence, number of divisor = $(2 + 1) \times (1 + 1) = 6$.
Therefore number of divisor besides unity and the given number = $6 - 2 = 4$.

Shortcut Approach - 15

If the sum of squares of two numbers is x and the square of their difference is y, then the product of the two numbers

$$= \left(\frac{x-y}{2}\right)$$

Illustration – 30

The sum of squares of two numbers is 40 and square of their difference is 20. Find the product of the two numbers

(a) 15 (b) 20 (c) 10 (d) 25

Shortcut Solution : (c)

$$\text{The product} = \frac{40-20}{2} = 10.$$

Shortcut Approach - 10

Two different numbers when divided by the same divisor, leaves remainder x and y respectively and when their sum is divided by the same divisor, remainder is z. Then the divisor $= x + y - z$

Illustration – 31

Two different numbers when divided by the same divisor, left remainder 10 and 15 respectively and when their sum was divided by the same divisor, remainder was 2. Find the divisor.

(a) 10 (b) 23 (c) 15 (d) 18

Shortcut Solution : (b)

Divisor $= x + y - z = 10 + 15 - 2 = 23.$

Shortcut Approach - 2

If N be a composite number such that $N = (x)^a (y)^b (z)^c$ .....; where $x, y, z,$ ..... are different prime numbers, then sum of divisors or factors of

$$z = \frac{x^{a+1}-1}{x-1} \times \frac{y^{b+1}-1}{y-1} \times \frac{z^{c+1}-1}{z-1} \times .....$$

Illustration – 32

What is the sum of the divisors of 60?

(a) 186 (b) 148 (c) 128 (d) 168

Shortcut Solution : (d)

$60 = 2^2 \times 3 \times 5$

Hence, sum of the divisor

$$= \frac{2^3-1}{2-1} \times \frac{3^2-1}{3-1} \times \frac{5^2-1}{5-1} = 168.$$

Shortcut Approach - 15

If the product of the numbers is x and the sum of these two numbers is y, then the numbers are given by

$$\text{The larger number} = \left(\frac{y+\sqrt{y^2-4x}}{2}\right) \text{ and}$$

$$\text{The smaller number} = \left(\frac{y-\sqrt{y^2-4x}}{2}\right)$$

Illustration – 33

The product of two numbers is 255 and the sum of these two numbers is 32. Find the smaller number.

(a) 12 (b) 15 (c) 16 (d) 17

Shortcut Solution : (b)

Here $x = 255, y = 32.$

Hence required number

$$= \frac{y-\sqrt{y^2-4x}}{2} = \frac{32-\sqrt{(32)^2-4\times 255}}{2}$$

$$= \frac{32-\sqrt{1024-1020}}{2} = \frac{32-2}{2} = 15.$$

Shortcut Approach - 15

If the product of two numbers is x and the difference between these two numbers is y, then the

$$\text{Large number} = \frac{\sqrt{y^2+4x}+y}{2} \text{ and}$$

$$\text{Smaller number} = \frac{\sqrt{y^2+4x}-y}{2}$$

Illustration – 34

The product of two numbers is 180 and the difference of these two numbers is 3. Find the sum of the two numbers.

(a) 27 (b) 26

(c) 25 (d) None of these

Shortcut Solution : (a)

Here $x = 180$ and $y = 3$

$$\text{Sum of two numbers} = \frac{\sqrt{y^2+4x}+y}{2} + \frac{\sqrt{y^2+4x}-y}{2}$$

$$= \sqrt{y^2+4x} = \sqrt{9+4\times 180}$$

$$= \sqrt{729} = 27.$$

LEVEL - 1

1. 24345 and 33334 are divided by certain number of three digits and the remainder is the same in both the cases. Find the divisor and the remainder.
(a) 103, 6 (b) 809, 3 (c) 101, 4 (d) 109, 5

2. What is the remainder when $1! + 2! + 3! + ... + 100!$ is divided by 7 ?
(a) 0 (b) 5 (c) 6 (d) 3

3. How many numbers, lying between 1 and 500, are divisible by 13?
(a) 40 (b) 38 (c) 41 (d) 46

4. A number when divided successively by 4 and 5 leaves remainder 1 and 4 respectively. When it is successively divided by 5 and 4, then the respective remainders will be:
(a) 1, 2 (b) 2, 3 (c) 3, 2 (d) 4, 1

5. Find the unit digit in the product $(2467)^{153} \times (341)^{72}$.
(a) 6 (b) 7 (c) 8 (d) 9

6. The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is:
(a) 0 (b) 4 (c) 6 (d) 7

7. A number A4571203B is divisible by 18. Find the value of A and B.
(a) 8, 4 (b) 6, 8 (c) 4, 6 (d) 6, 6

8. $55^3 + 17^3 - 72^3$ is divisible by
(a) both 3 and 13 (b) both 7 and 17
(c) both 3 and 17 (d) both 7 and 13

9. Sum of the digits of a given 2–digit number is 12. When its digits interchange their places, the number decreases by 54. Find the number.
(a) 93 (b) 84 (c) 75 (d) 66

10. Sum of the digits of a given 2-digit number is 13. When its digits interchange their places, the number increases by 9. Find the given number.
(a) 76 (b) 94 (c) 67 (d) 85

11. If the difference between a two-digit number and the number obtained by interchanging digits is 36, then find the difference of the digits of the original number.
(a) 1 (b) 2 (c) 3 (d) 4

12. If the product of first sixty positive consecutive integers be divisible by 8^n, where n is an integer, then the largest possible value of n is
(a) 18 (b) 19 (c) 17 (d) 16

13. The product of two terms is 4640 and their difference is 22. Find the sum of their reciprocals.
(a) $\frac{14}{2025}$ (b) $\frac{69}{2320}$ (c) $\frac{19}{3941}$ (d) $\frac{11}{1347}$

14. Ratio of the sum and difference of two numbers is 9 : 1. Find the ratio of these two numbers.
(a) 5 : 4 (b) 9 : 5 (c) 2 : 1 (d) 3 : 2

15. There is one number which is formed by writing one digit 6 times (e.g. 111111, 444444 etc.). Such a number is always divisible by:
(a) 7 and 11 (b) 11 and 13
(c) 7, 11 and 13 (d) None of these

16. Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 and 5?
(a) 4, 0 (b) 0, 4 (c) 0, 0 (d) 4, 4

17. What is the largest number of 4 digits which is completely divisible by 18?
(a) 9990 (b) 9984
(c) 9992 (d) None of these

18. What is the smallest number of 4-digits which is completely divisible by 22?
(a) 1008 (b) 1010 (c) 1002 (d) 1012

19. The least number which must be subtracted from 6709 to make it exactly divisible by 9 is:
(a) 2 (b) 3 (c) 4 (d) 5

20. If $(67^{67} + 67)$ is dividing by 68, the remainder is : **[SSC CGL 2009]**
(a) 61 (b) 67 (c) 63 (d) 66

21. What is the remainder when 4^{96} is divided by 6?
(a) 0 (b) 2 (c) 3 (d) 4

22. The remainder when 7^{84} is divided by 342 is :
(a) 0 (b) 1 (c) 21 (d) 340

23. The number 31131131131131131131 is
(a) divisible by 3 but not by 11
(b) divisible by 11 but not by 3
(c) divisible by both 3 and 11
(d) neither divisible by 3 nor by 11

24. If N is the sum of first 13,986 prime numbers, then N is always divisible by
(a) 6 (b) 4
(c) 8 (d) None of these

25. Find the remainder when $6^{6^{6^{\cdot^{\cdot^{\cdot^{66}}}}}}$ (100 times) divided by 10?
(a) 6 (b) 2 (c) 4 (d) 8

26. How many natural numbers are there which give a remainder of 41 after dividing 1997?
(a) 2 (b) 4
(c) 6 (d) None of these

GP_3386

27. If $653xy$ is divisible by 80 then the value of $x + y$ is
(a) 2 (b) 3 (c) 4 (d) 6

28. The last digit of the LCM of $(3^{2003} - 1)$ and $(3^{2003} + 1)$ is
(a) 8 (b) 2 (c) 4 (d) 6

29. The last digit in $(25_)^{32}$ and $(25_)^{33}$ both is 6. The missing digit is :
(a) 4 (b) 8 (c) 6 (d) 5

30. The last digit of $3^{3^{4n}} + 1$, is
(a) 0 (b) 4 (c) 8 (d) 2

31. The remainder when the number 123456789101112..... 484950 is divided by 16 is
(a) 3 (b) 4 (c) 5 (d) 6

32. If two numbers when divided by a certain divisor give remainder 35 and 30 respectively and when their sum is divided by the same divisor, the remainder is 20, then the divisor is
(a) 40 (b) 45 (c) 50 (d) 55

33. If two numbers are each divided by the same divisor, the remainders are respectively 3 and 4. If the sum of the numbers be divided by the same divisor, the remainder is 2. The divisor is :
[SSC CGL Tier I 2017]
(a) 9 (b) 7 (c) 5 (d) 3

34. Find the sum of the factors of 90.
(a) 214 (b) 234 (c) 224 (d) 226

35. Which of the following is/are true?
(i) $43^3 - 1$ is divisible by 11
(ii) $56^2 + 1$ is divisible by 19
(iii) $50^2 - 1$ is divisible by 17
(iv) $(729)^5 - 729$ is divisible by 5
(a) (i) and (ii) (b) (iii) and (iv)
(c) (ii), (iii) and (iv) (d) (ii) and (iii)

36. Which one of the following statement is correct
(a) 12177 is divisible by 11 but not by 99
(b) 12177 is divisible by 9 but not by 99
(c) 12177 is divisible by 9, 11 and 99
(d) 12177 is divisible by 3 but not by 9

37. If p and $p + 1$ both are prime numbers then what can be said about $p - 1$?
(a) it is a prime number
(b) It is a composite number
(c) Its square is a composite number.
(d) It has only 1 factor

38. How many fractions in the form of p/q are between $\frac{1}{2}$ and $\frac{1}{4}$.
(a) 1 (b) More than 1 but finite
(c) Infinite (d) More than 1 but less than 100

39. Which one of the following is definitely true?
(a) (+ve)(+ve)(−ve) = +ve
(b) (−ve)(−ve)(+ve) = −ve
(c) $(-ve)^4 - (+ve)(-ve) = +ve$
(d) $(-ve)^5 + (+ve)^2 = -ve$

40. A number N is divisible by 3 and 4 but not by 9 then which one of the following cannot be an integer?
(a) N/6 (b) N/42 (c) N/18 (d) N/21

41. How many numbers between 100 and 1000 contain at least one 8?
(a) 242 (b) 252 (c) 260 (d) 248

42. Find the last two digits of 7^{1252}
(a) 07 (b) 01 (c) 49 (d) 43

43. Find the unit digit of
$7^7 + 77^{77} + 777^{777}$
(a) 7 (b) 5 (c) 3 (d) 1

44. Find the highest power of 20 in 100!.
(a) 49 (b) 97 (c) 24 (d) 48

45. If N is sum of factorials of all the prime numbers less than 100, find the last two digits of N
(a) 48 (b) 58 (c) 68 (d) 78

46. Find the highest power of 10 in 50! + 60! + 70!
(a) 12 (b) 14 (c) 16 (d) 17

47. Find the highest power of 10 in 50! × 60! × 70!
(a) 40 (b) 42 (c) 38 (d) 44

48. Find the value of $1(1!) + 2(2!) + 3(3!) + \ldots 20(20!)$
(a) 20! − 1 (b) 21! − 1 (c) 22! − 2 (d) 21!

49. n^3 is odd. Which of the following statement (s) is (are) true?
(i) n is odd (ii) n^2 is odd
(iii) n^2 is even
(a) (i) only (b) (ii) only
(c) (i) and (ii) only (d) (i) and (iii) only

50. If $n^2 = 123456787654321$, what is n
(a) 12344321 (b) 1235789
(c) 11111111 (d) 1111111

51. $5^6 - 1$ is divisible by
(a) 5 (b) 13
(c) 31 (d) None of these

52. The remainder obtained when a prime number greater than 6 is divided by 6 is
(a) 1 or 3 (b) 1 or 5 (c) 3 or 5 (d) 4 or 5

53. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is
(a) 18 (b) 26
(c) 31 (d) None of these

54. $2^{73} - 2^{72} - 2^{71}$ is equal to
(a) 2^{69} (b) 2^{70} (c) 2^{71} (d) 2^{72}

55. How many times does the digit 6 appear when you count from 11 to 100?
(a) 9 (b) 10 (c) 19 (d) 20

56. The last digit of 3^{40} is
(a) 1 (b) 3 (c) 7 (d) 9

57. Find the unit place $(327)^{640}$
(a) 1 (b) 3 (c) 7 (d) 9

58. The unit digit in the sum of $(124)^{372} + (124)^{373}$ is :
(a) 5 (b) 4 (c) 2 (d) 0

59. Find the last digit of $32^{32^{32}}$
(a) 4 (b) 8 (c) 6 (d) 2

60. The unit digit of the expression
$$\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$$
(a) 7 (b) 9
(c) 8 (d) None of these

61. Find the unit digit of 111!.
(a) 0 (b) 1 (c) 5 (d) 3

62. Find the unit digit of the product of all the prime number between 1 and 99999
(a) 9 (b) 7
(c) 0 (d) None of these

63. Find unit digit of $3 \times 5 \times 7 \times 11 \times \ldots\ldots\ldots \times 211$.
(a) 1 (b) 5 (c) 0 (d) 3

64. If 78 * 3945 is divisible by 11 where * is a digit, then * is equal to :
(a) 1 (b) 0 (c) 3 (d) 5

65. Which of the following number is not divisible by 18?
(a) 54036 (b) 50436 (c) 34056 (d) 65043

66. A 4-digit number is formed by repeating a 2-digit number such of 2525, 3232, etc. Any number of this form is always exactly divisible by :
(a) 7 (b) 11
(c) 13 (d) Smallest 3- digit prime number

67. Which of the following number will not completely divide the $(29)^{37} + (17)^{37}$?

68. $(49)^{15} - 1$ is exactly divisible by :
(a) 50 (b) 51 (c) 29 (d) 8

69. The sum of first 20 odd natural numbers is equal to :
(a) 210 (b) 300 (c) 400 (d) 240

70. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by :
(a) 11 (b) 16 (c) 25 (d) 30

71. A number when divided by 5 leaves remainder 3. What is the remainder when the square of the same number is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

72. Two numbers, when divided by 17, leaves remainder 13 and 11 respectively. If the sum of those two numbers is divided by 17, the remainder will be :
(a) 13 (b) 11 (c) 7 (d) 4

73. When a number is divided by 387, the ramainder obtained is 48. If the same number is divided by 43, the remainder obtained will be:
(a) 0 (b) 3 (c) 5 (d) 35

74. Which of the following numbers will always divide a six – digit number of the form xy xy xy (where $1 \le x \le 9, 1 \le y \le 9$)?
(a) 1010 (b) 10101 (c) 11011 (d) 11010

75. In a division, the divisor is 12 times the quotient and 5 times the remainder. If the remainder is 36, then the dividend is :
(a) 2706 (b) 2726 (c) 2736 (d) 2262

76. For any integral value of n, $3^{2n} + 9n + 5$ when divided by 3 will leave the remainder
(a) 1 (b) 2 (c) 0 (d) 5

77. The remainder obtained when $23^3 + 31^3$ is divided by 54
(a) 0 (b) 1
(c) 3 (d) Can't be determined

78. If the sum of two numbers is 78 and their product is 1352, then find the larger number.
(a) 52 (b) 26 (c) 25 (d) 62

79. If the product of two numbers is 600 and their difference is 10, the find the smaller number.
(a) 30 (b) 20 (c) 10 (d) 40

80. Number of zeroes at the end in the product $1 \times 2 \times 3 \times 4 \times \ldots\ldots \times 25$ is
(a) 5 (b) 6 (c) 10 (d) 25

81. Find the number of zeros at the end of the product $10 \times 20 \times 30 \times \ldots\ldots\ldots \times 1000$
(a) 100 (b) 120 (c) 124 (d) 125

82. Find the number of zeros at the end of the product 100! + 200!
(a) 24 (b) 25
(c) 49 (d) None of these

83. Find the number of zeros at the end of the product 100! × 200!
(a) 49 (b) 24
(c) 73 (d) None of these

84. Find the number of zeros at the end of the product
$1 \times 3 \times 5 \times 7 \times 9 \times 11 \times \ldots\ldots\ldots\ldots \times 99 \times 101$
(a) 24 (b) 5 (c) 2 (d) 0

85. Find the number of zeros at the end of the product $5 \times 10 \times 15 \times \ldots\ldots\ldots \times 75$
(a) 11 (b) 15 (c) 10 (d) 18

86. Find the number of zeros at the end of the product 25! × 32! × 45!
(a) 10 (b) 23 (c) 22 (d) 7

87. Find the number of zeros at the end of the product $21 \times 22 \times 23 \times \ldots\ldots\ldots\ldots \times 59 \times 60$
(a) 14 (b) 4 (c) 10 (d) 12

GP_3386

88. Find the number of zeroes at the end of the product $1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \ldots\ldots\ldots\ldots 10^{10!}$
(a) 51 (b) 10
(c) 5! + 10! (d) None of these

89. Find the number of zeroes at the end of the product $2^2 \times 5^4 \times 4^2 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
(a) 98 (b) 90 (c) 94 (d) 100

90. $(1+2+3+4+\ldots\ldots+15)^n$ will always be divisible by 81 if
(a) n is an integer
(b) n is an odd number
(c) n is an even number
(d) n is a multiple of 4

91. Which of the following can be the product of two 3-digit number **3 and **6 ?
(a) 9798 (b) 1110128 (c) 870938 (d) 870936

92. If the number 2304ab is completely divisible by 80, then what will be the value of a + b?
(a) 4 (b) 9 (c) 6 (d) 8

93. In which of the following form 5 or prime numbers greater than 5 can be represented if k is a natural number?
(a) $6k \pm 1$ (b) $7k \pm 1$ (c) $11k \pm 1$ (d) $17k \pm 1$

94. A number leaves 5 as remainder when it is divided by 9. What will be the possible remainder when same number is divided by 18?
(a) Only 5 (b) Only 14
(c) 5 or 14 (d) Can not be calculated

95. The remainder when $9^{19} + 6$ is divided by 8 is
(a) 2 (b) 3 (c) 5 (d) 7

96. $19^5 + 21^5$ is divisible by
(a) Only 10 (b) Only 20
(c) Both 10 and 20 (d) Neither 10 nor 20

97. If k is a positive integer, then every square integer is of the form
(a) Only 4k (b) 4k or 4k + 3
(c) 4k + 1 or 4k + 3 (d) 4k or 4k + 1

98. $7^{12} - 4^{12}$ is exactly divisible by which of the following number?
(a) 34 (b) 33 (c) 36 (d) 35

99. N is a 1001 digits number consisting of 1001 sevens. What is the remainder when N is divided by 1001?
(a) 7 (b) 700
(c) 777 (d) None of these

100. The number of digits in the product of $5^{72} \times 8^{27}$ is :
(a) 77 (b) 75
(c) 99 (d) None of (a), (b), (c)

101. $2^{73} - 2^{72} - 2^{71}$ is same as :
(a) 2^{72} (b) 2^{71}
(c) 2^{70} (d) None of these

102. What is the remainder of $\frac{6^{36}}{215}$?
(a) 0 (b) 1
(c) 2 (d) None of these

103. The sum of two numbers is 18. The greatest product of these two numbers can be :
(a) 17 (b) 81
(c) 80 (d) can't be determined

104. p is a prime number and $(p^2 + 3)$ is also a prime number. The number of numbers that p can assume is :
(a) 3 (b) 2 (c) 1 (d) can't say

105. The smallest number which must be added to 803642 in order to obtain a multiple of 9 is :
(a) 1 (b) 2 (c) 3 (d) 4

106. Which one of the following statements is correct?
(a) There can be a real number which is both rational and irrational
(b) The sum of two irrational numbers is always irrational
(c) For any real numbers x and y, $x < y \Rightarrow x^2 < y^2$
(d) Every integer is a rational number

107. Which one of the following statements is not correct?
(a) Every integer is a rational number
(b) Every natural number is an integer
(c) Every natural number is a real number
(d) Every real number is a rational number

108. Let n be a positive integer. If $\frac{1}{n}$ has a terminating decimal expansion, then which one of the following is true?
(a) n is of the form 5^x, where x is a positive integer
(b) n is of the form 2^y, where y is non-positive integer
(c) n is of the form $2^x.5^y$ for some non-negative integers x and y
(d) n is of the form 10^z for some positive integer z

109. If a = 0.1818181818..... and b = 0.3030030003....then $(a + b)$ is :
(a) a rational number
(b) perfect number
(c) an irrational number
(d) both (a) and (c)

110. The remainder of $\frac{39^{93!}}{40}$ is :
(a) 0 (b) 1 (c) 39 (d) 13

LEVEL - 2

1. What is the greatest odd number of 4-digits which is completely divisible by 17?
(a) 9996 (b) 9979 (c) 9981 (d) 9991

2. The numbers 1 to 29 are written side by side as follows 1234567891011.....2829 If the number is divided by 9, then what is the remainder?
(a) 3 (b) 1
(c) 0 (d) None of these

3. The integers 1, 2, ..., 40 are written on a blackboard. The following operation is then repeated 39 times; In each repetition, any two numbers, say a and b, currently on the blackboard are erased and a new number a + b – 1 is written. What will be the number left on the board at the end?
(a) 820 (b) 821 (c) 781 (d) 819

4. There are two integers 34041 and 32506, when divided by a three-digit integer n, leave the same remainder. What is the value of n?
(a) 298 (b) 307
(c) 461 (d) can't be determined

5. A certain number when successively divided by 3 and 5 leaves remainder 1 and 2. What is the remainder if the same number be divided by 15?
(a) 7 (b) 8 (c) 9 (d) 10

6. A number is *interesting* if on adding the sum of the digits of the number and the product of the digits of the number, the result is equal to the number. What fraction of numbers between 10 and 100 (both 10 and 100 included) is interesting?
(a) 0.1 (b) 0.11 (c) 0.16 (d) 0.22

7. Find the different divisors of 37800, excluding unity.
(a) 95 (b) 94 (c) 93 (d) 92

8. Find the unit digit of following product
(7) (77) (777) (7777)....(77777...99 times 7).
(a) 1 (b) 5 (c) 7 (d) 3

9. If a two digit number 'ab' = a! + b! then how many such two digit numbers exist?
(a) 0 (b) 1 (c) 2 (d) 3

10. Let N = 1! + 2! + 3! + 4! + ... + 200! Which one about N is true?
(i) N is even (ii) N is odd
(iii) N/2 is Odd
(a) Only (i) (b) Only (ii)
(c) Only (i) & (iii) (d) Only (iii)

11. Find the remainder when 3^N is divided by 11 where N = 1111....1111 (1 is used 111 times).
(a) 2 (b) 5 (c) 3 (d) 10

12. Find the remainder on dividing $(9^{16} + 6)$ by 8.
(a) 5 (b) 7 (c) 2 (d) 3

13. Find the remainder of $\left[\frac{777777....100\text{digits}}{13}\right]$
(a) 1 (b) 7 (c) 3 (d) 5

14. If p is a prime number more than 3 then $p^2 - 1$ is divisible by.
(i) 8 (ii) 6 (iii) 18 (iv) 24
(a) Only (i) & (ii) (b) Only (iii)
(c) Only (i) (d) Only (i),(ii),(iv)

15. Sum of the series $1^2 - 2^2 + 3^2 - 4^2 + ... + 2001^2 - 2002^2 + 2003^2$ is:
(a) 2007006 (b) 1005004
(c) 200506 (d) None of these

16. Find the unit digit of $(624)^{50}$.
(a) 9 (b) 6 (c) 4 (d) 3

17. Find the unit place $(235)^{215} + (314)^{326} + (6736)^{213} + (3167)^{112}$
(a) 5 (b) 6 (c) 7 (d) 8

18. The last digit of the expression $4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + ... + 4^{99} + 9^{100}$ is :
(a) 0 (b) 3
(c) 5 (d) None of these

19. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 ..... + 99^3$.
(a) 0 (b) 1 (d) 2 (d) 5

20. The unit digit of
$2^{3^4} \times 3^{4^5} \times 4^{5^6} \times 5^{6^7} \times 6^{7^8} \times 7^{8^9}$ is :
(a) 0
(b) 5
(c) Can't be determined
(d) None of these

21. The unit digit of the expression $(1!)^{1!} + (2!)^{2!} + (3!)^{3!} + ...+ (100!)^{100!}$
(a) 0 (b) 1 (c) 2 (d) 7

22. Find the unit digit of the expression $888^{9235!} + 222^{9235!} + 666^{2359!} + 999^{9999!}$.
(a) 5 (b) 9
(c) 3 (d) None of these

23. The digit in unit's place of the product $(2464)^{1793} \times (615)^{317} \times (131)^{491}$ is
(a) 0 (b) 2 (c) 3 (d) 5

GP_3386

24. A six-digit number is formed by repeating a three-digit number ; for example, 256, 256 or 678, 678 etc. Any number of this form is always exactly divisible by :
(a) 7 only (b) 11 only (c) 13 only (d) 1001

25. Which of the following will not completely divide $(3^{41} + 7^{82})$?
(a) 4 (b) 52 (c) 17 (d) 26

26. Find the number of zeros at the end of the product $10^1 \times 10^2 \times 10^3 \times 10^4 \times \ldots\ldots\ldots \times 10^{10}$
(a) 10 (b) 55 (c) 50 (d) 45

27. Find the number of zeros at the end of the product $100^1 \times 99^2 \times 98^3 \times 97^4 \times \ldots\ldots\ldots \times 1^{100}$
(a) 970 (b) 1124 (c) 875 (d) 975

28. Find the number of zeros at the end of the product $(8^{123} - 8^{122} - 8^{121})\,(3^{223} - 3^{222} - 3^{221})$
(a) 1 (b) 2 (c) 0 (d) 3

29. Find the number of zeros at the end of the product $2 \times 4 \times 6 \times 8 \times 10 \times \ldots\ldots\ldots \times 200$
(a) 49 (b) 24 (c) 25 (d) 50

30. Find the number of zeros at the end of the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots\ldots\ldots \times 100^{100}$
(a) 1050 (b) 1150 (c) 1200 (d) 1300

31. Find the number of zeros at the end of the product $1^5 \times 2^5 \times 3^5 \times \ldots\ldots\ldots\ldots \times 32^5$
(a) 25 (b) 30 (c) 32 (d) 35

32. If $2 \times 4 \times 6 \times 8 \times 10 \times 12 \times \ldots\ldots\ldots \times 60$ will be divisible by 10^n, then what can be the maximum value of n?
(a) 12 (b) 10 (c) 14 (d) 15

33. Consider the following statements
I. 7710312401 is divisible by 11.
II. 173 is a prime number.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

34. N is a number which when divided by 10 gives 9 as the remainder,
when divided by 9 given 8 as the remainder,
when divided by 8 given 7 as the remainder,
when divided by 7 given 6 as the remainder,
when divided by 6 given 5 as the remainder,
when divided by 5 given 4 as the remainder,
when divided by 4 given 3 as the remainder,
when divided by 3 given 2 as the remainder,
when divided by 2 given 1 as the remainder.
What is N?
(a) 2519 (b) 841 (c) 839 (d) 2521

Hints and Solutions

Level 1

1. (c) Use Shortcut Approach–1

2. (b) $7! + 8! + 9! + 10! + ... + 100 = 7.6! + 8.7.6! + 9.8.7.6! + ... + 100!$ is completely divisible by 7 as each of the terms contain at least one 7 in it.
Now, $1! + 2! + 3! + 4! + 5! + 6!$
$= 1 + 2 + 6 + 24 + 120 + 720 = 873$
which leaves a remainder of 5 when divided by 7.

3. (b) Number divisible by 13, 26, 39, ... 494
Let n be the total numbers $494 = 13 + (n-1) \times 13$
$\Rightarrow n = 38$

4. (b) Complete remainder $= d_1 r_2 + r_1$
$= 4 \times 4 + 1 = 17$
Now, 17 when divided successively by 5 and 4
$\therefore$ The remainders are 2, 3.

5. (b) Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.
Now, 7^4 gives unit digit 1.
$\therefore 7^{153}$ gives unit digit $(1 \times 7) = 7$. Also 1^{72} gives unit digit 1.
Hence, unit's digit in the product $= (7 \times 1) = 7$.

6. (b) Unit digit in 7^4 is 1. So, unit digit in 7^{92} is 1.
$\therefore$ Unit digit in 7^{95} is 3.
Unit digit in 3^4 is 1.
$\therefore$ Unit digit in 3^{56} is 1.
$\therefore$ Unit digit in 3^{58} is 9.
$\therefore$ Unit digit in $(7^{95} - 3^{58}) = (13 - 9) = 4$.

7. (b) The number is divisible by 18 i.e., it has to be divisible by 2 and 9.
$\therefore$ B may be 0, 2, 4, 6, 8.
$A + 4 + 5 + 7 + 1 + 2 + 0 + 3 + B = A + B + 22$.
A + B could be 5, 14 (as the sum can't exceed 18, since A and B are each less than 10).
So, A and B can take the values of 6, 8.

8. (c) $55^3 + 17^3 - 72^3$
$= (55)^3 + (17)^3 - (55 + 17)^3$
$= 55^3 + 17^3 - [(55)^3 + (17)^3 + 3 \times 55 \times 17 \times 72]$
$= -3 \times 55 \times 17 \times 72$

9. (a) Use Shortcut Approach–5

10. (c) Use Shortcut Approach–6

11. (d) Use Shortcut Approach–7

12. (a) Product of first sixty consecutive integers = 60!
$8 = 2 \times 2 \times 2 = 2^3$
highest power of 2 is 60!
$= \left[\frac{60}{2}\right] + \left[\frac{60}{2^2}\right] + \left[\frac{60}{2^3}\right] + \left[\frac{60}{2^4}\right] + \left[\frac{60}{2^5}\right]$
$= 30 + 15 + 7 + 3 + 1 = 56$
highest power of 8 or $(2^3) = \left[\frac{56}{3}\right] = 18$

13. (b) Use Shortcut Approach–4

14. (a) Use Shortcut Approach–8

15. (c) Since 111111 is divisible by each one of 7, 11 and 13, so each one of given type of numbers is divisible by each one of 7, 11, and 13, as we may write, $222222 = 2 \times 111111$, $333333 = 3 \times 111111$, etc.

16. (a) Since the given number is divisible by 5, so 0 or 5 must come in place of \$. But, a number ending with 5 is never divisible by 8. So, 0 will replace \$.
Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.
Hence, digits in place of * and \$ are 4 and 0 respectively.

17. (a) Use Shortcut Approach–2

18. (d) Use Shortcut Approach–3

19. (c) On dividing 6709 by 9, we get remainder = 4.
$\therefore$ Required number to be subtracted = 4.

20. (d) Use Shortcut Approach–9

21. (d) Let us divide the different powers of 4 by 6 and find the remainder.
So remainder for $4^1 = 4, 4^2 = 4, 4^3 = 4, 4^4 = 4, 4^5 = 4, 4^6 = 4$ and so on.
Hence remainder for any power of 4 will be 4 only.

22. (b) $\frac{7^{84}}{342} = \frac{(7^3)^{28}}{342} = \frac{(343)^{28}}{342}$
By remainder theorem, $\frac{(343)^{28}}{342}$ will have the same remainder as $\frac{1^{28}}{342}$ i.e. the remainder is 1.

GP_3386

Alternative :

$$\frac{(343)^{28}}{342} = \frac{(342+1)^{28}}{342}$$

$$= \frac{(342)^{28} + {}^{28}C_1(342)^{27} + ... + {}^{28}C_{27}342 + 1}{342}$$

Clearly, 1 is the remainder.

23. (d) 311 is repeated seven times in the number, 311 is not divisible by 3 but 311 repeated twice is not divisible by 3, but divisible by 11. Similarly 311 repeated thrice is divisible by 3, but not by 11.
As 311 is repeated seven times, which is neither multiple of 2 nor 3.
So, number is not divisible by 3 or 11.

24. (d) N will be an odd number because N is sum of one even number (b) and 13985 odd numbers.
Hence, N will not be divisible by an even number.

25. (a) 6^n (where n is a natural number) will always leaves the remainder 6 when divide by 10.

26. (c) Let us assume that the quotient is Q and divisor is D. Using the condition given in question,
$1997 = QD + 41$
$\Rightarrow$ QD = 1956. Now we will factorize 1956 in two parts such that D (divisor) is more than 41.
Required natural numbers are 163, 326, 489, 652, 978 and 1956.

27. (d) Since $80 = 8 \times 10$ or $80 = 16 \times 5$
Thus y (*i.e.,* unit digit) must be zero.
$\therefore$ $653xy = 653x0$, where $653x0$ must be divisible by 16 or $653x$ is divisible by 8.
Thus the last 3-digit number 53x will be divisible by 8.
Hence, at $x = 6$, we get the required result.
$\therefore$ $x + y = 6 + 0 = 6$

28. (c) The given numbers are two consecutive even numbers, so their HCF = 2
Now, using LCM × HCF = Product of two numbers
$LCM \times 2 = (...6) \times (...8)$
It can be seen now that the unit digit of LCM = 4

29. (c) The last digit in the number must be 6: for only numbers ending in 6, when raised to any power, result in another no. ending in 6.

30. (d) $3^{3^{4n}} + 1 = (27)^{4n} + 1$
Since last digit of $27^{4n} = 1$
last digit of $3^{3^{4n}} + 1 = 1 + 1 = 2$

31. (d) The remainder when a number is divided by 16 is given by the remainder of the last 4 digits divided by 16 (because 10000 is a multiple of 16. This principle is very similar in logic to why we look at last 2 digits for divisibility by 4 and the last 3 digits for divisibility by 8.) Thus, the required answer would be the remainder of 4950/16 which is 6.

32. (b) Divisor $= r_1 + r_2 - r_3 = 35 + 30 - 20 = 45$

33. (c) Use Shortcut Approach–16

34. (b) Use Shortcut Approach–17

35. (b) $(50^2 - 1) = (50 + 1)(50 - 1)$
$= (17 \times 3) \times (7 \times 7)$
hence divisible by 17.
and $(729)^5 - 729 = 729(729^4 - 1)$
$= 729(729^2 - 1)(729^2 + 1)$
$= (729)(729 - 1)(729 + 1)(729^2 + 1)$
$= 729 \times 728 \times 730 \times (729^2 + 1)$
Hence it is divisible by 5.

36. (c) 12177 is divisible by 99 hence it is divisible by 9 and 11 too.

37. (d) p and p + 1 are primes hence p must be 2 then p – 1 = 1 and it is neither prime nor composite number. And it has only one factor.

38. (c) Between any two rational number there exist infinite number of rational numbers.

39. (c) Only option (c) is correct.

40. (c) As per the given condition N must be divisible by 12, so N must be in the form of 12k where k is not divisible by 3 (As N is not divisible by 9). Hence N/18 cannot be an integer.

41. (c) Number of numbers between 100 and 199 that contains at least one 8 is 20 similarly in every 100 we will have 20 numbers except the numbers between 800 to 899. Hence total number of numbers is $100 + 8 \times 20 = 260$

42. (b) Last two digits of exponent of 7 is the easiest one and has cyclicity of 4 only.
$7^1 = 07$ $7^2 = 49$ $7^3 - 43$ $7^4 - 01$
Hence we will divide the given exponent/ power 1252 by 4 since remainder is 0. Hence last point of cycle is the answer.
Required last two digits is 01

43. (a) The unit digit of $7^7 + 77^{77} + 777^{777}$ is same as the unit digit of $7^7 + 77^{77} + 777^{777}$

As we have seen that cyclicity of unit digit of exponent of 7 is 4, hence unit digit of 7^7 is 3. Then unit digit of 7^{77} is same as unit digit of $7^1 = 7$, and unit digit of 7^{777} is same as unit digit of 7^1 or 7. Hence unit digit of given expression must be unit digit of 3 + 7 + 7 and is equal to 7.

44. (c) Since $20 = 2^2 \times 5$

As we have seen in previous question highest power of 2 and 5 in 100! are 97 and 24 respectively.

Since highest power of 2 in 100! is 97 hence that of 2^2 is [97/2] = 48.

Now highest power of 22 is 48 and that of 5 is 24 hence highest power of 20 in 100! is 24.

45. (c) Since last two digits of all the factorials more than 9 is 00 hence required last two digits is given by

$2! + 3! + 5! + 7! = 2 + 6 + 120 + 5040 = 5168$,

Hence required last two digits is 68.

46. (a) Since highest power of 10 in 50!is less than that of 60!and 70!hence required highest power of 10 is same as that of 50! Which is equal to 12.

For further explanation – since highest power of 10 in 50! is 12 hence we can write $50! = P \times 10^{12}$

Similarly highest power of 10 in 60! is 14 hence we can write $60! = Q \times 10^{14}$

Similarly highest power of 10 in 70! is 16 hence we can write $70! = R \times 10^{16}$

Hence we can write $50! + 60! + 70!$

$= P \times 10^{12} + Q \times 10^{14} + R \times 10^{16}$

$= 10^{12}(p + Q \times 10^2 + R \times 10^4)$

Hence highest power of 10 is 12.

47. (b) Similar to above explanation –

$50! \times 60! \times 70!$

$= (P \times 10^{12}) + (Q \times 10^{14}) + (R \times 10^{16})$

$= (PQR)(10^{42})$ hence highest power of 10 is 42.

48. (b) $1(1!) = (2-1)! = 2! - 1!$

$2(2!) = (3-1)2! = 3! - 2!$

$3(3!) = (4-1)3! = 4! - 3!$

$20(20!) = (21-1)20! = 21! - 20!$

Hence sum = 21! – 1

49. (c) If n^3 is odd, then n and n^2 will also be odd.

50. (c) Try to solve this question by finding the pattern. Since $121 = 11^2$ (Two 1's)

$12321 = 111^2$ (Three 1's)

$1234321 = 1111^2$ (Four 1's) hence on the same logic.

$123456787654321 = n^2$ then n should have EIGHT 1's.

51. (c) $(5^6 - 1) = (5^3)^2 - (1)^2 = (125)^2 - (1)^2$

$= (125 + 1)(125 - 1) = 126 \times 124$

$= 31 \times 4 \times 126$ is divisible by 31

52. (b) We can solve this question by taking some general values of prime numbers and then solve, or else since prime number more than 3 must be in the form of either 6k –1 or 6k+1 hence required remainder must be either 1 or 5

53. (b) Odd numbers are not divisible by 2 and there are 50 odd numbers less than 100.

Out of these 50 odd numbers there are 17 numbers which are divisible by 3.

Out of remaining 33 there are 7 numbers which are divisible by 5.

Hence, numbers which are not divisible by 2, 3, 5 = (50 – 17 – 7) = 26

54. (c) $2^{73} - 2^{72} - 2^{71} = 2^{71}\left(2^2 - 2^1 - 1\right) = 2^{71}$

55. (c) From 1 to 100 a particular digit (say 6) appear 20 times.

And same digit appear once from 1 to 10 hence from 10 to 100 it appears 19 times.

56. (a)

3^1	3^2	3^3	3^4
UNIT DIGIT ↓ 3	9	27	81
3	9	7	1

We know 1 raised to any power (natural number) ends with 1.

So, $3^{40} = (3^4)^{10} = 81^{10}$

Unit digit = 1.

57. (a) The unit digit of $(327)^{640}$ will be same as the unit digit of 7^{640}.

7^1	7^2	7^3	7^4
UNIT DIGIT ↓ 7	49	343	2401
7	9	3	1

So, $7^{640} = (7^4)^{(160)} = (2401)^{(160)}$

Hence unit digit = 1.

58. (d) $4^1 \xrightarrow{\text{UNIT DIGIT}} 4$

$4^2 \longrightarrow 6$

$4^3 \longrightarrow 4$

$4^4 \longrightarrow 6$

. .

. .

. .

(ODD NUMBER)

$4 \longrightarrow 4$

(EVEN NUMBER)

$4 \longrightarrow 6$

Hence, unit digit $124^{372} + 124^{373}$ = Unit digit of $(6+4) = 0$.

59. (c) $(\text{EVEN})^{4k} \xrightarrow{\text{UNIT DIGIT}} 6$ or 0.

Any even number (except multiples of 10) raised to a multiple of 4 gives unit digit 6.

$32^{32^{32}} \longrightarrow$ Multiple of 4

$32 \longrightarrow$ Even

Hence, unit digit is 6.

60. (c) $$\frac{1\times2\times3\times4\times5\times6\times7\times8\times9\times10}{100}$$

$$= \frac{1\times2\times3\times2^2\times5\times2\times3\times7\times2^3\times3^2\times2\times5}{2^2\times5^2}$$

$= 1\times2^6\times3^4\times7$

Hence, unit digit of the given expression will be same as the unit digit of $(1\times4\times1\times7) = 8$.

61. (a) $111! = 1\times2\times3\times4\times\ldots\ldots\ldots\times111$

Since 111! contains even numbers and multiples of 5. We know, product of an even number and a multiple of 5 gives the unit digit 0.

Hence unit digit of $111! = 0$.

62. (c) Prime numbers between 1 to 99999 are 2, 3, 5, 7, 11, ...

We know,

$5\times(\text{Even number}) \xrightarrow{\text{Unit Digit}} 0$

Or,

Multiples of $5\times(\text{Even number}) \longrightarrow 0$

Since there is five and an even prime 2 in the product, so unit digit of the product will be 0.

63. (b) Required set – {3, 5, 7, 11, 13, ..., 211}

We know,

$5\times(\text{Even number}) \xrightarrow{\text{UNIT DIGIT}} 0$

and

$5\times(\text{Odd number}) \xrightarrow{\text{UNIT DIGIT}} 5$

Hence, unit digit of $(3\times5\times7\times11\times\ldots\ldots\times211) = 5$.

64. (d) If the difference of alternating sum of digits of a number is a multiple of 11 or 0, then the number will be divisible by 11.

O E O E O E O

7 8 * 3 9 4 5

Sum of digits :

Odd places, $5 + 9 + * + 7 = 21 + *$

Even places, $4 + 3 + 8 = 15$

Odd places – Even places

$\Rightarrow 21 + * - 15 = 6 + *$

$6 + * = 11$ or 0

$6 + * = 0$, $* = -6$ [which is not possible]

$6 + * = 11$, $* = 5$

Hence, $* = 5$.

65. (d) A number will be divisible by 18, if it is divisible by 2 and 9 both.

Here, 65043 is an odd number. So this is not divisible by 2. Hence 18 will not divide the number 65043.

66. (d) Let the 2 digit number be ab, then 4-digit number abab $= 1000a + 100b + 10a + b$

$= 1010a + 101b$

$= 101(10a + b)$

Hence, this number is always divisible by 101 which is smallest 3-digit prime number.

67. (b) $(29)^{37} + (17)^{37}$

$29 + 17 = 46$

[$a^n + b^n$ is always divisible by $(a+b)$ when n is an odd number]

So, this expression will be completely divisible by 46 and its factors. (Divisible by 2, 23, 46)

68. (d) $(49)^{15} - 1 = 49^{15} - 1^{15}$

$(49 - 1) = 48$, $49^{15} - 1^{15}$ is exactly divisible by 48 and its factors. Hence this expression is completely divisible by 8.

$A^n - B^n$ is always divisible by $(A - B)$ when n is odd.

69. (c) First 20 odd natural numbers are {1, 3, 5, 7, ..., 39}

$$\text{Sum} = \frac{20}{2}[1+39] = 400$$

Sum of n number in AP $= \frac{n}{2}$[1st term + Last term]

Alternate Solution

The sum of first n consecutive odd numbers $= n^2$.

So sum, $1 + 3 + \ldots\ldots\ldots + 39 = (20)^2 = 400$.

70. (d) $3^{25} + 3^{26} + 3^{27} + 3^{28}$

$= 3^{25}(1 + 3 + 3^2 + 3^3)$

$= 3^{25} \times 40$

$= 3^{25} \times 4 \times 10 = 3^{24} \times 30 \times 4$

Hence, the given expression will be completely divisible by 30.

71. (d) Let the number 'N' be $5a + 3$

$N = 5a + 3$

$N^2 = (5a + 3)^2$

$\Rightarrow N^2 = 25a^2 + 9 + 30a$

$N^2 = 25a^2 + 30a + 9$

First 2 terms ($25a^2$ and $30a$) are multiples of 5 hence, remainder when N^2 is divided by 5 = remainder when 9 is divided by 5 which is 4.

72. (c) $P = 17a + 13$

$Q = 17b + 11$

$P + Q = 17a + 17b + 13 + 11$

$\Rightarrow P + Q = 17(a + b) + 24$

$17(a + b)$ is divisible by 17. So, remainder when P + Q is divided by 17 $\Rightarrow \frac{24}{17}$, Remainder 7.

73. (c) $N = 387Q + 48$

$N = (43 \times 9)Q + (43 \times 1) + 5$

Hence, remainder obtained when N is divided by 43 = 5.

74. (b) xyxyxy

$= xy \times 10000 + xy \times 100 + xy$

$= xy(10101)$

Hence, 10101 always divide the number.

75. (c) Remainder ⟶ Quotient

36

$36 \times 5 = 180$ | Divisor ↓

$\frac{36 \times 5}{12} = 15$

Dividend = (Quotient) × (Divisor) + Remainder

$= (180 \times 15) + 36 = 2736$.

76. (b) $3^{2n} + 9n + 5$

Clearly, the first two terms of the expression is divisible by 3. Hence remainder

$\frac{5}{3} = 2$.

77. (a) $23^3 + 31^3$

We know, $a^n + b^n$ is always divisible by $(a + b)$, where n is odd.

So, $23^3 + 31^3$ is divisible by 54, hence remainder is 0.

78. (a) Use Shortcut Approach–18

79. (b) Use Shortcut Approach–19

80. (b) Use Shortcut Approach–12

81. (c) $10 \times 20 \times 30 \times 40 \times \ldots\ldots\ldots\ldots \times 1000$.

$= (10 \times 1) \times (10 \times 2) \times (10 \times 3) \times (10 \times 4) \times \ldots\ldots\ldots \times (10 \times 100)$

$= 10^{100}(1 \times 2 \times 3 \times 4 \times \ldots\ldots\ldots\ldots \times 100)$

$= 10^{100} \times 100!$

100 zeros

5	100
5	20
	4

$20 + 4 = 24$ zeros

Hence, total number of trailing zeros in the expression

$= 100 + 24 = 124$.

82. (a) 100! + 200!

100! ↓

5	100
5	20
	4

200! ↓

5	200
5	40
5	8
	1

$40 + 8 + 1 = 49$ trailing zeros in 200!

$20 + 4 = 24$ trailing zeros in 100!.

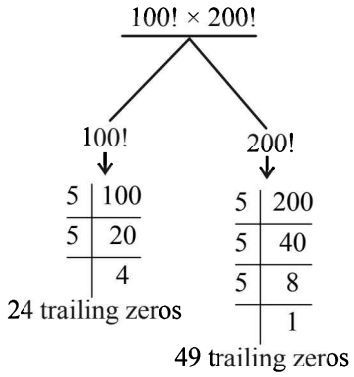
If we add two numbers with 'm' trailing zeros and 'n' trailing zeros, the resultant number will end with 'm' zeros if (m < n) or 'n' zeros if (m > n).

Here, $24 < 49$

Hence 24 trailing zeros.

GP_3386

83. (c)



If two numbers with 'm' and 'n' trailing zeros are multiple together, the resultant number will end with (m + n) zeros.

Hence, number of trailing zeros =49 + 24 = 73.

84. (d) $1 \times 3 \times 5 \times 7 \times \ldots\ldots\ldots\ldots \times 99 \times 101$

In this expression every term is an odd term. So, (2 × 5) pair is not possible here, hence number of trailing zeros in the expression would be zero.

85. (a) $5 \times 10 \times 15 \times \ldots\ldots\ldots \times 75$

$= 5 \times 5 \times 2 \times 5 \times 3 \times \ldots\ldots\ldots\ldots \times 5 \times 15$

$= 5^{15} \times 15!$

Highest power of 2 which divides 15!

2	15
2	7
2	3
	1

$\Rightarrow 7 + 3 + 1 = 11.$

Highest power of 5 which divides 15!

5	15
	3

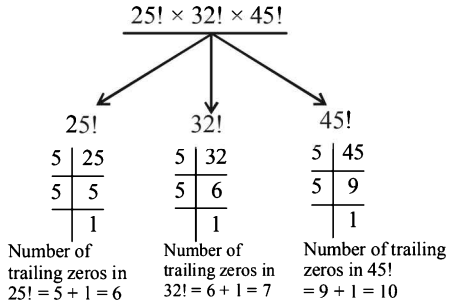
$\Rightarrow 3$

Here (power of 5's) > (power of 2's)

So, number of trailing zeros

= number of (2 × 5) pairs = 11.

86. (b)



Hence, number of trailing zeros in 25! × 32! × 45! = 6 + 7 + 10 = 23.

87. (c) $21 \times 22 \times 23 \times \ldots\ldots\ldots\ldots \times 59 \times 60$

$$= \frac{1 \times 2 \times 3 \times 4 \times \ldots\ldots\ldots\ldots \times 20 \times 21 \times 22 \times \ldots\ldots\ldots \times 60}{1 \times 2 \times 3 \times 4 \times \ldots\ldots\ldots\ldots \times 20}$$

$$= \frac{60!}{20!}$$

Hence, Number of trailing zeros in 60! = 14

Number of trailing zeros in 20! = 4

Required number of trailing zeros = 14 – 4 = 10.

88. (c) Every second term has power of 2's.

(Power of 2's) > (Power of 5's)

So, we have to count number of 5's

$5^{5!}$ contains 5! Fives and $10^{10!}$ contains 10! fives. $[10^{10!} = (2 \times 5)^{10!}]$

Hence, number of 5's in the product = 5! + 10!.

89. (a) $2^2 \times 5^4 \times 4^2 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$

$= 2^2 \times 5^4 \times 2^4 \times 2^8 \times 5^8 \times 2^{10} \times 3^{10} \times 3^{12} \times 5^{12} \times 2^{42} \times 2^{32} \times 5^{16} \times 2^{18} \times 5^{40} \times 5^{18}$

$= 2^{116} \times 3^{22} \times 5^{98}$

Number of (2 × 5) pairs = 98

Hence, number of trailing zeroes = 98.

90. (d) Required sum $= \left(\frac{15 \times 16}{2}\right)^n = 3^n 5^n 2^{3n}$

$\because 81 = 3^4$, so n should be the multiple of 4.

91. (c) The product of the two smallest numbers of three digits = 100 × 100 = 10,000 which is a five digit number. The product of the two smallest numbers of four digits is 10,00,000 which is a seven digits number. So the product of two numbers having three digits will be the number having 5 or 6 digits and the unit place digit in the given number will be 8. Hence (c) is correct.

92. (d) It is clear that b will be zero. The last three digit 4a0 will be divisible by 8 if a = 0 or 8. So, a + b = 0 or 8.

93. (a) Any prime number greater than or equal to 5 can be written in the form of 6k ± 1, where k is a natural number.

94. (c) Let, the number is 9x + 5, if x is an even number then 9x will be divisible by 18 and the remainder will be 5. If x is an odd number then after dividing previous remainder 5 the required remainder will be 9 + 5 = 14.

95. (d) Required remainder

$= (1)^{19} + 6 = 7$

$[\because 8 = 9 - 1$, so replaced 9 by 1]

96. (c) We can check divisibility of $19^5 + 21^5$ by 10 by adding the unit digits of 9^5 and 1^5 which

is equal to 9 + 1 = 10.
So, it must be divisible by 10.
Now, for divisibility by 20 we add 19 and 21 which is equal to 40. So, it is clear that it is also divisible by 20.
So, $19^5 + 21^5$ is divisible by both 10 and 20.

97. (d) If k is a positive integer, then every square integer is of the form 4k or 4k + 1, as every square number is either a multiple of 4 or exceeds multiple of 4 by unity.

98. (b) We know that, $(x^n - y^n)$ is divisible by $(x - y)$ for all n and is divisible by $(x + y)$ for even n.
$\therefore$ $(7^{12} - 4^{12})$ is divisible by (7 + 4) and (7 – 4)
$\Rightarrow$ $(7^{12} - 4^{12})$ is divisible by 11 and 3
$\therefore$ $(7^{12} - 4^{12})$ is divisible by 33.

99. (b) $1001 = 7 \times 11 \times 13$
We know that any digit written 6 times consecutively (like 111111, or 222222 etc.) will be divisible by 1001.
So, if $\underbrace{777777.....996 \text{ times}}$ 77777 will be divided by 100 then remainder will be same as 77777 is divided by 1001. Hence required remainder = 700

100. (b) $5^{72} \times 8^{27} = 5^{72} \times 2^{81} = (5^{72} \times 2^{72}) \times 2^9$
$= 512 \times 10^{72}$
Therefore there are total 75 = (3 + 72) digits in the above expression.

101. (b) $2^{73} - 2^{72} - 2^{71} = 2^{71}(2^2 - 2^1 - 1)$
$= 2^{71}(4 - 2 - 1) = 2^{71}$

102. (b) $\frac{6^{36}}{215} \to \frac{(6^3)^{12}}{215} \to \frac{(216)^{12}}{215} \to \frac{1^{12}}{215} \to 1$
is the remainder.

103. (b) a + b = 18
So maximum of (a × b) will be only when a = b
$\therefore$ a = b = 9
$\therefore$ maximum of a × b = 9 × 9 = 81

104. (c) Only one value p can assume, which is 2.
At p = 2, $p^2 + 3 = 7$, which is also prime.
Again at p = 3, 5, 7, 11, ...
$p^2 + 3$ = an even number which can not be a prime number.

105. (d) $\because$ 8 + 0 + 3 + 6 + 4 + 2 = 23
So, the required number = 27 – 23 = 4
Alternatively :
If we divide 803642 by 9 the remainder is 5.
Now the required number = 9 – remainder
= 9 – 5 = 4

106. (d) The same number can not be rational and irrational both, simultaneously. So (a) is wrong.
Again $(2+\sqrt{3})+(2-\sqrt{3}) = 4$
So, (b) is wrong, as irrational + irrational = rational.
Once again, $-3 < 2 \Rightarrow 9 > 4$

107. (d) As $\sqrt[3]{2}$ is a real number but not a rational number.

108. (c) $\frac{1}{2^x 5^y}$ will be a terminating decimal for non-negative integers x and y.

109. (c) Rational + Irrational = Irrational number

110. (b) $\because \frac{a^n}{a+1}$ gives remainder 1, when n is even.
Now since 93! is an even number, hence the remainder is 1.

Level-2

1. (b) Use Shortcut Approach–2

2. (a) Sum of the digits of the 'super' number
= 1 + 2 + 3 + ... upto last digit
= 45 × 3 + 30 = 165
165 when divided by 9 leave remainder 3.

3. (c) $1 + 2 + 3 + ... + 40 = \frac{40 \times 41}{2} = 820$
Since at each time any two numbers a and b are erased and a single new number $(a + b - 1)$ is writen. Hence, each one is subtracted and this process is repeated 39 times. Therefore, number left on the board at the end = 820 – 39 = 781.

4. (b) Let the common remainder be x. Then numbers $(34041 - x)$ and $(32506 - x)$ would be completely divisible by n.
Hence the difference of the numbers $(34041 - x)$ and $(32506 - x)$ will also be divisible by n
or $(34041 - x - 32506 + x) = 1535$ will also be divisible by n.
Now, using options we find that 1535 is divisible by 307.

5. (a) Use Shortcut Approach–11

6. (a) Let the numbers be the form $10x + y$
According to question
$10x + y = x + y + xy$
$9x = xy$
$\therefore$ $y = 9$
The numbers are 19, 29, 39, 49, 59, 69, 79, 89

GP_3386

and 99 total of 9 numbers

Hence the required fraction $= \frac{9}{91}$

$= 0.099 = 0.1$

9. (a) Use Shortcut Approach–14

8. (d) In this case we have 99 terms each ending with 7 hence required unit digit is same as that of 7^{99}. Unit digit of 7^{99} is same as 7^3 (as 99 when divided by 4 gives remainder 3) or 3.

9. (a) From the given condition $a! + b! = 10a + b$
Since $0! = 1, 1! = 1, 2! = 2, 3! = 6$ and $4! = 24$ since summation is a two digit number hence one of the number must be 4, from trial and error method no such two digit number exist.

10. (b) Since all the factorials except 1! Is even number hence required summation must be odd.

11. (c) Since remainder of $\left[\frac{3^1}{11}\right]$ is 3,

Remainder of $\left[\frac{3^2}{11}\right]$ is 9 (or -2)

Remainder of $\left[\frac{3^3}{11}\right]$ is 5

Remainder of $\left[\frac{3^4}{11}\right]$ is 4

Remainder of $\left[\frac{3^5}{11}\right]$ is 1

Since here cyclicity is of 5 hence we will divide the given power/exponent by 5 here exponent is 1111.....(111 times) which when divided by 5 gives remainder 1 hence remainder of $\left[\frac{3^{1111.....(111\text{ times})}}{11}\right]$ is same as $\left[\frac{3^1}{11}\right] = 3$

12. (b) Use Shortcut Approach–10

13. (c) Since 13 is a prime number hence any digit repeated 12 times (or 12k times) divisible by 13. Hence 77777..96 digits is divisible by 13.

Remainder of $\left[\frac{777777....100\text{ digits}}{13}\right]$

= remainder of $\left[\frac{7777}{13}\right]$ which is equal to 3

14. (d) Since $p > 3$ and a prime number hence
$p = 6k + 1$
$p^2 - 1 = (6k + 1)^2 - 1$
$= 36k^2 + 12 = 12k(3k + 1)$
If k is even $k = 2n$
$12k(3k + 1)$
$= 24n(6n + 1)$ and divisible by 24
If k is odd then $k = 2n + 1$
$12k(3k + 1) = 12(2n + 1)(6n + 2)$
$= 24(2n + 1)(3n + 1)$
And is divisible by 24
Hence it is divisible by 24 and all its factors.
We can prove similar property when
$p = 6k - 1$

15. (a) $1^2 - 2^2 + 3^2 - 4^2 + ... + 2001^2 - 2002^2 + 2003^2$.
$= 1^2 - \{(3^2 - 2^2) + (5^2 - 4^2) + ... + (2003^2 - 2002^2)\}$
$= 1^2 - \{5 + 9 + 13 + ... + 4005\} = 2007006$

16. (b) Use Shortcut Approach–13

17. (d) 5 (Any natural number) $\xrightarrow{\text{UNIT DIGIT}}$ 5.

4 (Even) $\longrightarrow$ 6

6 (Any natural number) $\longrightarrow$ 6

7 (Multiple of 4) $\longrightarrow$ 1.

Hence, unit digit of $(5^{215} + 4^{326} + 6^{213} + 7^{112})$
$= (5 + 6 + 6 + 1) = 18$
Unit digit of $18 = 8$.

18. (a)

4^{ODD} $\xrightarrow{\text{UNIT DIGIT}}$ 4

4^{EVEN} $\longrightarrow$ 6

9^{ODD} $\longrightarrow$ 9

9^{EVEN} $\longrightarrow$ 1

UNIT DIGIT

$4 + 9^2 + 4^3 + 9^4 + 4^5 + 9^6 + ...+ 4^{99} + 9^{100}$

$4 + 1 \quad 4 + 1 \quad 4 + 1 \quad 4 + 1$

$5 \quad 5 \quad 5 \quad 5$

Sum of unit digits = (5 + 5 + 5 + ... 50 terms)
Hence, unit digit is 0.

19. **(a)** Sum of the cubes of first n natural number

$$=\left[\frac{n(n+1)}{2}\right]^2$$

Hence, unit digit of $1^3+2^3+3^3+\ldots\ldots+99^3$

$$=\left[\frac{99(99+1)}{2}\right]^2=(99\times50)^2$$

Unit digit = 0.

20. **(a)** $(\text{Even number})\times5\longrightarrow$ unit digit 0.

$(\text{Odd number})\times5\longrightarrow$ unit digit 5.

Here, 2^{3^4}, 4^{5^6} and 6^{7^8} are even terms and 5^{6^7} is an odd multiple of 5. Hence the unit digit of the expression will be zero.

21. **(d)** $(1!)^{1!}\longrightarrow1\xrightarrow{\text{Unit Digit}}1$

$(2!)^{2!}\longrightarrow2^2\longrightarrow4$

$(3!)^{3!}\longrightarrow6^6\longrightarrow6$

$\left[6^{\text{Anything}}\xrightarrow{\text{Unit digit}}6\right]$

$$(4!)^{4!}\longrightarrow24^{24}\longrightarrow6\begin{pmatrix}4^{\text{Even}}\xrightarrow{\text{Unit digit}}6\\4^{\text{Odd}}\longrightarrow4\end{pmatrix}$$

$$(5!)^{5!}\longrightarrow(120)^{120}\longrightarrow0\left(0^{\text{Anything}}\longrightarrow0\right)$$

After 5! onwards every factorial ends with zero. Hence the unit digit of every term after (5!) 5! will be 0.

Required unit digit

$=(1+4+6+6+0+0+0+0\ldots\ldots+0)=17$

Unit digit is 7.

22. **(b)** $8^{4K}\xrightarrow{\text{UNIT DIGIT}}6$

$9^{4K}\longrightarrow1$

$2^{4K}\longrightarrow6$

$6^{(\text{Any natural number})}\longrightarrow6$

$$(888)^{(9235!)}+(999)^{(9999!)}+(222)^{(9235!)}+(666)^{(2359!)}$$

$$\downarrow\qquad\downarrow\qquad\downarrow\qquad\downarrow$$

$$6\qquad1\qquad6\qquad6$$

Unit digit

Hence, $(6+1+6+6)$ unit digit is 9.

23. **(a)**

$$(2464)^{(1793)}\times(615)^{(317)}\times(131)^{491}$$

$$\downarrow\qquad\qquad\downarrow\qquad\qquad\searrow$$

$$4\xrightarrow[\text{UNIT DIGIT}]{\text{(ODD NUMBER)}}4\qquad 5\xrightarrow[\text{UNIT DIGIT}]{\text{(Any natural no.)}}5\qquad 1\xrightarrow[\text{UNIT DIGIT}]{\text{(Any natural no.)}}1$$

Hence, unit digit of the expression

= Unit digit of $(4\times5\times1)=0$.

24. **(d)** Let the three digit number be abc then 6 digit number abcabc = 100000a + 10000b + 1000c + 100a + 10b + c

= 100100a + 10010b + 1001c

= 1(100a + 10b + c) 1001

So, this number will be always divisible by 1001 and its factors.

$1001=7\times11\times13$

Hence, divisible by 7, 11, 13, 77, 91, 143 and 1001.

25. **(c)** $3^{41}+7^{82}$

$$=3^{41}+(7^2)^{41}$$

$$=\underbrace{3^{41}+49^{41}}_{\downarrow}$$

$3+49=52$

Any number of the form 'A^n+B^n' is always divisible by 'A + B', if n is an odd number.

So, the given expression will be completely divisible by 52 and its factors, since 17 is not a factor of 52 .

26. **(b)** $10^1\times10^2\times10^3\times10^4\times10^5\times\ldots\ldots\times10^{10}$

$=10^{(1+2+3+\ldots\ldots+10)}$

$\left[\because a^m\times a^n=a^{m+n}\right]$

$$=10\left(\frac{10\times11}{2}\right)\left[\because\frac{n(n+1)}{2}=\text{sum of n consecutive natural numbers from 1 to n}\right]$$

$=10^{55}$

Hence, number of zeros at the end of the product = 55.

27. **(b)** In this expression every second term has power of 2's. It means power of 2's is more than power of 5's. So we have to count power of 5's only.

$100^1\times99^2\times98^3\times\ldots\ldots\times1^{100}$

$\Rightarrow$ Number of fives

$=1+6+11+\ldots\ldots+96$

$$\text{Number of terms}=\frac{[96-1]}{5}+1=20$$

$$\text{Sum of n terms} = \frac{n}{2}[\text{First term} + \text{Last term}]$$

$$= \frac{[96+1]}{2} \times 20$$

$= 970$

But, $100^1, 75^{26}, 50^{51}, 25^{76}$ contain $(1 + 26 + 51 + 76)$ fives extra.

Hence, total number of zeros at the end $= 1 + 26 + 51 + 76 + 970 = 1124$.

28. (b) $(8^{123} - 8^{122} - 8^{121})(3^{223} - 3^{222} - 3^{221})$

$= 8^{121}(8^2 - 8 - 1).\ 3^{221}(3^2 - 3 - 1)$

$= 8^{121} \times 3^{221} \times 55 \times 5$

$= 2^{363} \times 3^{221} \times 5 \times 11 \times 5$

$= 2^{361} \times 3^{221} \times 2 \times 5 \times 2 \times 5 \times 11$

Number of (2×5) pairs $= 2$

Hence, number of trailing zeros $= 2$.

29. (b) $2 \times 4 \times 6 \times 8 \times 10 \times \ldots\ldots\ldots \times 200$

$= (2 \times 1) \times (2 \times 2) \times (2 \times 3) \times \ldots\ldots\ldots \times (2 \times 100)$

$= 2^{100} \times (1 \times 2 \times 3 \times \ldots\ldots\ldots \times 100)$

$= 2^{100} \times 100!$

Clearly, number of 5's < number of 2's.

So, number of (2×5) pairs

= Highest power of 5 in 100!

5	100!
5	20
	4

$100! \Rightarrow 20 + 4 = 24$ Fives.

Hence, number of trailing zeros $= 24$.

30. (d) $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots\ldots\ldots\ldots \times 100^{100}$.

Every second term has power of 2's.

So, power of 2's > power of 5.

We have to count power of 5.

Number of 5's,

$$\underbrace{5 + 10 + 15 + \ldots\ldots\ldots\ldots + 100}_{\left[\frac{100+5}{2}\right] \times 20 = 1050} + \left[\begin{array}{l}\text{Number of additional} \\ \text{5's present in multiples} \\ \text{of 25}\end{array}\right]$$

$$\left[\begin{array}{l} 25^{25} \rightarrow 5^{25} \times 5^{25} \rightarrow 25 \text{ from here} \\ 50^{50} \rightarrow 5^{50} \times 5^{50} \times 2^{50} \rightarrow 50 \text{ from here} \\ 75^{75} \rightarrow 3^{25} \times 5^{75} \times 5^{75} \rightarrow 75 \text{ from here} \\ 100^{100} \rightarrow 2^{200} \times 5^{100} \times 5^{100} \rightarrow \\ \qquad\qquad 100 \text{ from here} \\ \qquad\qquad 250 \text{ from here} \end{array}\right]$$

Total number of 5's $= 1050 + 250$

$= 1300$

Hence, total number of trailing zeros $= 1300$.

31. (d) Here, (power of 2's) > (power of 5's)

So, we have to count the power of 5's only.

$$\left[\begin{array}{l} 5^5 \longrightarrow (1 \times 5)^5 \longrightarrow 1^5 \times 5^5 \xrightarrow{\text{Number of 5's}} 5 \\ 10^5 \longrightarrow (2 \times 5)^5 \longrightarrow 2^5 \times 5^5 \longrightarrow 5 \\ 15^5 \longrightarrow (3 \times 5)^5 \longrightarrow 3^5 \times 5^5 \longrightarrow 5 \\ 20^5 \longrightarrow (4 \times 5)^5 \longrightarrow 4^5 \times 5^5 \longrightarrow 5 \\ 25^5 \longrightarrow (5 \times 5)^5 \longrightarrow 5^5 \times 5^5 \longrightarrow 10 \\ 30^5 \longrightarrow (6 \times 5)^5 \longrightarrow 6^5 \times 5^5 \longrightarrow 5 \end{array}\right]$$

Total, $5 + 5 + 5 + 5 + 10 + 5 = 35$ number of 5's.

Hence, number of trailing zeros $= 35$.

32. (c) Use Shortcut Approach–12

33. (c) I. Any number in order to get completely divided by 11, the difference between the sum of even place digits and the sum of odd place digits must have equal to either 0 or the multiple of 11. In 7710312401, difference between sum of even place digits and the sum of odd place digit is 0 .

So, it is divisible by 11.

II. To check divisibility of 173, we can divide the number by all the prime numbers from 2 to 13. It is not divisible by 2, 3, 5, 7, 11 and 13. So, it is a prime number.

Hence, both statements I and II are correct.

34. (a) Answer will be LCM $(2, 3, 4, \ldots\ldots, 9) - 1$

$= 2520 - 1 = 2519$

CHAPTER

2

More At @Aj_ebooks

HCF and LCM

(Including Factors, Multiples and Prime Factorization)

FACTORS AND MULTIPLES

If one number '*a*' completely divides a second number '*b*' then 1st number '*a*' is said to be a **factor** of the 2nd number '*b*'.

For example; 3 completely divides 15, so 3 is a factor of 15; while 4 does not divide 15 completely, so 4 is not a factor of 15.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30

Factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

If a number '*a*' is exactly divisible by a number '*b*' then the 1st number '*a*' is said to be a **multiple** of 2nd number '*b*'.

For example; 35 is exactly divisible by 7, so 35 is a multiple of 7.

Multiple of a number '*b*' can be written down as '*nb*' where *n* is a natural number. So multiples of 5 are 5, 10, 15, 20, 25, ...

HIGHEST COMMON FACTOR (HCF) OR GREATEST COMMON DIVISOR (GCD)

The highest (i.e. largest) number that divides two or more given numbers is called the **highest common factor (HCF)** of those numbers.

Consider two numbers 12 and 15.

Factors of 12 are 1, 2, 3, 4, 6, 12.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30.

We have some common factors out of these factors of 12 and 30, which are 1, 2, 3, 6. Out of these common factors, 6 is the highest common factor. So, 6 is called the **Highest Common Factor (HCF)** of 12 and 30. HCF is also called **GCD**.

Methods to Find The HCF or GCD

There are two methods to find HCF of the given numbers

(i) Prime Factorization Method

When a number is written as the product of prime numbers, then it is called the **prime factorization** of that number.

For example: $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$. Here, $2 \times 2 \times 2 \times 3 \times 3$ or $2^3 \times 3^2$ is called prime factorization of 72.

To find the HCF of given numbers by this method, we perform the prime factorization of all the numbers and then check for the common prime factors. For every prime factor common to all the numbers, we choose the least index of that prime factor among the given numbers. The HCF is the product of all such prime factors with their respective least indices.

Illustration – 1

Find the HCF of $36x^3y^2$ and $24x^4y$.

Solution : $36x^3y^2 = 2^2.3^2.x^3.y^2$, $24x^4y = 2^3.3.x^4.y$. The least index of 2, 3, x and y in the numbers are 2, 1, 3 and 1 respectively. Hence the HCF = $2^2.3.x^3.y = 12x^3y$.

(ii) Division Method

To find the HCF of two numbers by division method, we divide the larger number by the smaller number. Then we divide the smaller number by the first remainder, then first remainder by the second remainder.. and so on, till the remainder becomes 0. The last divisor is the required HCF.

To find the HCF of more than two numbers by this method, first find the HCF of any two of the numbers and then find the HCF of this HCF and the third number and so on. The last HCF will be the required HCF.

GP_3386

Illustration – 2

Find the HCF of 288 and 1080 by the division method.

Solution :

$$\begin{array}{r} 288\overline{)1080}(3 \\ 864 \\ \hline 216\overline{)288}(1 \\ 216 \\ \hline 72\overline{)216}(3 \\ 216 \\ \hline 0 \end{array}$$

The last divisor 72 is the HCF of 288 and 1080.

Illustration – 3

Find the HCF of 1365, 1560 and 1755.

Solution :

$$\begin{array}{r} 1365\overline{)1560}(1 \\ 1365 \\ \hline 195\overline{)1365}(7 \\ 1365 \\ \hline 0000 \end{array}$$

Therefore, 195 is the HCF of 1365 and 1560

Again,

$$\begin{array}{r} 195\overline{)1755}(9 \\ 1755 \\ \hline 0000 \end{array}$$

$\therefore$ Required HCF = 195

Illustration – 4

The numbers 400,536 and 646; when divided by a number N, give the remainders of 22, 23 and 25 respectively. Find the greatest such number N.

Solution : N will be the HCF of (400 – 22), (536 – 23) and (646 – 25). Hence, N will be the HCF of 378, 513 and 621.

Hence, N = 27.

Illustration – 5

The HCF of two numbers is 12 and their product is 31104. How many such numbers are possible.

Solution : Let the numbers be 12x and 12y, where x and y are co-prime to each other.

Therefore, $12x \times 12y = 31104 \Rightarrow xy = 216$.

Now we need to find co-prime pairs whose product is 216.

$216 = 2^3 \times 3^3$. Therefore, the co-prime pairs will be (1, 216) and (8, 27). Therefore, (12, 12×216) and (8×12, 27×12) are two possible numbers.

LEAST COMMON MULTIPLE (LCM)

The least common multiple (LCM) of two or more numbers is the lowest number which is divisible by all the given numbers.

Consider two numbers 12 and 15.

Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132,... Multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, 135, 150,.....

Out of these series of multiples, we have some common multiples like 60, 120, 180, ..., etc. Out of these common multiples, 60 is the lowest, so 60 is called the **Lowest Common Multiple** (LCM) of 12 and 15.

Methods to Find The LCM

There are two methods to find the LCM.

(i) Prime Factorization Method

After performing the prime factorization of all the given numbers, we find the highest index of all the prime numbers among the given numbers. The LCM is the product of all these prime numbers with their respective highest indices because LCM must be divisible by all of the given numbers.

Illustration – 6

Find the LCM of 72, 288 and 1080.

Solution :

	72	=	$2^3 \times 3^2$
	288	=	$2^5 \times 3^2$
	1080	=	$2^3 \times 3^3 \times 5$
Hence,	LCM	=	$2^5 \times 3^3 \times 5^1 = 4320$

(ii) Division Method

To find the LCM we also use the division method in the following way:

First check whether any prime number that divides at least two of all the given numbers. If there is no such prime number, then the product of all these numbers is the required LCM, otherwise find the smallest prime number that divides at least two of the given numbers.

Consider the numbers, 5, 72, 196 and 240.

Here, we see that smallest prime number that divides at least two given numbers is 2.

Divide those numbers out of the given numbers by 2

which are divisible by 2 and write the quotient below it. The given number(s) that are not divisible by 2 write as it is below it and repeat this step till you do not find at least two numbers that are not divisible by any prime number.

2	5, 72, 196, 240
2	5, 36, 98, 120
2	5, 18, 49, 60
3	5, 9, 49, 30
5	5, 3, 49, 10
	1, 3, 49, 2

After that find the product of all divisors and the quotient left at the end of the division. This product is the required LCM.

Hence, LCM of the given numbers = Product of all divisors and the quotient left at the end.

$= 2 \times 2 \times 2 \times 3 \times 5 \times 3 \times 49 \times 2 = 35280$

Illustration – 7

On a traffic signal, traffic light changes its colour after every 24, 30 and 36 seconds in green, red and orange light. How many times in an hour only green and red light will change simultaneously.

Solution : LCM. of 24 and 30 = 120

So in 1 hr both green and red light will change simultaneously 3600/120 times = 30 times

LCM of 24, 30 and 36 is 360

Hence in 1 hr all three lights will change simultaneously 3600/360 times = 10 times

So in 1 hr only red and green lights will change 30 – 10 = 20 times simultaneously.

RULE FOR FINDING HCF AND LCM OF FRACTIONS

(I) HCF of two or more fractions

$$= \frac{\text{HCF of numerator of all fractions}}{\text{LCM of denominator of all fractions}}$$

(II) LCM of two or more fractions

$$= \frac{\text{LCM of numerator of all fractions}}{\text{HCF of denominator of all fractions}}$$

Illustration – 8

Find the HCF and LCM of $\frac{4}{5}, \frac{6}{11}, \frac{3}{5}$.

Solution:

$$\text{HCF} = \frac{\text{HCF of } 4, 6, 3}{\text{LCM of } 5, 11, 5} = \frac{1}{55}$$

$$\text{LCM} = \frac{\text{LCM of } 4, 6, 3}{\text{HCF of } 5, 11, 5} = \frac{12}{1} = 12$$

RULE FOR FINDING THE HCF/ LCM OF DECIMALS

First find (if necessary) the same number of decimal places in all the given numbers, then find their HCF/ LCM as if they are integers and marks off in the result as many decimal places as there are in each of the numbers.

Illustration – 9

Find the HCF of 2.8, 0.98 and 0.161.

Solution :

The given numbers are equivalent to 28000, 9800, 1610.

Now, $28000 = 2^2 \times 7 \times (2 \times 5)^3 = 2^5 \times 5^3 \times 7$

$9800 = 2 \times 7^2 \times (2 \times 5)^2 = 2^3 \times 5^2 \times 7^2$

$1610 = 7 \times 23 \times 2 \times 5 = 2 \times 5 \times 7 \times 23$

HCF of 28000, 9800 and 1610 $= 2 \times 5 \times 7 = 70$.

Hence required HCF = 0.70.

Illustration – 10

Find the LCM of 0.18, 2.4 and 60.

Solution :

The given numbers are equivalent to 18, 240 and 6000.

Now, $18 = 2 \times 3^2$

$240 = 2^3 \times 3 \times (2 \times 5) = 2^4 \times 3 \times 5$

$6000 = 2 \times 3 \times (2 \times 5)^3 = 2^4 \times 3 \times 5^3$

LCM of 18, 240 and 6000 $= 2^4 \times 3^2 \times 5^3 = 18000$

Hence, required LCM = 180.

RELATION BETWEEN ANY TWO GIVEN NUMBERS, THEIR HCF AND LCM

HCF × LCM = Product of the given two numbers

For example, HCF of 288 and 1080 is 72 and LCM of these two numbers is 4320. Clearly,

$72 \times 4320 = 288 \times 1080$

The above formula is applicable only for any two given numbers.

GP_3386

Rule for Finding the HCF/ LCM of the quantities which are in Different Units

The quantities whose HCF/ LCM to be found either they each of them has no unit (i.e. simply they are numbers) or each of them has same unit. So if they are not in the same unit, then first they are converted into same unit and then we find their HCF/ LCM.

Illustration – 11

Find the greatest weight which can be contained in 10 litres 857 millilitres and 15 litres 87 millilitres.

Solution :

10 litres 857 millilitres = 10857 millilitres
15 litres 87 millilitres = 15087 millilitres
Now, $10857 = 3 \times 11 \times 7 \times 47$
$15087 = 3 \times 47 \times 107$
HCF = $3 \times 47 = 141$ millilitres.

SHORTCUTS

Shortcut Approach - 1

For Finding HCF or GCD

To find the HCF of any number of given numbers, first find the difference between two nearest given numbers. Then find all factors (or divisors) of this difference. Highest factor which divides all the given numbers is the HCF.

Illustration – 12

Find the HCF of 12, 20 and 32.

Shortcut Solution :

Difference of nearest two numbers 12 and 20 = 20 – 12 = 8

All factors (or divisor) of 8 are 1, 2, 4 and 8.

1, 2 and 4 divides each of the three given numbers 12, 20 and 32. Out of 1, 2 and 4; 4 is the highest number. Hence, HCF = 4.

Shortcut Approach - 1

Using idea of co-prime, you can find the LCM by the following shortcut method:

LCM of 9, 10, 15 and 36 can be written directly as 9 × 10 × 2.

The logical thinking that behind it is as follows:

Step (i): If you can see a set of 2 or more co-prime numbers in the set of numbers of which you are finding the LCM, write them down by multiply them.

In the above situation, since we see that 9 and 10 are co-prime to each other, we start off writing the LCM by writing 9 × 10 as the first step.

Step (ii): For each of the other numbers, consider what prime factor(s) of it is/are not present in the product (if factorised into primes) taken in step 1. In case you see some prime factors of each of the other given numbers separately are not present in the product (if factorised into primes) taken in step 1, such prime factors will be multiplied in the product taken in step 1.

Prime factorisation of $9 \times 10 = 3 \times 3 \times 2 \times 5$
Prime factorisation of $15 = 3 \times 5$
Prime factorisation of $36 = 2 \times 2 \times 3 \times 3$
Here we see that both prime factors of 15 are present in the prime factorisation of 9 × 10 but one prime factor 2 of 36 is not present in the product taken in step 1. So to find the LCM of 9, 10, 15 and 36; we multiply the product taken in step 1 by 2.
Thus required product = $9 \times 10 \times 2 = 180$
Step (iii): The work of finding the HCF may sometimes be simplified by the following devices:

(a) Any obvious factor which is common to both numbers may be removed before the rule is applied. Care should however be taken to multiply this factor into the HCF of the quotients.
(b) If one of the numbers has a prime factor not contained in the other, it may be rejected.
(c) At any stage of the work, any factor of the divisor not contained in the dividend may be rejected. This is because any factor which divides only one of the two cannot be a portion of the required HCF.

Illustration – 13

Find the LCM of 5, 6, 12 and 18.

Shortcut Solution :

Here, 5 and 6 are co-prime numbers
Now prime factorisation of $(5 \times 6) = 5 \times 2 \times 3$
Prime factorisation of $12 = 2 \times 2 \times 3$
Prime factorisation of $18 = 2 \times 3 \times 3$
Since, one prime factor 2 out of all the prime factors of 12; and prime factor 3 among all the prime factors of 18 are not present in the prime factorisation product of 5 and 6.
$\therefore$ Required LCM = $(5 \times 6) \times 2 \times 3 = 180$

Illustration – 14

Find the HCF of 42237 and 75582.

Shortcut Solution :

$42237 = 9 \times 4693$

$75582 = 2 \times 9 \times 4199$

We may reject 2 which is not a common factor (by rule i). But 9 is a common factor. We, therefore, set it aside (by rule ii) and find the HCF of 4199 and 4693.

$$4199\overline{)4693}(1$$
$$\underline{4199}$$
$$494$$

494 is divisible by 2 but 4199 is not. We, therefore, divide 494 by 2 and proceed with 247 and 4199 (by rule iii).

$$247\overline{)4199}(17$$
$$\underline{247}$$
$$1729$$
$$\underline{1729}$$
$$0$$

The HCF of 4199 and 4693 is 247. Hence, the HCF of the original numbers is $247 \times 9 = 2223$.

Shortcut Approach - 1

The greatest number that will exactly divide x, y and z = HCF of x, y and z

Illustration – 15

What will be the greatest number that will exactly divide 96, 528 and 792?

Shortcut Solution :

$96 = 2^5 \times 3$

$528 = 2^4 \times 3 \times 11$

$792 = 2^3 \times 3^2 \times 11$

$HCF = 2^3 \times 3 = 24$

Hence, required greatest number = 24.

Shortcut Approach - 4

Least number which is exactly divisible by x, y and z = LCM of x, y and z

Illustration – 16

Find the least number which is exactly divisible by 52, 63 and 162.

Shortcut Solution :

$52 = 2^2 \times 13$

$63 = 3^3 \times 7$

$162 = 2 \times 3^4$

$LCM = 2^2 \times 3^4 \times 7 \times 13 = 29484$

Hence, required number = 29484.

Shortcut Approach - 1

The greatest number that will divide x, y and z leaving remainders a, b and c respectively= HCF of $(x - a)$, $(y - b)$ and $(z - c)$

Illustration – 17

Find the greatest number that will divide 29, 60 and 103 and will leave as remainders 5, 12 and 7 respectively.

(a) 24 **(b) 42**
(c) 48 **(d) 30**

Shortcut Solution : (a)

Required number = HCF of $(29 - 5)$, $(60 - 12)$ and $(103 - 7)$
= HCF of 24, 48 and 96

$24 = 2^3 \times 3$

$48 = 2^4 \times 3$

$96 = 2^5 \times 3$

$HCF = 2^3 \times 3 = 24$.

Hence required number = 24.

Shortcut Approach - 1

Greatest number that will divide x, y, z leaving the same remainder 'r' in each case
= HCF of $(x - r)$, $(y - r)$ and $(z - r)$

Illustration – 18

Find the greatest number which will divide 260, 720 and 1410 so as to leave the remainder 7 in each case.

(a) 33 **(b) 43**
(c) 32 **(d) 23**

Shortcut Solution : (d)

Required number
= HCF of $(260 - 7)$, $(720 - 7)$ and $(1410 - 7)$
= HCF of 253, 713 and 1403

Now, $253 = 11 \times 23$

$713 = 23 \times 31$

$1403 = 23 \times 61$

$HCF = 23$

Shortcut Approach - 1

Least number which when divided by x, y and z leaves the same remainder 'r' in each case
= (LCM of x, y and z) + r

GP_3386

Illustration – 19

What is the lowest number which when divided separately by 27, 42, 63 and 84 will in each case leave 21 as remainder?

(a) 777 **(b) 767**
(c) 707 **(d) 787**

Shortcut Solution : (a)

Required number = (LCM of 27, 42, 63 and 84) + 21
Now LCM = $2 \times 3 \times 3 \times 7 \times 3 \times 2$
$= 2^2 \times 3^3 \times 7 = 756$.

2	27,	42,	63,	84
3	27,	21,	63,	42
3	9,	7,	21,	14
7	3,	7,	7,	14
	3,	1,	1,	2

Hence required number = 756 + 21 = 777.

Shortcut Approach - 3

Greatest number that will divide x, y and z leaving the same remainder in each case (but remainder is not given)
= HCF of $|x-y|$, $|y-z|$ and $|z-x|$.

Illustration – 20

Find the greatest number that will divide 705, 1805 and 1475 leaving in each case the same remainder.

(a) 110 **(b) 120**
(c) 114 **(d) 115**

Shortcut Solution : (a)

Required number
= HCF of $|705-1805|$, $|1805-1475|$ and $|1475-705|$
= HCF of 1100, 330 and 770
Now, $1100 = 2^2 \times 5^2 \times 11$
$330 = 2 \times 3 \times 5 \times 11$
$770 = 2 \times 5 \times 7 \times 11$
$\therefore$ HCF $= 2 \times 5 \times 11 = 110$.

Shortcut Approach - 4

Least number which when divided by x, y and z leaves the remainder a, b and c respectively
= (LCM of x, y and z) – k, where $k = x-a = y-b = z-c$.

Illustration – 21

Find the least number which when divided by 24, 32 and 36 leaves the remainders 19, 27 and 31 respectively.

(a) 283 **(b) 823**
(c) 382 **(d) 238**

Shortcut Solution : (a)

Required number = (LCM of x, y and z) – K
Now LCM of 24, 32 and 36
$= 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3$
$= 2^5 \times 3^2 = 288$
and $K = x - a = 24 - 19 = 5$.

2	24,	32,	36
2	12,	16,	18
3	6,	8,	9
2	2,	8,	3
2	1,	4,	3
	1,	2,	3

Hence, required number = 288 – 5 = 283.

Shortcut Approach - 10

To find the n-digits greatest number divisible by x, y and z; follow the steps :
Step (i) : Find the LCM of x, y and z.
Let LCM of x, y and $z = L$.
Step (ii) : Find the remainder by dividing n-digits greatest number by L. Let the remainder = R.
Step (iii) : Required number = (n-digits greatest number) – R.

Illustration – 22

Find the greatest number of five digits divisible by 48, 60 and 64.

(a) 96000 **(b) 99940**
(c) 99840 **(d) 98940**

Shortcut Solution : (c)

LCM of 48, 60 and 64
$= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 4 = 960$

2	48,	60,	64
2	24,	15,	32
2	12,	15,	16
2	6,	15,	8
3	3,	15,	4
	1,	5,	4

Greatest five digits number = 99999

960) 99999 (104
960
3999
3840
159

Remainder = 159
∴ Required number = (Five digits greatest number) – 159
= 99999 – 159 = 99840

Shortcut Approach - 10

To find the *n*-digits greatest number, which when divided by *x*, *y* and *z* leaves the same remainder *K* in each case; follow the steps:
Step (i) : Find the LCM of *x*, *y* and *z*.
Let LCM *x*, *y* and *z* = *L*.
Step (ii) : Find the remainder by dividing *n*-digits greatest number by *L*.
Let the remainder = *R*.
Step (iii) : Required number = (*n*-digits greatest number) – *R* + *K*

Illustration – 23

Find the greatest 4-digits number, which when divided by 16, 24 and 36 leaves 4 as a remainder in each case.
(a) 9936 (b) 9932
(c) 9940 (d) 9904

Shortcut Solution : (c)

LCM of 16, 24 and 36 = $2^4 \times 3^2 = 144$
4-digit greatest number = 9999

144 | 9999 (69
864
1359
1296
63

Remainder, *R* = 63
Required number = 9999 – 63 + 4 = 9940.

Shortcut Approach - 10

To find the smallest *n*-digits number divisible by *x*, *y* and *z*; follow the steps :
Step (i) : Find the LCM of *x*, *y* and *z*.
Let LCM of *x*, *y* and *z* = *L*.
Step (ii) : Find the remainder by dividing *n*-digits smallest number by *L*.
Let the remainder = *R*.
Step (iii) : Required number = *n*-digits smallest number + (*L* – *R*)

Illustration – 24

Find the smallest of 4-digits number divisible by 12, 18, 21 and 28.
(a) 1006 (b) 1008
(c) 1080 (d) 1090

Shortcut Solution : (b)

LCM of 12, 18, 21 and 28 = $2^2 \times 7 \times 3^2 = 252$
Smallest 4-digits number = 1000
Remainder, *R* = 244
Required number
= 1000 + (*L* – *R*)
= 1000 + (252 – 244) = 1008

252) 1000 (3
756
244

Shortcut Approach - 10

To find smallest *n*-digits number which, when divided by *x*, *y* and *z* leaves *K* in each case; follow the steps :
Step (i) : Find the LCM of *x*, *y* and *z*.
Let LCM of *x*, *y* and *z* = *L*.
Step (ii) : Find the remainder by dividng *n*-digits smallest number by *L*.
Let the remainder = *R*.
Step (iii) : Required *n*-digits smallest number = *n*-digits smallest number + (*L* – *R*) + *K*.

Illustration – 25

Find the smallest 4-digits number which when divided by 4, 8 and 10; leaves 3 as remainder in each case.
(a) 1008 (b) 1040
(c) 1043 (d) 1084

Shortcut Solution : (c)

LCM of 4, 8 and 10 = $2^3 \times 5 = 40$
Smallest 4-digits number = 1000
Remainder, *R* = 0
Required number
= 1000 + (40 – 0) + 3
= 1043.

40) 1000 (25
80
200
200
0

Shortcut Approach - 14

Least number which when divided by x_1, x_2 and x_3 leaves the remainder a_1, a_2 and a_3 respectively = (LCM of x_1, x_2, x_3) – ($x_1 - a_1$) or ($x_2 - a_2$) or ($x_3 - a_3$)
Note that here, $x_1 - a_1 = x_2 - a_2 = x_3 - a_3$

Illustration – 26

Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10 respectively.
(a) 90 (b) 95
(c) 85 (d) 80

GP_3386

Shortcut Solution : (c)

LCM of 9, 10, 15 = 90
Required number = 90 – (9 – 4) = 85.

Shortcut Approach - 10

If product of two numbers and their HCF are given, then to find all possible pairs of numbers follow the steps :

Step- (i): Find the value of $\frac{\text{Product}}{(\text{HCF})^2}$.

Step-(ii): Find the all possible pairs of co-prime numbers, whose product is equal to the value obtained in step-1.

Step-(iii): Multiply each number of the possible pairs obtained in step-II by HCF.
The new pairs obtained in this step is the required pairs.

Illustration – 27

The product of two numbers is 3125 and their HCF is 25. Find the sum of the numbers.

(a) 150 (b) 75
(c) 100 (d) 125

Shortcut Solution : (c)

Step-(i): $\frac{\text{Product}}{(\text{HCF})^2} = \frac{3125}{(25)^2} = \frac{3125}{625} = 5.$

Step-(ii): All possible pairs of co-prime numbers = (1, 5)
Step-(iii) : (1 × 25, 5 × 25) = (25, 125)
Hence the two numbers are 25 and 125.
∴ Sum of the two numbers = 25 + 125 = 150.

Illustration – 28

The product of two numbers is 7168 and their HCF is 16. Find the numbers.

(a) (4, 1792), (28, 256) (b) (56, 128), (32, 224)
(c) (8, 896), (56, 128) (d) (16, 448), (64, 112)

Shortcut Solution : (d)

Step-(i): $\frac{\text{Product}}{(\text{HCF})^2} = \frac{7168}{(16)^2} = 28$

Step-(ii): All possible pairs of co-prime numbers = (1, 28) and (4, 7)
Step-(iii): (1 × 16, 28 × 16) and (4 × 16, 7 × 16)
or (16, 448) and (64, 112)
Hence the pair of numbers are (16, 448) and (64, 112).

LEVEL - 1

1. Find $GCD(2^{100}-1, 2^{120}-1)$.
(a) $2^{20}-1$ (b) $2^{40}-1$ (c) $2^{60}-1$ (d) $2^{10}-1$

2. Product of divisors of 7056 is
(a) $(84)^{48}$ (b) $(84)^{44}$
(c) $(84)^{45}$ (d) None of these

3. Find the sum of divisors of 544 which are perfect squares.
(a) 32 (b) 64 (c) 42 (d) 21

4. How many numbers are there between 200 and 800 which are divisible by both 5 and 7?
(a) 35 (b) 16
(c) 17 (d) Can't be determined

5. A hall is 13 metres 53 cm long and 8 metres 61cm broad is to be paved with minimum number of square tiles. The number of tiles required is:
(a) 123 (b) 77 (c) 99 (d) 57

6. The LCM of three different number is 120 which of the following cannot be their HCF
(a) 8 (b) 12 (c) 24 (d) 35

7. The least number of five digits which is exactly divisible by 12, 15 and 18, is:
(a) 10010 (b) 10051 (c) 10020 (d) 10080

8. The H.C.F. and L.C.M. of two numbers are 50 and 250 respectively. If the first number is divided by 2, the quotient is 50. The second number is:
(a) 50 (b) 100 (c) 125 (d) 250

9. Find the HCF of 132, 204 and 228. **(Bank Clerk 2009)**
(a) 24 (b) 12 (c) 36 (d) 48

10. Find the LCM of 8, 15, 24 and 72. **(Bank Clerk 2011)**
(a) 240 (b) 720 (c) 180 (d) 360

11. Find the HCF of 1365, 1560 and 1755. **(Bank PO 2015)**
(a) 195 (b) 13 (c) 15 (d) 65

12. Find the greatest possible rate at which a man should walk to cover a distance of 70 km and 245 km in exact number of days?
(a) 55 (b) 60 (c) 35 (d) 45

13. What will be the least number which when doubled will be excactly divisible by 12,18,21 and 30?
(a) 196 (b) 630 (c) 1260 (d) 2520

14. The greatest number which will divide 410, 751 and 1030 leaving a remainder 7 in each case is
(a) 29 (b) 31 (c) 17 (d) 37

15. Find the greatest number that divides 130, 305 and 245 leaving remainders 6, 9 and 17 respectively? **[SSC CHSL 2015]**
(a) 4 (b) 5 (c) 14 (d) 24

16. What is the greatest that divides 13850 and 17030 leaves a remainder 17? **[CDS 2012]**
(a) 477 (b) 159 (c) 107 (d) 87

17. Four runners started running simultaneously from a point on a circular track they took 200 sec, 300 sec, 360 sec and 450 sec to complete one round, after how much time do they meet at the starting point for the first time?
(a) 1800 sec (b) 3600 sec
(c) 2400 sec (d) 4800 sec

18. From 3 drums of milk, 279, 341 and 465 respectively are to be drawn out. To do it in a minimum time, the capacity of the measuring can be
(a) 271 (b) 61 (c) 111 (d) 31

19. The LCM and HCF of two numbers are 84 and 21 respectively. If the ratio of the two numbers is 1 : 4, then find the larger number.
(a) 27 (b) 44 (c) 105 (d) 84

20. HCF of two even perfect squares must be divisible by which of the following numbers –
(a) 3 (b) 4 (c) 6 (d) 8

21. The greatest possible length of a gauge which can be used to measure exactly the lengths 7 m 35 cm, 3 m 50 cm, 12 m 60 cm is –
(a) 7 cm (b) 14 cm (c) 35 cm (d) 70 cm

22. Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85 cm, 12m 95cm. **(SSC CGL Tier-I 2014)**
(a) 35 cm (b) 45 cm
(c) 1 m 15 cm (d) 90 cm

23. When 32, 60, 102 divided by a number n leaves same remainder in each case. Find the largest value of n.
(a) 7 (b) 2 (c) 14 (d) 3

24. Find the smallest number n that is completely divisible by all non zero digits (All digits from 1 to 9).
(a) 5040 (b) 1260 (c) 2520 (d) 630

GP_3386

25. Three numbers are in the ratio of 4 : 5 : 6 and their L.C.M. is 1200. Then their product is :
(a) 960000 (b) 721000 (c) 98000 (d) 92000

26. If two numbers are in the ratio of 3:4 and their LCM is 72 then the smaller of two numbers is –
(a) 12 (b) 18 (c) 32 (d) 24

27. HCF and LCM of four numbers a, b, c and d are 12 and 48. Which of the following can be value of a ?
(a) 4 (b) 16 (c) 24 (d) 60

28. How many two digits numbers exist which have only two factors.
(a) 21 (b) 23 (c) 25 (d) 27

29. In how many ways can 420 be written as a product of a prime and a composite factors?
(a) 2 (b) 3 (c) 4 (d) 5

30. The H.C.F. of two numbers is 23 and the other factors of their L.C.M. are 13 and 14. The larger of the two numbers is:
(a) 276 (b) 299 (c) 322 (d) 345

31. There are 4 numbers. The H.C.F. of each pair is 3 and the L.C.M. of all the 4 numbers is 116. What is the product of 4 numbers?
(a) 9782 (b) 9396 (c) 9224 (d) 9100

32. Find the greatest number that will divide 115, 149 and 183 leaving remainders 3, 5, 7 respectively.
(a) 14 (b) 16 (c) 18 (d) 20

33. The H.C.F. of two numbers is 8. Which one of the following can never be their L.C.M.?
(a) 24 (b) 48 (c) 56 (d) 60

34. An electric wire is sold only in multiples of 1 metre, and a customer required several lengths of wire, each 85cm long. To avoid any wastage and to minimise labour, he should purchase minimum lengths of
(a) 8.5 metres (b) 17 metres
(c) 1.7 metres (d) 1 metre

35. Find the lowest number which when subtracted from 3000, is exactly divisible by 7, 11 and 13
(a) 729
(b) 998
(c) Cannot be determined
(d) None of these

36. Find the side of the largest possible square slabs which can be paved on the floor of a room 2 m 50 cm long and 1 m 50 cm broad. Also find the number of such slabs to pave the floor.
(a) 25, 20 (b) 30, 15 (c) 50, 15 (d) 55, 10

37. Three bells chime at intervals of 48, 60 and 90 min, respectively. If all the three bells chime together at 10 : 00 am, at what time will all the three chime again that day?
(a) 1 : 00 pm (b) 2 : 00 pm
(c) 8 : 00 pm (d) 10 : 00 pm

38. Monica, Veronica and Rachat begin to jog around a circular stadium. They complete their revolutions in 42s, 56s and 63s respectively. After how many seconds will they be together at the starting point?
(a) 366
(b) 252
(c) 504
(d) Couldn't be determined

39. How many numbers are there between 4000 and 6000 which are exactly divisible by 32, 40, 48 and 60? **(SSC CHSL 2010)**
(a) 1 (b) 2 (c) 3 (d) 4

40. How many numbers are there between 4000 and 6000 which are exactly divisible by 32, 40, 48 and 60?
(a) 2 (b) 3 (c) 4 (d) 5

41. Four numbers are in the ratio of 10 : 12 : 15 : 18. If their HCF is 3, then find their LCM.
(a) 420 (b) 540 (c) 620 (d) 680

42. The H.C.F. of two numbers is 43 and their sum is 430. Total number of distinct pairs of two such numbers is :
(a) 5 (b) 2
(c) 6 (d) Data insufficient

43. Find the largest number which divides 1305, 4665 and 6905 leaving same remainder in each case. Also, find the common remainder. **[CDS 2009]**
(a) 1210, 185 (b) 1120, 185
(c) 1005, 175 (d) 1100, 175

44. Find the greatest number of five digits which is completely divisible by 12, 16, 20, 24 and 28. **[SSC CGL 2011]**
(a) 99120 (b) 99908 (c) 99899 (d) 99446

45. Find the greatest number of 4 digits which when divided by 8, 19, 12 and 15 leaves 6 remainder in each case. **[SSC CGL Tier -I 2016]**
(a) 9980 (b) 9726 (c) 9920 (d) 9626

46. The largest number which will divide the numbers 104, 221 and 377 leaving the same remainder in each case is :

(a) 56 (b) 13
(c) 39 (d) It does not exist

47. The L.C.M. of two numbers is 1020 and their H.C.F is 34 the possible pair of numbers is :
(a) 255, 34 (b) 102, 204
(c) 204, 170 (d) None of these

48. Four bells ring at an interval of 30 min, 1 hour, 1 hour 30 min and 1 hour 45 min. All bells started ring together at 12 noon. At what they will ring again?
(a) 12 O' clock mid – night
(b) 3 O' clock next morning
(c) 6 O' clock next morning
(d) 9 O' clock next morning

49. Find that number by which when 1050, 1250 and 1650 are divided respectively leaves 43, 31 and 7 as remainders.
(a) 47 (b) 51 (c) 53 (d) 57

50. The number nearest to 10000 which can be divided by each of 3, 4, 5, 6, 7 and 8 will be
(a) 9240 (b) 10080 (c) 9996 (d) 10000

51. What is the smallest positive integer which when divided by 4, 5, 8, 9 leaves remainder 3, 4, 7, 8 respectively? **(CDS 2010)**
(a) 119 (b) 319 (c) 359 (d) 719

52. Which is the largest number which can exactly divide each of the numbers 264, 312 and 456? **(Bank PO 2013)**
(a) 26 (b) 24 (c) 28 (d) 32

53. What will be the greatest number that divides 1356, 1868 and 2764 leaving 12 as remainder in each case? **(CDS 2007)**
(a) 64 (b) 124 (c) 156 (d) 260

LEVEL - 2

1. LCM of first 100 natural numbers is N. What is the LCM of first 105 natural numbers?
(a) $5! \times N$ (b) $10403\,N$
(c) $105N/103$ (d) $4\,N$

2. The last digit of the LCM of $(3^{2003} - 1)$ and $(3^{2003} + 1)$ is
(a) 8 (b) 2 (c) 4 (d) 6

3. The sum of two numbers is 462 and their highest common factor is 22. What is the maximum number of pairs that satisfy these conditions ?
(a) 1 (b) 3 (c) 5 (d) 6

4. If HCF of two odd numbers is 9 then their summation must be divisible by which of the following number –
(a) 10 (b) 12 (c) 15 (d) 18

5. Find the largest number n such that when 641, 881, and 1361 is divided by n leaves same remainder in each case.
(a) 120 (b) 240 (c) 60 (d) 30

6. The product of two numbers is 3468. What could be the maximum value of their HCF?
(a) 34 (b) 51 (c) 17 (d) 102

7. What is the smallest four digit number that gives same remainder when divided by any single digit prime number?
(a) 1031 (b) 1051 (c) 1011 (d) 1061

8. Find that smallest 6-digit number which when divided by 9, 12, 21 and 25 leaves no remainder **(Bank PO 2016)**
(a) 100005 (b) 100225 (c) 100800 (d) 100100

9. Find the least possible 5-digit number, which when divided by 10, 12, 16 and 18 leaves remainder 27. **(SSC CGL Tier-I 2017)**
(a) 10107 (b) 10100 (c) 10027 (d) 10507

10. A red light flashes 3 times per minute and a green light flashes 5 times in two minutes at a regular intervals. If both lights start flashing at the same time, how many times do they flash together in each hour?
(a) 30 (b) 24 (c) 20 (d) 60

11. How many pairs of factors of number 7200 have HCF 20?
(a) 10 (b) 44 (c) 53 (d) 18

12. If HCF of two numbers is 17 and their LCM is 222 then one of the two numbers may be –
(a) 51
(b) 34
(c) 37
(d) Data is inconsistence

13. Find the LCM of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$
(a) 12 (b) 12/5 (c) 14/3 (d) 18/7

14. If HCF of two odd numbers a and b is 7, then which of the following can be the sum of a and b.
(a) 21 (b) 28 (c) 35 (d) 45

15. Find the number of factors of 1200
(a) 30 (b) 24 (c) 32 (d) 36

16. In how many ways can 600 be expressed as a product of two of its factors?
(a) 8 (b) 10 (c) 12 (d) 15

17. How many two digit numbers have odd numbers of factors?
(a) 3 (b) 9 (c) 5 (d) 6

18. Find the number of composite factor of 1260.
(a) 32 (b) 31 (c) 33 (d) 30

19. Find the number of odd factors of 300300
(a) 128 (b) 256 (c) 512 (d) 64

20. How many factors of 900 are not perfect squares?
(a) 18 (b) 19 (c) 21 (d) 17

21. Three wheels can complete respectively 60, 36, 24 revolutions per minute. There is a red spot on each wheel that touches the ground at time zero. After how much time, all these spots will simultaneously touch the ground again?
(a) 5/2 seconds (b) 5/3 seconds
(c) 5 seconds (d) 7.5 seconds

22. The least number, which when divided by 2, 3, 4, 5 and 6, leaves in each case, a remainder 1, but when divided by 7 leaves no remainder. The number is
(a) 121 (b) 181 (c) 241 (d) 301

23. Find that smallest number which when divided by 16, 24 and 30 leaves remainder as 6, 14 and 20 respectively. **[RRB 2013]**
(a) 116 (b) 224 (c) 300 (d) 230

24. Which is the least number that must be subtracted from 1856, so that the remainder when divided by 7, 12 and 16 will leave the same remainder 4?
(a) 137 (b) 1361 (c) 140 (d) 172

25. The L.C.M. of two number is 630 and their H.C.F. is 9. If the sum of numbers is 153, their difference is
(a) 17 (b) 23 (c) 27 (d) 33

26. I have a certain number of beads which lie between 600 and 900. If 2 beads are taken away the remainder can be equally divided among 3, 4, 5, 6, 7 or 12 boys. The number of beads I have
(a) 729 (b) 842 (c) 576 (d) 961

27. One pendulum ticks 57 times in 58 seconds and another 608 times in 609 seconds. If they started simultaneously, find the time after which they will tick together.
(a) $\frac{211}{19}$ s (b) $\frac{1217}{19}$ s
(c) $\frac{1218}{19}$ s (d) $\frac{1018}{19}$ s

28. A man was engaged for a cetain number of days for ₹ 404.30 p but being absent for some days he was paid only ₹ 279.90p. His daily wages cannot exceed by:
(a) ₹ 29.10 p (b) ₹ 31.30 p
(c) ₹ 31.10 p (d) ₹ 31.41 p

29. Find the sum of odd factors of 544
(a) 16 (b) 18 (c) 20 (d) 22

30. Find the sum of all factors, sum of even factors and sum of odd factors of 360.
(a) 1170, 1092, 78 (b) 1136, 1080, 90
(c) 1000, 1142, 106 (d) 1370, 1728, 125

31. Find the number of factors, number of even factors and number of odd factors of 360.
(a) 24, 10, 18 (b) 24, 18, 6
(c) 30, 36, 6 (d) 12, 18, 10

32. Find the least number which is exactly divisible by 12, 15, 90, 108, 135 and 150.
(a) 1500 (b) 3000 (c) 2700 (d) 2250

33. The least number which when divided by 12, 16 and 18 leaves 5 as remainder in each case. Find the number. **[RBI Clerk 2009]**
(a) 139 (b) 149 (c) 159 (d) 169

34. What is the least number which when diminished by 7, is divisible by each one of 21, 28, 36 and 45? **[SSC CGL Tier - I 2015]**
(a) 1267 (b) 1157 (c) 1077 (d) 1347

35. Find the least number which when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.
(a) 2884 (b) 2256 (c) 865 (d) 3332

36. The HCF and LCM of two numbers m and n are respectively 6 and 210. If $m + n = 72$, then $\frac{1}{m} + \frac{1}{n}$ is equal to

(a) $\frac{1}{35}$ (b) $\frac{3}{35}$ (c) $\frac{5}{37}$ (d) $\frac{2}{35}$

37. The sum of two numbers is 1056 and their HCF is 66, find the number of such pairs.
(a) 6 (b) 2 (c) 4 (d) 8

38. The sum of HCF and LCM of two numbers is 403 and their LCM is 12 times their HCF. If one number is 93, then find the another number.
(a) 115 (b) 122 (c) 124 (d) 138

39. When in each box 5 or 6 dozens of apples were packed, three dozens were left. Therefore, bigger boxes were taken to pack 8 or 9 dozens of apples. However, still three dozens of apples remained. What was the least number of dozen of apples to be packed?
(a) 363 dozens (b) 315 dozens
(c) 345 dozens (d) 335 dozens

40. The quotient when L.C.M is divided by the H.C.F of a G.P. with first term 'a' and common ratio 'r' is :
(a) r^{n-1} (b) r^n
(c) $a^{-1} r^{n-2}$ (d) $(r^n - 1)$

41. The number of possible pairs of numbers, whose product is 5400 and HCF is 30 :
(a) 1 (b) 2 (c) 3 (d) 4

42. If the product of the HCF and the LCM of 3 natural numbers p, q, r, equals pqr, then p, q, r must be :
(a) such that (p, q, r) = 1
(b) prime number
(c) odd number
(d) such that (p, q) = (p, r) = 1

43. The sum of two numbers is 377. If the sum and the difference of their L.C.M. and H.C.F. are respectively 2665 and 2639 then the numbers are
(a) 144, 156 (b) 130, 143
(c) 117, 234 (d) 156, 221

44. Product of two numbers is 1440 and their HCF is 12. How many pairs of such numbers are possible?
(a) 2 (b) 3 (c) 1 (d) 4

45. The product of two whole numbers is 1500 and their HCF is 10. Find the numbers. **[CDS 2012]**
(a) (70, 80) (b) (50, 38)
(c) (10, 150) (d) (75, 20)

46. The sum of two numbers is 36 and their H.C.F. and L.C.M. are 3 and 105 respectively. What will be the sum of their reciprocal?

(a) $\frac{2}{35}$ (b) $\frac{3}{25}$ (c) $\frac{4}{35}$ (d) $\frac{2}{25}$

47. The L.C.M. of two multiples of 12 is 1056. If one of them is 132, then the other is
(a) 12 (b) 72 (c) 96 (d) 132

48. The numbers 400, 536 and 646; when divided by a number N, give the remainders of 22, 23 and 25 respectively. Find the greatest such number N. **(FCI 2011)**
(a) 40 (b) 45 (c) 27 (d) 22

49. What is the sum of the digits the least number which, when divided by 52, leaves 33 as remainder; when divided by 78, leaves 59 and when divided by 117, leaves 98 as reaminder? **[CDS 2010]**
(a) 17 (b) 18 (c) 19 (d) 21

50. Find the greatest number less than 900, which is divisible by 8, 12 and 28. **[RBI Clerk 2011]**
(a) 880 (b) 800 (c) 860 (d) 840

51. Find the largest number of 5 digits which when divided by 3, 5, 8, 12 leaves the same remainder 2 in each case. **[SSC CHSL 2016]**
(a) 99722 (b) 99958 (c) 99960 (d) 99962

GP_3386

Hints and Solutions

Level 1

1. (a) $(2^{100}-1)$ and $(2^{120}-1)$ will yield the *GCD* as $2^{20}-1$.

2. (c) $\because$ $7056 = 2^4 \times 3^2 \times 7^2$

$\therefore$ Number of factors/divisors of 7056

Product of factors $= (7056)^{45/2} = (84)^{45}$

3. (d) Sum of divisors of 544 which are perfect square is:

$(2^0+2^2+2^4)(17^0)=21$.

4. (c) In the given range, the last number which is divisible by both 5 and 7. i.e., 35 is 210 and the highest number is 770. So the total number of numbers between 200 and 800 which are divisible by both 5 and 7 is

$$\left(\frac{770-210}{35}\right)+1=17$$

5 (b) 13 m 53 cm = 1353 cm and 8 m 61 cm = 861 cm.

H.C.F. of 1353 and 861 is 123.

Now since minimum tiles are required for having the floor so area of the tile has to be the greatest so its size is greatest which is 123 cm.

$\therefore$ the number of square tiles

$$=\frac{\text{Area of the floor}}{\text{Area of the tile}}=\frac{1353\times861}{123\times123}=77$$

6. (d) LCM $= 2\times2\times2\times3\times5$

Hence, HCF = 4, 8 or 12

Only 35 from given options cannot be their HCF.

Note that LCM is always divisible by HCF.

7. (d) Least number of 5 digits is 10,000 L.C.M. of 12, 15 and 18 is 180.

On dividing 10000 by 180, the remainder is 100.

$\therefore$ Required number = 10000 + (180 – 100) = 10080.

8. (c) First number $= (50\times2)=100$. Second number

$$=\left(\frac{50\times250}{100}\right)=125.$$

9. (b) Use Shortcut Approach-1

10. (d) Use Shortcut Approach-2

11. (a) Use Shortcut Approach-1

12. (c) Rate = HCF of 70 and 245 = 35

13. (b) L.C.M. of 12, 18, 21, 30

$= 2\times3\times2\times3\times7\times5=1260$

$\therefore$ Required number $= (1260\div2)=630$.

14. (b) Required number = H.C. F of (410 – 7), (751 – 7) and (1030 – 7)

= H. C. F of 403, 744 and 1023 = 31

15. (a) Use Shortcut Approach-5

16. (b) Use Shortcut Approach-6

17. (a) Required time = LCM of 200, 300, 360, 450 sec = 1800 sec.

18. (d) To find the capacity we have to take the HCF of 279, 341 and 465.

$279 = 31\times9$

$341 = 31\times11$

$465 = 31\times3\times5$

$\Rightarrow$ HCF (279, 341, 465) = 31

Capacity of the measuring can be = 31 ml.

19. (d) Since HCF of two numbers is 21 hence numbers must be $21x$ and $21y$, given ratio is 1 : 4 hence $x : y = 1 : 4$, hence numbers are 21 and 84, and largest of these two is 84.

Alternatively

Since HCF of two numbers is 21 hence numbers must be a multiple of 21, out of the given option only 105 and 84 are multiple of 21, but 105 is more than the LCM 84 hence ruled out. Hence appropriate answer is 84.

20. (b) Even perfect square must be in the form of $2^2 \times n^2$ or $4k$, hence their HCF must be divisible by 4.

21. (c) The required largest length is the HCF of 735, 350 and 1260 i.e 35cm

22. (a) Use Shortcut Approach-3

23. (c) This is HCF type question. The required number will be HCF of the differences between any two numbers. Since $60 - 32 = 28$ and $102 - 60 = 42$ and HCF of 28 and 42 is 14 hence 14 is the largest such number.

24. (c) The required number must be LCM of (1, 2, 3, 4, 5, 6, 7, 8, and 9) = 2520

25. (a) Let three numbers be $4x$, $5x$ and $6x$, then their LCM would be $60x = 1200$ hence $x = 20$, then the three numbers are 80, 100 and 120 and their product is 960000.

26. (b) Let the two numbers as $3x$ and $4x$ then their LCM $= 12x = 72$ or $x = 6$ then smallest number $= 18$.

27. (c) Here two concepts are asked Concept 1- The numbers must be more than or equal to HCF and less than or equal to LCM, and numbers must be a multiple of HCF and factor of LCM. Only option (c) satisfies these conditions.

28. (a) Only prime numbers have two factors, so here we have to find two digit prime numbers. We know that there are 21 two digit prime numbers in all.

29. (c) Since $420 = 2^2 \times 3 \times 5 \times 7$ and it has four prime factors, but $420 = 2 \times 210 = 3 \times 70 = 5 \times 42 = 7 \times 60$, so required number of ways is 4.

30. (c) Clearly, the numbers are (23×13) and (23×14).

$\therefore$ Larger number $= (23 \times 14) = 322$.

31. (b) $\because$ Product of 'n' numbers = (HCF for each pair)^n × LCM of 'n' numbers $= (3)^4 \times 116 = 9396$

32. (b) Required number = HCF of (115 – 3), (149 – 5) and (183 – 7) = HCF of 112, 144 and 176 = 16

33. (d) H.C.F. of two numbers divides their L.C.M. exactly. 8 is not a factor of 60.

34. (b) Required length = LCM of 100 cm and 85 cm = 1700 cm = 17 m

35. (b) Required number = 3000 – maximum possible multiple of LCM of (7, 11 and 13) $= 3000 - 2(1001) = 998$

36. (c) HCF of 250 cm and 150 cm

$$150\overline{)250}(1$$
$$\underline{150}$$
$$100\overline{)150}(1$$
$$\underline{100}$$
$$50\overline{)100}(2$$
$$\underline{100}$$
$$\times$$

HCF = 50

$\therefore$ Number of slabs $= \dfrac{\text{Area of floor}}{\text{Area of slab}}$

$$= \frac{250 \times 150}{50 \times 50} = 15$$

37. (d) We have to find out the LCM of 48, 60 and 90.

2	48,	60,	90
3	24,	30,	45
2	8,	10,	15
5	4,	5,	15
2	4,	1,	3
2	2,	1,	3
3	1,	1,	3
	1,	1,	1

$\therefore$ LCM $= 2 \times 3 \times 2 \times 5 \times 2 \times 2 \times 3$

$$= 720 \text{ min} = \frac{720}{60} \text{h} = 12\text{h}$$

$\therefore$ Bell chime together again = 10 : 00 am + 12 h = 10 : 00 pm

38. (c) Required time = LCM of 42, 56 and 63s

LCM of 42, 56, 63 is

2	42,	56,	63
3	21,	28,	63
7	7,	28,	21
	1,	4,	3

$\therefore$ Required time $= 2 \times 3 \times 7 \times 4 \times 3 = 504$s

39. (d) Use Shortcut Approach-4

40. (c) LCM of 32, 40, 48 and 60 = 480

GP_3386

The number divisible by 480 between 4000 and 6000 are 4320, 4800, 5280 and 5760.
Hence, required number of numbers are 4.

41. (b) Let numbers be 10x, 12x and 15x and 18x.
Then, LCM = 180 x
As HCF = x
Hence, required LCM = 180 × 3 = 540

42. (b) (43 and 387) and (129, 301)
$43k + 43l = 430$
$\Rightarrow 43(k + l) = 430$
$\Rightarrow k + l = 10$
but k, l must be coprimes.
So $(k, l) = (1, 9)$ and $(3, 7)$

43. (b) Use Shortcut Approach-8

44. (a) Use Shortcut Approach-10

45. (b) Use Shortcut Approach-11

46. (c) The required number
= H.C.F. of (221–104), (377–221) and (377 – 104)
= H.C.F. of 117, 156, 273 = 39

47. (c) L.C.M. of 204 and 170 = 1020.

48. (d) The L.C.M of 30, 60, 90 and 105 min = 1260 min = 21 hours means bells will ring next morning at 9' O Clock.

49. (c) Required number will be H.C.F. of 1050 – 43, 1250 – 31, 1650 – 7 means, H.C.F. of 1007, 1219 and 1643.
The required H.C.F. will be 53

50. (b) The L.C.M. of given numbers will be 840.
After dividing 10000 by 840 the remainder will be 760
So, the required number will be
10000 – 760 or 10000 – 760 + 840 means 9240 or 10080. Clearly 10080 is much nearer to 10000 than 9240.

51. (c) Use Shortcut Approach-9

52. (b) Use Shortcut Approach-3

53. (a) Use Shortcut Approach -6

Level 1

1. (b) If we look at the numbers $100 < N \leq 105$, we see only 101 and 103 do not have their factors in N (because these are primes). So, obviously the new LCM will be $101 \times 103 \times N$.

2. (c) The given numbers are two consecutive even numbers, so their HCF = 2
Now, using LCM × HCF = Product of two numbers
LCM × 2 = (…6) × (…8)
It can be seen now that the unit digit of LCM = 4

3. (d) There are 6 such pairs :
(22, 440), (44, 418), (88, 374), (110, 352)
(176, 286), (220, 242)

4. (d) Since HCF of two numbers is 9 hence numbers must be $9x$ and $9y$ here x and y are co-prime to each other. Since both the numbers are odd hence both x and y must be odd. Their summation will be $9(x + y)$
but $x + y$ = odd + odd = Even. Hence $9(x + y)$ = 9 × Even. Hence it must be divisible by 18.

5. (b) This is HCF type question. The required value of n must be HCF of difference between any two pairs of numbers. N must be HCF of 881 – 641 = 240 and 1361 – 881 = 480. HCF of 240 and 480 is 240.

6. (a) If HCF of two numbers is h then the two numbers should be hx and hy, where x and y are co-prime to each other, then product of two numbers will be $(hx)(hy) = h^2xy$
$= 3468 = 2^2 3^1 17^2$ hence maximum value of h^2 is $2^2 17^2$ hence maximum value of h is 34

7. (b) Single digit prime numbers are 2, 3, 5 and 7. Out of them smallest is 2 that can give only 0 and 1 as remainders. Hence possible number must be in the form of k (LCM of 2, 3, 5, and 7) + 1 or $210k + 1$. From the given options only 1051 is in that format.

8. (c) Use Shortcut Approach-12

9. (a) Use Shortcut Approach-13

10. (a) Interval of flashing for the red light = $\frac{60}{3}$
= 20sec

Interval of flashing for the green light

$$= \frac{60 \times 2}{5} = 24\text{sec}$$

So they will flash together at an interval of 120 sec (LCM of 20 & 24) = 2min

Hence in one hour they will flash together for $\frac{60}{2} = 30$ times.

11. (c) Since HCF of two numbers are 20 hence numbers must be in the form of $20a$ and $20b$ where a and b are co-prime to each other. $20a$ and $20b$ are factors of 7200 hence a and b must be a factor of $7200/20 = 360 = 2^3 3^2 5^1$

Now condition is that a, and b are co-prime to each other and they are factors of 7200, similar type of questions we have solve in chapter 2 (factor theory) concept 7 and there we have seen a formula total number of pairs is $1 + (p + q + r) + 2(pq + pr + qr) + 4pqr$ here $p = 3$, $q = 2$ and $r = 1$ hence required number of such pairs are $1 + 6 + 22 + 24 = 53$

12. (d) HCF must be a factor of LCM but here in the given data 17 is not a factor of 222 hence data is inconsistence.

13. (a) Required LCM $= \frac{\{\text{LCM of } 1,2,3,4\}}{\{\text{HCF of } 5,2,3,4\}}$

14. (b) Let the two numbers be $7x$ and $7y$ where x and y are co-prime to each other and also they are odd numbers, then their sum is $7(x + y)$. Since x and y both are odd so their sum must be an even number and hence the summation of a and b must be an even number.

15. (a) Since $1200 = 12 \times 100 = 4 \times 3 \times 4 \times 25 = 2^4 3^1 5^2$ hence its number of factors is $(4 + 1)(1 + 1)(2 + 1) = 5 \times 2 \times 3 = 30$

16. (c) Since $600 = 6 \times 100 = 2 \times 3 \times 4 \times 25 = 2^3 3^1 5^2$ hence number of factors $= 4 \times 2 \times 3 = 24$ so number of ways 600 can be expressed as a product of two of its factors is $24/2 = 12$

17. (d) We know that only a perfect square has odd number of factors hence we have to find out 2 digit perfect squares. Smallest two digit perfect square is $16 = 4^2$ and largest is $81 = 9^2$. Hence total 6 such two digit numbers exist.

18. (b) Since $1260 = 2^2 3^2 5^1 7^1$ total number of factors of 1260 is $3 \times 3 \times 2 \times 2 = 36$ out of these 36 factors there are 4 prime factors 2, 3, 5 and 7 and one number 1 is neither prime nor composite. Hence total number of composite factors is $36 - 5 = 31$.

19. (d) Since 30030 is the product of 1st 6 prime numbers (You should memorize this)

Hence $300300 = 2^3 3^1 5^3 7^1 11^1 13^1$ so number of factors of 300300 is

$(3 + 1)(1 + 1)(3 + 1)(1 + 1)(1 + 1)(1 + 1)$

$= 4 \times 2 \times 4 \times 2 \times 2 \times 2 = 256$

And number of even factors of 300300 is

$(3)(1 + 1)$

$(3 + 1)(1 + 1)(1 + 1)(1 + 1)$

$= 3 \times 2 \times 4 \times 2 \times 2 \times 2 = 192$

Hence number of odd factors of 300300

$= 256 - 192 = 64$

Alternately

Since $300300 = 2^3 3^1 5^3 7^1 11^1 13^1$ to find number of odd factors power of 2 should be 0 hence number of odd factors of 300300 is same as factor of $= 3^1 5^3 7^1 11^1 13^1$ is

$(1 + 1)(3 + 1)(1 + 1)(1 + 1)(1 + 1) = 64$

20. (b) We know that $900 = 2^2 3^2 5^2$ total number of factors of $900 = 3 \times 3 \times 3 = 27$. For perfect square powers of prime factors must be even hence number of perfect square factors is $2 \times 2 \times 2 = 8$. Therefore number of non perfect square factors is $27 - 8 = 19$.

21. (c) A makes 1 rev. per sec

B makes $\frac{6}{10}$ rev per sec

C makes $\frac{4}{10}$ rev. per sec

In other words A, B and C take 1 , $\frac{5}{3}$ & $\frac{5}{2}$

seconds to complete one revolution.

$$\text{L.C.M of } 1, \frac{5}{3} \& \frac{5}{2} = \frac{\text{L.C.M. of } 1,5,5}{\text{H.C.F. of } 1,3,2} = 5$$

Hence, after every 5 seconds the red spots on all the three wheels touch the ground

22. (d) LCM (2, 3, 4, 5, 6) = 60

$\therefore$ Required number is of the form 60 k + 1

Least value of k for which 60k + 1 is divisible by 7 is k = 5

$\therefore$ Required number = 60 × 5 + 1 = 301

23. (d) Use Shortcut approach-14

24. (d) Suppose least no. be x

$1856 - x = n(\text{LCM of } 7,12,16) + 4$

or $1856 - x = n(336) + 4$

we should take n = 5 so that n(336) is nearest to 1856 and n (336) < 1856

$1856 - x = 1680 + 4 = 1684$

$x = 1856 - 1684 = 172$

25. (c) Let numbers be x and y.

$\because$ Product of two numbers = their (LCM × HCF)

$\Rightarrow xy = 630 \times 9$

Also, $x + y = 153$ (given)

since $x - y = = \sqrt{(x+y)^2 - 4xy}$

$$\Rightarrow x - y = \sqrt{(153)^2 - 4(630 \times 9)}$$

$$= \sqrt{23409 - 22680} = \sqrt{729} = 27$$

26. (b) LCM of the numbers = 420.

Hence there must be (420 × 2) + 2

= 842 beads.

27. (c) Time gap between two consecutive ticks

$\frac{58}{57}$ sec. and $\frac{609}{608}$ sec.

$\therefore$ Required time = LCM of $\frac{58}{57}$ and $\frac{609}{608}$

$$= \frac{\text{LCM of 58 and 609}}{\text{HCF of 57 and 608}} = \frac{1218}{19} \text{sec}$$

28. (c) His maximum daily wages must be the H.C.F. of 404.30 and 279.90 which is 31.10.

29. (b) $544 = 2 \times 2 \times 2 \times 2 \times 2 \times 17$

$= 2^5 \times 17$

Sum of odd factors = $(2^0)(17^0 + 17^1) = 18$

30. (a)

2	360
2	180
2	90
3	45
3	15
	5

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^3 \times 3^2 \times 5$

Sum of all factors = $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)(5^0 + 5^1)$

$= 15 \times 13 \times 6 = 1170$

Sum of even factors

$= (2^1 + 2^2 + 2^3) \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$

$= 14 \times 13 \times 6 = 1092$

Sum of odd factors

$= (2^0)(3^0 + 3^1 + 3^2)(5^0 + 5^1)$

$= 13 \times 6 = 78$

31. (b)

2	360
2	180
2	90
3	45
3	15
	5

$360 = 2^3 \times 3^2 \times 5$

Total number of factors

$= (3 + 1) \times (2 + 1) \times (1 + 1) = 24$

Number of even Factors

$= (3) \times (2 + 1) \times (1 + 1) = 18$

Number of odd factors $= 1 \times (2+1) \times (1+1)$
$= 3 \times 2 = 6.$

32. **(c)** Use Shortcut Approach-4

33. **(b)** Use Shortcut Approach-7

34. **(a)** Use Shortcut Approach-7

35. **(a)** LCM of 16, 18 and 20 = 720

$\therefore$ Required number = 720 k + 4

where, k is a natural number to be divisible by 7, (720 k + 4) will be a multiple of 7.

Smallest value of k = 4

$\therefore$ Required number = $720 \times 4 + 4 = 2884$

36. **(d)** We have, $m \times n = 6 \times 210 = 1260$

$$\therefore \quad \frac{1}{m}+\frac{1}{n}=\frac{m+n}{mn}=\frac{72}{1260}=\frac{4}{70}=\frac{2}{35}$$

37. **(c)** Let the numbers be 66a and 66b, where a and b are coprimes.

According to the question,

$66a + 66b = 1056$

$\Rightarrow \quad 66(a+b) = 1056$

$$\Rightarrow \quad (a+b)=\frac{1056}{66}=16$$

$\therefore$ Possible values of a and b are

$(a = 1, b = 15), (a = 3, b = 13)$

$(a = 5, b = 11), (a = 7, b = 9)$

$\therefore$ Numbers are

$(66 \times 1, 66 \times 15), (66 \times 3, 66 \times 13)$

$(66 \times 5, 66 \times 11), (66 \times 7, 66 \times 9).$

$\therefore$ Possible number of pairs = 4

38. **(c)** Let LCM = m, HCF = n,

According to the question,

$m = 12n,$ ...(i)

and $m + n = 403$ ...(ii)

$\Rightarrow \quad 12n + n = 403$ [from Eq. (i)]

$\Rightarrow \quad 13n = 403$

$$\therefore \quad n=\frac{403}{13}=31$$

$\therefore \quad m = 12 \times 31 = 372$

Let the another number = k

$\therefore \quad 93 \times k = 372 \times 31$

[as product of two numbers = HCF × LCM]

$$\Rightarrow \quad k=\frac{372\times 31}{93}=124$$

39. **(a)** Required number of apples

= (LCM of 5, 6, 8, 9) + 3

LCM of 5, 6, 8, 9 is

2	5, 6, 8, 9
3	5, 3, 4, 9
	5, 1, 4, 3

$\therefore \quad LCM = 2 \times 3 \times 5 \times 4 \times 3 = 360$

$\therefore$ Required number of apples

$= 360 + 3 = 363$

40. **(a)** The G.P. is as following :

$a, ar, ar^2, ar^3, \ldots\ldots$ etc.

then LCM of this G. P. = ar^{n-1}

and HCF of this G.P. = a

$$\therefore \quad \frac{LCM}{HCF}=\frac{ar^{n-1}}{a}=r^{n-1}$$

41. **(b)** $30k \times 30l = 5400$, where k and l are coprimes.

$k \times l = 6$

Now, since $6 = 1 \times 6$ and 2×3

Hence, there are two pairs viz, (30, 180) and (60 and 90).

42. **(a)** It is possible only when there is no common factor among p, q and r. Hence, the HCF of p, q, r = 1

and the LCM of p, q, r = p.q.r

$\therefore$ (HCF × LCM) of p, q, r = 1 × p. q. r = pqr

43. **(d)** Let, numbers be ax and bx where x is the G.C.F. of those numbers and a and b are co–prime numbers,

So, there L.C.M. will be abx

From the question, $(a + b)x = 337$ ...(i)

$abx + x = 2665$ ....(ii)

$abx - x = 2639$ ....(iii)

After subtracting (iii) from (ii), $2x = 26$ or $x = 13$

GP_3386

So, from (i) $a + b = \frac{377}{13} = 29$ and from (ii)

$$ab = \frac{2665 - 13}{13} = 204$$

After solving, a = 12, b = 17;

($\because$ a + b = 12 + 17; ab = 12 × 17)

So, numbers will be 12 × 13 and 17 × 13 means 156 and 221.

44. **(a)** Use Shortcut Approach-16

45. **(c)** Use Shortcut Approach-16

46. **(c)** Product of numbers = 3 × 105 = xy

The sum of numbers = 36 = x + y

$\therefore$ **The sum of their reciprocal**

$$= \frac{1}{x} + \frac{1}{y} = \frac{y + x}{xy} = \frac{36}{3 \times 105} = \frac{4}{35}$$

47. **(c)** Shortcut : $\because$ $\frac{1056}{132} = 8$,

$\therefore$ Second number = 12 × 8 = 96.

48. **(c)** Use Shortcut Approach-5

49. **(a)** Use Shortcut Approach-9

50. **(d)** Use Shortcut Approach-10

51. **(d)** Use Shortcut Approach-11

CHAPTER

3

Fractions and Decimals

More At @Aj_ebooks

FRACTIONS

A number which can be expressed in the form of $\frac{p}{q}$, where p and q are both integer but $q \neq 0$. In the fraction, p is called **numerator** and q is called **denominator**. For example :

$\frac{5}{7}, \frac{1}{8}, \frac{-3}{4}, \frac{5}{-7}, -\frac{3}{19}, -\frac{5}{9}, \frac{9}{4}, \frac{-12}{7}$, etc.

An integer such as 5 can be thought as having the denominator 1. Thus 7 can be also written as $\frac{7}{1}$. So each integer is also a fraction. In the fraction $\frac{5}{7}$, 5 is numerator and 7 is denominator.

The fraction bar may be horizontal $\left(\text{as in } \frac{2}{5}\right)$, oblique $\left(\text{as in } \frac{2}{7}\right)$ or diagonal $\left(\text{as in } \frac{2}{9}\right)$.

Note: The fraction $\frac{a}{b}$ is also used to represent the ratio $a : b$ and the division $a \div b$.

Note that

(i) If the numerator of a fraction is zero and denominator is not equal to zero, then the value of fraction is zero.

(ii) If the numerator of a fraction is non-zero and denominator is zero, then the value of the fraction is infinity.

(iii) If the numerator and denominator of any fraction is either multiplied or divided by the same number, then the value of the fraction remains unchanged.

Types of Fractions

(i) **Proper Fraction :** When the numerator of a fraction is less than its denominator, then the fraction is called a **proper fraction.**

For example : $\frac{1}{4}, \frac{2}{7}, \frac{23}{37}$ etc.

(ii) **Improper Fraction :** When the numerator of a fraction is greater than its denominator, then the fraction is called **improper fraction.**

For example : $\frac{25}{19}, \frac{7}{5}, \frac{81}{17}$ etc.

(iii) **Compound Fraction :** A fraction in which numerator or denominator or both are fraction(s), is called a **compound fraction**.

For example : $\frac{3/7}{5/11}, \frac{5/13}{16}, \frac{-8}{13/113}$ etc.

(iv) **Inverse Fraction :** If we interchange the numerator and denominator of a fraction, then the resultant fraction will be the inverse fraction of the original fraction.

For example : $\frac{5}{8}$ is the inverse fraction of $\frac{8}{5}$.

(v) **Mixed Fraction :** A combination of an integer and a fraction is called a **mixed fraction.**

For example : $5\frac{3}{7}, -1\frac{21}{106}$ etc.

Here $5\frac{3}{7} = 5 + \frac{3}{7}$ and $-1\frac{21}{106} = -\left(1 + \frac{21}{106}\right)$

(vi) **Continued Fraction :** It is difficult to define continued fraction. Simply we say that a fraction which contains additional fractions in its denominators, is called **continued fraction**.

GP_3386

For example :

$$3+\frac{1}{4+\frac{1}{3+\frac{2}{5}}},\ 2+\frac{1}{1+\frac{1}{\frac{1}{2}+\frac{1}{3+\frac{1}{4}}}}\ \text{etc.}$$

(vii) Like Fractions : Two or more fractions are called **like fractions**, if their denominators are equal.

For example : $\frac{5}{13}, \frac{7}{13}, \frac{12}{13}$ are like fractions.

(viii) Unlike Fractions : Two or more fractions are called **unlike fractions**, if their denominators are not equal.

For example : $\frac{5}{8}, \frac{6}{7}, \frac{13}{18}$ are unlike fractions.

Conversion of Improper Fraction into Mixed Fraction

To convert improper fraction into mixed fraction, divide the numerator by denominator. The quotient is the whole number, the remainder is the numerator of the fraction of the mixed number and the denominator remains the same.

For example :

$$\frac{27}{13}=2\frac{1}{13}$$

$$13\overline{)27}\ \overline{)2}$$
$$\underline{26}$$
$$1$$

To convert Mixed Fraction into Improper Fraction

To convert mixed fraction into improper fraction, multiply the whole number with the denominator of the fraction and then add the result to the numerator of the fraction. The final result is the numerator and the denominator remains the same.

For example : $7\frac{3}{5}=\frac{7\times5+3}{5}=\frac{38}{5}$

Thus improper fraction of the mixed fraction $7\frac{3}{5}$ is $\frac{38}{5}$.

Simplest/ Lowest/ Standard Form of a Fraction

Simplest form of a fraction is obtained by dividing its numerator and denominator by their greatest common factor (GCD or HCF).

Note that when a fraction is in its simplest form, then its numerator and denominator are co-prime numbers i.e. they have no common factor except 1.

EQUIVALENT FRACTIONS OF A FRACTION

By multiplying (or dividing) both numerator and denominator of a fraction by any same non-zero number we obtained an equivalent fraction of a fraction. You can find as many equivalent fraction as you want by multiplying (or dividing) both numerator and denominator of a fraction by different number each time.

For example :

$$\frac{3}{7}=\frac{3\times2}{7\times2}=\frac{6}{14}$$
$$\frac{3}{7}=\frac{3\times4}{7\times4}=\frac{12}{28}$$
$$\frac{3}{7}=\frac{3\times7}{7\times7}=\frac{21}{49}$$

Thus $\frac{6}{14}, \frac{12}{28}, \frac{21}{49}$ are equivalent fractions of $\frac{3}{7}$. In other words, we also say that $\frac{3}{7}$, $\frac{6}{14}, \frac{12}{28}$ and $\frac{21}{49}$ are equivalent fractions.

MULTIPLICATIVE INVERSE OF A NON-ZERO FRACTION

Multiplicative inverse of a non-zero fraction is a number which when multiplied with the given number gives 1. Reciprocal of a fraction is obtained by interchanging its numerator and denominator.

Thus multiplicative inverse of a non-zero fraction $\frac{a}{b}$ is $\frac{b}{a}$.

Here we can see that $\frac{a}{b}\times\frac{b}{a}=1$.

For example, multiplicative inverse of $\frac{6}{11}$ is $\frac{11}{6}$.

COMPARING FRACTIONS

Comparing Like Fractions : Fractions with same denominator are called *like fractions*. Like fraction with greater numerator is greater *e.g.* like fractions $\frac{3}{2}, \frac{9}{2}, \frac{7}{2}, \frac{19}{2}$ can be arranged in ascending order as

$\frac{3}{2} < \frac{7}{2} < \frac{9}{2} < \frac{19}{2}$ because $3 < 7 < 9 < 19$.

Comparing Unlike Fractions : Unlike Fractions can be compared by converting them into like fractions. This can be achieved by LCM method.

For example :

Compare $\frac{3}{4}$ and $\frac{2}{5}$

LCM of 4 and 5 is 20

Now $\frac{3}{4} = \frac{3\times5}{4\times5}$ $[\because 20 \div 4 = 5]$ and $\frac{2}{5} = \frac{2\times4}{5\times4}$ $[\because 20 \div 5 = 4]$

$= \frac{8}{20}$ and $\frac{15}{20}$

$\frac{3}{4}$ and $\frac{2}{5}$ are equivalent to $\frac{15}{20}$ and $\frac{8}{20}$ respectively and $15 > 8$.

$\therefore \frac{15}{20} > \frac{8}{20} \Rightarrow \frac{3}{4} > \frac{2}{5}$

These fractions can also be compared by converting them into decimal numbers

e.g. $\frac{3}{4} = 0.75$ and $\frac{2}{5} = 0.4$

Now $0.75 > 0.4$ $\therefore \frac{3}{4} > \frac{2}{5}$

ADDITION AND SUBTRACTION OF FRACTIONS

Addition of Like Fractions

For addition of like fractions we add numerators and keep the denominator as it is.

For example :

$\frac{3}{4} + \frac{9}{4} = \frac{3+9}{4} = \frac{12}{4}$

$\frac{1}{2} + \frac{5}{2} + \frac{9}{2} = \frac{1+5+9}{2} = \frac{15}{2}$

Subtraction of Like Fractions

For subtraction of like fractions we subtract numerators and keep the denominator as it is.

For example : $\frac{9}{5} - \frac{3}{5} = \frac{9-3}{5} = \frac{6}{5}$

Addition and Subtraction of Unlike Fractions

Addition and subtraction of unlike fractions can be achieved by **LCM method**.

(i) Find the LCM of the denominators of the fractions.

(ii) Divide the LCM by each denominator of the fractions and multiply the quotient with the respective numerators

(iii) Add the multiplication results and the sum is the numerator and the LCM is the denominator.

For example : (i) $\frac{3}{5} + \frac{5}{7} = \frac{7\times3+5\times5}{35} = \frac{21+25}{35} = \frac{46}{35}$

(ii) $\frac{3}{5} - \frac{7}{20} = \frac{3\times4-7\times1}{20} = \frac{12-7}{20} = \frac{5}{20} = \frac{1}{4}$

Multiplication of Fractions

Multiplication of two fractions can be achieved by simply multiplying their numerators and denominators.

For example : $\frac{2}{3} \times \frac{1}{4} = \frac{2\times1}{3\times4} = \frac{2}{12} = \frac{1}{6}$

$\frac{7}{9} \times \frac{2}{5} = \frac{7\times2}{9\times5} = \frac{14}{45}$

Division of Fractions

To divide a fraction F_1 by another fraction F_2, we simply multiply F_1 by **reciprocal of** F_2.

For example : $\frac{3}{2} \div \frac{5}{7} = \frac{3}{2} \times \frac{7}{5} = \frac{21}{10}$

$3\frac{1}{2} \div 4 = \frac{7}{2} \div 4 = \frac{7}{2} \times \frac{1}{4} = \frac{7}{8}$, etc.

Note that addition, subtraction, multiplication and division of mixed fractions are done by first converting them into improper fractions.

For example :

$$1\frac{3}{5} + 1\frac{8}{9} - 2\frac{4}{9} + 3\frac{1}{5} = \frac{8}{5} + \frac{17}{9} - \frac{22}{9} + \frac{16}{5}$$

$$= \frac{72+85-110+144}{45}$$

$$= \frac{301-110}{45} = \frac{191}{45} = 4\frac{11}{45}$$

Alternate Method :

$$1\frac{3}{5} + 1\frac{8}{9} - 2\frac{4}{9} + 3\frac{1}{5} = (1+1-2+3) + \left(\frac{3}{5} + \frac{8}{9} - \frac{4}{9} + \frac{1}{5}\right)$$

GP_3386

$$= 3 + \frac{27+40-20+9}{45}$$

$$= 3 + \frac{56}{45} = 3 + 1\frac{11}{45}$$

$$= 3 + 1 + \frac{11}{45} = 4\frac{11}{45}.$$

DECIMAL FRACTIONS

Fractions whose denominators are powers of 10 are known as **decimal fractions**. *e.g.* $\frac{2}{10}, \frac{7}{100}, \frac{999}{1000}$, etc. are decimal fractions. Any fraction with denominator in power of 10 can be expressed as decimal number

e.g.: $\frac{2}{10} = 0.2$ $\frac{7}{100} = 0.07$, $\frac{999}{1000} = 0.999$, etc.

Changing Fraction into Decimal Number : A fraction can be converted to decimal by dividing the numerator by its denominator,

For example: $\frac{3}{4} = 0.75$, $\frac{1}{100} = 0.01$

Changing Decimal Number into Fraction : To convert a given decimal number into fraction, take the given number without decimal as numerator and take the denominator as 10^n, where n is a natural number, which is equal to the number of digits after decimal in the given decimal number.

For example:

$$2.56 = \frac{256}{10^2} = \frac{256}{100} = \frac{64}{25} \text{(Simplest form)}$$

$$0.069 = \frac{0069}{10^3} = \frac{69}{1000} \text{ (Simplest form)}$$

$$25.2016 = \frac{252016}{10^4} = \frac{252016}{10000} = \frac{15751}{625}$$

(Simplest form)

Addition and Subtraction of Decimal Numbers : To add or subtract given decimal fractions, the numbers must be arranged one below the other in such a way that decimal point must lie in the same column. Now they can be added or subtracted in usual way.

Note that if we add any number of zeros to the right of a decimal number, its value does not changed

For example : 2.3 = 2.300, 0.06 = 0.06000 etc.

Illustration – 1

Find the sum of 54.6032, 0.532 and 2.50024

Solution :
$$\begin{array}{r} 54.6032 \\ 0.532 \\ +2.50024 \\ \hline 57.63544 \end{array}$$

Illustration – 2

Subtract 20.3428 from 125.89

Solution :
$$\begin{array}{r} 125.8900 \\ -20.3428 \\ \hline 105.5472 \end{array}$$

Note that in addition and subtraction of decimal numbers, we put some zero(s) to the right of the numbers if required to make the number of digits after decimal equal in each number, so that the addition and subtraction becomes convenient.

Multiplication of Decimal Numbers:

(a) If we multiply a decimal fraction by 10^n, where n is a natural number, shift the decimal to the right as many places as is the power of 10.

(b) If we multiply a decimal number by a whole number, follow the usual process. There is no change in the decimal places in the product from the right.

(c) If we multiply a decimal number by another decimal number, first multiply them without considering the decimal point. Then place the decimal point in the product in such a way that the number of decimal places from right in the product is equal to the sum of the number of decimal places from right in the two numbers multiplied.

Illustration – 3

Find the value :

(i) 77.254 × 100

(ii) 24.926 × 21

(iii) 33.213 × 1.36

Solution : (i) $77.254 \times 100 = 77.254 \times 10^2 = 7725.4$

(ii) $24.926 \times 21 = 523.446$

(iii) 33.213×1.36

First calculate $33213 \times 136 = 4516968$

Now count decimal places of 33.213 (3 decimal places) and in 1.36 (2 decimal places) i.e., total 5 decimal places.

$\therefore$ $33.213 \times 1.36 = 45.16968$

Note that decimal places are counted from right.

Division of Decimal Numbers:

(a) When we divide a decimal number by 10^n, where n is a natural number, shift the decimal to the left as many places as is the power of 10.

(b) When we divide a decimal number by a whole number, divide it in usual manner and in the quotient put the decimal point as many places as in the dividend from the right.

(c) When we divide a decimal number by another decimal number, first convert the decimal fractions into the whole numbers by multiplying with suitable power of 10 by both numerator and denominator (i.e., dividend and divisor) and then divide it in the usual manner.

Illustration – 4

Find the value :

(i) 74.268 ÷ 100 (ii) 32.46 ÷ 2

(iii) 24.36 ÷ 0.26

Solution : (i) $74.268 \div 100 = 74.268 \div 10^2 = 0.74268$

(ii) $\frac{32.46}{2}$

First divide 3246 by 2

$\frac{3246}{2} = 1623$, Now put the decimal point in 1623 as many places as in dividend 32.46. Hence

$\frac{32.46}{2} = 16.23$

(iii) $24.36 \div 0.26 = \frac{24.36}{0.26} = \frac{2436}{26}$

(Multiplying Numerator and Denominator by 10^2 or 100)

$\therefore \quad 24.36 \div 0.26 = \frac{2436}{26} = 93.6923$ (approx)

ROUNDING OFF (APPROXIMATION) OF DECIMAL NUMBERS

In some decimal numbers, there are large number of digits after decimal place.

For example : 4.5073291307856.

Many time in such cases, we want to consider the decimal number only upto a certain place after decimal. For this

- We will take the number upto the required place without any change, if the next digit of the required place is less than 5 (this is called **rounding down**).
- We will take the number upto the required place by increasing the required place digit by 1, if the next digit of the required place is 5 or greater than 5 (this is called **rounding up**).

Illustration – 5

Rounding off the following numbers upto 3 decimal places:

(i) 5.67325698

(ii) 0.0045286

(iii) 62.586631

Solution : (i) 5.673 [as the fourth place digit (2) after decimal is less than 5].

(ii) 0.005 [as the fourth place digit (5) after decimal is 5]

(iii) *62.587 [as the fourth place digit (6) after decimal is greater than 5]*

SPECIAL TYPES OF DECIMAL NUMBERS

(i) Non-terminating Recurring Decimal Numbers:

If in a decimal number a digit or a set of digits repeats itself again and again but never come to end, then the decimal number is called **non-terminating recurring decimal number.**

For example:

20.3333....., 6.541541541....., 0.25101010..... etc.

It can also be expressed by putting a bar over the first set of repeating digit(s). Hence

$20.3333..... = 20.\overline{3}$, $6.541541541..... = 6.\overline{541}$, $0.25101010..... = 0.25\overline{10}$ etc.

Any given non-terminating recurring decimal number can be converted into fraction by using the following formula :

Required Fraction

$$= \frac{\left(\text{Given number without decimal and bar}\right) - \left(\begin{array}{c}\text{Number formed by recurring digits}\\ \text{without decimal}\end{array}\right)}{\begin{array}{l}\text{9's equal to number of recurring digits followed}\\ \text{by 0s equal to number of non-recurring digits}\\ \text{after the decimal point}\end{array}}$$

Required Fraction

$$= \frac{\left(\text{Given number without decimal and bar}\right) - \left(\begin{array}{c}\text{Number formed by non-recurring digits}\\ \text{without changing its order}\end{array}\right)}{\begin{array}{l}\text{9's equal to number of recurring digit(s) followed}\\ \text{by 0s equal to number of non-recurring digits after}\\ \text{the decimal point}\end{array}}$$

GP_3386

Illustration – 6

Convert the following numbers into fraction :

(a) $21.3\overline{162}$ **(b)** $0.5\overline{42}$ **(c)** $2.\overline{45}$ **(d)** $0.\overline{321}$

(e) $0.00\overline{21}$

Solution : (a) Required fraction

$$= \frac{213162 - 213}{9990} = \frac{212949}{9990}$$

(b) Required fraction $= \frac{542 - 5}{990} = \frac{537}{990}$

(c) Required fraction $= \frac{245 - 2}{99} = \frac{243}{99}$

(d) Required fraction $= \frac{321}{999}$

(e) Required fraction $= \frac{21}{9900}$

(ii) Non-terminating Non-recurring Decimal Numbers :

Those decimal numbers in which digits after decimal never come to end but there is no single digit or a set of digits which repeats continuously after decimal are called **non-terminating non-recurring decimal numbers.**

For example : 2.010010001....., 0.211311141111...., 52.1223334444.... etc.

USEFUL DECIMAL REPRESENTATION OF SOME FRACTIONS

$\frac{1}{2} = 0.5$, $\frac{1}{3} = 0.\overline{3}$, $\frac{1}{4} = 0.25$, $\frac{1}{5} = 0.2$,

$\frac{1}{6} = 0.1\overline{6} = 0.167$ (approx.), $\frac{1}{8} = 0.125$, $\frac{1}{9} = 0.\overline{1}$,

$\frac{1}{10} = 0.1$, $\frac{1}{20} = 0.05$, $\frac{1}{25} = 0.04$, $\frac{1}{50} = 0.02$,

$\frac{2}{3} = 0.\overline{6} = 0.67$ (approx.)

Shortcut Approach - 2

To compare two fractions $\frac{a}{b}$ and $\frac{c}{d}$, follow as

$\frac{a}{b}$ $\quad\quad$ $\frac{c}{d}$

ad $\quad\quad$ bc [Write the product ad and bc on their numerator side a and c respectively]

If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$

If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$

Illustration – 7

Compare $\frac{5}{6}$ and $\frac{8}{9}$

Shortcut Solution : (b)

$\frac{5}{6}$ $\quad\quad$ $\frac{8}{9}$

$45 \quad < \quad 48$

$\therefore \frac{5}{6} < \frac{8}{9}$

Shortcut Approach - 2

If in any given sequence of fractions, the difference in consecutive numerators is equal or greater than the difference in consecutive denominators, then the fraction having larger numerator is larger and the fraction having smaller numerator is smaller.

Illustration – 8

Arrange the following fractions in decreasing order:

$\frac{3}{5}, \frac{5}{7}, \frac{6}{8}, \frac{9}{11}$

(a) $\frac{3}{5}, \frac{5}{7}, \frac{6}{8}, \frac{9}{11}$ **(b)** $\frac{3}{5}, \frac{5}{7}, \frac{9}{11}, \frac{6}{8}$

(c) $\frac{9}{11}, \frac{3}{5}, \frac{5}{7}, \frac{6}{8}$ **(d)** $\frac{9}{11}, \frac{6}{8}, \frac{5}{7}, \frac{3}{5}$

Shortcut Solution : (d)

$$\left(\begin{matrix}\text{Difference in} \\ \text{consecutive numerator}\end{matrix}\right) = \left(\begin{matrix}\text{Difference in} \\ \text{consecutive denominator}\end{matrix}\right) = 2$$

Hence the fraction having greater numerator is greater.

$\therefore$ Decreasing order :

$\frac{9}{11} > \frac{6}{8} > \frac{5}{7} > \frac{3}{5}$

Illustration – 9

Which of the following is the smallest fraction :

$\frac{2}{9}, \frac{5}{11}, \frac{8}{13}, \frac{11}{15}$

(a) $\frac{2}{9}$ (b) $\frac{5}{11}$ (c) $\frac{8}{13}$ (d) $\frac{11}{15}$

Shortcut Solution : (a)

Difference in consecutive numerator = 3

Difference in consecutive denominator = 2 $\because$ (3 > 2)

Therefore, the fraction with smallest numerator is smallest.

Thus smallest fraction $= \frac{2}{9}$.

Shortcut Approach - 2

If in any given sequence of fractions, numerator is increasing by a definite value and denominator is also increasing by a different definite value but increase in the value of numerator is less than increase in the value of denominator, then

(i) **If $\frac{\text{increase in numerator}}{\text{increase in denominator}}$ > First fraction, the fraction with greatest numerator is greatest.**

(ii) **If $\frac{\text{increase in numerator}}{\text{increase in denominator}}$ < First fraction, the fraction with greater numerator is least.**

Illustration – 10

Which one of the following fractions is the greatest?

$\frac{1}{5}, \frac{3}{8}, \frac{5}{11}, \frac{7}{14}$

(a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{11}$ (d) $\frac{7}{14}$

Shortcut Solution : (d)

Here numerator increases by a constant value 2 and denominator increases by a constant value 3.

Now, $\frac{\text{Increase in numerator}}{\text{Increase in denominator}} = \frac{2}{3}$

First fraction = $\frac{1}{5}$

Since $\frac{2}{3} > \frac{1}{5}$; Hence $\frac{7}{14}$ is the greatest fraction.

Illustration – 11

Which one of the following fractions is the greatest?

$\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{5}{6}$ (d) $\frac{6}{7}$

Shortcut Solution : (d)

Here numerator increases by a constant value 1 and denominator increases by a constant value 1.

Now, $\frac{\text{Increase innumerator}}{\text{Increase in denominator}} = \frac{1}{1} = 1$

Since, $1 > \frac{3}{4}$.

Hence $\frac{6}{7}$ is the greatest fraction.

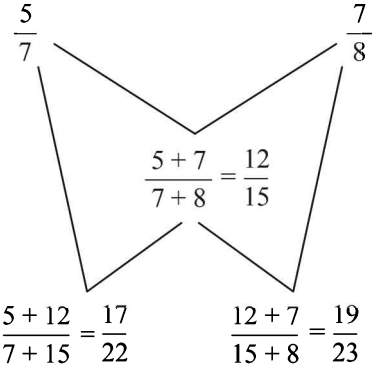
Shortcut Approach - 2

A fraction between any two given fractions $\frac{x_1}{y_1}$ and $\frac{x_2}{y_2}$ is $\frac{x_1 + x_2}{y_1 + y_2}$. Using this you can find any number of fractions between any two given fractions.

Illustration – 12

Find any three fractions between $\frac{5}{7}$ and $\frac{7}{8}$.

Shortcut Solution :



Hence, three fractions between $\frac{5}{7}$ and $\frac{7}{8}$ are $\frac{17}{22}, \frac{12}{15}$ and $\frac{19}{23}$.

GP_3386

Shortcut Approach - 2

If a man spends $\frac{x_1}{y_1}$ part of his salary on food, $\frac{x_2}{y_2}$ part of his salary on clothing, $\frac{x_3}{y_3}$ part of his salary on education and so on. After these expenditures,

Balance salary = (Salary) $\times \left[1-\left(\frac{x_1}{y_1}+\frac{x_2}{y_2}+\frac{x_3}{y_3}+.....\right)\right]$

Illustration – 13

A man spends $\frac{1}{5}$ of his salary on rent, $\frac{2}{5}$ of his salary on food and $\frac{3}{10}$ of his salary on education. Still ₹10,000 has left with him. Find his salary.

(a) ₹50,000 (b) ₹60,000
(c) ₹75,000 (d) ₹1,00,000

Shortcut Solution : (d)

$$\text{Balance Salary} = (\text{Salary}) \times \left[1-\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{10}\right)\right]$$

$$\Rightarrow 10,000 = (\text{Salary}) \times \left[1-\frac{2+4+3}{10}\right]$$

$$\Rightarrow 10,000 = (\text{Salary}) \times \frac{1}{10}$$

$\therefore$ Salary = 10×10000 = ₹1,00,000

Shortcut Approach - 6

If a man spends $\frac{x_1}{y_1}$ part of his salary on food, $\frac{x_2}{y_2}$ part of his remaining salary on clothing, $\frac{x_3}{y_3}$ part of his remaining salary on education and so on. After these expenditures,

Balance salary

$$= (\text{Salary}) \times \left[\left(1-\frac{x_1}{y_1}\right) \times \left(1-\frac{x_2}{y_2}\right) \times \left(1-\frac{x_3}{y_3}\right) \times .....\right]$$

Illustration – 14

A man spends $\frac{1}{5}$ of his salary on rent, $\frac{2}{5}$ of his remaining salary of food and $\frac{3}{10}$ of his remaining salary on education. Still ₹10,000 left with him. Find his salary.

(a) ₹ 26,500 (b) ₹ 28,000
(c) ₹ 29,600 (d) ₹ 29,800

Shortcut Solution : (d)

Balance salary

$$= (\text{Salary}) \times \left[\left(1-\frac{x_1}{y_1}\right) \times \left(1-\frac{x_2}{y_2}\right) \times \left(1-\frac{x_3}{y_3}\right) \times .....\right]$$

$$\Rightarrow 10,000 = (\text{Salary}) \times \left[\left(1-\frac{1}{5}\right) \times \left(1-\frac{2}{5}\right) \times \left(1-\frac{3}{10}\right)\right]$$

$$\Rightarrow 10,000 = (\text{Salary}) \times \frac{4}{5} \times \frac{3}{5} \times \frac{7}{10}$$

$$\therefore \text{Salary} = 10,000 \times \frac{5}{4} \times \frac{5}{3} \times \frac{10}{7}$$

= ₹29,761.90 = ₹29,800 (approx.)

Shortcut Approach - 2

For any single activity,

(i) Part done + Part remained = 1
(ii) Total amount

$$\text{Total amount} = \frac{\text{Amount spent}}{\text{Balance part}} = \frac{\text{Amount remained}}{\text{Part remained}}$$

Illustration – 15

In a flag post, $\frac{1}{5}$ is red, $\frac{1}{3}$ is green and the rest is white. If the white part is 7 metres long, find the length of the flag post.

Shortcut Solution :

$$\text{Length of the flag post} = \frac{\text{Amount remained}}{\text{Part remained}}$$

$$= \frac{7}{1-\left(\frac{1}{5}+\frac{1}{3}\right)} = \frac{7}{1-\frac{8}{15}} = \frac{7}{\frac{7}{15}} = 7 \times \frac{15}{7} = 15 \text{ metres}$$

Shortcut Approach - 2

The denominator of a fraction (or rational number) is 'D' more than its numerator. If the numerator is increased by x and the denominator is decreased by y, we get P, then the fraction (or rational number) is given by $\frac{x-P(D-y)}{x+(yP-D)}$.

Illustration – 16

The denominator of a fraction is 2 more than its numerator. If the numerator is increased by 9 and the denominator is decreased by 5, we get 7. Find the fraction.

(a) $\frac{3}{5}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{7}{9}$ **(d)** $\frac{5}{7}$

Shortcut Solution : (d)

Required fraction $= \frac{x-P(D-y)}{x+(yP-D)}$

Here, $D=2, x=9, y=5$ and $P=7$.

$\therefore$ Required fraction $= \frac{9-7(2-5)}{9+(5\times 7-2)}$

$= \frac{9+21}{9+33} = \frac{30}{42} = \frac{5}{7}$.

GP_3386

LEVEL - 1

1. Which one of the following is largest among $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$? **(SSC CGL 2010)**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{6}$

2. Which of the following fractions is smallest? **(CDS 2014)**

(a) $\frac{5}{2}$ (b) $\frac{7}{3}$ (c) $\frac{9}{4}$ (d) $\frac{11}{5}$

3. Find the value of

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{100}\right).$$

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{50}$ (d) $\frac{2}{5}$

4. If $\sqrt{24} = 4.899$, the value of $\sqrt{\frac{8}{3}}$ is

(a) 0.544 (b) 1.333 (c) 1.633 (d) 2.666

5. If $\sqrt{15625} = 125$, then the value of $\sqrt{15625} + \sqrt{156.25} + \sqrt{1.5625}$ is

(a) 1.3875 (b) 13.875 (c) 138.75 (d) 156.25

6. Simplify : $\dfrac{1}{1+\dfrac{\frac{2}{3}}{1+\frac{2}{3}+\dfrac{\frac{8}{9}}{1-\frac{2}{3}}}}$

(a) $\frac{11}{13}$ (b) $\frac{13}{15}$ (c) $\frac{13}{11}$ (d) $\frac{15}{13}$

7. $\sqrt{0.01+\sqrt{0.0064}} = ?$

(a) 0.3 (b) 0.03

(c) $\sqrt{0.18}$ (d) None of these

8. $\sqrt{\dfrac{0.081\times0.324\times4.624}{1.5625\times0.0289\times72.9\times64}}$ is equal to

(a) 0.024 (b) 0.24 (c) 2.4 (d) 24

9. Find the value of

$$\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\frac{1}{5\times6}+.....+\frac{1}{9\times10}$$

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$ (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

10. What will be the value of p + q when $0.\overline{43}$ will be written in p/q format?

(a) 134 (b) 142 (c) 146 (d) 132

11. Which of the following is a prime number?

(a) 187 (b) 221 (c) 391 (d) 307

12. If A = 4.3333….. and B = 3.6666666….. where as A + B = p/q then what could be the value of p + q?

(a) 9 (b) 7 (c) 6 (d) 11

13. A fraction between $\frac{2}{5}$ and $\frac{4}{9}$ is **(SSC CGL Tier-II 2015)**

(a) $\frac{3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{1}{2}$

14. How many fractions in the form of p/q are between $\frac{1}{2}$ and $\frac{1}{4}$.

(a) 1
(b) More than 1 but finite
(c) Infinite
(d) More than 1 but less than 100

15. Which of the following fractions lie between $\frac{3}{5}$ and $\frac{2}{3}$? **(SSC CGL Tier -I 2016)**

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{1}{15}$ (d) $\frac{5}{8}$

16. Find the summation of $1.\overline{23}$ and $1.2\overline{3}$.
(a) 2441/990 (b) 2231/990
(c) 1332/990 (d) 1221/990

17. Two positive integers differ by 4 and sum of their reciprocals is $\frac{10}{21}$. Then one of the numbers is
(a) 1 (b) 3 (c) 5 (d) 21

18. What will be the simplest form of $0.0579\overline{2}$?
(a) $\frac{579}{9000}$ (b) $\frac{5213}{90000}$ (c) $\frac{1903}{30000}$ (d) $\frac{193}{3000}$

19. The value of $0.\overline{45}+0.4\overline{5}+0.45$ will be
(a) $\frac{2693}{1980}$ (b) $\frac{946}{990}$ (c) $\frac{9406}{9990}$ (d) $\frac{946}{999}$

20. What will be the value of $2.1\overline{3}+3.1\overline{2}$?
(a) $5.2\overline{5}$ (b) $5.\overline{2}$ (c) $5.2\overline{3}$ (d) 5.25

21. The smallest possible number with three places of decimal is
(a) 0.012 (b) 0.123
(c) 0.111 (d) None of these

22. The value of $1.\overline{2}\times 0.\overline{03}$ is
(a) $0.\overline{04}$ (b) $0.0\overline{36}$ (c) $1.\overline{13}$ (d) $0.0\overline{37}$

23. The ascending order of $\frac{3}{4},\frac{5}{12},\frac{11}{18}$ and $\frac{17}{24}$ is
(a) $\frac{17}{24},\frac{11}{18},\frac{5}{12},\frac{3}{4}$ (b) $\frac{3}{4},\frac{17}{24},\frac{11}{18},\frac{5}{12}$
(c) $\frac{5}{12},\frac{11}{18},\frac{17}{24},\frac{3}{4}$ (d) $\frac{11}{18},\frac{17}{24},\frac{5}{12},\frac{3}{4}$

24. Arrange the following fractions in decreasing order: $\frac{3}{5},\frac{7}{9},\frac{11}{13}$. **(Bank Clerk 2015)**
(a) $\frac{11}{13},\frac{7}{9},\frac{3}{5}$ (b) $\frac{11}{13},\frac{3}{5},\frac{7}{9}$
(c) $\frac{3}{5},\frac{11}{13},\frac{7}{9}$ (d) $\frac{7}{9},\frac{3}{5},\frac{11}{13}$

25. Which of the fraction is largest among $\frac{2}{5},\frac{5}{11},\frac{8}{17},\frac{11}{23}$? **(SSC CGL 2011)**
(a) $\frac{2}{5}$ (b) $\frac{11}{23}$ (c) $\frac{5}{11}$ (d) $\frac{8}{17}$

26. The product of two fractions is $\frac{14}{15}$ and when they divided the resultant is $\frac{35}{24}$. The larger among the fraction is
(a) $\frac{7}{4}$ (b) $\frac{7}{6}$ (c) $\frac{7}{3}$ (d) $\frac{4}{5}$

27. What will obtained when sum of $5\frac{3}{4}$ and $1\frac{2}{3}$ is divided by its difference?
(a) $\frac{79}{59}$ (b) $\frac{79}{49}$ (c) $\frac{89}{49}$ (d) $\frac{49}{89}$

28. Represent 45 min. in the fraction of one day
(a) $\frac{1}{40}$ (b) $\frac{1}{32}$ (c) $\frac{1}{60}$ (d) $\frac{1}{24}$

29. The difference between L.C.M. and H.C.F. of 0.08 and 16 will be
(a) 15.2 (b) 1.52
(c) 15.92 (d) None of these

30. The H.C.F. of 2.4, 0.18, 0.027 and 15 will be
(a) 0.3 (b) 0.03
(c) 0.003 (d) None of these

31. 1/4th of number of boys and 3/8th of number of girls participated in annual sports of the school. What fractional part of total number of students participated?
(a) 32% (b) 20%
(c) 36% (d) Data inadequate

32. Find the value of $\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}$.
(a) $\frac{10}{11}$ (b) $\frac{5}{11}$ (c) $\frac{9}{11}$ (d) $\frac{7}{11}$

33. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=?$
(a) 2 (b) 5 (c) 4 (d) 6

34. If the numerator of a fraction is increased by 200% and the denominator of the fraction is increased by 150%, the resultant fraction is 9/35. What is the original fraction?
(a) $\frac{3}{10}$ (b) $\frac{2}{15}$
(c) $\frac{3}{16}$ (d) None of there

GP_3386

35. The denominator of a rational number is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, the result is 2. The fraction is:

(SSC CGL Tier -II 2016)

(a) $\frac{3}{5}$ (b) $\frac{5}{8}$ (c) $\frac{7}{10}$ (d) $\frac{8}{11}$

36. The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes eight times the numerator, then find the fraction

[SSC CGL Tier-II 2013]

(a) $\frac{3}{7}$ (b) $\frac{4}{8}$ (c) $\frac{2}{7}$ (d) $\frac{3}{8}$

37. If the fraction a /b is positive, then which of the following must be true?

(a) $a>0$ (b) $b>0$ (c) $ab>0$ (d) $a+b>0$

38. Express $0.82\overline{68}$ as a fraction.

(a) $\frac{3093}{4950}$ (b) $\frac{3043}{4850}$ (c) $\frac{4093}{4950}$ (d) $\frac{3039}{4950}$

39. Tanya gives to each of four girls $\frac{1}{12},\frac{5}{18},\frac{7}{30},\frac{7}{48}$ of the apples in a basket and has only just enough apples to be able to do so without dividing an apple. The minimum number of apple had she?

(a) 250 (b) 720
(c) 750 (d) None of these

40. Mallika travelled $\frac{5}{16}$ of her journey by coach and $\frac{7}{20}$ by rail, she walked the remaining 10 km, how far did she go altogether?

(a) $27\frac{17}{29}$ (b) $33\frac{7}{27}$ (c) $19\frac{7}{27}$ (d) $29\frac{17}{27}$

41. The expression $33.33 \div 1.1$ simplifies as :

(a) 33.3 (b) 303 (c) 33.0 (d) 30.3

42. Simplified value of

$$\frac{8.73\times8.73\times8.73+4.27\times4.27\times4.27}{8.73\times8.73-8.73\times4.27+4.27\times4.27}$$ is :

(a) 11 (b) 12 (c) 13 (d) 14

43. The next number of the sequence $\frac{1}{2},\frac{3}{4},\frac{5}{8},\frac{7}{16},\ldots$ is

(a) $\frac{9}{24}$ (b) $\frac{9}{32}$ (c) $\frac{10}{24}$ (d) $\frac{11}{32}$

44. What is value of the expression $\sqrt{7\sqrt{7\sqrt{7\sqrt{7\sqrt{\ldots\ldots\ldots\infty}}}}}$?

(a) 49 (b) 7 (c) 1 (d) 0

45. If $X=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $Y=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, then find value of $\sqrt{\frac{X}{Y}}+\sqrt{\frac{Y}{X}}$?

(a) 8 (b) 12 (c) 10 (d) 0

46. If $X=\frac{2\sqrt{10}}{\sqrt{2}+\sqrt{5}}$, then find value of $\frac{X+\sqrt{2}}{X-\sqrt{2}}+\frac{X+\sqrt{5}}{X-\sqrt{5}}$?

(a) 2 (b) 1 (c) 3 (d) 4

47. If $3^{(a+8)}=27^{(2a+1)}$, then find 'a'?

(a) 1 (b) 0 (c) –1 (d) 0.5

48. If 738A6A is divisible by 11, then the value of A is

(a) 6 (b) 3 (c) 9 (d) 1

49. Given that $\sqrt{4096}=64$, the value of $\sqrt{4096}+\sqrt{40.96}+\sqrt{0.004096}$ is-

(a) 70.4 (b) 70.464 (c) 71.104 (d) 71.4

50. If x=1001, then find the value of $\sqrt[3]{x(x^2+3x+3)+1}$?

(a) 999 (b) 1000 (c) 1001 (d) 1002

51. Evaluate $\sqrt{6+\sqrt{6+\sqrt{6+}}}$ -------∞

(a) –2 (b) 3 (c) 6 (d) – 6

52. If 4x = 18 y, then the values of $\left(\frac{x}{y}-1\right)$ is :

(a) 1/3 (b) 7/2 (c) 2/3 (d) 3/2

53. The value of $\frac{1\frac{1}{4}\div1\frac{1}{2}}{\frac{1}{15}+1-\frac{9}{10}}$ is

(a) 3 (b) 6 (c) 5/2 (d) 5

54. $\dfrac{(0.87)^4-(0.13)^4}{0.87\times0.87+0.13\times0.13}=?$

(a) 1 (b) 0.87 (c) 0.13 (d) 0.74

55. Which of the following fractions are in ascending order?

(a) $\frac{11}{52},\frac{7}{26},\frac{4}{13}$ (b) $\frac{7}{26},\frac{4}{13},\frac{11}{52}$,

(c) $\frac{4}{13},\frac{7}{26},\frac{11}{52}$, (d) $\frac{7}{26},\frac{11}{52},\frac{4}{13}$,

56. If A:B = 3:4, B:C = 5:7 and C:D = 8:9, then A:D is equal to

(a) 3 : 7 (b) 7 : 3 (c) 21 : 10 (d) 10 : 21

57. What is the unit's digit of the expression $29^{28} - 28^{29}$?

(a) 3 (b) 7 (c) 4 (d) 5

58. If $\sqrt{6}\times\sqrt{15}=x\sqrt{10}$, then the value of x is

(a) 3 (b) ± 3 (c) $\sqrt{3}$ (d) $\sqrt{6}$

59. What is the value of (x) in the following equation

$$\frac{(x)^{0.7}}{36}=\frac{9}{(x)^{1.3}}$$

(a) 17 (b) 19 (c) 16 (d) 18

60. The product of 2 numbers is 1575 and their quotient is 9/7. Then the sum of the numbers is–

(a) 74 (b) 78 (c) 80 (d) 90

61. The value of $\left[(81)^{3.6}\times(9)^{2.7}/(81)^{4.2}\times(3)\right]$ is–

(a) 3 (b) 6 (c) 9 (d) 8.2

62. When $(67^{67}+67)$ is divided by 68, the remainder is-

(a) 1 (b) 63 (c) 66 (d) 67

63. If $a + 1/a + 2 = 0$, then the value of $(a^{37} - 1/a^{100})$ is-

(a) 0 (b) –2 (c) 1 (d) 2

64. If $x = 64$ and $\sqrt[3]{x}+\sqrt[3]{y}=\sqrt[3]{216}$, then y = ?

(a) 27 (b) 64 (c) 8 (d) 125

65. $\frac{1}{8}$ part of a pencil is black and $\frac{1}{2}$ part of the remaining is white. If the remaining part is blue and length of this blue part is $3\frac{1}{2}$ cm, then find the length of the pencil. **[CDS 2009]**

(a) 6 cm (b) 7 cm (c) 8 cm (d) 9 cm

LEVEL - 2

1. The value of

$$\left[35.7-\left(3+\frac{1}{3+\frac{1}{3}}\right)-\left(2+\frac{1}{2+\frac{1}{2}}\right)\right]$$ is

(a) 30 (b) 34.8 (c) 36.6 (d) 41.4

2. The simplified value of

$$\frac{\left(1+\frac{1}{1+\frac{1}{100}}\right)\left(1+\frac{1}{1+\frac{1}{100}}\right)-\left(1-\frac{1}{1+\frac{1}{100}}\right)\left(1-\frac{1}{1+\frac{1}{100}}\right)}{\left(1+\frac{1}{1+\frac{1}{100}}\right)+\left(1-\frac{1}{1+\frac{1}{100}}\right)}$$ is

(a) 100 (b) $\frac{200}{101}$ (c) 200 (d) $\frac{202}{100}$

3. $99\frac{1}{7}+99\frac{2}{7}++99\frac{3}{7}+99\frac{4}{7}+99\frac{5}{7}+99\frac{6}{7}$

is equal to

(a) 603 (b) 600 (c) 598 (d) 597

4. One third of the boys and half of the girls of a college take part in a camp. If a total of 500 students take part in the camp among which 300 are boys, then how many students are there in the college?

(a) 1300 (b) 700 (c) 900 (d) 1100

5. A, B, C and D purchase a gift worth ₹ 60. A pays 1/2 of what others are paying B pays 1/3rd of what others are paying and C pays 1/4th of what others are paying. What is the amount paid by D?

(a) ₹ 14 (b) ₹ 15 (c) ₹ 16 (d) ₹ 13

6. Sum of three fractions is $2\frac{11}{24}$. If the greatest fraction, is divided by the smallest fraction the result is 7/6 which is greater than the middle fraction by 1/3. Find all the three fractions.

(a) $\frac{3}{5},\frac{4}{7},\frac{2}{3}$ (b) $\frac{7}{8},\frac{5}{6},\frac{3}{4}$

(c) $\frac{7}{9},\frac{2}{3},\frac{3}{5}$ (d) $\frac{7}{8},\frac{7}{9},\frac{7}{10}$

7. A man reads 3/8 of a book on a day and 4/5 of the remainder on the second day. If the number of pages still unread are 40, then how many pages did the book contain?

(a) 300 (b) 500 (c) 320 (d) 350

8. A man reads $\frac{3}{8}$ of a book on a day and $\frac{4}{5}$ of the remainder on the second day. If the number of pages still unread are 40, then how many pages did the book contain? **[SSC CGL Tier-II 2015]**

(a) 300 (b) 500 (c) 320 (d) 350

9. Abhishek, Bobby and Charlie start from the same point and travel in the same direction around an Island which is 6 km in circumference. Abhishek travels at the rate of 3, Bobby at the rate of $2\frac{1}{2}$ and Charlie at the rate of $1\frac{1}{4}$ km/ hour. In how many hours will they come together again?

(a) 6 hrs (b) 12 hrs (c) 24 hrs (d) 15 hrs

10. If $a + b + c + d = 4$, then the value of

$$\frac{1}{(1-a)(1-b)(1-c)}+\frac{1}{(1-b)(1-c)(1-d)}+$$

$$\frac{1}{(1-c)(1-d)(1-a)}+\frac{1}{(1-d)(1-a)(1-b)}$$

(a) 1 (b) 4 (c) 0 (d) 5

11. If $x=4+\sqrt{5}$, then the value of $\sqrt{x}+1/\sqrt{x}$ is

(a) $\sqrt{3}$ (b) $\sqrt{10}$ (c) $2\sqrt{6}$ (d) 6

12. If m : n = 2 : 3, then the value of $\frac{3m+5n}{6m-n}$ is equal to

(a) 3/7 (b) 7/3 (c) 5/3 (d) 3/5

13. For all real values of x and y if (4x – y) is divisible by 3, then $(4x^2 + 7xy – 2y^2)$ is always divisible by

(a) 12 (b) 3 (c) –11 (d) 1

14. The simplified form of

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\ldots\left(1-\frac{1}{49}\right) \text{ is}$$

(a) $\frac{1}{7}$ (b) $\frac{1}{49}$ (c) $\frac{1}{14}$ (d) $\frac{1}{48}$

15. Find the value of

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots.+\frac{1}{\sqrt{99}+\sqrt{100}}$$

is

(a) 8 (b) 6 (c) 10 (d) 9

16. A person gave ₹ 500 to his eldest son. Then the gave $\frac{1}{10}$ part of his total wealth to his second son and the amount of money received by his third son equal to the total amount received by his first and second son. How much money the person had? **(SSC CGL Tier -I 2016)**

(a) 1250 (b) 1750 (c) 1500 (d) 1000

17. $\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm of the pole is still in the mud. Find the full length of the pole. **[CDS 2014]**

(a) 1850 (b) 1050 (c) 2000 (d) 1500

18. A farmer divides his herd of n cows among his four sons, so that the first son gets one-half the herd, the second one-fourth, the third son $\frac{1}{5}$ and the fourth son 7 cows. Then the value of n is; **[SSC CGL Tier I 2012]**

(a) 100 (b) 120 (c) 140 (d) 160

Hints and Solutions

Level 1

1. (d) Use Shortcut approach - 2

2. (a) Use Shortcut approach - 2

3. (c) Given expression

$$= \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times.....\times\frac{99}{100} = \frac{2}{100} = \frac{1}{50}$$

4. (c) $\sqrt{\frac{8}{3}} = \sqrt{\frac{8\times3}{3\times3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = 1.633.$

5. (c) Given expression

$$= \sqrt{15625} + \sqrt{\frac{15625}{100}} + \sqrt{\frac{15625}{10000}}$$

$$= \left(125 + \frac{125}{10} + \frac{125}{100}\right) = (125 + 12.5 + 1.25)$$

$$= 138.75$$

6. (b) Given exp. $= \dfrac{1}{1+\dfrac{\frac{2}{3}}{\frac{5}{3}+\frac{\frac{8}{9}}{(1/3)}}} = \dfrac{1}{1+\dfrac{2/3}{\frac{5}{3}+\frac{8}{9}\times3}}$

$$= \frac{1}{1+\frac{2/3}{(13/3)}} = \frac{1}{1+\frac{2}{13}} = \frac{13}{15}$$

7. (a) Given expression

$$= \sqrt{0.01+0.08} = \sqrt{0.09} = 0.3$$

8. (a) Given exp. $= \sqrt{\dfrac{81\times324\times4624}{15625\times289\times729\times64}}$

$$= \frac{9\times18\times68}{125\times17\times27\times8} = \frac{3}{125} = 0.024.$$

9. (b) Given expression

$$= \left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+$$

$$.....+\left(\frac{1}{9}-\frac{1}{10}\right)$$

$$= \left(\frac{1}{2}-\frac{1}{10}\right) = \frac{4}{10} = \frac{2}{5}$$

10. (b) Since 0.43 = 43/99 hence p + q = 43 + 99 = 142.

11. (d) Since 187 = 11 × 17, 221 = 13 × 17, 391 = 17 × 23, while 307 is a prime number.

12. (a) The value of A + B = 4 + 3 + (0.33333 … + 0.666666….) = 8 = 8/1 = p/q hence p + q = 8 + 1 = 9.

13. (a) Use Shortcut approach-4

14. (c) Between any two rational numbers there exist infinite number of rational numbers.

15. (d) Use Shortcut approach-4

16. (a) $1.\overline{23}$ = (123 – 1) / 99 = 122/99

And $1.2\overline{3}$ = (123 – 12)/90 = 111/90

Hence required summation is 122/99 + 111/90 = (1220 + 1221)/990 = 2441/990

17. (b) Let one number be x, then second number will be (x + 4)

From the given condition $\frac{1}{x}+\frac{1}{x+4}=\frac{10}{21}$

After solving we will get x = 3

18. (b) Required value $= \dfrac{5792-579}{90000} = \dfrac{5213}{90000}$

19. (a) $\frac{45}{99}+\frac{41}{90}+\frac{45}{100} = \frac{4500+4510+4455}{11\times9\times10\times10}$

$$= \frac{13465}{9900} = \frac{2693}{1980}$$

20. (a) Required value $= 2+\dfrac{13-1}{90}+3+\dfrac{12-1}{90}$

$$= 5+\frac{23}{90} = 5.2\overline{5}$$

21. **(d)** The smallest number is .001

22. **(d)** $1.\overline{2} \times 0.\overline{03} = \frac{11}{9} \times \frac{3}{99} = \frac{1}{27} = 0.\overline{037}$

23. **(c)** $\frac{5}{12} < \frac{11}{18} < \frac{17}{24} < \frac{3}{4}$

24. **(a)** Use Shortcut approach-2

25. **(b)** Use Shortcut approach-3

26. **(b)** $xy = \frac{14}{15}$ and $\frac{x}{y} = \frac{35}{24}$,

After multiplying, $x = \frac{7}{6}$ and $y = \frac{4}{5}$, clearly

$\frac{7}{6} > \frac{4}{5}$

27. **(c)** Total $= \frac{23}{4} + \frac{5}{3} = \frac{69+20}{12} = \frac{89}{12}$,

difference $= \frac{69-20}{12} = \frac{49}{12}$

Required value $= \frac{89}{12} \div \frac{49}{12} = \frac{89}{49}$

28. **(b)** 45 min. $= \frac{45}{60}$ hour $= \frac{45}{60 \times 24}$ day $= \frac{1}{32}$ day

29. (c) $8 = 2^3$, $1600 = 2^6 \times 5^2$

Here L.C.M. $= (2^6 \times 5^2) \div 100 = 16$

and H.C.F. $= 2^3 \div 100 = 0.08$

So, Required difference $= 15.92$

30. **(c)** $2400 = 2^5 \times 3 \times 5^2$; $180 = 2^2 \times 3^2 \times 5$

$27 = 3^3$; $15000 = 2^3 \times 3 \times 5^4$

So, required H.C.F. $3 \div 10^3 = 0.003$

31. (d) Total number of students participate

$= \left(\frac{1}{4}B + \frac{3}{8}G\right)$

$\therefore$ Required percentage

$= \left(\frac{\frac{1}{4}B + \frac{3}{8}G}{(B+G)} \times 100\%\right)$

Clearly, given data is inadequate.

32. **(b)** $\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99}$

$= \left[\frac{1}{3} + \frac{1}{15}\right] + \frac{1}{35} + \frac{1}{63} + \frac{1}{99}$

$= \left[\frac{6}{15} + \frac{1}{35}\right] + \frac{1}{63} + \frac{1}{99}$

$= \frac{45}{105} + \frac{1}{63} + \frac{1}{99} = \left[\frac{3}{7} + \frac{1}{63}\right] + \frac{1}{99}$

$= \frac{28}{63} + \frac{1}{99} = \frac{4}{9} + \frac{1}{99}$

$\frac{44+1}{99} = \frac{45}{99} = \frac{5}{11}$

33. **(a)** $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}$

LCM of 2, 4, 7, 14 and 28 = 28

$= \frac{28+14+7+4+2+1}{28} = \frac{56}{28} = 2$

34. **(d)** Let the original fraction be $\frac{x}{y}$.

Numerator is increased by 200%.

$\therefore$ Numerator = x + 200% of x

$= x + \frac{200x}{100} = \frac{100x + 200x}{100} = 300x/100$

And denominator of the fraction is increased by 150%.

Denominator $= y + \frac{150y}{100} = \frac{100y + 150y}{100}$

$= \frac{250y}{100}$

Then, according to the question,

$\frac{300x/100}{250y/100} = \frac{9}{35} \Rightarrow \frac{300x}{250y} = \frac{9}{35}$

$\therefore \quad \frac{x}{y} = \frac{9}{35} \times \frac{250}{300} = \frac{3}{14}$

35. **(b)** Use Shortcut approach-8

36. **(a)** Use Shortcut approach-8

37. **(c)** If the fraction $\frac{a}{b}$ is positive, then ab > 0 must be true.

38. **(c)** $0.8\overline{268} = \frac{8268-82}{9900} = \frac{8186}{9900} = \frac{4093}{4950}$

39. **(b)** The required value = LCM of 12, 18, 30, 48 = 720

40. **(d)** Let the total journey be x kms then,

$$\frac{5x}{16} + \frac{7x}{20} + 10 = x \Rightarrow x = 29\frac{17}{27}$$

41. **(d)** $33.33 \div 1.1 = \frac{33.33}{1.1} = \frac{33.33}{1.10}$

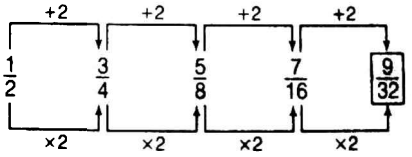
$$= \frac{3333}{110} = 30.3$$

42. **(c)** $\because \frac{a^3+b^3}{a^2+b^2-ab}$

$$= \frac{(a+b)(a^2+b^2-ab)}{(a^2+b^2-ab)} = (a+b)$$

Hence, the required number = 8.73 + 4.27 = 13

43. **(b)**



Hence, the next number of the sequence $= \frac{9}{32}$

44. **(b)** Let $x = \sqrt{7\sqrt{7\sqrt{7\sqrt{\cdots\cdots\infty}}}}$

$$\Rightarrow x = \sqrt{7x}$$

Squaring both sides and solving for x, we get $x^2 - 7x = 0$ or $x = 0, 7$. But x can't be zero, so $x = 7$.

45. **(c)** $\sqrt{\frac{X}{Y}} + \sqrt{\frac{Y}{X}} = \frac{X+Y}{\sqrt{XY}}$.

Putting $X = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $Y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ we get

$(\sqrt{3}+\sqrt{2})^2 + (\sqrt{3}-\sqrt{2})^2 = 10$.

46. **(a)**

$$X = \frac{2\sqrt{10}}{\sqrt{2}+\sqrt{5}} \Rightarrow \frac{X}{\sqrt{2}} = \frac{2\sqrt{5}}{\sqrt{2}+\sqrt{5}} \text{ and } \frac{X}{\sqrt{5}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{5}}$$

Applying Componendo and Dividendo on both relations and adding, we get

$$\frac{X+\sqrt{2}}{X-\sqrt{2}} + \frac{X+\sqrt{5}}{X-\sqrt{5}} = 2.$$

47. **(a)** $3^{(a+8)} = 27^{(2a+1)} \Rightarrow 3^{(a+8)} = 3^{3(2a+1)}$.

Equating the powers, we get (a+8)=3(2a+1) or 'a' = 1.

48. **(c)** As 738A6A is divisible by 11, therefore, the sum of digits at even places = sum of digits at odd places

$$\Leftrightarrow A + A + 3 = 6 + 8 + 7 \Rightarrow A = 9$$

49. **(b)** $\sqrt{4096} + \sqrt{40.96} + \sqrt{0.004096}$

$= 64 + 6.4 + 0.064 = 70.464$

50. (d) $\sqrt[3]{x(x^2+3x+3)+1} = \sqrt[3]{x^3+3x^2+3x+1}$

$= \sqrt[3]{(x+1)^3} = (x+1)$

So, for 'x' = 1001, the value will be 1002.

51. **(b)** $\therefore$ Two consecutive factor of $6 = 3 \times 2$

$\therefore \sqrt{6+\sqrt{6+\sqrt{6+}}}$ ------- $\propto = +3, -2$

52. **(b)** $\therefore 4x = 18y \Rightarrow \frac{x}{y} = 18/4 = 9/2 \Rightarrow \frac{x}{y} - 1 = 7/2$

53. **(d)** $\frac{1\frac{1}{4} \div 1\frac{1}{2}}{\frac{1}{15}+1-\frac{9}{10}} = \frac{\frac{5}{4} \div \frac{3}{2}}{\frac{1}{15}+1-\frac{9}{10}} = \frac{\frac{5}{4} \times \frac{2}{3}}{\frac{2+30-27}{30}}$

$$= \frac{5}{6} \times 6 = 5$$

54. **(d)** Expression $= \frac{(0.87)^4 - (0.13)^4}{0.87 \times 0.87 + 0.13 \times 0.13}$

$$= \frac{\left[(0.87)^2 + (0.13)^2\right]\left[(0.87)^2 - (0.13)^2\right]}{(0.87)^2 + (0.13)^2}$$

$= (0.87)^2 - (0.13)^2 = 1 \times 0.74 = 0.74$

55. **(a)** LCM of 13, 26, 52 = 52

Now, 11/52 = 11/52 ; 7/26 = 14/52

4/13 = 16/52

Obviously, $\frac{11}{52}, \frac{14}{52}, \frac{16}{52}$

$\therefore \frac{11}{52} < \frac{7}{26} < \frac{4}{13}$

56. **(d)** $A:D = \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}$

$= \frac{3}{4} \times \frac{5}{7} \times \frac{8}{9} = \frac{10}{21} = 10:21$

57. **(b)** Dividing each power by 4 and putting its remainders as powers, we get the expression as $29^4 - 28^1 = 1 - 8 = 7$ as 28^{29} will be greater in value than 29^{28}.

58. **(a)** $\sqrt{6} + \sqrt{15} = x\sqrt{10}$

$\sqrt{3} \times \sqrt{2} \times \sqrt{3} \times \sqrt{5} = x \times \sqrt{5} \times \sqrt{2} = \sqrt{9} = +3$

59. **(d)** $\frac{(x)^{0.7}}{36} = \frac{9}{(x)^{1.3}} \Rightarrow x^{0.7+1.3} = 36 \times 9$

$x^2 = 324 \Rightarrow x = \sqrt{324} = 18$

60. **(c)** Let the numbers be x and y.
According to question
$xy = 1575$ and $x/y = 9/7$

$\frac{xy}{x/y} = 1575/9/7 \Rightarrow y^2 = 1225$

$y = 35$ and $x = 45$
The sum of the numbers $= 45+35 = 80$

61. **(c)** $\left[(81)^{3.6} \times (9)^{2.7}\right] / \left[(81)^{4.2} \times (3)\right]$

$= \left[(3)^{14.4} \times (3)^{5.4}\right] / \left[(3)^{16.8} \times (3)\right]$
$= 3^{14.4+5.4-16.8-1} = 3^2 = 9$

62. **(c)** $(67^{67}+67)/68 \Rightarrow (67^{67}+67)/(67+1)$
$= [(67^{67}+1)+66]/(67+1)$; Remainder $= 66$
Because $(67^{67}+1)$ is always divisible by $(67+1)$

63. **(b)** $a + 1/a + 2 = 0$
$\Rightarrow a^2 + 1 + 2a = 0$
$\Rightarrow a + 1 = 0 \Rightarrow a = -1$
$\therefore a^{37} - 1/a^{100} = (-1) - (1) = -2$

64. **(c)** $\sqrt[3]{x} + \sqrt[3]{y} = \sqrt[3]{216} \Rightarrow \sqrt[3]{64} + \sqrt[3]{y} = 6$

$\Rightarrow 4 + \sqrt[3]{y} = 6 \Rightarrow \sqrt[3]{y} = 2 \Rightarrow y = 8$

65. **(c)** Use Shortcut approach-7

Level 1

1. **(a)** Given expression

$= 35.7 - \left(3 + \frac{1}{\frac{10}{3}}\right) - \left(2 + \frac{1}{\frac{5}{2}}\right)$

$= 35.7 - \left(3 + \frac{3}{10}\right) - \left(2 + \frac{2}{5}\right)$

$= 35.7 - \left(\frac{33}{10} + \frac{12}{5}\right)$

$= 35.7 - \frac{57}{10} = 30.$

2. **(b)** Given exp.

$= \frac{a^2 - b^2}{a+b} = a - b = \left(1 + \frac{1}{1 + \frac{1}{100}}\right) - \left(1 - \frac{1}{1 + \frac{1}{100}}\right)$

$= 2 \times \frac{1}{(101/100)} = 2 \times \frac{100}{101} = \frac{200}{101}$

3. **(d)** Required sum

$= \left(99\frac{1}{7} + 99\frac{6}{7}\right) + \left(99\frac{2}{7} + 99\frac{5}{7}\right) + \left(99\frac{3}{7} + 99\frac{4}{7}\right)$

$= 199 \times 3 = 597$

4. **(a)** 300 boys and 200 girls are taking part in the camp and

$\frac{1}{3} \times (\text{total boys}) = 300$, so total boys = 900 and

$\frac{1}{2} \times (\text{total girls}) = 200$, so total girls = 400

So, total students $= 900 + 400 = 1300$

5. **(d)** Let A, B, C and D pay ₹ x, ₹ y, ₹ z and ₹ a.
According to the question,

$x = \frac{1}{2}(y + z + a)$ ...(i)

$y = \frac{1}{3}(x + z + a)$ ...(ii)

$$z = \frac{1}{4}(x + y + a) \quad ...(iii)$$

Also, $x + y + z + a = 60$

Now, put the value of $x + y + a = 4z$

Then, $4z + z = 60 \Rightarrow 5z = 60 \Rightarrow z = 12$

Similarly, on putting the value of

$x + z + a = 3y$, we get

$3y + y = 60 \Rightarrow 4y = 60 \Rightarrow y = 15$

Again, on putting the value of

$y + z + a = 2x$, we get

$2x + x = 60 \Rightarrow 3x = 60 \Rightarrow x = 20$

Now, $x + y + z + a = 60$

On putting the value of x, y and z, we get

$12 + 15 + 20 + a = 60 \Rightarrow a = ₹13$

6. **(b)** Let the greatest, middle and smallest fractions be x, y and z respectively in decreasing order.

According to the question,

$$\frac{\text{Greatest fraction}}{\text{Smallest fraction}} = \frac{7}{6} \Rightarrow \quad \frac{x}{z} = \frac{7}{6}$$

$$\Rightarrow \quad x = \frac{7z}{6} \quad ...(i)$$

$$\text{and} \quad y = \frac{7}{6} - \frac{1}{3} = \frac{7-2}{6} = \frac{5}{6} \quad ...(ii)$$

$$\text{Now, } x + y + z = 2\frac{11}{24} \quad ...(iii)$$

Substitute the value of x and y from eqs. (i), (ii) and put in eq. (iii), we get

$$\Rightarrow \quad \frac{7}{6}z + \frac{5}{6} + z = \frac{59}{24}$$

$$\Rightarrow \quad \frac{7z + 5 + 6z}{6} = \frac{59}{24} \Rightarrow 13z = \frac{59}{4} - 5 = \frac{39}{4}$$

$$\Rightarrow \quad z = \frac{39}{4 \times 13} \Rightarrow z = \frac{3}{4}$$

On putting the value of z in Eqs. (i), we get

$$x = \frac{7}{6}z = \frac{7}{6} \times \frac{3}{4} = \frac{7}{8}$$

7. **(c)** Let the number of pages be x.

$$\text{Number of pages read in first day} = \frac{3x}{8}$$

$$\text{Remaining pages} = x - \frac{3x}{8} = \frac{5}{8}x$$

Number of pages read on second day

$$= \frac{4}{5} \times \frac{5}{8}x = \frac{1}{2}x$$

$$\text{Now, remaining pages} = x - \left(\frac{3x}{8} + \frac{x}{2}\right)$$

$$= \left(x - \frac{7x}{8}\right) = \left(\frac{8x - 7x}{8}\right) = \frac{x}{8}$$

According to the question,

$$\frac{x}{8} = 40 \Rightarrow x = 40 \times 8 \Rightarrow x = 320$$

8. **(c)** Use Shortcut approach-7

9. **(c)** Time taken for each of three persons A, B and C is respectively $\frac{6}{3}, \frac{6}{2\frac{1}{2}}$ and $\frac{6}{1\frac{1}{4}}$ hrs

i.e., $\frac{2}{1}, \frac{12}{5}$ and $\frac{24}{5}$ hrs.

So, it is required to find the LCM of

$$\frac{2}{1}, \frac{12}{5}, \frac{24}{5} = \frac{24}{1}\text{hrs} = 24 \text{ hours}$$

10. **(c)** $$\frac{1}{(1-a)(1-b)(1-c)} + \frac{1}{(1-b)(1-c)(1-d)} +$$

$$\frac{1}{(1-c)(1-d)(1-a)} + \frac{1}{(1-d)(1-a)(1-b)}$$

$$\frac{1-d+1-a+1-b+1-c}{(1-a)(1-b)(1-c)(1-d)}$$

$$= \frac{4-(a+b+c+d)}{(1-a)(1-b)(1-c)(1-d)}$$

$$= \frac{4-4}{(1-a)(1-b)(1-c)(1-d)} = 0$$

11. **(b)** Let $x = 4 + \sqrt{15} = \frac{8 + 2\sqrt{15}}{2} = \left[\frac{\sqrt{5} + \sqrt{3}}{\sqrt{2}}\right]^2$.

$$\text{So, } \sqrt{x} + \frac{1}{\sqrt{x}} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{5} + \sqrt{3}}$$

$$= \frac{10+2\sqrt{15}}{\sqrt{2}(\sqrt{5}+\sqrt{3})} = \sqrt{10}$$

12. **(b)** $\frac{m}{n} = \frac{2}{3}$ (given)

Now, divide the N^r and D^r of the given expression by n.

$$\frac{3\left(\frac{m}{n}\right)+5}{6\left(\frac{m}{n}\right)-1} = \frac{3\left(\frac{2}{3}\right)+5}{6\left(\frac{2}{3}\right)-1} = \frac{7}{3}$$

13. **(b)** $4x^2 + 7xy - 2y^2 = (4x - y)(x + 2y)$

$(4x - y)$ is divisible by 3

$4x^2 + 7xy - 2y^2$ is also divisible by 3

14. **(b)** $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right).....\left(1-\frac{1}{49}\right)$

$$= \frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times.....\frac{47}{48}\times\frac{48}{49} = \frac{1}{49}$$

15. **(d)** $\frac{1}{\sqrt{1}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{1}} + \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} = \sqrt{2}-\sqrt{1}$

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} \ldots \frac{1}{\sqrt{99}+\sqrt{100}}$$

$$= \sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+....+\sqrt{100}-\sqrt{99}$$

$$= \sqrt{100}-\sqrt{1} = 10-1 = 9$$

16. **(a)** Use Shortcut approach-5

17. **(b)** Use Shortcut approach-7

18. **(c)** Use Shortcut approach-5

CHAPTER

4

Square and Square Roots, Cube and Cube Roots, Indices and Surds

SQUARES

When a number is multiplied by itself, then we get the square of the number.

For example : Square of $5 = 5 \times 5 = (5^2) = 25$.

Square of 2 and 3 digits numbers are very useful for calculations.

For this it is advised to learn the square of 1 to 30 as given in the table.

Number	Square	Number	Square
1	1	16	256
2	4	17	289
3	9	18	324
4	16	19	361
5	25	20	400
6	36	21	441
7	49	22	484
8	64	23	529
9	81	24	576
10	100	25	625
11	121	26	676
12	144	27	729
13	169	28	784
14	196	29	841
15	225	30	900

Properties of Square of a Number

(i) No square number ends in 2, 3, 7 or 8 i.e. unit place digit in a square number can never be 2, 3, 7 or 8.

(ii) If a number is even then its square is also even. If a number is odd then its square is also odd.

(iii) Number of zeroes at the end of a perfect square number is always even. In other words we can say that numbers ending with odd numbers of zeroes are never perfect squares.

(iv) The difference of squares of two consecutive numbers will always be equal to the sum of the numbers i.e.

$a^2 - b^2 = a + b$, where a and b are natural numbers and a is greater than b.

For example : $16^2 - 15^2 = 16 + 15 = 31$

(v) Square of any number is always a positive number.

Methods of Finding Square of a Number

I : Vedic Method (called Yavadunam Method)

This method is used for any number which is near to a base (some natural number power of 10 e.g. 10, 100, 1000 etc.). This method can be easily understand through the following examples :

(i) $(98)^2 = ?$

Nearest base = 100

$(98)^2 = \left| 98 + (98-100) = 96 \right| \left| (98-100)^2 = 4 = 04 \right| = 9604$

(Number of digits should be equal to the number of zeroes in the base 100. For this put one zero before 4.)

(ii) $(87)^2 = ?$

100 is Nearest base

Diffrence = 100 – 87 = 13

So, $(87)^2 \Rightarrow \underset{(75)}{|87 - 13 + 1|}, |(13)^2 = 1(69)| = 7569$

(iii) $(104)^2 = ?$

Nearest base = 100

$(104)^2 = |104 + (104-100) = 108 | (104-100)^2 = 16| = 10816$

(iv) $(1002)^2 = ?$

Nearest base = 1000

$|(1002-1000)^2 = 4 = 004|$

$= 1004004$

(v) $(9999)^2 = ?$

Nearest base = 10000

$(9999)^2 = |9999 + (9999 - 10000) = 9998|$

$|(9999-10000)^2 = 1 = 0001|$

$= 99980001$

(vi) $(100015)^2 = ?$

$(100015)^2 = |100015 + (100015 - 100000) = 100030|$

$|(100015-100000)^2 = 225 = 00225|$

$= 10003000225$

II : Multiplication Method

Using this method, the square of any 2-digits number can be calculated. You can understand this method very easily through the following examples :

(i) $(87)^2 = |(7)^2 = (4)(9)|2 \times 8 \times 7 + 4 = (11)(6)|(8)^2 + 11 = (75)| = 7569$

Carry Carry

(ii) $(79)^2 = |(9)^2 = (8)(1)|2 \times 7 \times 9 + 8 = (13)(4)|(7)^2 + 13 = (62)| = 6241$

Carry Carry

(iii) $(79)^2 = |(9)^2 = 8\ (1)|2 \times 7 \times 9 + 8 = 13\ (4)|(7)^2 + 13 = (62)| = 6241$

This can be also written in one short as :

$(79)^2 = |8\ (1)|13\ (4)|\ (62)\ | = 6241$

III : Duplex Method

This method is used for any number. This method can be easily understand through the following examples :

(i) Square of a two digits number : $(69)^2 = ?$

(69)

6	69	9
$(6)^2 = 36$	$2 \times (6 \times 9) = 108$	$(9)^2 = 81$
$36 + 11 = (47)$	$108 + 8 = 11(6)$	$8\ (1)$

$(69)^2 = 4761$

(ii) Square of a three digits number : $(427)^2 = ?$

(427)

4	42	427	27	7
$(4)^2 = 16$	$2 \times (4 \times 2) = 16$	$(2)^2 + 2 \times (4 \times 7) = 60$	$2 \times (2 \times 7) = 28$	$(7)^2 = 49$
$16 + 2 = (18)$	$16 + 6 = 2(2)$	$60 + 3 = 6(3)$	$28 + 4 = 3(2)$	$4(9)$

$(427)^2 = 182329$

(iii) Square of four digits number : $(1638)^2 = ?$

(1638)

1	16	163	1638	638	38	8
$(1)^2 = 1$	$2 \times (1 \times 6) = 12$	$(6)^2 + 2 \times (1 \times 3) = 42$	$2 \times (1 \times 8) + 2 \times (6 \times 3) = 52$	$(3)^2 + 2 \times (6 \times 8) = 105$	$2 \times (3 \times 8) = 48$	$(8)^2 = 64$
$1 + 1 = 2$	$12 + 4 = 1(6)$	$42 + 6 = 4(8)$	$52 + 11 = 6(3)$	$105 + 5 = 11(0)$	$48 + 6 = 5(4)$	$6(4)$

$(1638)^2 = 2683044$

The calculation for the first row and the second row for the above examples are as follows :

	AB (Two digits number)		
First Row	A	AB	B
Second Row	$(A)^2$	$2 \times (A \times B)$	$(B)^2$

	ABC (Three digits number)				
First Row	A	AB	ABC	BC	C
Second Row	$(A)^2$	$2 \times (A \times B)$	$(A)^2 + 2 \times (B \times C)$	$2 \times (B \times C)$	$(C)^2$

	ABCD (Four digits number)						
First Row	A	BC	BCD	$ABCD$	BCD	CD	D
Second Row	$(A)^2$	$2 \times (B \times C)$	$(C)^2 + 2 \times (B \times D)$	$2 \times (C \times D) + 2 \times (B \times C)$	$(C)^2 + 2 \times (B \times D)$	$2 \times (C \times D)$	$(D)^2$

IV : Algebraic Method

This method is used to find the square of any two digits number. In this method, we use the following identity of algebra :

$(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

This method can be easily understand using the following examples :

(i) $(64)^2 = (60 + 4)^2 = (60)^2 + 2 \times 60 \times 4 + (4)^2$

$= 3600 + 480 + 16 = 4096.$

(ii) $(87)^2 = (90 - 3)^2 = (90)^2 - 2 \times 90 \times 3 + (3)^2$

$= 8100 - 540 + 9 = 7569.$

Note that in the above examples 'a' is the multiple of 10 nearest to the given number and 'b' is the number required to be added or subtracted from 'a' so that sum or difference of 'a' and 'b' respectively will be the given number.

V : Number Consisting of Is Only

First count 1s in the given number. Let it be n. Then write down counting from 1 to n in the ascending order. Thereafter write counting in descending order till 1.

For examples : (i) $(11)^2 = 121$ (ii) $(111)^2 = 12321$

(iii) $(1111)^2 = 1234321$

VI : Number Consisting 0 and Is Only

You can easily understand this method through the following examples :

(i) $(101)^2 = 10201$

(ii) $(10101)^2 = 102030201$

(iii) $(1010101)^2 = 1020304030201$

(iv) $(1001)^2 = 1002001$

(v) $(1001001)^2 = 1002003002001$

VII : Number Ending in 5

A number ending in 5 can be written as $(10x + 5)$ and $(10x + 5)^2 = x(x + 1)|25.$

Let us understand this through the following examples :

(i) $(35)^2 = 3 \times 4 \mid 25 = 1225$

(ii) $(65)^2 = 6 \times 7 \mid 25 = 4225$

(iii) $(145)^2 = 14 \times 15 \mid 25 = 21025$

(iv) $(10005)^2 = 1000 \times 1001 \mid 25 = 100100025$

VIII : Number Ending in 25

A number ending in 25 can be written as $(100x + 25)$

and $(100x+25)^2 = \left|x\left(x+\frac{1}{2}\right)\right| 625$

Let us understand through the following examples :

(i) $(625)^2 = 6 \times 6.5 \mid 0625 = 39 \mid 0625 = 390625$

(ii) $(1625)^2 = 16 \times 16.5 \mid 0625 = 264 \mid 0625 = 2640625$

(iii) $(1725)^2 = 17 \times 17.5 \mid 0625 = 297.5 \mid 0625 = 2975 \mid 625 = 2975625$

(iv) $(1925)^2 = 19 \times 19.5 \mid 0625 = 370.5 \mid 0625 = 3705 \mid 625 = 3705625$

Note that when $x\left(x+\frac{1}{2}\right)$ is a decimal number, then in the square of the given number we reject the decimal from the value of the product $x\left(x+\frac{1}{2}\right)$ and 0 from 0625 as in the case of example (iii) and (iv).

IX : Numbers Around 50

If $x = 50 \pm a$, then $x^2 = 25 \pm a \mid a^2$, where a^2 is always written in two digits number.

Let us understand this method through the following examples :

(i) $(54)^2 = ?$

$54 = 50 + 4 \Rightarrow (54)^2 = 25 + 4 \mid (4)^2 = 29 \mid 16 = 2916$

(ii) $(48)^2 = ?$

$48 = 50 - 2 \Rightarrow (48)^2 = 25 - 2 \mid (2)^2 = 23 \mid 04 = 2304$

(iii) $(47)^2 = ?$

$47 = 50 - 3 \Rightarrow (47)^2 = 25 - 3 \mid (3)^2 = 22 \mid 09 = 2209$

Note that in the above examples, we write 04 and 09 for 4 and 9 respectively to write it as two digits numbers.

X : Numbers Around 500

If $x = 500 \pm a$, then $x^2 = 250 \pm a \mid a^2$, where a^2 is always written in the three-digits number.

Let us understand this method through the following examples :

(i) $(503)^2 = ?$

$503 = 500 + 3 \Rightarrow (503)^2 = 250 + 3 \mid (3)^2 = 253 \mid 009 = 253009$

(ii) $(496)^2 = ?$

$496 = 500 - 4 \Rightarrow (496)^2 = 250 - 4 \mid (4)^2 = 246 \mid 016 = 246016$

Note that : In the above examples, we write 009 and 016 for 9 and 16 respectively to write it as three digits numbers.

XI : Numbers Near the Base

This method is used for any number which is near to a base (so natural number power of 10 e.g., 10, 100, 1000 etc or $10^1, 10^2, 10^3$ etc.).

If $a = (10)^n \pm b$, then $a^2 = a \pm b \mid b^2$, where b^2 is always written in 'n' digits number.

Let us understand this method through the following examples :

(i) $(93)^2 = ?$

$93 = (10)^2 - 7 \Rightarrow (93)^2 = 93 - 7 \mid (7)^2 = 86 \mid 49 = 8649.$

(ii) $(102)^2 = ?$

$102 = (10)^2 + 2 \Rightarrow (102)^2 = 102 + 2 \mid (2)^2 = 104 \mid 04 = 10404.$

(iii) $(1006)^2 = ?$

$1006 = (10)^3 + 6 \Rightarrow (1006)^2 = 1006 + 6 \mid (6)^2 = 1012 \mid 036 = 1012036.$

(iv) $(99984)^2 = ?$

$99984 = (10)^5 - 16 \Rightarrow (99984)^2 = 99984 - 16 \mid (16)^2 = 99968 \mid 00256 = 9996800256.$

(v) $(87)^2 = ?$

$87 = (10)^2 - 13 \Rightarrow (87)^2 = 87 - 13 \mid (13)^2 = 74 \mid 169 = 75 \mid 69 = 7569$

Note that in this example (v) b^2 is 3 digits number 169, but we need it as a two digits number as the value of $n = 2$. For this, we add hundredth digit '1' of 169 to its previous part 74.

XII : $x^2 = (x + a)(x - a) + a^2$

To find the square by using the formula, $x^2 = (x + a)(x - a) + a^2$, we choose a number '$a$' so that $(x + a)$ or $(x - a)$ become an easy number;

Let us understand this method through the following examples :

(i) $(43)^2 = (43 + 3)(43 - 3) + (3)^2 = 46 \times 40 + 9 = 1840 + 9 = 1849$

(ii) $(76)^2 = (76 + 4)(76 - 4) + (4)^2 = 80 \times 72 + 16 = 5760 + 16 = 5776$

XIII : Square of a Mixed Number

If a mixed number is in the form of $x\frac{1}{2}$, then

$\left(x\frac{1}{2}\right)^2 = x(x+1) \mid \frac{1}{4}$, where x is a positive integer.

GP_3386

This method is used for only those mixed numbers which are in the form of $x\frac{1}{2}$.

Let us understand this method through the following examples :

(i) $\left(6\frac{1}{2}\right)^2 = 6\times 7 \mid \frac{1}{4} = 42\frac{1}{4}$.

(ii) $\left(9\frac{1}{2}\right)^2 = 9\times 10 \mid \frac{1}{4} = 90\frac{1}{4}$.

(iii) $\left(20\frac{1}{2}\right)^2 = 20\times 21 \mid \frac{1}{4} = 420 \mid \frac{1}{4} = 420\frac{1}{4}$.

XIV : Square of Decimal Numbers

To find the square of any decimal number, write the square of the number ignoring the decimal number and then place the decimal number after twice the place of the original number starting from right side. Let us understand through the following examples :

(i) $(4.7)^2 = ?$
$(47)^2 = 2209$
Here decimal in 4.7 after one digit from right. Therefore, the decimal in the value of the square of 47 will be after two digits from right.
$\therefore (4.7)^2 = 22.09$

(ii) $(6.21)^2 = ?$
$(621)^2 = 385641$
$\therefore (6.21)^2 = 38.5641$

PYTHAGOREAN TRIPLETS

If in a triplet of three numbers, sum of square of the two smaller numbers is equal to square of the third number, then the triplet of three numbers is called **Pythagorean triplets**.

For example : 3, 4, 5 because $3^2 + 4^2 = 5^2$.

Some other Pythagorean triplets are

(i) 5, 12, 13 (ii) 8, 15, 17
(iii) 7, 24, 25 (iv) 6, 8, 10
(v) 10, 24, 26 (vi) 16, 30, 34
(vii) 20, 21, 29 (viii) 9, 12, 15
(ix) 9, 40, 41 (x) 11, 60, 61

SQUARE ROOTS

Square root of a number is that number, the square of which is equal to the given number. There are two types of square roots of a number, positive and negative. Square root is also known as under-root or 2nd root.

Symbolically, square root of a positive number 'n' is written as $\sqrt{n}$, $\sqrt[2]{n}$ or $(n)^{\frac{1}{2}}$.

We know that $4 \times 4 = 16$ [or $(+4)\times(+4) = 16$]
And $(-4)\times(-4) = 16$.

$\therefore \sqrt{16} = \pm 4$, $\sqrt[2]{16} = \pm 4$ and $(16)^{\frac{1}{2}} = \pm 4$

Here +4 (i.e. 4) is called positive square root of 16 and – 4 is called negative square root of 16.

Also, $\sqrt{a}\times\sqrt{a} = a$

Note that normally, we consider only positive square root of a number.

If the number whose square root is to be find out is a perfect square, then its square root is an integer otherwise square root is not an integer.

Properties of Square Roots

(i) If a number has a natural number square root, then its unit digit must be 0, 1, 4, 5, 6 or 9.

Unit digit of square number	0	1	4	5	6	9
Unit digit of square root	0	1 or 9	2 or 8	5	4 or 6	3 or 7

(ii) If unit digit of a number be 2, 3, 7 or 8; then it does not have a natural number square root.

(iii) If a number ends in an odd number of zeros, then it does not have a square root. If a square number is followed by an even number of zeros, it has a square root in which the number of zeros in at the end is half the number of zeros in the number.

(iv) Negative number has no square root.

Methods to find Square Root of a Perfect Square Number

Method-I : Prime Factorisation Method

(i) Express the given number into prime factors by successive division.

(ii) Make pairs of prime factors such that both the factors in each pair are equal.

(iii) Take one factor from each pair.

(iv) Find the product of factors obtained in step (iii). The product thus obtained is the required square root.

Illustration – 1

Show that 63504 is a perfect square. Also, find the number whose square is 63504.

Solution :

Resolving 63504 into prime factors, we obtain

$63504 = 2\times2\times2\times2\times3\times3\times3\times3\times7\times7$

Grouping the factors in pairs of equal factors, we obtain

$63504 = (2\times2)\times(2\times2)\times(3\times3)\times(3\times3)\times(7\times7)$

Clearly, no factor is left over in grouping the factors in pairs of equal factors, So, 63504 is a perfect square.

Again,

$63504 = (2\times2\times3\times3\times7)\times(2\times2\times3\times3\times7)$

[Grouping first factors in each group]

$= 252\times252 = 252^2$

Thus, 63504 is the square of 252.

2	63504
2	31752
2	15876
2	7938
3	3969
3	1323
3	441
3	147
7	49
7	7
	1

Method-II : Division Method

This method can be easily understood with the help of the following example:

Let us find the square root of 54756.

Step-(i) : Place bars over every pair of digits starting with the unit digit. Also place a bar on one digit (5) not forming a pair on the extreme left. Each pair and remaining one digit (if any) on the extreme left is called a period. Here we find three periods.

	2	3	4
2	$\overline{5}$	$\overline{47}$	$\overline{56}$
	4		
43	1	47	
	1	29	
464		18	56
		18	56
			0

$\therefore \sqrt{54756} = 234$

Step-(ii) : The first period has only one digit 5. Think the largest number whose square is less than or equal to the first period. Clearly the largest number whose square is less than 5 is 2. Take this number 2 as divisor and quotient both and put the quotient above the first period i.e. 5.

Step-(iii) : Subtract the product of divisor and quotient from the first period. Here, on subtracting, we get 1 as remainder. Bring down the next period i.e. 47 to the right of the remainder. By doing this, we get 147 as new dividend.

Step-(iv) : For the next divisor double the previous quotient i.e. 2. We take this i.e. 4 as the left most digit of the new divisor. Now, 3 is the largest digit such that $43\times3 = 129$, which is just less than new dividend 147. So, take 43 as the new divisor and 3 as the next digit of the quotient.

Step-(v) : Now subtract 129 from 147. Here on subtracting, we get 18 as new remainder. Bring down the next period 56 to the right of 18. By doing this we get 1856 as the new dividend.

Step-(vi) : For the next divisor double the quotient 23. We take this i.e. 46 as the left most digit of the new divisor. Now 4 is the largest digit such that $464\times4 = 1856$, which is equal to the new dividend 1856. So, we take 464 as the new divisor and 4 as the next digit of the quotient. Subtract 1856 from the new dividend 1856. Here, on subtracting, we get 0 as remainder. Now no period left to bring down to make new dividend, therefore, the last quotient is 234. This last quotient is the required square root.

Note that, number of periods = Number of digits in the square root.

Illustration – 2

Find the square root of 24336.

(a) 144 (b) 154 (c) 156 (d) 166

Solution : (c)

	1	5	6
1	$\overline{2}$	$\overline{43}$	$\overline{36}$
	1		
25	1	43	
	1	25	
306		18	36
		18	36
			0

$\sqrt{24336} = 156$

Illustration – 3

Find the square root of 12327121.

(a) 3511 (b) 3601 (c) 3519 (d) 3611

GP_3386

Solution : (a)

	3 5 1 1
3	12 32 71 21 9
65	3 32 3 25
701	7 71 7 01
7021	7021 7021
	0

$\sqrt{12327121} = 3511$

Square Root of Numbers Which are not Perfect Squares.

Division method can also be applied for finding square root of numbers which are not perfect square numbers. Method is explained with the following illustrative examples.

Illustration – 4

Find square root of 3 upto 3 decimal places.

Solution :

	1.732
1	3.00 00 00 1
27	2 00 1 89
343	11 00 10 29
3462	71 00 69 24
	1 76

Here we have added 3 pairs of zeros after decimal. One pair each for 1 digit after decimal point.

$\therefore \sqrt{3} = 1.732$ upto three decimal places.

Square Root of Decimal Numbers....

To find the square root of a decimal number, first count the number of digits after decimal in the given number, if it is not even, then put a 0 (zero) at the extreme right, so that there will be an even num.ber of digits after decimal point.

Now, periods are marked by putting the bar over it as marked for finding the square root of a perfect square number starting from right hand side before the decimal and from the left hand side after the decimal point.

For example, $\overline{1}\ \overline{46}\ \overline{96} . \overline{71}\ \overline{29}$

Direction of marking periods

After marking the periods, follow the same steps as for finding the square root of a perfect square number without considering the decimal point. In the final answer, put decimal point after as many number of digits from right as the number of periods after point in the given number.

	1 2 1 . 2 3
1	1 46 96 . 71 29 1
22	46 44
241	296 241
2422	5571 4844
24243	72729 72729
	0

$\sqrt{14696.7129} = 121.23$

Square Root of a Fraction

To find the square root of a fraction, we find the square root of numerator and denominator both separately.

For example: $\sqrt{\frac{1369}{1521}} = \frac{\sqrt{1369}}{\sqrt{1521}} = \frac{37}{39}$

To Find Smallest and Largest *n*-digit Number Which is a Perfect Square

This can be understand through the following illustrations.

Illustration – 5

Find the smallest 6-digit number which is a perfect square.

Solution : Smallest 6-digit number = 100000.

	3 1 7
3	10 00 00 9
61	1 0 0 6 1
627	3 9 0 0 4 3 8 9
	−4 8 9

Since $626 \times 6 = 3756$
$627 \times 7 = 4389$
Now, $3756 < 3900 < 4389$
$\Rightarrow 626 \times 6 < 3900 < 627 \times 7$
Clearly smallest 6-digit number which is perfect square will be 489 greater than 100000.
Therefore, required number = 100000 + 489 = 100489.

Illustration – 6

Find the greatest 7-digits number which is a perfect square.

Solution : Greatest 7-digits number = 9999999

	3 1 6 2
3	9 99 99 99 9
61	9 9 6 1
626	3 8 9 9 3 7 5 6
6322	1 4 3 9 9 1 2 6 4 4
	1 7 5 5

Clearly greatest 7-digits number which is a perfect square will be 1430 less than 9999999.
Therefore, required number = 9999999 – 1755 = 9998244.

CUBES

If a number is multiplied itself two times, then the result of this multiplication is called the **cube** of that number.
For examples :
Cube of $3 = 3 \times 3 \times 3 = 27$
Cube of $4 = 4 \times 4 \times 4 = 64$
Cube of any number a is denoted by $(a)^3$ or a^3.
$\therefore (3)^3 = 27, (4)^3 = 64$.

Test of Perfect Cube

In a prime factorisation of a perfect cube, every number occurs three times or a multiple of three times.
For example:
(i) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2)^6$
64 is a perfect cube.
(ii) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = (3)^6$
729 is a perfect cube.

Illustration – 7

Examine (i) 243 (ii) 26136 are perfect cubes.

Solution :
(i) $243 = (3 \times 3 \times 3) \times 3 \times 3$
Here occurrence of 3 is not in multiple of '3' so 243 is not a perfect cube number.
(ii) $21636 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 11 \times 11$
In 21636, prime number 11 does not occur in group of three, so 21636 is not a perfect cube number.

Properties of Cube of a Number

1. If a number is negative then its cube is also negative.
Example : (i) $(-2)^3 = (-2) \times (-2) \times (-2) = -8$
(ii) $(-5)^3 = (-5) \times (-5) \times (-5) = -125$
This shows that negative numbers can also be perfect cubes.

2. Cube of an even number is also even.
Example : (i) $(2)^3 = 2 \times 2 \times 2 = 8$
(ii) $(6)^3 = 6 \times 6 \times 6 = 216$.

3. Cube of an odd number is also odd.
Example : (i) $(3)^3 = 3 \times 3 \times 3 = 27$
(ii) $(7)^3 = 7 \times 7 \times 7 = 343$.

Relation between Unit Digit of a Number and its Perfect Cube
Observe the following table

x	1	2	3	4	5	6	7	8	9
x^3	1	8	27	64	125	216	343	512	729

It can be observed from the table that if the unit digit of a number is 1, 4, 5, 6, 9. then unit digit of cube is also 1, 4, 5, 6, 9. If the unit digit of number is 2 or 8 then the unit digit in cube is 8 or 2 respectively.
3 and 7 form similar pair as 2 and 8.
Example :
(i) Unit place digit of cube of 41 is 1 because unit place digit of 41 is 1 .

GP_3386

(ii) Unit place digit of cube of 76 is 6 because unit place digit of 76 is 6 .

(iii) Unit place digit of cube of 92 is 8 because unit place digit of 92 is 2.

CUBE ROOTS

If x is a perfect cube such that $x = y^3$, then y is called cube root of x and it is denoted by $\sqrt[3]{x} = y$

For example, $216 = (6)^3 \therefore \sqrt[3]{216} = 6$

Method to Find Cube Root of a Perfect Cube Number

Cube root of a perfect cube number is find out by its prime factorisation. For this follow the steps :

(i) Find prime factors of the given number.

(ii) Group the factors in triplets such that all three factors in each triplet are same.

(iii) Choose one factor from each group and take the product. The product is the cube root of the given number.

Illustration – 8

Find cube root of 32768.

Solution : $32768 = (2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\therefore \sqrt[3]{32768} = 2\times2\times2\times2\times2 = 32$

Cube Root of Rational Numbers Whose Numerators and Denominators are Perfect Cubes

If p and q, where $q \neq 0$, are perfect cubes then

$$\sqrt[3]{\frac{p}{q}} = \frac{\sqrt[3]{p}}{\sqrt[3]{q}}$$

i.e cube root of quotient of two perfect cubes is the quotient of cube roots of perfect cubes. Here we see that $\sqrt[3]{p}$ and $\sqrt[3]{q}$ are integers and $\sqrt[3]{q} \neq 0$. Therefore $\frac{\sqrt[3]{p}}{\sqrt[3]{q}}$ is a rational number.

Example: (i) $\sqrt[3]{\frac{125}{64}} = \frac{\sqrt[3]{125}}{\sqrt[3]{64}} = \frac{5}{4}$

(ii) $\sqrt[3]{\frac{-729}{1331}} = -\frac{\sqrt[3]{729}}{\sqrt[3]{1331}} = -\frac{9}{11}$

Properties of Cube Root

(i) Cube root of 0, 1 and –1 are respectively 0, 1 and –1.

(ii) Cube root of a positive number is positive and cube number root of negative number is negative.

Relation Between Unit Digit of a Perfect Cube Number and Unit Digit of its Cube Root:

The following table shows the relationship between the last digit (unit digit) of a perfect cube number and last digit (unit digit) of its cube number root.

Last digit of a perfect cube no.	1	2	3	4	5	6	7	8	9	0
Last digit of cube root	1	8	7	4	5	6	3	2	9	0

INDICES AND SURDS

Indices

When a number is multiplied by itself, it gives the square of the number. i.e., $a \times a = a^2$

(Example $5 \times 5 = 5^2$)

If the same number is multiplied by itself twice we get the cube of the number i.e., $a \times a \times a = a^3$

(Example $4 \times 4 \times 4 = 4^3$)

In the same way $a \times a \times a \times a \times a = a^5$ and $a \times a \times a \times$ ... upto n times $= a^n$

In a^n, 'n' is called power, index or exponent and 'a' is called base. a^n is known as n^{th} power of 'a' and is read as 'a' raised to the power 'n' or 'a' to the power 'n'.

Basic Rule of Indices (or Power)

(i) $a^m \times a^n = a^{m+n}$

(Example: $5^3 \times 5^4 = 5^{3+4} = 5^7$)

(ii) $\frac{a^m}{a^n} = a^{m-n}$, if $m > n$

$\left(\text{Example}: \frac{6^5}{6^2} = 6^{5-2} = 6^3\right)$

$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $m < n$

$\left(\text{Example}: \frac{4^3}{4^8} = \frac{1}{4^{8-3}} = \frac{1}{4^5}\right)$

and $\frac{a^m}{a^n} = a^0 = 1$, if $m = n$

$\left(\text{Example}: \frac{3^4}{3^4} = 3^{4-4} = 3^0 = 1\right)$

(iii) $\left(a^m\right)^n = a^{mn} = \left(a^n\right)^m$

$\left(\text{Example}: \left(6^2\right)^4 = 6^{2\times4} = 6^8 = \left(6^4\right)^2\right)$

(iv) $(ab)^n = a^n . b^n$ $\left(\text{Example}: (6\times4)^3 = 6^3 \times 4^3\right)$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\text{Example}: \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4}\right)$

and $\left(\frac{b}{a}\right)^n = \left(\frac{a}{b}\right)^n$ $\left(\text{Example} \left(\frac{3}{5}\right)^{-4} = \left(\frac{5}{3}\right)^4\right)$

(v) $a^{-n} = \frac{1}{a^n}$ $\left(\text{Example}: 5^{-3} = \frac{1}{5^3}\right)$

(vi) $\frac{1}{a^{-n}} = a^n$ $\left(\text{Example}: \frac{1}{5^{-3}} = 5^3\right)$

(vii) For any non-zero real number a, $a^0 = 1$

Indices

If $a^n = b$, where 'a' is a non-zero rational number and 'n' is a positive integer (i.e. natural number), then $a^n = b$ can be also written as $a = (b)^{1/n}$ or $\sqrt[n]{b}$.

For example: $2^5 = 32 \Rightarrow 2 = (32)^{1/5}$ or $\sqrt[5]{32}$

Here $\sqrt[n]{b}$ is called n^{th} root of b. So, 5th root of 32 i.e. $\sqrt[5]{32}$ or $(32)^{1/5} = 2$.

If $\sqrt[n]{b}$ is an irrational number then it is called surd or order n. Here, the sign $\left(\sqrt{\ }\right)$ is called **radical sign**, 'b' is called **radicand**.

Note that in case of square root of a number, order of the surd is not shown. For example, sqaure root of 8 is written as $\sqrt{8}$. In the surd $\sqrt{8}$, order is not shown. In this case order of is always taken as 2.

Basic Rule of Surds

(i) $\sqrt[n]{a} = (a)^{1/n}$

(ii) $(\sqrt[n]{a})^m = (a)^{m/n} = \sqrt[n]{a^m}$

Special case : $(\sqrt[n]{a})^n = a$

(iii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

(iv) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

(vi) $\sqrt[n]{a} = \sqrt[n\times m]{a^m}$

Illustration – 9

Find the value of :

(i) $(\sqrt[4]{5})^{12}$ **(ii)** $(\sqrt[7]{21})^{14}$ **(iii)** $(\sqrt[5]{27})^5$

Solution :

(i) $(\sqrt[4]{5})^{12} = (5)^{12/4} = (5)^3 = 125$

(ii) $(\sqrt[7]{21})^{14} = (21)^{14/7} = (21)^2 = 441$

(iii) $(\sqrt[5]{27})^5 = (27)^{5/5} = (27)^1 = 27.$

GP_3386

Illustration – 10

Find the value of

(i) $\sqrt[3]{128}$ **(ii)** $\sqrt[4]{121}\times\sqrt[4]{121}$

Solution : (i) $\sqrt[3]{128}=\sqrt[3]{64\times2}=\sqrt[3]{64}\times\sqrt[3]{2}$

$=\sqrt[3]{(4)^3}\cdot\sqrt[3]{2}=(4)^{3/3}\cdot\sqrt[3]{2}=4\sqrt[3]{2}$

(ii) $\sqrt[4]{121}\times\sqrt[4]{121}=\sqrt[4]{121\times121}=\sqrt[4]{(11)^2\times(11)^2}$

$=\sqrt[4]{(11)^4}=(11)^{4/4}=(11)^1=11$

Illustration – 11

Find the value of

(i) $\dfrac{\sqrt[4]{3888}}{\sqrt[4]{48}}$ **(ii)** $\dfrac{\sqrt[3]{11616}}{\sqrt[3]{1452}}$

Solution :

(i) $\dfrac{\sqrt[4]{3888}}{\sqrt[4]{48}}=\sqrt[4]{81}=\sqrt[4]{(3)^4}$

$=(3)^{4/4}=(3)^1=3$

(ii) $\dfrac{\sqrt[3]{11616}}{\sqrt[3]{1452}}=\sqrt[3]{\dfrac{11616}{1452}}=\sqrt[3]{8}=\sqrt[3]{(2)^3}$

$=(2^3)^{1/3}=(2)^{3/3}=(2)^1=2$

Illustration – 12

Find the value of

(i) $\sqrt[2]{\sqrt[3]{729}}$ **(ii)** $\sqrt[2]{\sqrt[4]{256}}$

Solution : (i) $\sqrt[2]{\sqrt[3]{729}}=\sqrt[2\times3]{729}=\sqrt[6]{(3)^6}$

$=(3)^{6/6}=(3)^1=3.$

(ii) $\sqrt[2]{\sqrt[4]{256}}=\sqrt[2\times4]{256}=\sqrt[8]{(2)^8}$

$=(2)^{8/8}=(2)^1=2.$

Illustration – 13

If $x^{\sqrt{x}}=\left(\sqrt{x}\right)^x$ **then x is**

(a) 3 **(b) 2**
(c) 4 **(d) None of these**

Solution : (c) $x^{\sqrt{x}}=\left(\sqrt{x}\right)^x\Rightarrow x^{\sqrt{x}}=x^{\frac{1}{2}x}$

$\Rightarrow\sqrt{x}=\dfrac{1}{2}x\Rightarrow x=\dfrac{1}{4}x^2\Rightarrow x=4$

Illustration – 14

If $4^{\sqrt{x}^{\sqrt{x}}}=256$ **then the value of x is**

(a) 2 **(b) 16** **(c) 4** **(d)** $\sqrt{2}$

Solution : (c) $4^{\sqrt{x}^{\sqrt{x}}}=256=4^4$

$\Rightarrow\sqrt{x}^{\sqrt{x}}=4=2^2\Rightarrow\sqrt{x}=2\Rightarrow x=4$

Illustration – 15

Which is smaller, $\sqrt[3]{5}$ **or** $\sqrt[4]{6}$ **?**

Solution : $\sqrt[3]{5}=(5)^{\frac{1}{3}}$ and $\sqrt[4]{6}=(6)^{\frac{1}{4}}$

LCM of 3 & 4 = 12

Now convert each into power of 12 i.e

$(5)^{\frac{1}{3}}=(5)^{\frac{4}{12}}$ and $(6)^{\frac{1}{4}}=(6)^{\frac{3}{12}}$

$\therefore\quad 5^{\frac{4}{12}}=\left(5^4\right)^{\frac{1}{12}}=(625)^{\frac{1}{12}}$ and

$6^{\frac{3}{12}}=\left(6^3\right)^{\frac{1}{12}}=(216)^{\frac{1}{12}}$

Now $(216)<(625)$

$\therefore\quad (216)^{\frac{1}{12}}<(625)^{\frac{1}{12}}$

$\therefore\quad \sqrt[4]{6}<\sqrt[3]{5}$

Illustration – 16

Find k if $\sqrt[3]{\sqrt[4]{x^2}}=x^k$

Solution : $\sqrt[3]{\sqrt[4]{x^2}}=x^k\Rightarrow\sqrt[3]{x^{\frac{2}{4}}}=x^k$

$\Rightarrow x^{\frac{2}{4}\times\frac{1}{3}}=x^k\Rightarrow x^{\frac{1}{6}}=x^k\Rightarrow k=\dfrac{1}{6}$

Illustration – 17

Simplify : $\left[\left\{\left(\dfrac{-1}{5}\right)^{-2}\right\}^2\right]^{-1}$

Solution : Given expression can be written as

$$\left[\left(\frac{-1}{5}\right)^{-2}\right]^{2\times(-1)} \quad [\text{using } (x^m)^n = x^{m\times n}]$$

$$\Rightarrow \left[\left(\frac{-1}{5}\right)^{-2}\right]^{-2} = \left(\frac{-1}{5}\right)^{(-2)\times(-2)} = \left(\frac{-1}{5}\right)^{4}$$

$$= \frac{(-1)^4}{(5)^4} \left[\text{using } \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\right]$$

$$= \frac{1}{625}$$

Illustration – 18

Simplify : $\left(6^{-1} - 8^{-1}\right)^{-1} + \left(2^{-1} - 3^{-1}\right)^{-1}$

Solution : $\left(6^{-1} - 8^{-1}\right)^{-1} + \left(2^{-1} - 3^{-1}\right)^{-1}$

$$= \left(\frac{1}{6} - \frac{1}{8}\right)^{-1} + \left(\frac{1}{2} - \frac{1}{3}\right)^{-1} = \left(\frac{4-3}{24}\right)^{-1} + \left(\frac{3-2}{6}\right)^{-1}$$

$$= \left(\frac{1}{24}\right)^{-1} + \left(\frac{1}{6}\right)^{-1} \left[\because \text{using } n^{-1} = \frac{1}{n}\right]$$

$= 24 + 6 = 30$

Comparison of Surds

To compare two or more surds, follow the steps :

Step-(i) : Find the LCM of orders of the given surds. Let the LCM = l.

Step-(ii) : Convert each surd into a surd of order l. The surd with smaller radicand is smaller and with greater radicand is greater.

Illustration – 19

Which of the following surds is smallest?

$\sqrt[4]{6}, \sqrt{3}, \sqrt[3]{4}$

Solution : Order of surds $4\sqrt{6}$, $\sqrt{3}$ and $\sqrt[3]{4}$ are 4, 2 and 3 respectively.

LCM of 4, 2 and 3 = 12.

Now we convert each surd in a surd of order 12.

$$\sqrt[4]{6} = \sqrt[4\times3]{(6)^3} = \sqrt[12]{216}$$

$$\sqrt{3} = \sqrt[12]{(3)^{12}} = \sqrt[12]{531441}$$

$$\sqrt[3]{4} = \sqrt[3\times4]{(4)^4} = \sqrt[12]{256}$$

Now we compare the radicand to compare the surds :

$216 < 256 < 531441$.

$$\therefore \quad \sqrt[12]{216} < \sqrt[12]{256} < \sqrt[12]{531441}$$

$$\Rightarrow \sqrt[4]{6} < \sqrt[3]{4} < \sqrt{3}$$

Hence smallest surd = $\sqrt[4]{6}$

Illustration – 20

Which of the following is the greatest surd?

$\sqrt[3]{2}, \sqrt[6]{3}, \sqrt[9]{4}$

Solution : LCM of 3, 6, 9 = 18

$$\sqrt[3]{2} = \sqrt[3\times6]{(2)^6} = \sqrt[18]{64}$$

$$\sqrt[6]{3} = \sqrt[6\times3]{(3)^3} = \sqrt[18]{27}$$

$$\sqrt[9]{4} = \sqrt[9\times2]{(4)^2} = \sqrt[18]{16}$$

Now, $\sqrt[18]{16} < \sqrt[18]{27} < \sqrt[18]{64} \Rightarrow \sqrt[9]{4} < \sqrt[6]{3} < \sqrt[3]{2}$

Hence greatest surd = $\sqrt[3]{2}$

Illustration – 21

Arrange the following surds in decreasing order?

$\sqrt[4]{3}, \sqrt[6]{10}, \sqrt[12]{25}$

Solution : LCM of 4, 6, 12 = 12

$$\sqrt[4]{3} = \sqrt[4\times3]{(3)^3} = \sqrt[12]{27}$$

$$\sqrt[6]{10} = \sqrt[6\times2]{(10)^2} = \sqrt[12]{100}$$

$$\sqrt[12]{25} = \sqrt[12]{25}$$

$\because 100 > 27 > 25$

$\therefore \sqrt[12]{100} > \sqrt[12]{27} > \sqrt[12]{25}$

$\Rightarrow \sqrt[6]{10} > \sqrt[4]{3} > \sqrt[12]{25}$

SHORTCUTS

Shortcut Approach - 1

Easier method to find the square root of 3 or 4 digits number:

GP_3386

Step-(i) : Square root of 3 or 4 digits number is always a 2 digits number. Ten's digit of the square root is the largest digit whose square is less than or equal to the number form from the digit(s) remains after leaving the two digits from right from the given number.
Step-(ii) :Guess the unit digit of the square root on the basis of unit digit of the given number as explained in the following examples.

(i) $\sqrt{3481} = ?$

$(5)^2 < 34 < 36 \Rightarrow$ Ten's digit of $\sqrt{3481} = 5$
Since unit digit of 3481 = 1
Therefore, unti digit of $\sqrt{3481}$ is 1 or 9.
Thus $\sqrt{3481}$ is 51 or 59.
Now 51 is close to 50 and 59 is close to 60.
Now $(50)^2 = 2500$ and $(60)^2 = 3600$.
But 3481 is closer to 3600.
$\therefore \sqrt{3481} = 59$

(ii) $\sqrt{324} = ?$

$(1)^2 < 3 < (2)^2 \Rightarrow$ Ten's digit of 324 is 1.
Since unit digit of 324 = 4.
Therefore, unit digit of $\sqrt{324}$ is 2 or 8.
Thus $\sqrt{324}$ = 12 or 18.
Now, 12 is close to 10 and 18 is close to 20.
Now $(10)^2 = 100$ and $(20)^2 = 400$.
But 324 is closer to 400.
$\therefore \sqrt{324} = 18$

Shortcut Approach - 3

Approximate Square Root of a number :
Divide the given number by a divisor around approximate square root of the number. Find the average of the divisor and quotient. This average is the approximate square root of the given number.

(i) $\sqrt{850} = ?$
Take 30 as divisor.
$850 \div 30 = 28.33$
Average of 30 and 28.33 = 29.17.
Hence, approximate square root of 850 = 29.17.

(ii) $\sqrt{1171} = ?$
Take 35 as divisor.
$1171 \div 35 = 33.46$

Average of 35 and 33.46 $= \dfrac{35 + 33.46}{2} = 34.23$

Hence approximate square root of 1171 = 34.23.
Note that, we can take divisor as any other number nearby the square root of 1171. If we take 30 as divisor, then
$1171 \div 30 = 39.03$.

Average of 30 and 39.03 $= \dfrac{30 + 39.03}{2} = 34.52$

Hence approximate square root of 1171 = 34.52.

Shortcut Approach - 3

If in a given number, number of digits is n and if n is even, then square root of that number will have $\dfrac{n}{2}$ digits and if n is odd, then number of digits will be $\dfrac{n+1}{2}$.

Illustration – 22

How many digits are there in the square root of 9126441

Shortcut Solution :

Since number of digits in 9126441 is 7 and 7 is odd number.

$\therefore$ Number of digits in square root $= \dfrac{7+1}{2} = 4$ digits.

Shortcut Approach - 3

To calculate cube of a number the following two formulae are used

(i) $(a + b)^3 = a^3 + 3ab\,(a - b) + b^3$

(ii) $(a - b)^3 = a^3 - 3ab\,(a - b) - b^3$

You can easily understand it by the following illustrations.

Illustration – 23

Find the cube of 13.

Shortcut Solution :

$(13)^3 = (10 + 3)^3$
$= (10)^3 + 3 \times 10 \times 3\,(10 + 3) + (3)^3$
$= 1000 + 90 \times 13 + 27 = 1027 + 1170 = 2197.$

Illustration – 24

Find the cube of 18.

Shortcut Solution :

$(18)^3 = (20-2)^3$
$= (20)^3 - 3 \times 20 \times 2(20-2) - (2)^3$
$= 8000 - 120 \times 18 - 8$
$= 8000 - 2160 - 8 = 5832.$

Shortcut Approach - 3

Cube Root By Successive Subtraction:

If we subtract 1, 7, 19, 37, 61, 91 ..... from a given number, till we get zero then the number of subtractions will give the cube root of the given number. The numbers 1, 7, 19, 37 ..... are obtained by putting m = 1, 2, 3, 4..... in

$[1 + m \times (m-1) \times 3]$

Example : 64 – 1 = 63, 63 – 7 = 56, 56 – 19 = 37, 37 – 37 = 0.

Here number of subtractions till we get zero is 4.

Therefore $\sqrt[3]{64} = 4$

Cube root of any number upto 3-digits is a single digit number. We can easily find the cube root of any perfect cube number upto 3 digits. Using the relation between unit digit of a perfect cube number and unit digit of its cube root. Cube root of any 4, 5 or 6 digits number is a 2-digits number.

To find the cube root of any 4, 5 or 6 digits number follow the steps :

(i) Put a slash before the last three digits from right of given perfect cube number. 103823 can represent as 103 / 823, 1331 can represent as 1 / 331.

(ii) The number on right side of slash will give us the unit place digit of cube root and that on left hand side will give ten's digit of cube root.

(iii) The unit place digit of cube root can be found by looking at unit place digit of the given number and the relation between unit digit of a perfect cube number and unit digit of its cube root . For example, cube root of 287496 will have 6 at units place in its cube root because the number itself has 6 in its unit place.

(iv) To find the ten's place digit of the cube root, we take the number which lies to the left of the slash. Now we find two perfect cubes between which the left hand part i.e., number before slash, lies. For example in 287/496, left part 287 lies between 216 (cube of 6) and 343 (cube of 7).

(v) Now out of the numbers 6 and 7 obtained above we take the smaller number as ten's place digit of the cube root. Thus cube root of 287496, which is a perfect cube, is 66.

Thus by simple observation we will be able to find cube root of any given perfect cube number upto 6 digits.

Illustration – 25

Find cube root of 681472.

Shortcut Solution :

(a) Represent 681472 as 681 / 472.
(b) The cube ends in 2, so the cube root ends with 8
(c) Left part 681 lies between 512 (cube of 8) and 729 (cube of 9).
(d) Smaller number is 8, so cube root of 681472 is 88.

Illustration – 26

Find cube root of 1157625.

Shortcut Solution :

(a) Represent 1157625 as 1157 / 625.
(b) Number ends with 5, so the cube root ends with 5 (Table-2)
(c) Left part 1157 lies between 1000 (cube of 10) and 1331 (cube of 11).
(d) Smaller number is 10. So cube root of 1157625 is 105.

Shortcut Approach - 3

Finding cube root by Approximation

Divide the given number by a divisor around approximate cube root of the number. Divide the quotient by the same divisor again. Take average of the two divisors and final quotient. This is approximate cube root of the number.

Illustration – 27

Find cube root of 800.

Shortcut Solution :

Take 10 as divisor $800 \div 10 = 80$ and $80 \div 10 = 8$

Average of 10, 10, 8 is $28 \div 3 = 9.3$

$\therefore \quad \sqrt[3]{800} = 9.3$

Illustration – 28

Find cube root of 230.

Shortcut Solution :

Take 6 as divisor. $230 \div 6 = 38.3$ and $38.3 \div 6 = 6.38$

(rounded off downwards)
Average of 6, 6 and 6.38 is 6.126

$\therefore \quad \sqrt[3]{230} = 6.126$

Shortcut Approach - 7

If x is a positive real number and a, b and c are real number; then

(i) $\left(\frac{x^b}{x^c}\right)^a \cdot \left(\frac{x^c}{x^a}\right)^b \cdot \left(\frac{x^a}{x^b}\right)^c = 1$

(ii) $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

(iii) $\left(\frac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \cdot \left(\frac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \cdot \left(\frac{x^c}{x^a}\right)^{(c^2+a^2+ca)} = 1$

Illustration – 29

Find the value of the followings :

(i) $\left(\frac{6^3}{6^5}\right)^2 \cdot \left(\frac{6^5}{6^2}\right)^3 \cdot \left(\frac{6^2}{6^3}\right)^5$

(ii) $\left(\frac{23^5}{23^8}\right)^{5+8} \cdot \left(\frac{23^8}{23^9}\right)^{8+9} \cdot \left(\frac{23^9}{23^5}\right)^{9+5}$

(iii) $\left(\frac{31^7}{31^{10}}\right)^{(7^2+10^2+7\times10)} \cdot \left(\frac{31^{10}}{31^{11}}\right)^{(10^2+11^2+10\times11)} \cdot \left(\frac{31^{11}}{31^7}\right)^{(11^2+7^2+11\times7)}$

Shortcut Solution :

(i) 1 (ii) 1 (iii) 1

Shortcut Approach - 3

If the sum of a number and its square is x, then the number is given by $\left[\frac{\sqrt{1+4x}-1}{2}\right]$.

Illustration – 30

If the sum of a number and its square is 90, what is the number?

(a) 5 (b) 6 (c) 8 (d) 9

Shortcut Solution :

$$\text{Required number} = \frac{\sqrt{1+4x}-1}{2} = \frac{\sqrt{1+4\times90}-1}{2}$$

$$= \frac{\sqrt{361}-1}{2} = \frac{19-1}{2} = 9.$$

Shortcut Approach - 3

The difference between the squares of two consecutive numbers is the sum of the two consecutive numbers and it is always an odd number.

Illustration – 31

Find the value of $25^2 + 21^2 + 19^2 - 24^2 - 20^2 - 18^2$.

(a) 120 (b) 127 (c) 132 (d) 141

Shortcut Solution : (b)

$25^2 + 21^2 + 19^2 - 24^2 - 20^2 - 18^2$

$= (25^2 - 24^2) + (21^2 - 20^2) + (19^2 - 18^2)$

$= (25+24) + (21+20) + (19+18) = 49 + 41 + 37 = 127.$

Shortcut Approach - 10

A number is divided by a certain number N_1 and gives a remainder R. If the same number is divided by another number N_2, where $N_1 > N_2$ and N_1 is divisible by N_2, then the new remainder is obtained by dividing R by N_2.

Illustration – 32

A number when divided by 1404 gives a remainder 93. What remainder would be obtained by dividing the same number by 39?

Shortcut Solution :

Required number = 15

$$\begin{array}{r} 39\overline{)\,93\,(2} \\ \underline{78} \\ 15 \end{array}$$

Shortcut Approach - 10

If the sum of two numbers be x and difference is y, then the difference of their squares is xy.

Illustration – 33

The sum of two numbers is 160 and their difference is 39. The difference of their square is

(a) 3645 (b) 6240 (c) 7642 (d) 6724

Shortcut Solution : (b)

Required number = $xy = 160 \times 39 = 6240$

Shortcut Approach - 10

If the difference between the squares of two consecutive numbers is x, then the numbers are $\left(\frac{x-1}{2}\right)$ and $\left(\frac{x+1}{2}\right)$.

Illustration – 34

The difference between the squares of two consecutive numbers is 59. Find the numbers.

(a) 30, 31 (b) 28, 29 (c) 28, 31 (d) 29, 30

Shortcut Solution : (d)

Required numbers $= \left(\frac{x-1}{2}\right)$ and $\left(\frac{x+1}{2}\right)$

$= \frac{59-1}{2}$ and $= \frac{59+1}{2} = 29$ and 30.

Shortcut Approach - 10

If the two consecutive numbers are x and y, then the difference of their squares is given by $(x+y)$.

Illustration – 35

Two consecutive numbers are 75 and 76. Find the difference of their squares.

(a) 151 (b) 149 (c) 141 (d) 139

Shortcut Solution : (a)

Required number = $x + y = 75 + 76 = 151$

Shortcut Approach - 10

If the product of the two numbers is x and the sum of their squares is y, then

(i) the sum of the two numbers is given by $\sqrt{y+2x}$ and

(ii) the difference of the two numbers is given by $\sqrt{y-2x}$.

Illustration – 36

The product of two numbers is 224. The sum of their squares is 452. Find the difference of the two numbers.

(a) 15 (b) 24 (c) 2 (d) 3

Shortcut Solution : (c)

Required difference = $\sqrt{y-2x}$

$= \sqrt{452 - 2 \times 224} = \sqrt{4} = 2.$

GP_3386

LEVEL - 1

1. Which of the following numbers is not a perfect square?
(a) 29241 (b) 7853300
(b) 103684 (d) 15625

2. Which one of the following sentence/s are correct.
(i) If a perfect square is divisible by 99 it must be divisible by 121
(ii) If a perfect square is divisible by 52 it must be divisible by 169
(iii) If a perfect square is divisible by 28 it must be divisible by 343
(a) Only (i) (b) Only (i) and (ii)
(c) Only (i) and (iii) (d) All three

3. Find the square root of $\sqrt{2304}$
(IBPS PO-2007)
(a) 48 (b) 42
(c) 47 (d) None of these

4. Find the square root of $\sqrt{5184}$
(Bank Clerk-2007)
(a) 78 (b) 71
(c) 72 (d) None of these

5. Find the square root of $28-\sqrt{300}$
(a) $\sqrt{3}-5$ (b) $\sqrt{3}+5$
(c) $5-\sqrt{2}$ (d) $4-\sqrt{3}$

6. Find the square root of $5+2\sqrt{6}$
(a) $\sqrt{3}-\sqrt{2}$ (b) $\sqrt{5}+\sqrt{2}$
(c) $\sqrt{3}+\sqrt{2}$ (d) $\sqrt{3}+\sqrt{5}$

7. Find the approximate square root of 624.
(SSC CGL 2009)
(a) 23.98 (b) 24.98
(c) 25.98 (d) None of these.

8. Find the Approximate square root of 1550.
(SSC CHSL-2009)
(a) 37. 375 (b) 38. 375
(c) 39. 375 (d) None of these.

9. Which one of the following is equal to $\left(3+\sqrt{8}\right)+\frac{1}{3-\sqrt{8}}$?
(a) $\left(3+\sqrt{8}\right)$ (b) 6
(c) $2\sqrt{8}$ (d) $2\left(3+\sqrt{8}\right)$

10. Which one of the following is equal to $\frac{1}{5-\sqrt{3}}+\frac{1}{5+\sqrt{3}}$?
(a) $2\left(5+\sqrt{3}\right)$ (b) $\sqrt{3}/8$
(c) $5/8$ (d) None of these

11. Which one of the following is equal to $\frac{1}{4-\sqrt{2}}$?
(a) $\frac{2}{7}-\frac{\sqrt{2}}{14}$ (b) $\frac{2}{7}+\frac{\sqrt{2}}{14}$
(c) $-\frac{2}{7}+\frac{\sqrt{2}}{14}$ (d) $\frac{2}{14}+\frac{\sqrt{2}}{14}$

12. Which one of the following is equal to $\frac{5\sqrt{5}-1}{5\sqrt{5}+1}$?
(a) $\frac{63}{62}+\frac{5\sqrt{5}}{62}$ (b) $\frac{63}{62}-\frac{5\sqrt{5}}{62}$
(c) $\frac{62}{63}-\frac{5\sqrt{5}}{63}$ (d) $\frac{62}{63}+\frac{5\sqrt{5}}{63}$

13. Which one of the following is equal to $\frac{8\sqrt{7}+7\sqrt{8}}{8\sqrt{7}-7\sqrt{8}}$?
(a) $\frac{\left(840+56\sqrt{56}\right)}{52}$ (b) $\frac{\left(840-56\sqrt{56}\right)}{52}$
(c) $\frac{\left(840+56\sqrt{56}\right)}{48}$ (d)None of these

14. Simplify $\frac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}}$
(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{3}$

15. Find square root of $16-2\sqrt{20}-2\sqrt{28}+2\sqrt{35}$

(a) $\left(\sqrt{5}-\sqrt{7}-\sqrt{4}\right)$ (b) $\left(-\sqrt{5}-\sqrt{7}+\sqrt{4}\right)$

(c) $\left(-\sqrt{5}+\sqrt{7}-\sqrt{4}\right)$ (d) $\left(-\sqrt{5}+\sqrt{7}+\sqrt{4}\right)$

16. How many digits are there in square root of 6594624. **(IBPS Clerk 2008)**

(a) 7 (b) 6 (c) 5 (d) 4

17. How many digits are there in square root of 20820969. **(IBPS Clerk 2009)**

(a) 4 (b) 5 (c) 6 (d) 8

18. Find the value of x if $\sqrt{x^{\sqrt{x^{\sqrt{x^{\cdots\infty}}}}}}=9$

(a) $\sqrt{3}$ (b) $3^{\frac{1}{4}}$ (c) $3^{\frac{1}{6}}$ (d) $3^{\frac{4}{9}}$

19. If $x = 3^{1/3} + 3^{1/3}$, then the value of $3x^3 - 9x$ would be

(a) $72-18\sqrt[3]{3}$ (b) $72+18\sqrt[3]{3}$

(c) $36-18\sqrt[3]{3}$ (d) $36+18\sqrt[3]{3}$

20. If $x^4/x^9 = 5^{-5}$ then x is equal to

(a) 5 (b) 4

(c) 9 (d) None of these

21. The value of is $\sqrt{7+\sqrt{7-\sqrt{7+\sqrt{7-\infty}}}}$

(a) 1 (b) 2 (c) 3 (d) 4

22. $2-\dfrac{\sqrt{6407522209}}{\sqrt{3600840049}}=?$

(a) 0.666039 (b) 0.666029

(c) 0.666009 (d) None of these

23. $\sqrt{110.25}\times\sqrt{0.01}\div\sqrt{0.00025}-\sqrt{42025}$ equal to

(a) 0.75 (b) 0.50 (c) 0.64 (d) 0.73

24. What number should replace the question mark in the image below?

6	12	?
48	96	192

(a) 18 (b) 20 (c) 22 (d) 24

25. $x^2 - 3y^2 = 1376$. How many integer solutions exist for the given equation?

(a) One (b) Two (c) Four (d) Zero

26. What number must be added to the expression $16a^2 - 12a$ to make it a perfect square?

(a) 9/4 (b) 11/2 (c) 13/2 (d) 16

27. If the sum of a number and its square is 110, what is the number? **(SSC CGL Tire-1 2012)**

(a) 10 (b) 21

(c) 11 (d) None of these.

28. If the sum of a number and its square is 240, what is the number? **(SSC CGL Tire-I 2013)**

(a) 13 (b) 14

(c) 15 (d) None of these

29. A number when divided by 357, gives a remainder 39. What remainder would be obtained by dividing the same number by 17. **(SSC CGL Tire-I 2014)**

(a) 6 (b) 3 (c) 5 (d) 8

30. A number when divided by 899, gives a remainder 63. What remainder would be obtained by dividing the same number by 29. **(SSC CGL Tire-I 2015)**

(a) 0 (b) 3 (c) 5 (d) 8

31. If $\sqrt{24} = 4.899$, the value of $\sqrt{\dfrac{8}{3}}$ is

(a) 0.544 (b) 1.333 (c) 1.633 (d) 2.666

32. Which one among $2^{1/2}, 3^{1/3}, 4^{1/4}, 6^{1/6}$ and $12^{1/12}$ is the largest?

(a) $2^{1/2}$ (b) $3^{1/3}$ (c) $4^{1/4}$ (d) $6^{1/6}$

33. Find the square root of $7 + \sqrt{48}$.

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $3+\sqrt{2}$ (d) $3-\sqrt{2}$

34. $\left(\dfrac{1}{64}\right)^0+(64)^{-1/2}-(32)^{4/5}$ is equal to

(a) $-15\dfrac{7}{8}$ (b) $16\dfrac{1}{8}$ (c) $-14\dfrac{7}{8}$ (d) $17\dfrac{1}{8}$

35. $\sqrt{\dfrac{0.324\times0.081\times4.624}{1.5625\times0.0289\times72.9\times64}}=?$

(a) 24 (b) 2.40

(c) 0.024 (d) None of these

36. $\sqrt{(0.798)^2+0.404\times0.798+(0.202)^2}+1=?$

(a) 0 (b) 2 (c) 1.596 (d) 10.404

GP_3386

37. Find the square root of $7-2\sqrt{10}$.

(a) $\sqrt{5}+\sqrt{2}$ (b) $-\sqrt{5}-\sqrt{2}$

(c) $\pm(\sqrt{5}-\sqrt{2})$ (d) $\pm(\sqrt{5}+\sqrt{2})$

38. If $y=\sqrt{42-\sqrt{42-\sqrt{42.....\infty}}}$, the value of $y=?$

(a) 5 (b) 6 (c) 7 (d) 14

39. Arrange the following in descending order.

$\sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}, \sqrt[12]{12}$

(a) $\sqrt[4]{4} > \sqrt[12]{12} > \sqrt[6]{6} > \sqrt[3]{3}$

(b) $\sqrt[12]{12} > \sqrt[6]{6} > \sqrt[4]{4} > \sqrt[3]{3}$

(c) $\sqrt[3]{3} > \sqrt[4]{4} > \sqrt[6]{6} > \sqrt[12]{12}$

(d) $\sqrt[4]{4} > \sqrt[6]{6} > \sqrt[3]{3} > \sqrt[12]{12}$

40. If $y=\sqrt{12+\sqrt{12+\sqrt{12+.....\infty}}}$, then $y=?$

(a) 3 (b) 5 (c) 6 (d) 4

41. $\sqrt{3\sqrt{3\sqrt{3.....}}}$ is equal to

(a) $\sqrt{3}$ (b) 3 (c) $2\sqrt{3}$ (d) $3\sqrt{3}$

42. The largest number among 2^{50}, 3^{40}, 4^{30} and 5^{20} is

(a) 2^{50} (b) 3^{40} (c) 4^{30} (d) 5^{20}

43. The largest number among

$\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$ is

(a) $\sqrt{7}-\sqrt{5}$ (b) $\sqrt{5}-\sqrt{3}$

(c) $\sqrt{9}-\sqrt{7}$ (d) $\sqrt{11}-\sqrt{9}$

44. Find the value of the following:

(Bank PO 2011)

$$\left(\frac{13^3}{13^6}\right)^2 \cdot \left(\frac{13^6}{13^2}\right)^3 \cdot \left(\frac{13^2}{13^3}\right)^6$$

(a) 11 (b) 111

(c) 1 (d) None of thses

45. Find the value of the following.

(Bank PO 2012)

$$\left(\frac{(121)^7}{(121)^8}\right)^{7+8} \cdot \left(\frac{(121)^8}{(121)^9}\right)^{8+9} \cdot \left(\frac{(121)^9}{(121)^7}\right)^{9+7}$$

(a) 121 (b) 1

(c) 11 (d) None of these

46. If $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$, then x will be equal to

(a) –2 (b) 2 (c) –1 (d) 1

47. The simplified form of $\left[\left(\sqrt[5]{x^{-3/5}}\right)^{-5/3}\right]^5$ is

(a) x^5 (b) x^{-5} (c) x (d) $\frac{1}{x}$

48. Find the value of the following.

(Bank PO 2013)

$$\left(\frac{(49)^3}{(49)^9}\right)^{(3^2+9^2+3\times 9)} \cdot \left(\frac{(49)^9}{(49)^{27}}\right)^{(9^2+(27)^2+9\times 27)} \cdot \left(\frac{(49)^{27}}{(49)^3}\right)^{((27)^2+(3)^2+27\times 3)}$$

(a) 1 (b) 49

(c) $(49)^2$ (d) None of these.

49. The value of

$\left[x^{(b-c)}\right]^{b+c}\left[x^{(c-a)}\right]^{(c+a)}\left[x^{(a-b)}\right]^{(a+b)}$ is

(a) 0 (b) 1

(c) x (d) $x^{a^2+b^2+c^2}$

50. The sum of two number be 100 and their difference is 37. The difference of their square is: **(SSC CHSL 2014)**

(a) 2700 (b) 3700 (c) 3900 (d) 2900

51. The sum of two number is 291 and their difference is 89, the difference of their square is **(CDS 2015)**

(a) 25799 (b) 25699 (c) 25599 (d) 25899

52. The difference between the squares of two consecutive number is 31, find the numbers. **(CDS 2016)**

(a) 15, 16 (b) 14, 15

(c) 13, 14 (d) None of these

53. The difference between the squares of two consecutive number 201, find the numbers. **(Bank PO 2015)**

(a) 101, 102 (b) 102, 103

(c) 100, 101 (d) None of these

54. Two consecutive numbers are 171 and 172, then the difference of their squares. **(SSC CGL Tier-I 2015)**

(a) 345 (b) 343
(c) 353 (d) None of these

55. Two consecutive numbers are 1178 and 1179 then the difference of their squares. **(SSC CGL Tier-I 2016)**
(a) 2357 (b) 2457
(c) 2557 (d) None of these

56. The product of two numbers is 3976. The sum of their squares is 8177. Find the difference of the two numbers. **(SSC CGL Tier-I 2016)**
(a) 17 (b) 18 (c) 16 (d) 15

57. The product of two numbers is 5429. The sum of their squares is 11642. Find the sum of the two numbers. **(SSC CGL Tier-I 2017)**
(a) 151 (b) 152 (c) 150 (d) 153

58. If $x^{\frac{1}{3}} + y^{\frac{1}{3}} = z^{\frac{1}{3}}$, then $\{(x+y-z)^3 + 27xyz\}$ will be equal to
(a) –1 (b) 1 (c) 0 (d) 27

59. $\sqrt[3]{333+\sqrt[3]{987+\sqrt[3]{2197}}}$ is equal to
(a) 21 (b) 18 (c) 7 (d) 3

60. The smallest perfect square number which can be divided by 21, 36 and 66 is
(a) 214344 (b) 214434 (c) 213444 (d) 231444

61. If $1^3 + 2^3 + ... + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + ... (0.99)^3$ is approximately equal to
(a) 0.2695 (b) 2.695 (c) 3.695 (d) 0.3695

62. Find the cube of 19. **(SSC CGL - 2009)**
(a) 6559 (b) 6659 (c) 6759 (d) 6859

63. Find the cube of 28. **(SSC CGL - 2010)**
(a) 23952 (b) 22952
(c) 21952 (d) None of these

64. What is the least value of A for which $90 \times A$ is a perfect cube?
(a) 200 (b) 300 (c) 500 (d) 600

65. Find the greatest 8 digit number which is a perfect square. **(FCI – 2009)**
(a) 99970001 (b) 99980001
(c) 99990001 (d) None of these

66. Find the greatest 9 digit number which is a perfect square. **(FCI – 2010)**
(a) 999970884 (b) 999960884
(c) 999950884 (d) None of these

67. The sum of the digits of that smallest number which when multiplied with 1800 becomes a perfect cube number is
(a) 2 (b) 3 (c) 6 (d) 8

68. Find the cube root of 729, by successive substraction method. **(RRB - 2009)**
(a) 9 (b) 11
(c) 13 (d) None of these.

69. Find the cube root of 343, by successive substraction method. **(RRB - 2010)**
(a) 9 (b) 8
(c) 7 (d) None of these.

70. If P = 124, then $\sqrt[3]{P(P^2+3P+3)+1} = ?$
(a) 5 (b) 7 (c) 123 (d) 125

71. Find the cube root of 3400. **(Bank Clerk - 2010)**
(a) 15.037 (b) 14.037
(c) 16.037 (d) 13.037

72. Find the cube root of 1750. **(Bank Clerk - 2011)**
(a) 15.05 (b) 14.05 (c) 13.05 (d) 12.05

73. If m and n are natural numbers, then $\sqrt[m]{n}$ is
(a) always irrational
(b) irrational unless n is the mth power of an integer
(c) irrational unless m is the nth power of an integer
(d) irrational unless m and n are coprime

74. Consider the following in respect of the numbers $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[6]{6}$

I. $\sqrt[6]{6}$ is the greatest number

II. $\sqrt{2}$ is the smallest number.

Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

75. If $a = 2+\sqrt{3}$, then what is the value of $(a^2 + a^{-2})$?
(a) 12 (b) 14 (c) 16 (d) 18

76. $\left(\sqrt{56+\sqrt{56+\sqrt{56+.....}}}\right) \div 2^2 = ?$
(a) 0 (b) 1
(c) 2 (d) 8

77. Find the value of
$(47)^2 + (31)^2 + (21)^2 - (46)^2 - (30)^2 - (20)^2$ **(Bank PO 2013)**

GP_3386

(a) 190 (b) 195 (c) 205 (d) 215

78. Find the value of
$(91)^2 + (81)^2 + (71)^2 - (90)^2 - (80)^2 - (70)^2$
(Bank PO 2014)
(a) 480 (b) 485 (c) 481 (d) 483

79. If $a^x = b^y = c^z$ and abc = 1, then what is the value of xy + yz + zx?
(a) 1 (b) 3 (c) 0 (d) 5

80. $\sqrt[3]{103823} = ?$
(a) 49 (b) 51 (c) 45 (d) 47

81. If n^2 is a perfect cube, then which of the following statements is always true?
(a) n is odd
(b) n is even
(c) n^3 is a perfect square
(d) n is a perfect cube

82. If $n^2 = 123.45654321$, which of the following is the exact value of n.
(a) 11.1001 (b) 11.1101
(c) 11.1111 (d) 11.1011

83. $N^2 = 12345678987654321$. Find N.
(a) 101010101 (b) 11111
(c) 111111111 (d) 1000000001

84. If p, q, r be integers such that $p^2 = q^2 . r$ then :
(a) p is an even number
(b) q is an even number
(c) r is an even number
(d) r is a perfect square

85. What is the least number which must be subtracted from 369 to make it a perfect cube?
(a) 8 (b) 26 (c) 2 (d) 25

86. $5^{x-1} + 5^x + 5^{x+1} = 775$, then the value of x for every positive integer x, is :
(a) 1 (b) 3
(c) 2 (d) Can't be determined

LEVEL - 2

1. If L is the LCM of the two numbers $2^4 \times 3^5 \times 5^2 \times 7^4$ and $\sqrt{2^5} \times 3^3 \times 5^2 \times 7^2$ then what minimum number should be multiplied with either of the number so that L becomes a perfect square?
(a) 12 (b) 6
(c) 18 (d) 24

2. Which one of the following is equal to
$$\frac{1}{2-\sqrt{3}} + \frac{1}{3-\sqrt{8}} + \frac{1}{4-\sqrt{15}} ?$$
(a) $9 + \sqrt{3} + \sqrt{8} + \sqrt{15}$
(b) $9 - \sqrt{3} + \sqrt{8} + \sqrt{15}$
(c) $9 + \sqrt{3} - \sqrt{8} + \sqrt{15}$
(d) $9 + \sqrt{3} + \sqrt{8} - \sqrt{15}$

3. Find $x^2 - y^2$ if $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
(a) $16\sqrt{15}$ (b) $\sqrt{15}$
(c) $2\sqrt{15}$ (d) $4\sqrt{15}$

4. Find the value of
$$\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{2+\sqrt{3}} + \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{6}+\sqrt{5}}$$
(a) $\sqrt{6}-1$
(b) $\sqrt{6}+1$
(c) $\sqrt{5}+1$
(d) $\sqrt{6}+\sqrt{5}+\sqrt{4}+\sqrt{3}+\sqrt{2}+1$

5. Simplify:
$$\sqrt{\left[\left\{\sqrt{4\sqrt{3\sqrt{6\sqrt{25^{72}}}}}\right\} \times \left\{\sqrt{2\sqrt{3\sqrt{4\sqrt{9^{48}}}}}\right\}\right]}$$
(a) 75 (b) 45
(c) 1/5 (d) 15

6. If $2^a = 3^b = 5^c = 30^d$, then which one of the following is a correct relation between a, b, c and d.
(a) $\frac{1}{d} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (b) $\frac{2}{d} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$
(c) $\frac{3}{d} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ (d) $d = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

7. The value of

$$\left[\frac{1}{\sqrt{9}-\sqrt{8}}\right]-\left[\frac{1}{\sqrt{8}-\sqrt{7}}\right]+\left[\frac{1}{\sqrt{7}-\sqrt{6}}\right]$$

$$-\left[\frac{1}{\sqrt{6}-\sqrt{5}}\right]+\left[\frac{1}{\sqrt{5}-\sqrt{4}}\right] \text{ is}$$

(a) 6 (b) 5 (c) –7 (d) –6

8. If $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$, then the value of $p + q + r$ would be

(a) 0 (b) 1
(c) 2 (d) $x + y + z$

9. $M = \sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$ and

$N = \sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$. What is the value of $\frac{M-N}{M+N}$?

(a) 0 (b) 1
(c) –1 (d) None of these

10. If $x = 3+\sqrt{8}$, then $\left(x^2+\frac{1}{x^2}\right) = ?$

(a) 34 (b) 24 (c) 38 (d) 36

11. If $2^x = 4^y = 8^z$, $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$, find the value of z.

(a) $\frac{7}{24}$ (b) $\frac{1}{24}$ (c) $\frac{5}{48}$ (d) $\frac{7}{48}$

12. If $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = a + b\sqrt{35}$, then the value of (a – b) is

(a) 5 (b) 6
(c) 8 (d) None of these

13. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$, then the value of a and b is

(a) a = 11, b = – 6 (b) a = 6, b = 11
(c) a = –11, b = 6 (d) a = – 11, b = – 6

GP_3386

Hints and Solutions

Level 1

1. (b) Instead of finding out the square roots of the numbers just check with the rule of square, if 7853300 is a perfect square then 78533 has to be a perfect square but we know that a number that ends with 3 can not be a perfect.

2. (b) If a perfect square is divisible by $99 = 3^2 \times 11^1$ then it must be divisible by $11^2 = 121$ hence statement (i) is correct, similarly statement (ii) is also correct.
If a perfect square is divisible by $28 = 2^2 \times 7^1$ then it must be divisible by $7^2 = 49$ but not necessary divisible by $7^3 = 343$.

3. (a) Use Shortcut Approach–1

4. (c) Use Shortcut Approach–1

5. (a) Let the square root of $28 - \sqrt{300}$ is $p - \sqrt{q}$, then by squaring both the sides

$p^2 + q - 2p\sqrt{q} = 28 - \sqrt{300}$,

equating rational and irrational parts

$p^2 + q = 28$ and $2p\sqrt{q} = \sqrt{300}$

$= 10\sqrt{3} = 2 \times 5 \times \sqrt{3}$

Hence we can assume that $p = 5$ and $q = 3$.

so required square roots are $\pm\left(5 - \sqrt{3}\right)$

6. (c) Let the square root of $5 + 2\sqrt{6}$ is $\sqrt{p} + \sqrt{q}$ then by squaring both the sides we will get

$p + q + 2\sqrt{pq} = 5 + 2\sqrt{6}$.

Now equating the rational and irrational terms.

$p + q = 5$ and $pq = 6$ then after solving these equations we will get $p = 3, q = 2$

Hence required square root is $\sqrt{3} + \sqrt{2}$

7. (b) Use Shortcut Approach–2.

8. (c) Use Shortcut Approach–2.

9. (d) Consider

$$\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} = \frac{3+\sqrt{8}}{9-8} = 3+\sqrt{8}$$

Hence $\left(3+\sqrt{8}\right) + \frac{1}{3-\sqrt{8}} = 2\left(3+\sqrt{8}\right)$

10. (d) Consider

$$\frac{1}{5-\sqrt{3}} = \frac{1}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}}$$

$$= \frac{5+\sqrt{3}}{22} = \frac{5}{22} + \frac{\sqrt{3}}{22}$$

Then

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$$

$$= \frac{5-\sqrt{3}}{22} = \frac{5}{22} - \frac{\sqrt{3}}{22}$$

$$\frac{1}{5-\sqrt{3}} + \frac{1}{5+\sqrt{3}} = 2 \times \frac{5}{22} = \frac{5}{11}$$

11. (b) The following expression is $1/\left(4-\sqrt{2}\right)$ when multiplied with its conjugate in numerator and denominator we will get

$$\frac{1}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}} = \frac{4+\sqrt{2}}{4^2-2}$$

$$= \frac{4+\sqrt{2}}{14} = \frac{2}{7} + \frac{\sqrt{2}}{14}$$

12. (b) Multiply the numerator and denominator by conjugate of denominator i.e., $5\sqrt{5}-1$

$$\frac{5\sqrt{5}-1}{5\sqrt{5}+1} \times \frac{5\sqrt{5}-1}{5\sqrt{5}-1} = \frac{125+1-10\sqrt{5}}{125-1}$$

$$= \frac{126}{124} - \frac{10\sqrt{5}}{124} = \frac{63}{62} - \frac{5\sqrt{5}}{62}$$

13. (d) Multiply the numerator and denominator by conjugate of denominator i.e., $8\sqrt{7}+7\sqrt{8}$

$$=\frac{8\sqrt{7}+7\sqrt{8}}{8\sqrt{7}-7\sqrt{8}}\times\frac{8\sqrt{7}+7\sqrt{8}}{8\sqrt{7}+7\sqrt{8}}$$

$$=\frac{448+392+112\sqrt{56}}{448-392}=\frac{\left(840+112\sqrt{56}\right)}{56}$$

$$=15+2\sqrt{56}$$

14. (b) Since square root of $3+2\sqrt{2}$ is $1+\sqrt{2}$ and that of $3-2\sqrt{2}$ is $1-\sqrt{2}$

Hence

$$\frac{\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}}{\sqrt{3+2\sqrt{2}}+\sqrt{3-2\sqrt{2}}}=\frac{\left\{\left(1+\sqrt{2}\right)-\left(1-\sqrt{2}\right)\right\}}{\left\{\left(1+\sqrt{2}\right)+\left(1-\sqrt{2}\right)\right\}}$$

$$=\frac{2\sqrt{2}}{2}=\sqrt{2}$$

15. (b) Let the required square root be $\sqrt{a}+\sqrt{b}-\sqrt{c}$

Then

$$16-2\sqrt{20}-2\sqrt{28}+2\sqrt{35}=\left(\sqrt{a}+\sqrt{b}-\sqrt{c}\right)^2$$

$$=a+b+c+2\sqrt{ab}-2\sqrt{ac}-2\sqrt{bc}$$

Now equating the rational and irrational part we will get $a+b+c=16$,

$ab=35$, $bc=20$ and $ac=28$

Since, $ab=35$ so we can assume that $a=7$ and b = 5 then $c=4$

Required square root is $\pm\left(\sqrt{5}+\sqrt{7}-\sqrt{4}\right)$

16. (d) Use Shortcut Approach–3.

17. (a) Use Shortcut Approach–3.

18. (d) From the given information we can say that

$$\left(\sqrt{x}\right)^9=9=3^2 \text{ hence } \sqrt{x}=3^{\frac{2}{9}} \text{ or } x=3^{\frac{4}{9}}$$

19. (a) $x=3^{1/3}+3^{1/3}=2.3^{1/3}$

$$3x^3-9x=3\left(2.3^{1/3}\right)^3-9.2.3^{1/3}$$

[Just put the value of x]

$$=3.\left(2^3.3\right)-18.3^{1/3}=72-18.3^{1/3}$$

20. (a) From the given condition $\frac{1}{x^5}=\frac{1}{5^5}$, $x=5$

21. (c) Let $\sqrt{7+\sqrt{7-\sqrt{7+\sqrt{7-\ldots\infty}}}}=x$

$\sqrt{7+\sqrt{7-x}}=x$. Squaring both sides,

$7+\sqrt{7-x}=x^2$

$\sqrt{7-x}=x^2-7$

Squaring both sides again, $7-x=(x^2-7)^2$

From the given options only option (c) satisfy the equation.

22. (a) Numerator = 6407522209

$=64\times10^8+2\times8\times10^4\times47+47^2=(80047)^2$

Similarly denominator

$=3600840049=(60007)^2$

Hence given expression is

$2-(80047)/(60007)=0.666039$

23. (b) $\sqrt{110.25}\times\sqrt{0.01}\div\sqrt{0.0025}-\sqrt{420.25}$

$=10.5\times0.1+0.05-20.5=0.50$

24. (d) Here, each number in the second row is obtained by multiplying the corresponding number of the first row by 8.

$\therefore$ The number that should replace the question mark = 192/8 = 24

25. (d) $3y^2=x^2-1376$

As we can see L.H.S. is definitely a multiple of 3 and in R.H.S. 1376 leaves a remainder of 2 when divided by 3.

There are three possibilities for x in R.H.S:

(i) If x is multiple of 3, so is x^2, and R.H.S. will leave a remainder of 1 when divided by 3.

(ii) If x is of the form $3m+1$, x^2 will be of the form $3n+1$ and R.H.S will leave a remainder of 2. $m, n\in N$

(iii) If x is of the form $3m+2$, x^2 will be of the form $3n+1$ and R.H.S. will leave a remainder of 2. $m, n\in N$ So R.H.S. can never be a multiple of 3, while L.H.S. is always a multiple of 3. Hence no real solution exists.

26. (a) $16a^2-12a=(4a)^2-2(4a)(3/2)$

$\therefore$ The number is $(3/2)^2=(9/4)$.

27. (a) Use Shortcut Approach–8.

28. (c) Use Shortcut Approach–8.

29. (c) Use Shortcut Approach–10

30. (a) Use Shortcut Approach–10

GP_3386

31. (c) $\sqrt{\frac{8}{3}} = \sqrt{\frac{8\times3}{3\times3}} = \frac{\sqrt{24}}{3} = \frac{4.899}{3} = 1.633.$

32. (b) In this question it is adv[isable to raise all the numbers to the power of 12, so the numbers become,

$(2^{1/2})^{12}, (3^{1/3})^{12}, (4^{1/4})^{12}, (6^{1/6})^{12}$

or $2^6, 3^4, 4^3, 6^2$ or $64, 81, 64, 36$

So, $3^{1/3}$ is the largest.

33. (b) Given $7+\sqrt{48} = 7+4\sqrt{3}$ let its square root is $p+\sqrt{q}$

Then $\left(p+\sqrt{q}\right)^2 = p^2+q+2p\sqrt{q}$

$= 7+4\sqrt{3}$ equating rational and irrational part.

$p^2+q = 7$ and $2p\sqrt{q} = 4\sqrt{3}$

or $p\sqrt{q} = 2\sqrt{3}$ or $p = 2$ and $q = 3$

Hence required square root is $2+\sqrt{3}$

34. (c) $\left(\frac{1}{64}^{0}\right) + (64)^{-1/2} - (32)^{\frac{4}{5}}$

$= 1+\frac{1}{8}-16 = \frac{-119}{8} = -14\frac{7}{8}$

35. (c) Give expression

$= \sqrt{\frac{324\times81\times4624}{15625\times289\times729\times64}}$

(sum of decimal places being equal in nume. and deno.)

$= \frac{18\times9\times68}{125\times17\times27\times8} = \frac{3}{125} = 0.024$

36. (b) $\sqrt{(0.798)^2+0.404\times0.798+(0.202)^2}+1$

$= \sqrt{(0.798)^2+2\times0.202\times0.798+(0.202)^2}+1$

$= \sqrt{(0.798+0.202)^2}+1$

$= 1.000 + 1 = 1 + 1 = 2$

37. (c) $7-2\sqrt{10} = 5+2-2\sqrt{5\times2}$

$\Rightarrow 7-2\sqrt{10} = (\sqrt{5})^2+(\sqrt{2})^2-2\sqrt{5}.\sqrt{2}$

$\Rightarrow 7-2\sqrt{10} = (\sqrt{5}-\sqrt{2})^2$

Thus the $\sqrt{7-2\sqrt{10}} = \pm(\sqrt{5}-\sqrt{2})$

38. (b) $y = -\sqrt{42-\sqrt{42-\sqrt{42.....\infty}}}$

Squaring both sides, we get

$y^2 = -42\sqrt{42-\sqrt{42-\sqrt{42.....}}}$

$\Rightarrow y^2 = 42-y$

$\Rightarrow y^2+y-42 = 0$

$\Rightarrow y^2+7y-6y-42 = 0$

$y(y+7)-6(y+7) = 0$

$y = 6$ and $y = -7$

but $y \geq 0$, hence $y = 6$.

Shortcut :-
If $y = \sqrt{x-\sqrt{x-\sqrt{x.....\infty}}}$ and $x = n(n+1)$ then $y = n$.
Here $42 = 6\times7$
Hence, $y = 6$

39. (c) $\sqrt[3]{3}\ \sqrt[4]{4}\ \sqrt[6]{6}\ \sqrt[12]{12}$

$\downarrow\ \downarrow\ \downarrow\ \downarrow$

$3^{\frac{1}{3}}\ 4^{\frac{1}{4}}\ 6^{\frac{1}{6}}\ 12^{\frac{1}{12}}$

LCM of $(3, 4, 6, 12) = 12$.

$3^{\frac{1}{3}\times12} = 3^4 = 81$

$4^{\frac{1}{4}\times12} = 4^3 = 64$

$6^{\frac{1}{6}\times12} = 6^2 = 36$

$12^{\frac{1}{12}\times12} = 12^1 = 12$

$81 > 64 > 36 > 12$

$3^{1/3} > 4^{1/4} > 6^{1/6} > 12^{1/12}$.

40. (d) $y = \sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12+.....\infty}}}}$

$y^2 = 12+\sqrt{12+\sqrt{12+\sqrt{12+.....}}}$

(Squaring both sides)

$\Rightarrow y^2 = 12+y$

$\Rightarrow y^2-y-12 = 0$

$\Rightarrow y^2-4y+3y-12 = 0$

$\Rightarrow y(y-4)+3(y-4) = 0$

$\Rightarrow y = 4$ or -3

But $y \geq 0$.

Hence, $y = 4$.

Shortcut :-
If $y = \sqrt{x+\sqrt{x+\sqrt{x+.....}}}$ and $x = n(n+1)$, then. $y = n+1$
Here, $12 = 3\times4$
Hence, $y = 4$.

41. **(b)** Let, $x = \sqrt{3\sqrt{3\sqrt{3.....}}}$

$\Rightarrow \quad x^2 = 3\sqrt{3\sqrt{3...}}$ (Squaring both sides)

$\Rightarrow \quad x^2 = 3x$

$\Rightarrow \quad x^2 - 3x = 0$

$\Rightarrow \quad x(x-3) = 0$

$x = 0$ or $x = 3$

But x is clearly positive

Hence, $x = 3$.

Shortcut : -

$\sqrt{n\sqrt{n\sqrt{n\sqrt{n..\infty}}}} = n$

42. **(b)** $2^{50} = 2^{5\times10} = (2^5)^{10} = (32)^{10}$;

$3^{40} = 3^{4\times10} = (3^4)^{10} = (81)^{10}$

$4^{30} = 4^{3\times10} = (4^3)^{10} = (64)^{10}$;

$5^{20} = 5^{2\times10} = (5^2)^{10} = (25)^{10}$

So, the largest will be 3^{40}.

43. **(b)** If $b-a = d-c = f-e$ .....then putting smaller value in

$\sqrt{b}-\sqrt{a}, \sqrt{d}-\sqrt{c}, \sqrt{f}-\sqrt{e}$

we get the largest difference.

So, $\sqrt{5}-\sqrt{3}$ here will be the largest

Trick : $\sqrt{7}-\sqrt{5} = \dfrac{2}{\sqrt{7}+\sqrt{5}}$

$$\sqrt{11}-\sqrt{9} = \frac{2}{\sqrt{11}+\sqrt{9}} \text{ etc.}$$

$\because$ Numerator is same, so that number will be largest whose denominator is the smallest.

44. **(c)** Use Shortcut Approach–7.

45. **(b)** Use Shortcut Approach–7.

46. **(c)** $\left(\frac{3}{5}\right)^3\left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$

$\Rightarrow \quad \left(\frac{3}{5}\right)^{3-6} = \left(\frac{3}{5}\right)^{2x-1} \Rightarrow 2x-1 = -3$

$\Rightarrow \quad 2x = -2$

$\therefore \quad x = -1$

47. **(c)**

$$\left[\left(\sqrt[5]{x^{-3/5}}\right)^{-\frac{5}{3}}\right]^5$$

$\Rightarrow x^{\left(-\frac{3}{5}\right)\times\frac{1}{5}\times\left(-\frac{5}{3}\right)\times5} \Rightarrow x^1 = x$

48. **(a)** Use Shortcut Approach–7.

49. **(b)** $x^{b^2-c^2+c^2-a^2+a^2-b^2} = x^0 = 1$

50. **(b)** Use Shortcut Approach–11

51. **(d)** Use Shortcut Approach–11

52. **(a)** Use Shortcut Approach–12.

53. **(c)** Use Shortcut Approach–12.

54. **(b)** Use Shortcut Approach–13.

55. **(a)** Use Shortcut Approach–13.

56. **(d)** Use Shortcut Approach–14.

57. **(c)** Use Shortcut Approach–14.

58. **(c)** If $a+b+c = 0$, then $a^3+b^3+c^3 = 3abc$

After putting $a = x^{\frac{1}{3}}, b = y^{\frac{1}{3}}, c = -z^{\frac{1}{3}}$

$x+y-z = -3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$

or, $(x+y-z)^3 = -27xyz$

or, $(x+y-z)^3 + 27xyz = 0$

59. **(c)** $\sqrt[3]{333+\sqrt[3]{987+\sqrt[3]{2197}}}$

$= \sqrt[3]{333+\sqrt[3]{987+13}}$

$= \sqrt[3]{333+\sqrt[3]{1000}}$

$= \sqrt[3]{333+10} = \sqrt[3]{343} = 7$

60. **(c)** The L.C.M. of given numbers $= 2^2\times3^2\times11\times7$

So, required number $= 2^2\times3^2\times11^2\times7^2$

$= 213444$

61. **(b)** Required sum $= (0.11)^3\times2025$

$= 2.695$ (approximately)

62. **(d)** Use Shortcut Approach–4.

63. **(c)** Use Shortcut Approach–4

64. **(b)** $90 = 3^2\times2\times5$;

So required number will be $3\times2^2\times5^2 = 300$

65. **(b)** Use Shortcut Approach–3

66. **(c)** Use Shortcut Approach–3.

67. **(c)** $1800 = 2^3\times3^2\times5^2$, so we have to multiply 1800 by $3\times5 = 15$ to get perfect cube. Here sum of digits $= 1+5 = 6$

68. **(a)** Use Shortcut Approach–5.

69. **(c)** Use Shortcut Approach–5.

70. **(d)** $\sqrt[3]{P(P^2+3P+3)+1}$

$= \sqrt[3]{P^3+3P^2+3P+1}$

$= \sqrt[3]{(P+1)^3} = P+1$

$\because \quad P = 124 \Rightarrow P+1 = 125$

71. **(a)** Use Shortcut Approach–6.

72. **(d)** Use short cut Approach–6.

GP_3386

73. **(b)** If m and n are natural numbers, then $\sqrt[m]{n}$ is irrartional unless n is mth power of an integer.

74. **(d)** $\sqrt{2}, \sqrt[3]{3}, \sqrt[6]{6}$

Taking LCM of 2, 3 and 6 = 12

Now, $\sqrt{2} = (2)^{1/2} = (2)^{6/12} = \sqrt[12]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = (3)^{1/3} = (3)^{4/12} = \sqrt[12]{3^4} = \sqrt[12]{81}$

$\sqrt[6]{6} = (6)^{1/6} = (6)^{2/12} = \sqrt[12]{6^2} = \sqrt[12]{36}$

So, neither I nor II are correct.

75. **(b)** Given that, $a = 2 + \sqrt{3}$

Then $\frac{1}{a} = 2 - \sqrt{3}$ [by conjugate property]

Now, we have, $a^2 + a^{-2} = \left(a + \frac{1}{a}\right)^2 - 2$

$= \left(2 + \sqrt{3} + 2 - \sqrt{3}\right)^2 - 2$

$= (4)^2 - 2 = 16 - 2 = 14$

76. **(c)** Here, $56 = 7 \times 8$

$\Rightarrow \sqrt{56 + \sqrt{56 + \sqrt{56 + \ldots\ldots}}} = 8$

$\therefore \quad ? = \frac{\sqrt{56 + \sqrt{56 + \sqrt{56 + \ldots\ldots}}}}{(2)^2}$

$= \frac{8}{4} = 2$

77. **(b)** Use Shortcut Approach–9.

78. **(d)** Use Shortcut Approach–9.

79. **(c)** Here, $a^x = b^y = c^z$

Let $a^x = b^y = c^z = K$

$\therefore \quad a = K^{\frac{1}{x}}, b = K^{\frac{1}{y}}$ and $c = K^{\frac{1}{z}}$

$\Rightarrow \quad abc = K^{\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)}$

$\because \quad abc = 1$ [given]

$\therefore \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ or $xy + yz + zx = 0$

80. **(d)** $? = \sqrt[3]{103823} = \sqrt[3]{(47)^3} = [(47)^3]^{\frac{1}{3}} = 47^{3 \times \frac{1}{3}} = 47$

81. **(d)** If n^2 is a perfect cube, then n is a perfect cube.

82. **(c)** $11.1111 \times 11.1111 = 123.45654321$

83. **(c)** $(111111111)^2 = 12345678987654321$

84. **(d)** Options (a), (b) and (c) are irrelevent.

as $(25)^2 = (5)^2 \times 25$

or $(27)^2 = (9)^2 \times 9$

or $(6)^2 = (4)^2 \times 9$

or $(6)^2 = (3)^2 \times 4$ etc.

85. **(b)** **Go through option.** If you subtract 2, 8 or 25 from 369, then you will not obtain a perfect cube number. So (b) is the correct answer.

Alternatively : If you write down the nearest perfect cube such that $k \leq 369$, where k is a perfect cube, then

$7^3 = 343$, which is just below 369.

So $369 - 343 = 26$

86. **(b)** $5^{x-1} + 5^x + 5^{x+1} = 775$

$\Rightarrow \quad 5^{x-1}(1 + 5 + 5^2) = 775$

$\Rightarrow \quad 5^{x-1}(31) = 775$

$\Rightarrow \quad 5^{x-1} = 25 = 5^2$

$\Rightarrow \quad x - 1 = 2 \Rightarrow x = 3$

Level 1

1. **(b)** Given numbers are $2^4 \times 3^5 \times 5^2 \times 7^4$ and $2^5 \times 3^3 \times 5^2 \times 7^2$ then their LCM $L = 2^5 \times 3^5 \times 7^4 \times 5^2$ to make it perfect square we need power of 2 as 6 and that of 3 as 6.

2. **(a)** $\frac{1}{2 - \sqrt{3}} + \frac{1}{3 - \sqrt{8}} + \frac{1}{4 - \sqrt{15}}$

$= \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} + \frac{1}{3 - \sqrt{8}} \times \frac{3 + \sqrt{8}}{3 + \sqrt{8}}$

$+ \frac{1}{4 - \sqrt{15}} \times \frac{\left(4 + \sqrt{15}\right)}{4 + \sqrt{15}}$

$= \left(2 + \sqrt{3}\right) + \left(3 + \sqrt{8}\right) + \left(4 + \sqrt{15}\right)$

$= 9 + \sqrt{3} + \sqrt{8} + \sqrt{15}$

3. **(a)** Since $x^2 - y^2 = (x + y)(x - y)$

$x + y = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} + \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}}$

$$= \frac{\left\{\left(\sqrt{5}+\sqrt{3}\right)^2 + \left(\sqrt{5}-\sqrt{3}\right)^2\right\}}{5-3} = \frac{16}{2} = 8$$

$$x - y = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \left\{\frac{\left(\sqrt{5}+\sqrt{3}\right)^2 - \left(\sqrt{5}-\sqrt{3}\right)^2}{5-3}\right\}$$

$$= \frac{4\sqrt{15}}{2} = 2\sqrt{15}$$

4. (a) Since $\frac{1}{\sqrt{2}+1} = \sqrt{2}-1$ similarly

$$\frac{1}{\sqrt{3}+\sqrt{2}} = \sqrt{3}-\sqrt{2} \text{ and so on}$$

Hence

$$\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{2+\sqrt{3}}$$

$$+ \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{6}+\sqrt{5}}$$

$$= \left(\sqrt{2}-1\right) + \left(\sqrt{3}-\sqrt{2}\right) + \left(2-\sqrt{3}\right)$$

$$+ \left(\sqrt{5}-2\right) + \left(\sqrt{6}-\sqrt{5}\right) = \sqrt{6}-1$$

5. (b) $\sqrt{4\sqrt{3\sqrt{6\sqrt{25^{72}}}}} = 5^{\frac{2\times 72}{4\times 3\times 6}} = 5^2 = 25$

and $\sqrt{2\sqrt{3\sqrt{4\sqrt{9^{48}}}}} = 3^{\frac{2\times 48}{4\times 3\times 2}} = 3^4 = 81$

Hence required value is

$\sqrt{81\times 25} = \sqrt{2025} = 45$

6. (a) Let $2^a = 3^b = 5^c = 30^d = k$

Taking log we will get alog

$2 = \log k$ or $\log 2 = \frac{\log k}{a}$

Similarly $\log 3 = \frac{\log k}{b}$ $\log 5 = \frac{\log k}{c}$

$$\log 30 = \frac{\log k}{d} = \log 2 + \log 3 + \log 5$$

or $\frac{\log k}{d} = \frac{\log k}{a} + \frac{\log k}{b} + \frac{\log k}{c}$

or $\frac{1}{d} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$

7. (b) By rationalization we have

$$\left[\frac{1}{\sqrt{9}-\sqrt{8}}\right] = \left[\frac{1}{\sqrt{9}-\sqrt{8}}\right] \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{\sqrt{9}+\sqrt{8}}{9-8}$$

$$= \sqrt{9}+\sqrt{8}$$

Similarly, $\left[\frac{1}{\sqrt{8}-\sqrt{7}}\right] = \sqrt{8}+\sqrt{7}$ and

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$$

and so on. The given expression

$= (\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) +$
$(\sqrt{5}+\sqrt{4})$

$= \sqrt{9}+\sqrt{4} = 3+2 = 5.$

8. (a) $x^{1/p} = y^{1/q} = z^{1/r}$ and $xyz = 1$ (given)

$x^{1/p} = y^{1/q} = z^{1/r} = k$ [constant]

$x^{1/p} = k \Rightarrow x = k^p$, same as others two terms

$= y = k^q, z = k^r.$

multiply $x, y, z = xyz = k^p k^q k^r = k^{p+q+r}$

$\Rightarrow 1 = k^0 = k^{p+q+r}$

$\Rightarrow p+q+r = 0$

9. (a) $M = \sqrt{3-\sqrt{5}+\sqrt{9-4\sqrt{5}}}$

$$= \sqrt{3-\sqrt{5}+\sqrt{((\sqrt{5})^2 - 2\times 2\times \sqrt{5} + 2^2)}}$$

$$= \sqrt{3-\sqrt{5}+\sqrt{(\sqrt{5}-2)^2}}$$

$$= \sqrt{3-\sqrt{5}+\sqrt{5}-2} = \sqrt{1} = 1$$

$$N = \sqrt{\sqrt{7}-1-\sqrt{11-4\sqrt{7}}}$$

$$= \sqrt{\sqrt{7}-1\sqrt{(\sqrt{7})^2 - (2\times 2\times \sqrt{7}) + (2)^2}}$$

$$= \sqrt{\sqrt{7}-1-\sqrt{(\sqrt{7}-2)^2}}$$

$$= \sqrt{\sqrt{7}-1-\sqrt{7}+2} = \sqrt{1} = 1$$

GP_3386

Hence, $\frac{M-N}{M+N}=\frac{1-1}{1+1}=0.$

10. (a) $x+\frac{1}{x}=3+\sqrt{8}+\frac{1}{3+\sqrt{8}}$

$=\frac{(3+\sqrt{8})^2+1}{(3+\sqrt{8})}=\frac{9+8+6\sqrt{8}+1}{(3+\sqrt{8})}$

$=\frac{18+6\sqrt{8}}{(3+\sqrt{8})}=\frac{6(3+\sqrt{8})}{(3+\sqrt{8})}=6,$

$\left(x+\frac{1}{x}\right)^2=6^2$

$x^2+\frac{1}{x^2}+2.x.\frac{1}{x}=36,\ x^2+\frac{1}{x^2}=36-2=34$

11. (d) $2^x = 4^y = 8^z$

$\downarrow \quad \downarrow \quad \downarrow$

$2^x \quad 2^{2y} \quad 2^{3z}$

Base is equal and so, power will be equal.

$x=2y=3z.$

$y=\frac{3z}{2}, 4y=6z$

$x=3z, 2x=6z$

So, $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$

$\Rightarrow \frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z}=\frac{24}{7}$

$\Rightarrow \frac{3}{6z}=\frac{24}{7}$

$\therefore \quad z=\frac{7}{48}$

12. (d) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}=a+b\sqrt{35}$

Rationalising the denominator on left hand side by multiplying by $\sqrt{7}-\sqrt{5}$.

$\therefore \quad \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{(\sqrt{7}-\sqrt{5})\times(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})\times(\sqrt{7}-\sqrt{5})}$

$=\frac{(\sqrt{7})^2+(\sqrt{5})^2-2\sqrt{7\times5}}{(\sqrt{7})^2-(\sqrt{5})^2}=\frac{7+5-2\sqrt{35}}{7-5}$

$=\frac{12-2\sqrt{35}}{2}=6-\sqrt{35}$

$\therefore \quad a=6$ and $b=-1$

or $\quad a-b=6-(-1)=7$

13. (a) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$=\frac{35+14\sqrt{3}-20\sqrt{3}-24}{49-48}=11-6\sqrt{3}$

$\because \quad \frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

$\therefore \quad a+b\sqrt{3}=11-6\sqrt{3}$

or $\quad a=11$ and $b=-6$

CHAPTER

5

Fundamental Operations, VBODMAS Rule, Algebraic Identities, Approximation and Simplification

(Including Basic Calculations)

FUNDAMENTAL OPERATIONS

1. Addition

(i) Sum for two positive numbers is a positive number.
For example: $(+5)+(+2)=+7$ i.e. $5+2=7$

(ii) Sum for two negative numbers is a negative number.
For example: $(-5)+(-3)=-8$ i.e. $-5-3=-8$

(iii) Sum of positive and a negative number is the difference between their magnitudes and give the sign of the number with greater magnitude.
For example: $(-3)+(+5)=2$ and $(-7)+(+2)=-5$
i.e. $-3+5=2$ and $-7+2=-5$

2. Subtraction

Subtraction of two numbers is same as the sum of a positive and a negative number.
For example :
$(+9)-(+2)=(+9)+(-2)=7$ i.e. $9-2=9+(-2)=7$
$(-3)-(-5)=(-3)+5=+2$ i.e. $-3-(-5)=-3+5=2$

> **Note :** In subtraction of two negative numbers, sign of second number will change and become positive.

3. Multiplication

(i) Product of two positive numbers is positive.
(ii) Product of two negative numbers is positive.
(iii) Product of a positive number and a negative number is negative.
(iv) Product of more than two numbers is positive or negative depending upon the presence of number of negative quantities.
If the number of negative numbers is even then product is positive and if the number of negative numbers is odd then product is negative.
For example : $(-3)\times(+2)=-6$ i.e. $-3\times2=-6$
$(-5)\times(-7)=+35$ i.e. $-5\times(-7)=35$
$(-2)\times(-3)\times(-5)=-30$
$(-2)\times(-3)\times(+5)=+30$

(v) Product of any number and zero (0) is always zero (0).

1. Addition

(i) If both the dividend and the divisor are of same sign, then quotient is always positive.
(ii) If the dividend and the divisor are of different sign, then quotient is negative,
For example : $(-36)\div(+9)=-4, (-35)\div(-7)=+5$
(iii) When zero (0) is divided by any non-zero number, the quotient is always zero (0).
(iv) Division by zero (0) is not defined.

BRACKETS

They are used for the grouping of things or entities. The various kind of brackets are:
(i) '–' is known as *line (or bar) bracket* or *vinculum*.
(ii) ( ) is known as *parenthesis, common bracket* or *small bracket*.
(iii) { } is known as *curly bracket, brace or middle bracket*.
(iv) [ ] is known as *rectangular bracket or big bracket*.

'VBODMAS' RULE

A given series of calculations or operations is done in a specific order as each letter of VBODMAS in order represent.
V → Vinculum or bar –
B → Brackets and order of operation of brackets is ( ), { }, [ ]
O → Of (Calculation is done the same as multiplication)
D → Division

GP_3386

M → Multiplication
A → Addition
S → Subtraction
So, first of all we solve vinculum then the inner most brackets moving outwards. Then we perform 'of' which means multiplication, then division, addition and subtraction.

- Addition and subtraction can be done together or separately as required.
- Between any two brackets if there is not any sign of addition, subtraction and division it means we have to do multiplication

$(20 \div 5)(7 + 3 \times 2) + 8 = 4(7 + 6) + 8$
$= 4 \times 13 + 8 = 52 + 8 = 60$

Illustration – 1

Find the value of

$$\left[5 - \left\{6 - (5 - \overline{4 - 3})\right\}\right] \text{ of } \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \div \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$$

(a) $\frac{9}{5}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{4}{7}$

Solution : (a)

$$= \left[5 - \left\{6 - (5 - \overline{4 - 3})\right\}\right] \text{ of } \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}} \div \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}}$$

$$= [5 - \{6 - (5 - 1)\}] \text{of } \frac{\frac{3}{2}}{\frac{1}{2}} \div \frac{\frac{5}{6}}{\frac{1}{6}}$$

$$= \{5 - (6 - 4)\} \text{ of } \left(\frac{3}{2} \times \frac{2}{1}\right) \div \left(\frac{5}{6} \times \frac{6}{1}\right)$$

$$= (5 - 2) \text{ of } 3 \div 5 = 3 \text{ of } 3 \div 5 = 3 \times \frac{3}{5} = \frac{9}{5}$$

Note that if there is a minus (–) sign before the bracket, then while removing bracket, sign of each term will be changed i.e. sign of positive become negative and negative become positive.

ALGEBRAIC IDENTITIES

Consider the equality $(x + 2)(x + 3) = x^2 + 5x + 6$
Let us evaluate both sides of this equality for some value of variable x say $x = 4$
LHS $= (x + 2)(x + 3) = (4 + 2)(4 + 3) = 6 \times 7 = 42$
RHS $= (4)^2 + 5 \times 4 + 6 = 16 + 20 + 6 = 42$
So for $x = 4$, LHS = RHS
Let us calculate LHS and RHS for $x = -3$
LHS $= (-3 + 2)(-3 + 3) = 0$
RHS $= (-3)^2 + 5(-3) + 6 = 9 - 15 + 6 = 0$
$\therefore$ For $x = -3$, LHS = RHS
If we take any value of variable x, we can find that LHS = RHS
Such an equality which is true for every value of the variable present in it is called an **identity**. Thus $(x + 2)(x + 3) = x^2 + 5x + 6$, is an identity.
Identities differ from equations in the following manners.
An **equation** is a statement of equality of two algebraic expression involving one or more variables and it is true for certain values of the variable.
For example: $4x + 3 = x - 3$ ... (1)
$\Rightarrow \quad 3x = -6 \Rightarrow x = -2$
Thus equality (1) is true only for $x = -2$, no other value of x satisfy equation (1).

Standard Identities

(i) $(a + b)^2 = a^2 + 2ab + b^2$
(ii) $(a - b)^2 = a^2 - 2ab + b^2$
(iii) $a^2 - b^2 = (a + b)(a - b)$
(iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
(v) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Some More Identities

We have dealt with identities involving squares. Now we will see how to handle identities involving cubes.

(i) $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

or $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

or $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iii) $a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$

(iv) $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$

(v) $a^3 + b^3 + c^3 - 3abc = (a+b+c)$

$(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

APPROXIMATION

To calculate the value of complex mathematical expression a lot of time is consumed in the examination hall. To save time, we use approximation method. In this method we calculate the nearest (round off) value of the given expression. Find value obtained by this method is not equal to the exact result, but it is very close to the exact result (either a little less or little more).

Basic Rules of Approximation

(i) Take the nearest round off value of the numbers given in the expression.
For example: 2899.27 can be approximated as 2900.

(ii) To multiply larger numbers, some numbers are increased to its nearest round off value and others are decreased to its nearest round off value accordingly, so that the calculation becomes easy.
For example: 898×463 is approximated as 900×460.

(iii) When we divide one number by other, then we can increase or decrease both the numbers accordingly.
For example: 541032 ÷ 20134 can be approximated as 540000 ÷ 20000

Illustration – 2

Find the approximate value of
$234 \div 17 + 15.3 \times 18 - 13 \times 3.7$
(a) 250 (b) 220 (c) 245 (d) 235

Solution : (a) $234 \div 17 + 15.3 \times 18 - 13 \times 3.7$
$= 225 \div 15 + 15 \times 18 - 13 \times 4$
$= 15 + 270 - 52 = 15 + 270 - 50 = 235.$

Illustration – 3

What approximate value should come in place of the question mark?
$11^3 + 0.8^3 + 12^3 + 1.1^3 + 1.2^3 = ?$
(a) 3063 (b) 3060 (c) 3066 (d) 3068

Solution : (a) $11^3 + 0.8^3 + 12^3 + 1.1^3 + 1.2^3$
$= 1331 + 1 + 1728 + 1.331 + 1.728$
$= 1332 + 1728 + 1 + 2 = 3063.$

Illustration – 4

What approximate value should come in place of the question mark (?) in the following question?

$$\frac{256}{\sqrt{17}} + \frac{190}{16} = ?$$

(a) 68 (b) 76 (c) 78 (d) $\frac{446}{16}$

Solution : (b) $\frac{256}{\sqrt{17}} + \frac{190}{16} = \frac{256}{4} + 12 = 64 + 12 = 76.$

SIMPLIFICATION

If different operations are done according to 'VBODMAS' rule and using algebraic identities to evaluate any given expression or express a complex expression into a simple expression is called **simplification.**

Illustration – 5

Simplify:
$287 \times 287 + 269 \times 269 - 2 \times 287 \times 269$
(a) 534 (b) 446 (c) 354 (d) 324

Solution : (d) Given Exp. $= a^2 + b^2 - 2ab$,
where $a = 287$ and $b = 269$
$= (a - b)^2 = (287 - 269)^2$
$= (18)^2 = 324.$

Illustration – 6

If $\sqrt{3} = 1.732$ and $\sqrt{2} = 1.414$, the value of $\frac{1}{\sqrt{3}+\sqrt{2}}$ is
(a) 0.064 (b) 0.308 (c) 0.318 (d) 2.146

Solution : (c)

$$\frac{1}{\sqrt{3}+\sqrt{2}} = \frac{1}{(\sqrt{3}+\sqrt{2})} \times \frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \left(\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)$$
$$= (\sqrt{3}-\sqrt{2}) = (1.732 - 1.414) = 0.318$$

Illustration – 7

Find the value of $\sqrt{0.01 + \sqrt{0.0064}}$
(a) 0.3 (b) 0.03
(c) $\sqrt{0.18}$ (d) None of these

Solution : (a) Given expression
$= \sqrt{0.01 + 0.08} = \sqrt{0.09} = 0.3$

Illustration – 8

The value of $\frac{\frac{1}{2} \div \frac{1}{2} \text{ of } \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} \text{ of } \frac{1}{2}}$ is
(a) 2/3 (b) 2 (c) 4/3 (d) 3

Solution : (a) $\frac{\frac{1}{2} \div \frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} = \frac{\frac{1}{2} \times 2 \times \frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

GP_3386

Illustration – 9

Simplify : $5\sqrt[3]{250}+7\sqrt[3]{16}-14\sqrt[3]{54}$

(a) $-2\sqrt[3]{2}$ **(b)** $-3\sqrt[3]{2}$ **(c)** $3\sqrt[3]{2}$ **(d)** $2\sqrt[3]{3}$

Solution : (b) $5\sqrt[3]{250}+7\sqrt[3]{16}-14\sqrt[3]{54}$

$= 5\sqrt[3]{125\times 2}+7\sqrt[3]{8\times 2}-14\sqrt[3]{27\times 2}$

$= 5\times 5\sqrt[3]{2}+7\times 2\sqrt[3]{2}-14\times 3\times\sqrt[3]{2}$

$= (25+14-42)\sqrt[3]{2} = -3\sqrt[3]{2}$

Illustration – 10

The value of $4-[6-\{6-(5-\overline{4-3})\}]$ **is**

(a) 4 **(b)** 1 **(c)** 0 **(d)** 5

Solution : (c)

$4-[6-\{6-(5-\overline{4-3})\}] = 4-[6-\{6-(5-1)\}]$

$= 4-[6-\{6-4\}]$

$= 4-[6-2] = 4-4 = 0$

Illustration – 11

Simplify : $12\frac{1}{2}-\left[8\frac{1}{2}+\{6-(7-\overline{4-2})\}\right]$

(a) 3 **(b)** 4 **(c)** 6 **(d)** 8

Solution : (a) $12\frac{1}{2}-\left[8\frac{1}{2}+\{6-(7-\overline{4-2})\}\right]$

$= 12\frac{1}{2}-\left[8\frac{1}{2}+\{6-(7-2)\}\right]$

$= 12\frac{1}{2}-\left[8\frac{1}{2}+1\right] = 12\frac{1}{2}-9\frac{1}{2} = 3.$

Illustration – 12

$\frac{1}{4}+\left[\frac{1}{4}-\left\{\frac{1}{4}+\left(\frac{1}{4}-\overline{\frac{1}{4}-\frac{1}{4}}\right)\right\}\right] = ?$

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$ **(c)** 1 **(d)** 0

Solution : (d) $\frac{1}{4}+\left[\frac{1}{4}-\left\{\frac{1}{4}+\left(\frac{1}{4}-\overline{\frac{1}{4}-\frac{1}{4}}\right)\right\}\right]$

$= \frac{1}{4}+\left[\frac{1}{4}-\left\{\frac{1}{4}+\left(\frac{1}{4}-0\right)\right\}\right] = \frac{1}{4}+\left[\frac{1}{4}-\left\{\frac{1}{4}+\frac{1}{4}\right\}\right]$

$= \frac{1}{4}+\left[\frac{1}{4}-\frac{2}{4}\right] = \frac{1}{4}+\left[-\frac{1}{4}\right] = \frac{1}{4}-\frac{1}{4} = 0$

SHORTCUTS

Shortcut Approach - 1

Application of the formula :

$(a+b)^2 = a^2+2ab+b^2$

(or)

$(a+b)^2 = a^2+b^2+2ab$

Illustration – 13

Find the value of

$0.63\times 0.63+2\times 0.63\times 0.37+0.37\times 0.37$

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

Shortcut Solution : (a)

Let $a = 0.63$ and $b = 0.37$

Then $0.63\times 0.63+2\times 0.63\times 0.37+0.37\times 0.37$

$= a\times a+2\times a\times b+b\times b = a^2+2ab+b^2$

$= (a+b)^2 = (0.63+0.37)^2 = (1)^2 = 1$

Shortcut Approach - 1

Application of the fomrula :

$(a-b)^2 = a^2-2ab+b^2$

(or)

$(a-b)^2 = a^2+b^2-2ab$

Illustration – 14

$\frac{2.31\times 2.31+1.06\times 1.06-2\times 2.31\times 1.06}{0.25} = ?$

(a) 5.50 **(b)** 6.25 **(c)** 3.15 **(d)** 4.60

Shortcut Solution : (b)

Let $a = 2.31$ and $b = 1.06$

Then $\frac{2.31\times 2.31+1.06\times 1.06-2\times 2.31\times 1.06}{0.25}$

$= \frac{a\times a+b\times b-2\times a\times b}{0.25}$

$= \frac{a^2+b^2-2ab}{0.25} = \frac{(a-b)^2}{0.25}$

$= \frac{(2.31-1.06)^2}{0.25} = \frac{1.25\times 1.25}{0.25} = 6.25$

Shortcut Approach - 1

Application of the formula :

$a^2-b^2 = (a+b)(a-b)$

Illustration – 15

$$\frac{\frac{3}{8}\times\frac{3}{8}-\frac{1}{8}\times\frac{1}{8}}{\frac{3}{8}+\frac{1}{8}}=?$$

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{1}{4}$

Shortcut Solution : (d)

Let $a=\frac{3}{8}$ and $b=\frac{1}{8}$

Then $$\frac{\frac{3}{8}\times\frac{3}{8}-\frac{1}{8}\times\frac{1}{8}}{\frac{3}{8}+\frac{1}{8}}=\frac{a\times a-b\times b}{a+b}=\frac{a^2-b^2}{a+b}$$

$$=\frac{(a+b)(a-b)}{a+b}=a-b=\frac{3}{8}-\frac{1}{8}=\frac{2}{8}=\frac{1}{4}.$$

Shortcut Approach - 1

Application of the formula :
$(a+b)^2+(a-b)^2=2(a^2+b^2)$

Illustration – 16

$$\frac{(73+51)^2+(22)^2}{(73)^2+(51)^2}=?$$

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

Shortcut Solution : (b)

$$\frac{(73+51)^2+(22)^2}{(73)^2+(51)^2}=\frac{(73+51)^2+(73-51)^2}{(73)^2+(51)^2}$$

Let $a=73$ and $b=51$

$\therefore$ Given expression $=\frac{(a+b)^2+(a-b)^2}{a^2+b^2}$

We know that $(a+b)^2+(a-b)^2=2(a^2+b^2)$

$\Rightarrow$ $\frac{(a+b)^2+(a-b)^2}{a^2+b^2}=2$ $\therefore$ Given expression $=2$.

Shortcut Approach - 1

Application of the formula:
$(a+b)^2-(a-b)^2=4ab$

Illustration – 17

Evaluate : $(100+87)^2-(100-87)^2$.

(a) 34800 (b) 12500 (c) 10000 (d) 27200

Shortcut Solution : (a)

Let $100=a$ and $87=b$.

$\therefore$ $(100+87)^2-(100-87)^2=(a+b)^2-(a-b)^2$
$=4ab=4\times100\times87=34800.$

Shortcut Approach - 1

Application of the formula :
$(a+b)^3=a^3+3a^2b+3ab^2+b^3$
$(a+b)^3=a^3+b^3+3ab\,(a+b)$

Illustration – 18

Evaluate : $2.37\times2.37\times2.37+3\times2.37\times2.37\times1.63+3\times2.37\times1.63\times1.63+1.63\times1.63\times1.63$.

(a) 16 (b) 32 (c) 48 (d) 64

Shortcut Solution : (d)

Let $a=2.37$ and $b=1.63$.

$2.37\times2.37\times2.37+3\times2.37\times2.37\times1.63+3\times2.37\times1.63\times1.63+1.63\times1.63\times1.63$

$=a\times a\times a+3\times a\times a\times b+3\times a\times b\times b+b\times b\times b$

$a^3+3a^2b+3ab^2+b^3=(a+b)^3=(2.37+1.63)^3=(4)^3=64.$

Shortcut Approach - 1

Application of the formula :
$(a-b)^3=a^3-3a^2b+3ab^2-b^3$
or
$(a-b)^3=a^3-b^3-3ab\,(a-b)$

Illustration – 19

Simplify :

$$\frac{8.76\times8.76\times8.76-(2.26)^3-3\times8.76\times2.26(8.76-2.26)}{(8.76)^2-2\times8.76\times2.26+2.26\times2.26}$$

(a) 1.2 (b) 6.5 (c) 3.4 (d) 7.7

Shortcut Solution : (b)

Let $a=8.76$ and $b=2.26$

$$\frac{8.76\times8.76\times8.76-(2.26)^3-3\times8.76\times2.26(8.76-2.26)}{(8.76)^2-2\times8.76\times2.26+2.26\times2.26}$$

The given expression

$$=\frac{a\times a\times a-b^3-3\times a\times b(a-b)}{a^2-2\times a\times b+b\times b}=\frac{(a-b)^3}{(a-b)^2}$$

$=a-b=8.76-2.26=6.5.$

Shortcut Approach - 1

Application of the formula :
$a^3+b^3=(a+b)\,(a^2-ab+b^2)$

GP_3386

Illustration – 20

Evaluate : $\dfrac{6.13\times6.13\times6.13+2.12\times2.12\times2.12}{6.13\times6.13-6.13\times2.12+2.12\times2.12}$

(a) 8 (b) 8.25 (c) 8.5 (d) 8.75

Shortcut Solution : (b)

Let $a = 6.13$ and $b = 2.12$

$$\frac{6.13\times6.13\times6.13+2.12\times2.12\times2.12}{6.13\times6.13-6.13\times2.12+2.12\times2.12}$$

The given expression $= \dfrac{a^3+b^3}{a^2-ab+b^2}$

We know that $a^3+b^3=(a+b)(a^2-ab+b^2)$

$$\Rightarrow \frac{a^3+b^3}{a^2-ab+b^2}=a+b=6.13+2.12=8.25.$$

Shortcut Approach - 1

Application of the formula :
$a^3-b^3=(a-b)(a^2+ab+b^2)$

Illustration – 21

$$\frac{(6.7)^3-0.064}{6.7\times6.7+6.7\times0.4+0.16}=?$$

(a) 3.3 (b) 4.3 (c) 5.3 (d) 6.3

Shortcut Solution : (d)

Let $a = 6.7$ and $b = 0.4$. Then

$$\frac{(6.7)^3-0.064}{6.7\times6.7+6.7\times0.4+0.16}=\frac{(6.7)^3-(0.4)^3}{(6.7)^2+6.7\times0.4+(0.4)^2}$$

$$=\frac{a^3-b^3}{a^2+a\times b+b^2}$$

We know that $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\Rightarrow \frac{a^3-b^3}{a^2+ab+b^2}=a-b=6.7-0.4=6.3.$$

Shortcut Approach - 10

Application of the formula :
$a^3+b^3+c^3=3abc$, **if** $a+b+c=0$

Illustration – 22

$(12)^3-25\times25\times25+13\times13\times13=?$

(a) 12100 (b) 24200 (c) 26500 (d) – 11700

Shortcut Solution : (d)

$(12)^3-25\times25\times25+13\times13\times13$

$=(12)^3-(25)^3+(13)^3=(12)^3+(-25)^3+(13)^3$

Now, $12+(-25)+13=12-25+13=0$

$\therefore$ Given expression $=3\times12\times(-25)\times13=-11700$

Shortcut Approach - 11

Application of the formula:

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

or $a^3+b^3+c^3-3abc$

$$=\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

Illustration – 23

Find the value of :

$$\frac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$$

(a) 10 (b) 5 (c) 20 (d) 15

Shortcut Solution : (a)

$$\because \quad \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}=(a+b+c)$$

$\therefore \quad a+b+c=1.5+4.7+3.8=10$

Shortcut Approach - 11

Application of the formula:

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Illustration – 24

If $x+y+z=12$ and $x^2+y^2+z^2=62$, then find the value of $xy+yz+zx$.

(a) 36 (b) 41 (c) 74 (d) 93

Shortcut Solution : (b)

$x+y+z=12$

$(x+y+z)^2=(12)^2$

$\Rightarrow x^2+y^2+z^2+2xy+2yz+2zx=144$

$\Rightarrow xy+yz+zx=41$

Shortcut Approach - 11

To simplify a continued Fraction :
You can easily understand this through the following example,

$$5+\cfrac{1}{6+\cfrac{1}{6+\cfrac{1}{10}}}$$

To simplify a continued fraction, begin at the bottom and work upwards as follow :

$$5+\cfrac{1}{6+\cfrac{1}{6+\cfrac{1}{10}}}$$

First take up the lowest complex fraction $\cfrac{1}{6+\cfrac{1}{10}}$

Multiply the numerator and denominator by 10 and thus we get $\frac{10}{61}$.

Next multiply the numerator and denominator of the fraction $\cfrac{1}{6+\cfrac{10}{61}}$ **by 61 and thus we get** $\frac{61}{376}$.

Hence, the given fraction $= 5+\frac{61}{376}$ **or** $5\frac{61}{376}$

Illustration – 25

Simplify : $\cfrac{1}{2+\cfrac{1}{3+\cfrac{3}{4+\cfrac{1}{3}}}}$

(a) $\frac{16}{43}$ **(b)** $\frac{71}{94}$ **(c)** $\frac{48}{109}$ **(d)** $\frac{55}{93}$

Shortcut Solution : (c)

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{3}{4+\cfrac{1}{3}}}}=\cfrac{1}{2+\cfrac{1}{3+\cfrac{9}{13}}}=\cfrac{1}{2+\cfrac{13}{48}}=\frac{48}{109}$$

Shortcut Approach - 10

To find the value of $\sqrt{x+\sqrt{x+\sqrt{x+.....}}}$ **, find the factors of x, such that the difference between the factors is 1, then the larger factor will be the result.**

Illustration – 26

Find the value of $\sqrt{30+\sqrt{30+\sqrt{30+....}}}$

(a) 2 **(b)** 3 **(c)** 5 **(d)** 6

Shortcut Solution : (d)

The factors of 30 with difference 1 are 5 and 6. Here 6 is the larger factor.

Hence $\sqrt{30+\sqrt{30+\sqrt{30+....}}}=6$.

Shortcut Approach - 10

To find the value of $\sqrt{x-\sqrt{x-\sqrt{x-.....}}}$ **find the factors of x, such that the difference between the factors is 1, then the smaller factor will be the result.**

Illustration – 27

Find the value of $\sqrt{56-\sqrt{56-\sqrt{56-.....}}}$

(a) 7 **(b)** 6 **(c)** 5 **(d)** 4

Shortcut Solution : (a)

The factors of 56 with difference 1 are 7 and 8. Here 7 is the smaller factor.

Hence $\sqrt{56-\sqrt{56-\sqrt{56-.....}}}=7$.

Shortcut Approach - 11

$$\sqrt{x\sqrt{x\sqrt{x}.....\ n \text{ times}}}=(x)^{\frac{2^n-1}{2^n}}$$

Illustration – 28

Find the value of $\sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}$

(a) $(3)^{\frac{1}{16}}$ **(b)** 3^2 **(c)** 3 **(d)** $(3)^{\frac{15}{16}}$

Shortcut Solution : (d)

Here $x=3$ and $n=4$

$$\therefore \sqrt{3\sqrt{3\sqrt{3\sqrt{3}}}}=(3)^{\frac{2^4-1}{2^4}}=(3)^{\frac{15}{16}}$$

Shortcut Approach - 10

$$\sqrt{x\sqrt{x\sqrt{x\sqrt{x}.....\infty}}}=x$$

Illustration – 29

Find the value of $\sqrt{8\sqrt{8\sqrt{8\sqrt{8}.....\infty}}}$

(a) 5 **(b)** 6 **(c)** 7 **(d)** 8

Shortcut Solution : (d)

$$\sqrt{8\sqrt{8\sqrt{8\sqrt{8}.....\infty}}}=8$$

LEVEL - 1

1. Solve: $\frac{(835+378)^2+(835-378)^2}{835\times835+378\times378}$

[SSC CGL 2009]

(a) 1 (b) 2 (c) 3 (d) 4

2. Solve: $\frac{(999+588)^2-(999-588)^2}{999\times588}$

[SSC CGL Tier II 2013]

(a) 4 (b) 3 (c) 2 (d) 1

3. The value of $\frac{(0.96)^3-(0.1)^3}{(0.96)^2+(0.096)+(0.01)}$ is

[SSC CGL 2011]

(a) 0.86 (b) 1.06 (c) 0.95 (d) 0.97

4. Find the sum $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$

(a) $2008-\frac{1}{2008}$ (b) $2007-\frac{1}{2008}$

(c) $2008-\frac{1}{2007}$ (d) $2007-\frac{1}{2007}$

5. $\frac{(m-n)^3+(n-r)^3+(r-m)^3}{6(m-n)(n-r)(r-m)}=?$

[CDS 2013]

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{6}$

6. If x = 0.5, then which of the following has the smallest value?

(a) $2^{\frac{1}{x}}$ (b) $\frac{1}{x}$ (c) $\frac{1}{x^2}$ (d) 2^x

7. If m # n = m + n + mn, then 2 # 3 =

(a) 0 (b) 6 (c) 11 (d) 20

8. Given m # n = m + n + mn. If for any m, there is a number q such that m # q = m, then q =

(a) 0 (b) 1

(c) –1 (d) doesn't exist.

9. If m < n, then

(a) m . n < n . m

(b) m . m > n . n

(c) m . n . n < n . m . m

(d) m . m . m < n . n . n

10. $\sqrt{110.25}\times\sqrt{0.01}\div\sqrt{0.0025}-\sqrt{420.25}$ equal to

(a) 0.75 (b) 0.50 (c) 0.64 (d) 0.73

11. The value of the expression 7777 + 7777 × 7777 × (5/77) × (11/35) is:

(a) 1234321 (b) 12344321

(c) 77777 (d) None of these

12. 16 × 16 + 128 + 4 × 4 = ? **[Bank clerk 2014]**

(a) 20 (b) 100 (c) 400 (d) 500

13. If $x*y = x^2 + y^2 - xy$, then value of 9*11 is

(a) 93 (b) 103 (c) 60.5 (d) 121

14. If * means adding six times of second number into first number, then find the value of (1*2)*3.

(a) 121 (b) 31 (c) 93 (d) 91

15. If $a * b * c = \sqrt{\frac{(a+2)(b+3)}{(c+1)}}$, then the value of (6 * 15 * 3) is

(a) 6 (b) 3

(c) 4 (d) Can't be determined

16. The value of

$$\left[35.7-\left(3+\frac{1}{3+\frac{1}{3}}\right)-\left(2+\frac{1}{2+\frac{1}{2}}\right)\right]$$ is

(a) 30 (b) 34.8 (c) 36.6 (d) 41.4

17. If $(X+(1/X))=4$, then the value of X^4+1/X^4 is

(a) 124 (b) 64

(c) 194 (d) Can't be determined

18. Find the value of

$$\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{100}\right).$$

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{50}$ (d) $\frac{2}{5}$

19. Evaluate $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$

(a) 2 (b) 3 (c) 4 (d) 5

20. Find the value of

$$\frac{0.5\times0.5+0.1\times0.1-0.2\times0.5}{0.4}$$ **[CDS 2013]**

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.6

21. $2.002+7.9\{(2.8-6.3(3.6-1.5)+15.6)\}=?$

(a) 2.002 (b) 4.2845

(c) 40.843 (d) 42.845

22. $9-1\frac{2}{9}$ of $3\frac{3}{11}\div5\frac{1}{7}$ of $\frac{7}{9}=?$

(a) $\frac{5}{4}$ (b) 8 (c) $8\frac{32}{81}$ (d) 9

23. Find the value of * in the following.

$$1\frac{2}{3}\div\frac{2}{7}\times\frac{*}{7}=1\frac{1}{4}\times\frac{2}{3}\div\frac{1}{6}$$

(a) 0.006 (b) $\frac{1}{6}$ (c) 0.6 (d) 6

24. When $\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$ is divided by $\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)$, the result is :

(a) $2\frac{1}{18}$ (b) $3\frac{1}{6}$ (c) $3\frac{3}{10}$ (d) $5\frac{1}{10}$

25. The value of

$$3\div\left[(8-5)\div\left\{(4-2)+\left(2+\frac{8}{13}\right)\right\}\right]$$ is

(a) $\frac{15}{17}$ (b) $\frac{13}{17}$ (c) $\frac{15}{19}$ (d) $\frac{13}{19}$

26. Arranging the following in ascending order $2^{10000}, 10^{3000}, 3^{6000}, 7^{4000}$ we get

(a) $3^{6000}<10^{3000}<2^{10000}<7^{4000}$

(b) $2^{10000}<7^{4000}<10^{3000}<3^{6000}$

(c) $10^{3000}<3^{6000}<7^{4000}<2^{10000}$

(d) $7^{4000}<3^{6000}<2^{10000}<10^{3000}$

27. $\frac{998^2-997^2-45}{98^2-97^2}=?$ **[SSC CGL Tier I 2017]**

(a) 1995 (b) 195 (c) 95 (d) 10

28. $\left(\frac{4}{3}+\frac{3}{4}\right)^2-\left(\frac{4}{3}-\frac{3}{4}\right)^2=?$ **[SSC CGL 2010]**

(a) 1 (b) 2 (c) 3 (d) 4

29. If a = 331, b = 336 and c = – 667, then the value of $a^3+b^3+c^3-3abc$ is

(a) 1 (b) 6 (c) 3 (d) 0

30. If $a+b+c=6$ and $a^2+b^2+c^2=20$, then the value of $a^3+b^3+c^3-3abc$ is **[SSC CGL Tier II 2015]**

(a) 64 (b) 70 (c) 72 (d) 76

31. If $x^4+\frac{1}{x^4}=119$ and $x>1$, then the value of $x^3-\frac{1}{x^3}$ is

(a) 54 (b) 18 (c) 72 (d) 36

32. If 'a' be a positive number, then the least value of $a+\frac{1}{a}$ is

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

33. If $x=\sqrt{a}+\frac{1}{\sqrt{a}}$, $y=\sqrt{a}-\frac{1}{\sqrt{a}}$, then the value of $x^4+y^4-2x^2y^2$ is

(a) 16 (b) 20 (c) 10 (d) 5

34. $(y-z)^3+(z-x)^3+(x-y)^3$ is equal to

(a) $3(y-z)(z+x)(y-x)$

(b) $(x-y)(y+z)(x-z)$

(c) $3(y-z)(z-x)(x-y)$

(d) $(y-z)(z-x)(x-y)$

35. If $p=\frac{5}{18}$, then $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ is equal to

(a) $\frac{4}{27}$ (b) $\frac{5}{27}$ (c) $\frac{8}{27}$ (d) $\frac{10}{27}$

36. If $\frac{4}{9}$ of $\frac{3}{10}$ of $\frac{5}{8}$ of a number is 45, what is the number?

(a) 450 (b) 550

(c) 560 (d) None of these

37. $69012-20167+(51246\div6)=?$

(a) 57385 (b) 57286

(c) 57476 (d) None of these

GP_3386

38. $6\frac{5}{6}\times5\frac{1}{3}+17\frac{2}{3}\times4\frac{1}{2}=?$

(a) $112\frac{1}{3}$ (b) 663

(c) 240 (d) None of these

39. $\frac{\sqrt{29241}}{\sqrt{361}}\times5\frac{2}{9}=?$

(a) 47 (b) 49 (c) 46 (d) 45

40. 45% of 1500 + 35% of 1700 = ?% of 3175.

(a) 50 (b) 45

(c) 30 (d) None of these

41. $17\frac{2}{3}$ of $180+\frac{1}{4}$ of $480=?$

(a) 3180 (b) 3420 (c) 3200 (d) 3300

42. $\frac{5}{7}$ of $1596+3015=?-2150$

(a) 7200 (b) 48000 (c) 5300 (d) 6300

43. $6.595\times1084+2568.34-1708.34=?$

(a) 6,000 (b) 12,000 (c) 10,000 (d) 8,000

44. 35% of 1478 + 29% of 3214 = ?

(a) 1600 (b) 1250 (c) 1300 (d) 1450

45. Which of the following has fractions in ascending order?

(a) $\frac{2}{5},\frac{3}{5},\frac{1}{3},\frac{4}{7},\frac{5}{6},\frac{6}{7}$ (b) $\frac{1}{3},\frac{2}{5},\frac{3}{5},\frac{5}{6},\frac{4}{7},\frac{6}{7}$

(c) $\frac{1}{3},\frac{2}{5},\frac{3}{5},\frac{4}{7},\frac{5}{6},\frac{6}{7}$ (d) $\frac{1}{3},\frac{2}{5},\frac{4}{7},\frac{3}{5},\frac{5}{6},\frac{6}{7}$

46. $27\frac{3}{11}+118\frac{2}{5}-32\frac{5}{22}=11\frac{6}{11}+?$

(a) $113\frac{9}{10}$ (b) $111\frac{9}{11}$

(c) $90\frac{9}{10}$ (d) None of these

47. ?% of 360 = 129.6

(a) 277 (b) 36 (c) 64 (d) 72

48. The value of $3\frac{1}{3}+33\frac{1}{3}+.....+333333\frac{1}{3}$ is :

(a) 370368 (b) 370370

(c) 370369 (d) 370371

49. The value of

$$\left(2-\frac{1}{3}\right)\times\left(2-\frac{3}{5}\right)\times\left(2-\frac{5}{7}\right)\times...\times\left(2-\frac{999}{1001}\right)$$

(a) $\frac{1001}{5}$ (b) 999 (c) 1003 (d) $\frac{1003}{3}$

50. Calculate the number of digits in the product of $4^{11111}\times5^{22222}$

(a) 22222 (b) 22220

(c) 22221 (d) 22223

51. Find the value of $\sqrt[3]{0.\overline{037}}$

(a) $0.\overline{3}$ (b) $0.\overline{6}$ (c) $0.\overline{37}$ (d) $0.\overline{7}$

52. If the number p is 5 more than q and the sum of the squares of p and q is 55, then the product of p and q is

(a) 10 (b) –10 (c) 15 (d) – 15

53. $1.\overline{27}$ in the form $\frac{p}{q}$ is equal to

(a) $\frac{127}{100}$ (b) $\frac{73}{100}$ (c) $\frac{14}{11}$ (d) $\frac{11}{14}$

54. If x = 101, find the value of $x(x^2-3x+3)$

(a) 1000001 (b) 100001

(c) 100000 (d) 10001

55. If $a+b+c=3$ and $a^2+b^2+c^2=6$, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$, find abc

(a) $\frac{3}{2}$ (b) 2 (c) $\frac{5}{2}$ (d) 3

56. The value of $\frac{(0.32)^3+(0.45)^3-(0.77)^3}{81(0.32)(0.45)(0.77)}$ will be

(a) 1 (b) 0 (c) $-\frac{1}{27}$ (d) $\frac{1}{27}$

57. After solving $\sqrt{\left(6-4\sqrt{3}+\sqrt{16-8\sqrt{3}}\right)}$, we will get

(a) $\sqrt{3}+1$ (b) $\sqrt{3}-1$

(c) 2 (d) $4-2\sqrt{3}$

58. After simplifying

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right).....\left(1-\frac{1}{n}\right)$$, we get

(a) $\frac{1}{n}$ (b) $\frac{2}{n}$

(c) $\frac{2(n-1)}{n}$ (d) $\frac{2}{n(n+1)}$

59. If $\frac{a}{b}+\frac{b}{a}=1$, then the value of a^3+b^3 is

[SSC CGL 2011, SSC FCI 2012]

(a) 0 (b) 1 (c) 2 (d) 3

60. The value of

$$\frac{0.051\times0.051\times0.051+0.041\times0.041\times0.041}{0.051\times0.051-0.051\times0.041+0.041\times0.041}$$

is

(a) 0.092 (b) 1.002 (c) 0.0092 (d) 1.0002

61. If $(x+y)=8$ and $xy=15$, then the value of (x^3-y^3) is :

(a) 120 (b) 60 (c) 98 (d) 82

62. If $(x+y+z)=6$ and $(xy+yz+zx)=11$, then the value of $(x^3+y^3+z^3-3xyz)$ is :

(a) 81 (b) 54

(c) 18 (d) none of these

63. If $(x+y+z)=0$, then the value of $(x^3+y^3+z^3)$ is :

(a) $3x^3y^3z^3$ (b) $3xyz$

(c) xyz (d) $3(x+y+z)$

64. If $a^{1/3}+b^{1/3}+c^{1/3}=0$, then :

(a) $a+b+c=0$ (b) $a+b+c=3abc$

(c) $a^3+b^3+c^3=0$ (d) $(a+b+c)^3=27abc$

65. If $(x+y)=13$ and $xy=36$, then the value of (x^3+y^3) is :

(a) 369 (b) 936

(c) 793 (d) None of these

66. If $x+y=13$ and $xy=40$, then the value of (x^2+y^2) is :

(a) 69 (b) 80

(c) 89 (d) None of these

67. If $\left(x+\frac{1}{x}\right)=2$, then $\left(x^2+\frac{1}{x^2}\right)$ is equal to :

(a) 2 (b) 4 (c) 5 (d) 6

68. If $\left(x+\frac{1}{x}\right)=2\sqrt{3}$, then the value of $\left(x^3+\frac{1}{x^3}\right)$ is :

(a) $12\sqrt{3}$ (b) 18

(c) $18\sqrt{3}$ (d) None of these

69. $\sqrt{6+\sqrt{6+\sqrt{6+...}}}$ is equal to

[SSC CGL 2011]

(a) 3 (b) 4 (c) 5 (d) 6

70. $\sqrt{3\sqrt{3\sqrt{3...}}}$ is equal to **[SSC LDC 2010]**

(a) $\sqrt{3}$ (b) 3 (c) $2\sqrt{3}$ (d) $3\sqrt{3}$

71. If $\sqrt{7\sqrt{7\sqrt{7\sqrt{7...}}}}=(343)^{y-1}$, then y will be equal to **[SSC CGL 2010]**

(a) 2/3 (b) 1 (c) 4/3 (d) 3/4

72. $\left[\frac{(0.1)^2-(0.01)^2}{0.0001}\right]+1$ is equal to

[SSC CGL 2010]

(a) 1010 (b) 110 (c) 101 (d) 100

73. The value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ will be

[SSC CGL Tier I 2017]

(a) 2 (b) $2^{15/32}$ (c) $2^{31/32}$ (d) 4

74. $\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12.....}}}}=?$

[SSC CHSL 2016]

(a) 3 (b) 4 (c) 6 (d) 12

75. The value of $\sqrt{20+\sqrt{20+\sqrt{20+.....}}}$ is

[CDS 2012]

(a) 4 (b) 5

(c) 6 (d) greater than 6

76. $\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2.....}}}}=?$ **[SSC CHSL 2016]**

(a) 0 (b) 1 (c) 2 (d) 3

77. $\left(\sqrt{56+\sqrt{56+\sqrt{56+.....}}}\right)\div 2^2=?$

[SSC CGL Tier I 2017]

(a) 0 (b) 1 (c) 2 (d) 8

GP_3386

LEVEL - 2

1. If $R=\dfrac{30^{65}-29^{65}}{30^{64}+29^{64}}$, then

(a) $0<R\leq 0.1$ (b) $0.1<R\leq 0.5$

(c) $0.5<R\leq 1.0$ (d) $R>1.0$

2. Find the value of

$$\frac{(a-b)^2}{(b-c)(c-a)}+\frac{(b-c)^2}{(a-b)(c-a)}+\frac{(a-c)^2}{(a-b)(b-c)}.$$

[CDS 2015]

(a) 0 (b) 1 (c) 2 (d) 3

3. If $\dfrac{x^2+y^2+z^2-64}{xy-yz-zx}=-2$ and $x+y=3z$, then the value of z is

(a) 2 (b) 3

(c) 4 (d) None of these

4. Simplify : $\dfrac{1}{1+\dfrac{\frac{2}{3}}{1+\frac{2}{3}+\frac{\frac{8}{9}}{1-\frac{2}{3}}}}$

(a) $\dfrac{11}{13}$ (b) $\dfrac{13}{15}$ (c) $\dfrac{13}{11}$ (d) $\dfrac{15}{13}$

5. Find the value of $1+\dfrac{2}{3+\dfrac{4}{5+\frac{6}{7}}}$

[SSC CGL Tier – I 2017]

(a) $1\dfrac{82}{151}$ (b) $2\dfrac{31}{151}$ (c) $2\dfrac{49}{159}$ (d) $1\dfrac{49}{150}$

6. If $\dfrac{97}{19}=a+\dfrac{1}{b+\frac{1}{c}}$ where a, b and c are positive integers, then what is the sum of a, b and c?

(a) 16 (b) 20

(c) 9 (d) Cannot be determined

7. If $a+b+c=0$, the value of

$\left(\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}\right)$ is

(a) 2 (b) 3 (c) 4 (d) 5

8. If $\dfrac{x^2+y^2+z^2-64}{xy-yz-zx}=-2$ and $x+y=3z$, then the value of z is

[SSC CGL Tier I 2017]

(a) 2 (b) 3

(c) 4 (d) None of these

9. If $a+b+c=3, a^2+b^2+c^2=6$ and $\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=1$, where a, b, c are all non-zero, then 'abc' is equal to **[SSC CGL Tier I 2014]**

(a) $\dfrac{2}{3}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$

10. If $x+\dfrac{1}{x}=2$, then $x^{2013}+\dfrac{1}{x^{2014}}=?$

(a) 0 (b) 1 (c) –1 (d) 2

11. If $5a+\dfrac{1}{3a}=5$, then the value of $9a^2+\dfrac{1}{25a^2}$ is

(a) $\dfrac{51}{5}$ (b) $\dfrac{29}{5}$ (c) $\dfrac{52}{5}$ (d) $\dfrac{39}{5}$

12. If $x+\dfrac{1}{x}=\sqrt{3}$, then the value of $x^{18}+x^{12}+x^6+1$ is

(a) 0 (b) 1 (c) 2 (d) 3

13. If $x=997$, $y=998$, $z=999$, then the value of $x^2+y^2+z^2-xy-yz-zx$ will be

(a) 3 (b) 9 (c) 16 (d) 4

14. If $x^2+y^2+2x+1=0$, then the value of $x^{31}+y^{35}$ is

(a) –1 (b) 0 (c) 1 (d) 2

15. If $a^x=(x+y+z)^y$, $a^y=(x+y+z)^z$ and $a^z=(x+y+z)^x$, then the value of $x+y+z$ (given $a \neq 0$) is
(a) 0 (b) a^3 (c) 1 (d) a

16. The value of the expression $x^4-17x^3+17x^2-17x+17$ at $x=16$ is
(a) 0 (b) 1 (c) 2 (d) 3

17. If $m+\frac{1}{m-2}=4$, then what is value of
$$(m-2)^2+\frac{1}{(m-2)^2}=?$$
(a) -2 (b) 0 (c) 2 (d) 4

18. If $a^2=b+c, b^2=c+a, c^2=a+b$, then the value of $\frac{1}{1+a}+\frac{1}{b+1}+\frac{1}{1+c}$
(a) abc (b) $a^2b^2c^2$
(c) 1 (d) 0

19. If $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}=a\sqrt[3]{4}+b\sqrt[3]{2}+c$ and a, b, c are rational numbers, then $a+b+c$ is equal to
(a) 0 (b) 1 (c) 2 (d) 3

20. The value of
$$\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)\left\{\left(\sqrt[3]{3.5}\right)^2-\sqrt[3]{8.75}+\left(\sqrt[3]{2.5}\right)^2\right\} \text{ is:}$$
[SSC CGL 2009]
(a) 5.375 (b) 1
(c) 6 (d) 5

21. If $x-y=1$, then $x^3-y^3-3(xy-2)$ is equal to
[SSC CHSL 2013]
(a) 7 (b) 6 (c) 10 (d) 4

22. $3\frac{3}{5}$ of $157.85+39\%$ of $1847=?-447.30$
(a) 1200 (b) 1500 (c) 1600 (d) 1800

23. Find the value of $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+.....\frac{1}{156}$
(a) $\frac{1}{39}$ (b) $\frac{10}{39}$ (c) $\frac{10}{13}$ (d) $\frac{1}{3}$

24. Find the value of
$$\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+...+\frac{1}{\sqrt{8}+\sqrt{9}}$$
(a) $3-\sqrt{3}$ (b) $3+\sqrt{3}$
(c) $\sqrt{3}-3$ (d) $2+\sqrt{3}$

25. Find the value of $\frac{1}{10}+\frac{1}{40}+\frac{1}{88}+.....+\frac{1}{598}$
(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{5}{13}$

26. The value of $9+99+999+.....+20^{th}$ term is :
(a) $\frac{10\left(10^{20}-1\right)}{9}$ (b) 209
(c) $\frac{-190}{9}$ (d) $\frac{10\left(10^{20}-1\right)}{9}-20$

27. $\frac{a+b}{a-b}=\frac{4}{1}$, find the value of $\frac{a}{b}$
(a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{5}{3}$ (d) $\frac{5}{4}$

28. $\frac{(100-1)(100-2)(100-3)....(100-200)}{100\times99\times98\times.....\times3\times2\times1}$ is equal to
(a) $\frac{100}{99\times98\times97\times.....\times3\times2\times1}$
(b) $-\frac{1}{99\times98\times97\times.....\times3\times2\times1}$
(c) 0
(d) $-\frac{2}{99\times98\times97\times.....\times3\times2\times1}$

29. If $x=1+\sqrt{2}+\sqrt{3}$, then the value of $\left(x+\frac{1}{x-1}\right)$ is
(a) $1+2\sqrt{3}$ (b) $2+\sqrt{3}$
(c) $3+\sqrt{2}$ (d) $2\sqrt{3}-1$

30. If $p-2q=4$, then the value of $p^3-8q^3-24pq-64$ is **[SSC CGL Tier I 2013]**
(a) 0 (b) 3 (c) -1 (d) 2

31. If $xy(x+y)=1$, then the value of $\frac{1}{x^3y^3}-x^3-y^3$.
[SSC CGL Tier I 2015]
(a) 3 (b) -3 (c) 1 (d) -1

32. If $(a-b)=3$, $(b-c)=5$ and $(c-a)=1$, then the value of $\frac{a^3+b^3+c^3-3abc}{a+b+c}$ is
[SSC CGL Tier II 2012]
(a) 17.5 (b) 20.5 (c) 10.5 (d) 35

GP_3386

33. If $4x+\frac{1}{3x}=5$, find the value of $9x^2+\frac{1}{16x^2}$

(a) $\frac{201}{16}$ (b) $\frac{145}{61}$

(c) $\frac{200}{17}$ (d) $\frac{201}{12}$

34. If $a^4+a^2b^2+b^4=12$, $a^2-ab+b^2=4$, then find the value of ab.

(a) $\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $-\frac{1}{2}$

35. If $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=1$ & $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=0$.

Find $\frac{p^2}{x^2}+\frac{q^2}{y^2}+\frac{r^2}{z^2}$.

(a) 0 (b) 1

(c) 2 (d) 3

36. If $a^2+a+1=0$ find the value of $(a^2-a+1)(a+1-a^2)$

(a) 1 (b) 0 (c) 4 (d) 2

37. If $\frac{1}{x+1}+\frac{2}{x+2}+\frac{3}{x+3}.....\frac{1007}{x+1007}=1249$

find, $\frac{x}{x+1}+\frac{x}{x+2}.....\frac{x}{x+1007}$

(a) –242 (b) 242

(c) –401 (d) 401

38. If $x=11$, find the value of $x^5-12x^4+13x^3-12x^2+12x-5$.

(a) 1728 (b) 121 (c) 1337 (d) 101

39. Find the minimum value of $x^2+\frac{1}{x^2-2}+5$, here $x>\sqrt{2}$

(a) 5 (b) 7 (c) 9 (d) 11

40. If x and y are positive real numbers and $xy=8$, then the minimum value of $2x+y$ is

(a) 9 (b) 17 (c) 10 (d) 8

41. If $a+\frac{1}{a}=1$, find the value of $a^{100}+a^{99}+a^{98}+a^{97}+a^{96}+a^{95}$

(a) 0 (b) 1 (c) 2 (d) 3

42. If $ax^2+bx+c=a(x-p)^2$, then the relation among a, b, c would be :

(a) $abc=1$ (b) $b^2=ac$

(c) $b^2=4ac$ (d) $2b=a+c$

43. If $a^2+b^2=c^2$, find the value of $\frac{a^6+b^6-c^6}{a^2b^2c^2}$

(a) 3 (b) –3 (c) 2 (d) –2

44. If $(a+b)^2=21+c^2$, $(b+c)^2=32+a^2$ and $(c+a)^2=28+b^2$, find $a+b+c=?$

(a) 7 (b) –9 (c) 8 (d) –8

45. If $a+b+c=6$, $a^2+b^2+c^2=14$ and $a^3+b^3+c^3=36$, then the value of abc is

(a) 3 (b) 6 (c) 9 (d) 12

46. If $x=2+\sqrt{3}$, find the value of

$\frac{x^6+x^4+x^2+1}{x^3}$

(a) 52 (b) 54 (c) 56 (d) 58

47. If $a^2+b^2+4c^2=2(a+b-2c)-3$ where a, b, c are real, then the value of $(a^2+b^2+c^2)$ is

(a) 3 (b) $3\frac{1}{4}$ (c) 2 (d) $2\frac{1}{4}$

48. If $a^2+a+1=0$, find the value of a^5+a^4+1

(a) –1 (b) 0

(c) 1 (d) –2

49. Find the value of $\frac{\left(\frac{6}{5}\right)^4+\left(\frac{5}{6}\right)^4+1}{\left(\frac{6}{5}\right)^2+\left(\frac{5}{6}\right)^2+1}$

(a) 1 (b) $\frac{25}{36}$ (c) $1\frac{121}{900}$ (d) $\frac{41}{30}$

50. The value of $\frac{\left(\frac{2}{3}\right)^4+\left(\frac{1}{3}\right)^{-4}+1}{\left(\frac{2}{3}\right)^4+\left(\frac{1}{3}\right)^{-4}-2}$ will be

(a) $\frac{103}{9}$ (b) $\frac{114}{9}$

(c) $\frac{137}{9}$ (d) $\frac{123}{9}$

51. The value of $\sqrt{\frac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}}$ is

(a) $\sqrt{6}-\sqrt{2}$ (b) $\sqrt{6}+\sqrt{2}$

(c) $\sqrt{6}-2$ (d) $2-\sqrt{6}$

52. If $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y=\frac{\sqrt{3}-1}{\sqrt{3}+1}$, then the value of x^2+y^2 is

(a) 14 (b) 13 (c) 15 (d) 10

53. The value of

$$\frac{1}{\sqrt{(12-\sqrt{140})}}-\frac{1}{\sqrt{(8-\sqrt{60})}}-\frac{2}{\sqrt{(10+\sqrt{84})}}$$ is

(a) 0 (b) 1 (c) 2 (d) 3

54. The square root of

$$\frac{(0.75)^3}{1-0.75}+\left(0.75+(0.75)^2+1\right)$$ is

(a) 4 (b) 3 (c) 2 (d) 1

55. Find the value of

$$\left(\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+.....+\frac{17}{5184}+\frac{19}{8100}\right)$$

(a) $\frac{1}{100}$ (b) $\frac{99}{100}$

(c) $\frac{101}{100}$ (d) $\frac{91}{100}$

56. $\frac{\frac{1}{3}.\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}.\frac{1}{4}-3.\frac{1}{3}.\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{5}.\frac{1}{5}}{\frac{1}{3}.\frac{1}{3}+\frac{1}{4}.\frac{1}{4}+\frac{1}{5}.\frac{1}{5}-\left(\frac{1}{3}.\frac{1}{4}+\frac{1}{4}.\frac{1}{5}+\frac{1}{5}.\frac{1}{3}\right)}$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{47}{60}$ (d) $\frac{49}{60}$

57. If $\left(x+\frac{1}{x}\right)=2$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ is:

(a) 2 (b) 4

(c) 8 (d) None of these

58. If $\left(x^3-\frac{1}{x^3}\right)=36$, then the value of $\left(x-\frac{1}{x}\right)$ is:

(a) 1 (b) 2 (c) 3 (d) 6

59. If the sum of two numbers is 3 and the sum of their squares is 12, then their product is equal to: **[SSC CGL 2009]**

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) $-\frac{3}{2}$ (d) $-\frac{2}{3}$

60. The sum of two numbers is 42 and their product is 437. Then the absolute difference between the numbers is: **[SSC CGL 2011]**

(a) 3 (b) 4 (c) 5 (d) 7

61. The product of two numbers is 120. The sum of their squares is 289. The difference of these two numbers is **[SSC CHSL 2014]**

(a) 9 (b) 7 (c) 8 (d) 6

62. The product of two consecutive odd numbers is 4899. What is the smaller number? **[Bank Clerk 2013]**

(a) 67 (b) 69 (c) 71 (d) 73

GP_3386

Hints and Solutions

Level 1

1. (b) Use Shortcut Approach-4

2. (a) Use Shortcut Approach-5

3. (a) Use Shortcut Approach-9

4. (a) This question should be solved by pattern method,

Consider the 1st term only

$$= \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} = \sqrt{1+1+\frac{1}{4}} = \sqrt{\frac{9}{4}}$$

$$= \frac{3}{2} = 2-\frac{1}{2}$$

Consider the 1st two terms

$$= \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}$$

$$= \frac{3}{2} + \sqrt{1+\frac{1}{4}+\frac{1}{9}} = \frac{3}{2} + \sqrt{\frac{49}{36}} = \frac{3}{2}+\frac{7}{6}$$

$$= \frac{8}{3} = 3-\frac{1}{3}$$

Following the same pattern we can conclude that if n terms are there then final result is $n-\frac{1}{n}$

So final result is 2008 –1/2008

5. (a) Use Shortcut Approach-10

6. (d) If we look at the options closely we will find 2^x is the smallest.

7. (c) From the given relationship

$2\#3 = 2+3+2\times 3 = 11$

8. (a) From the options we find that q = 0

9. (d) Since m < n from the options we will find option (d) is correct.

10. (b) $\sqrt{110.25}\times\sqrt{0.01}\div\sqrt{0.0025}-\sqrt{420.25}$

$= 10.5\times 0.1 \div 0.05 - 20.5 = 0.50$

11. (d) Given expression $7777+7777\times 7777\times(5/77)\times(11/35)$

$= 7777+(7777)\times(7777)\times(1/7)\times(1/7)$

$= (7777)+(1111)\times(1111)$

= unit digit is 7 + 1 = 8, none of the given result has 8 as unit digit hence option (d) is the best.

12. (c) Use Shortcut Approach-1

13. (b) $x*y = x^2+y^2-xy$

$9*11 = 9^2+11^2-9\times 11$

$= 81+121-99 = 103$

14. (b) $1*2 = 1+2\times 6 = 13$

$13*3 = 13+3\times 6 = 31$

15. (a) $6*15*3 = \sqrt{\frac{(6+2)(15+3)}{(3+1)}} = \sqrt{\frac{8\times 18}{4}} = 6$

16. (a) Given expression

$$= 35.7 - \left(3+\frac{1}{\frac{10}{3}}\right) - \left(2+\frac{1}{\frac{5}{2}}\right)$$

$$= 35.7 - \left(3+\frac{3}{10}\right) - \left(2+\frac{2}{5}\right)$$

$$= 35.7 - \frac{33}{10} - \frac{12}{5} = 35.7 - \frac{57}{10} = 30.$$

17. (c) $\left[X+\frac{1}{X}\right]^2 = X^2+\frac{1}{X^2}+2 = 16$

or $X^2+\frac{1}{X^2} = 14$

Now, $X^4+\frac{1}{X^4}+2 = 196$ or $X^4+\frac{1}{X^4} = 194.$

18. (c) Given expression

$$= \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times.....\times\frac{99}{100} = \frac{2}{100} = \frac{1}{50}$$

19. (b) $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}} = \frac{2\sqrt{6}+\sqrt{6}}{2\sqrt{6}-\sqrt{6}} = \frac{3\sqrt{6}}{\sqrt{6}} = 3$

20. (c) Use Shortcut Approach-2

21. (d) Given exp. $= 2.002+7.9(2.8-6.3\times 2.1+15.6)$

$= 2.002+7.9(2.8-13.23+15.6) = 2.002+7.9\times 5.17$

$= 2.002+40.843 = 42.845$

22. (b) Given exp. $= 9-\frac{11}{9}\text{ of }\frac{36}{11}\div\frac{36}{7}\text{of }\frac{7}{9}$

$= 9-4\div 4 = 9-1 = 8$

23. **(d)** Let $\frac{5}{3}\div\frac{2}{7}\times\frac{x}{7}=\frac{5}{4}\times\frac{2}{3}\div\frac{1}{6}$. Then,

$$\frac{5}{3}\times\frac{7}{2}\times\frac{x}{7}=\frac{5}{4}\times\frac{2}{3}\times 6 \Leftrightarrow \frac{5}{6}x=5$$

$$\Leftrightarrow x=\left(\frac{5\times 6}{5}\right)=6.$$

24. **(d)** $$\frac{\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)}{\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)}$$

$$=\frac{\left(\frac{30-15+12-10}{60}\right)}{\left(\frac{2}{5}+\frac{3}{5}\right)-\left(\frac{5}{9}+\frac{7}{18}\right)}=\frac{\left(\frac{17}{60}\right)}{1-\frac{17}{18}}$$

$$=\left(\frac{17}{60}\times 18\right)=\frac{51}{10}=5\frac{1}{10}$$

25. **(b)** $$3\div\left[(8-5)\div\left\{(4-2)\div\left(2+\frac{8}{13}\right)\right\}\right]$$

$$\Rightarrow 3\div\left[(3)\div\left(2\div\frac{34}{13}\right)\right]$$

$$\Rightarrow 3\div\left[\frac{3\times 34}{13\times 2}\right]\Rightarrow\frac{3\times 13\times 2}{3\times 34}=\frac{13}{17}$$

26. **(a)** $2^{10000}=(2^{10})^{1000}=(1024)^{1000}$
$(10)^{3000}=(10^3)^{1000}=(1000)^{1000}$
$3^{6000}=(3^6)^{1000}=(729)^{1000}$
$7^{4000}=(7^4)^{1000}=(2401)^{1000}$
$3^{6000}<10^{3000}<2^{10000}<7^{4000}$

27. **(d)** Use Shortcut Approach-3

28. **(d)** Use Shortcut Approach-5

29. **(d)** Here, $a+b+c=0$
$\Rightarrow 331+336-667=0$
$\therefore\ a^3+b^3+c^3-3abc=0$

30. **(c)** Use Shortcut Approach-11

31. **(d)** $$x^4+\frac{1}{x^4}=119$$

$$\Rightarrow\left(x^2+\frac{1}{x^2}\right)^2-2=119\ \Rightarrow\left(x^2+\frac{1}{x^2}\right)^2=121$$

$$\Rightarrow x^2+\frac{1}{x^2}=11\Rightarrow\left(x-\frac{1}{x}\right)^2+2=11$$

$$\Rightarrow x-\frac{1}{x}=3\Rightarrow\left(x-\frac{1}{x}\right)^3=27$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=27$$

$$\Rightarrow x^3-\frac{1}{x^3}=27+9=36$$

32. **(c)** The least value of $a+\frac{1}{a}$ is 2 where $a=1$.

33. **(a)** $x^4+y^4-2x^2y^2\Rightarrow(x^2-y^2)^2\Rightarrow[(x+y)(x-y)]^2$

$$\Rightarrow\left[\left(\sqrt{a}+\frac{1}{\sqrt{a}}+\sqrt{a}-\frac{1}{\sqrt{a}}\right)\times\left(\sqrt{a}+\frac{1}{\sqrt{a}}-\sqrt{a}+\frac{1}{\sqrt{a}}\right)\right]^2$$

$$\Rightarrow\left(2\sqrt{a}\times\frac{2}{\sqrt{a}}\right)^2\Rightarrow 16$$

34. **(c)** If $a+b+c=0$,
then, $a^3+b^3+c^3=3abc$
Here, $y-z+z-x+x-y=0$
$\Rightarrow(y-z)^3+(z-x)^3+(x-y)^3$
$=3(y-z)(z-x)(x-y)$

35. **(c)** $$27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$$

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2.\frac{1}{6}+3(3p)\left(\frac{1}{6}\right)^2$$

$$=\left(3P-\frac{1}{6}\right)^3=\left(3\times\frac{5}{18}-\frac{1}{6}\right)^3=\frac{8}{27}$$

36. **(d)** $$\text{Number}=\frac{45\times 9\times 10\times 8}{4\times 3\times 5}\Rightarrow 540$$

37. **(d)** $$?=48845+\frac{51246}{6}=48845+8541=57386$$

38. **(d)** $$?=\frac{41}{6}\times\frac{16}{3}+\frac{53}{3}\times\frac{9}{2}=\frac{41\times 16+53\times 9\times 3}{6\times 3}$$

$$=\frac{656+1431}{18}=\frac{2087}{18}=115\frac{17}{18}$$

39. **(a)** $$?=\frac{\sqrt{29241}}{\sqrt{361}}\times\frac{47}{9}=\frac{171}{19}\times\frac{47}{9}=47$$

GP_3386

40. (d) $45\%\text{ of }1500+35\%\text{ of }1700=?\%\text{ of }3175$

$$\therefore \quad ?=\frac{127000}{3175}=40$$

41. (d) $17\frac{2}{3}\text{ of }180+\frac{1}{4}\text{ of }480=?$

$$\text{or, } ?=\frac{53}{3}\text{of }180+\frac{1}{4}\text{of }480$$

$$=3180+120=3300$$

42. (d) $?=5\times230+3000+2150$

$$=1150+3000+2150=6300$$

43. (d) $?=6.6\times1080+2560-1700=7128+860$

$$\approx 8000$$

44. (d) $?=35\%\text{ of }1478+29\%\text{ of }3214$

$35\%\text{ of }(1500-22)+(30\%-1\%)\text{ of }3200$

$35\times15-8+30\times32-32$

$$=525+960-40=525+920=1445=1450$$

45. (d) $\frac{1}{3}=0.33;\ \frac{2}{5}=0.4;\frac{4}{7}=0.57;\ \frac{3}{5}=0.6;$

$$\frac{5}{6}=0.83\text{ and }\frac{6}{7}=0.85$$

46. (d) $?=(27+118-32-11)+\left(\frac{3}{11}+\frac{2}{5}-\frac{5}{22}-\frac{6}{11}\right)$

$$\text{or, } ?=102+\left(\frac{30+44-25-60}{110}\right)=101\frac{9}{10}$$

47. (b) $?\%\text{ of }360=129.6$

$$\text{or, } ?=\frac{129.6\times100}{360}=36$$

48. (b) $3\frac{1}{3}+33\frac{1}{3}+.....+333333\frac{1}{3}$

$$=3+\frac{1}{3}+33\frac{1}{3}+.....+333333+\frac{1}{3}$$

$$=3+33+333+.....+333333+\frac{1}{3}\times6$$

$$=3+33+333+3333+33333+333333+2$$

$$=370368+2=370370.$$

49. (d) $\left(2-\frac{1}{3}\right)\times\left(2-\frac{3}{5}\right)\times\left(2-\frac{5}{7}\right)\times$

$$.....\times\left(2-\frac{999}{1001}\right)$$

$$=\frac{5}{3}\times\frac{7}{5}\times\frac{9}{7}\times.....\times\frac{1003}{1001}=\frac{1003}{3}$$

50. (d) $4^{11111}\times5^{22222}$

$$=\left(2^2\right)^{11111}\times5^{22222}$$

$$=2^{22222}\times5^{22222} \qquad \left[\left(a^m\right)^n=a^{mn}\right]$$

$$=10^{22222} \qquad [a^m\times b^m=(ab)^m]$$

Hence, number of digits = 22222 + 1

= 22223.

51. (a) $0.\overline{037}=\frac{37}{999}=\frac{1}{27}$

$$\Rightarrow\sqrt[3]{0.\overline{037}}=\sqrt[3]{1/27}=\frac{1}{3}=0.333.....=0.\bar{3}.$$

52. (c) $p=5+q$

$\Rightarrow$ $p-q=5$

Also, $p^2+q^2=55$

$p^2+q^2=(p-q)^2+2pq$

$\Rightarrow$ $55=(5)^2+2pq$

$\Rightarrow$ $2pq=55-25\Rightarrow2pq=30\Rightarrow pq=15.$

Hence, the product of p and q = 15.

53. (c) $1.\overline{27}$

$$=1+0.\overline{27}=1+\frac{27}{99}$$

$$=1+\frac{3}{11}=\frac{14}{11}$$

Alternate

Let $x=1.272727$ ..... (i)

$100x=127.2727$ ..... (ii)

From (ii) – (i)

$127-1=100x-x\Rightarrow99x=126$

$$x=\frac{126}{99}=\frac{14}{11}.$$

54. (a) $x(x^2-3x+3)$

$=x^3-3x^2+3x$

$=x^3-3x^2+3x+1-1=(x-1)^3-1$

Substituting x = 101

$(101-1)^3+1=1000000+1=1000001.$

55. (a) We know,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Here, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1\ \Rightarrow\ ab+bc+ca=abc$

$\therefore$ $(3)^2=6+2(abc)\Rightarrow abc=\frac{3}{2}$

56. (c) When, $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Here, $a=0.32$, $b=0.45$ and $c=-0.77$

So, given data

$$= -\frac{1}{27} \times \frac{(0.32)^3 + (0.45)^3 + (-0.77)^3}{3(0.32)(0.45)(-0.77)}$$

$$= \frac{-1}{27} \times 1 = \frac{-1}{27}$$

57. (b) Required value

$$= \sqrt{6 - 4\sqrt{3} + \sqrt{(2\sqrt{3} - 2)^2}}$$

$$= \sqrt{4 - 2\sqrt{3}} = \sqrt{3} - 1$$

58. (a) Required value

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots \times \frac{n-1}{n} = \frac{1}{n}$$

59. (a) Use Shortcut Approach-8

60. (a) $\because \frac{a^3 + b^3}{a^2 - ab + b^2} = a + b = 0.051 + 0.041 = 0.092$

61. (c) $(x - y)^2 = (x + y)^2 - 4xy = 64 - 60 = 4$
$\Rightarrow (x - y) = 2$
$x^3 - y^3 = (x - y)^3 + 3xy(x - y) = 8 + 45\ (2)$
$x^3 - y^3 = 98$

62. (c) $(x^3 + y^3 + z^3 - 3xyz)$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= (x + y + z)[(x + y + z)^2 - 3(xy + yz + zx)]$
$= 6 \times [36 - 3 \times 11] = 6 \times 3 = 18$

63. (b) $x + y + z = 0$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz.$

64. (d) $\because x + y + z = 0$
$\Rightarrow x^3 + y^3 + z^3 = 3xyz$
$\therefore a^{1/3} + b^{1/3} + c^{1/3} = 0$

$\Rightarrow a + b + c = 3a^{1/3}b^{1/3}c^{1/3}$
$\therefore (a + b + c)^3 = 27\,abc$

65. (c) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
$2197 = x^3 + y^3 + 3 \times 36(13)$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = 2197 - 1404 = 793$

66. (c) $(x + y)^2 = x^2 + y^2 + 2xy$
$169 = x^2 + y^2 + 2 \times 40 \Rightarrow x^2 + y^2 = 89$

67. (a) $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2x.\frac{1}{x}$

$(2)^2 = x^2 + \frac{1}{x^2} + 2 \Rightarrow x^2 + \frac{1}{x^2} = 2$

68. (c) $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$(2\sqrt{3})^3 = x^3 + \frac{1}{x^3} + 3(2\sqrt{3})$

$\Rightarrow x^3 + \frac{1}{x^3} = 18\sqrt{3}$

69. (a) Use Shortcut Approach-14
70. (b) Use Shortcut Approach-17
71. (c) Use Shortcut Approach-17
72. (d) Use Shortcut Approach - 3
73. (c) Use Shortcut Approach-16
74. (b) Use Shortcut Approach-14
75. (b) Use Shortcut Approach-14
76. (b) Use Shortcut Approach-15
77. (c) Use Shortcut Approach-14

Level 2

1. (d) Since $a^n - b^n$
$= (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots\ldots + b^{n-1})$
Hence, $30^{65} - 29^{65} = (30 - 29)$
$(30^{64} + 30^{63}.29 + 30^{62}.29^2 + \ldots\ldots\ldots + 29^{64})$
The expression in the second factor is greater than $30^{64} + 29^{64}$ that is denominator that means numerator is bigger than denominator, hence R >1

2. (d) Use Shortcut Solution-10

3. (c) Given : $x^2 + y^2 + z^2 - 64$
$= -2(xy - yz - zx)$ ...(i)
Now, $[x + y + (-z)]^2$
$= x^2 + y^2 + z^2 + 2(xy - yz - zx)$
$\Rightarrow (3z - z)^2 = x^2 + y^2 + z^2 + 2(xy - yz - zx)$
$\Rightarrow -2(xy - yz - zx) = (x^2 + y^2 + z^2) - (2z)^2$ ...(ii)
From (i) and (ii), we get:
$(2z)^2 = 64 \Leftrightarrow 4z^2 = 64 \Leftrightarrow z^2 = 16 \Leftrightarrow z = 4.$

4. (b) Given exp. $= \dfrac{1}{1 + \dfrac{\frac{2}{3}}{\frac{5}{3} + \frac{\frac{8}{9}}{(1/3)}}} = \dfrac{1}{1 + \dfrac{2/3}{\frac{5}{3} + \frac{8}{9} \times 3}}$

$= \dfrac{1}{1 + \dfrac{2/3}{(13/3)}} = \dfrac{1}{1 + \dfrac{2}{13}} = \dfrac{13}{15}$

5. (a) Use Shortcut Approach-13

GP_3386

6. (a) $\frac{97}{19}=5+\frac{2}{19}$. Also, $\frac{19}{2}$ can be written as $9+\frac{1}{2}$. So the values of a, b and c are 5, 9 and 2 respectively. Hence, the sum of a, b and c is 16.

7. (b) If $a+b+c=0$
then $a^3+b^3+c^3=3abc$
Dividing both sides by abc

$$\frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc}=\frac{3abc}{abc}$$

$$\Rightarrow \quad \frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=3$$

8. (c) Use Shortcut Approach-12

9. (b) Use Shortcut Approach-12

10. (d) $x+\frac{1}{x}=2 \Rightarrow x+\frac{1}{x}-2=0$

$\Rightarrow x^2-2x+1=0\,;\ (x-1)^2=0\,;\ x=1$

Now, $x^{2013}+\frac{1}{x^{2014}}=1+1=2$

11. (d) $5a+\frac{1}{3a}=5$

Multiply both sides by $\frac{3}{5}$

$$\frac{3}{5}\left(5a+\frac{1}{3a}\right)=5\times\frac{3}{5} \Rightarrow 3a+\frac{1}{5a}=3$$

Squaring sides

$$9a^2+\frac{1}{25a^2}+2\times3a\times\frac{1}{5a}=9$$

$$\Rightarrow 9a^2+\frac{1}{25a^2}=9-\frac{6}{5}=\frac{39}{5}$$

12. (a) $x+\frac{1}{x}=\sqrt{3}$

Cubing both sides,

$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\left(\sqrt{3}\right)^3$$

$$\Rightarrow x^3+\frac{1}{x^3}+3\sqrt{3}=3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3}=0$$

Now $x^{18}+x^{12}+x^6+1$
$=x^{12}(x^6+1)+1(x^6+1)=(x^{12}+1)(x^6+1)$

$$=(x^{12}+1).x^3\left(x^3+\frac{1}{x^3}\right)=0$$

13. (a) $x^2+y^2+z^2-xy-yz-zx$

$$=\frac{2}{2}(x^2+y^2+z^2-xy-yz-zx)$$

$$=\frac{1}{2}(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$=\frac{1}{2}(x^2+y^2-2xy+y^2+z^2-2yz+x^2+z^2-2xy)$$

$$=\frac{1}{2}[(x-y)^2+(y-z)^2+(z-x)^2]$$

$$=\frac{1}{2}[(997-998)^2+(998-999)^2+(999-997)^2]$$

$$=\frac{1}{2}[1^2+1^2+2^2]=\frac{1}{2}\times6=3$$

14. (a) $x^2+y^2+2x+1=0$
$\Rightarrow (x+1)^2+y^2=0$
$\Rightarrow x+1=0 \Rightarrow x=-1$ and $y=0$
$\therefore x^{31}+y^{35}=-1$

15. (d) $a^x=(x+y+z)^y$
$a^y=(x+y+z)^z$
$a^z=(x+y+z)^x$
Multiplying corresponding terms,
$a^x\,.\,a^y.\,a^z=(x+y+z)^{x+y+z}$
$\Rightarrow a^{x+y+z}=(x+y+z)^{x+y+z} \Rightarrow a=x+y+z$

16. (b) $x^4-17x^3+17x^2-17x+17$
$=x^4-16x^3+16x^2-16x-x^3+x^2-x+17$
When $x=16$,
Expression $=16^4-16^4+16^3-16^2-16^3+16^2$
$-16+17=1$

17. (c) $m+\frac{1}{m-2}=4$

$m^2-6m+9=0 \Rightarrow (m-3)(m-3)=0$
$\Rightarrow m=3$
$\Rightarrow m-2=1$

Now $(m-2)^2+\frac{1}{(m-2)^2}=1^2+\frac{1}{1^2}=2$

18. (c) $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$ .....(i)

Given that,
$a^2=b+c \Rightarrow a+a^2=a+b+c$

$$a(a+1)=a+b+c \Rightarrow a+1=\frac{a+b+c}{a}$$

$$\frac{1}{a+1}=\frac{a}{a+b+c}$$

Similarly,

$\frac{1}{b+1}=\frac{b}{a+b+c}$ and $\frac{1}{c+1}=\frac{c}{a+b+c}$

Put in eq. (i)

$\therefore \quad \frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}$

$=\frac{a+b+c}{a+b+c}=1$

19. (a) $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}=a\sqrt[3]{4}+b\sqrt[3]{2}+c$

$\Rightarrow \frac{1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}=a.2^{\frac{2}{3}}+b.2^{\frac{1}{3}}+c$

$\Rightarrow \frac{\left(2^{\frac{1}{3}}-1\right)}{\left(2^{\frac{1}{3}}-1\right)\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}+1\right)}$

$=a.2^{\frac{2}{3}}+b.2^{\frac{1}{3}}+c$

$\Rightarrow \frac{2^{\frac{1}{3}}-1}{2-1}=a.2^{\frac{2}{3}}+b.2^{\frac{1}{3}}+c$

$[\because (a-b)(a^2+ab+b^2)=a^3-b^3]$

$\Rightarrow a=0, b=1, c=-1$

$a+b+c=0+1-1=0$

20. (c) Use Shortcut Approach-8

21. (a) Use Shortcut Approach-7

22. (d) $?=\frac{18}{5}\times 160+\frac{40}{100}\times 1850+450$

$=576+740+450=1760=1800$

23. (b) $\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+.....+\frac{1}{156}$

$=\frac{1}{3\times 4}+\frac{1}{4\times 5}+\frac{1}{5\times 6}+.....+\frac{1}{12\times 13}$

$=\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+...+\frac{1}{12}-\frac{1}{13}$

$=\frac{1}{3}-\frac{1}{13}=\frac{10}{39}$

24. (a) $\frac{1}{\sqrt{3}+\sqrt{4}}=\frac{1}{\sqrt{4}+\sqrt{3}}$

$=\frac{1}{\sqrt{4}+\sqrt{3}}\times\frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}-\sqrt{3}}=\frac{\sqrt{4}-\sqrt{3}}{(\sqrt{4})^2-(\sqrt{3})^2}=\sqrt{4}-\sqrt{3}$

Similarly, $\frac{1}{\sqrt{4}+\sqrt{5}}=\sqrt{5}-\sqrt{4}$;

$\frac{1}{\sqrt{6}+\sqrt{7}}=\sqrt{7}-\sqrt{6}$

$\frac{1}{\sqrt{7}+\sqrt{8}}=\sqrt{8}-\sqrt{7}$; $\frac{1}{\sqrt{9}+\sqrt{8}}=\sqrt{9}-\sqrt{8}$

So, $\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+.....+\frac{1}{\sqrt{8}+\sqrt{9}}$

$=\sqrt{9}-\sqrt{8}+\sqrt{8}-\sqrt{7}+\sqrt{7}-\sqrt{6}$

$+.....+\sqrt{4}-\sqrt{3}$

$=\sqrt{9}-\sqrt{3}=3-\sqrt{3}$

25. (b) $\frac{1}{10}+\frac{1}{40}+\frac{1}{88}+.....+\frac{1}{598}$

$=\frac{1}{2\times 5}+\frac{1}{5\times 8}+\frac{1}{8\times 11}+.....+\frac{1}{23\times 26}$

$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+....+\frac{1}{23}-\frac{1}{26}\right]$

$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{26}\right]=\frac{1}{3}\times\frac{12}{26}=\frac{2}{13}$

Shortcut

$\frac{1}{\text{Diff between two numbers}}\left[\frac{1}{\text{first term}}-\frac{1}{\text{last term}}\right]$

$\Rightarrow \frac{1}{3}\left[\frac{1}{2}-\frac{1}{26}\right]=\frac{2}{13}$

26. (d) $9+99+999+..... +20^{th}$ term

$=(10-1)+(10^2-1)+(10^3-1)+.....+(10^{20}-1)$

$=10+10^2+10^3+.....+10^{20}-(1+1+1+$

$.....+20\text{ term})$

$=\frac{10(10^{20}-1)}{(10-1)}-20 \quad =\frac{10(10^{20}-1)}{9}-20$

27. (c) $\frac{a+b}{a-b}=\frac{4}{1}$

Using componendo and dividendo,

$\frac{a+b+a-b}{a+b-a+b}=\frac{4+1}{4-1}\Rightarrow\frac{a}{b}=\frac{5}{3}$

GP_3386

28. (c)

$$\frac{(100-1)\times(100-2)\times(100-3)\times....\times(100-200)}{100\times98\times97\times....\times3\times2\times1}$$

$$=\frac{99\times98\times97\times...\times2\times1\times0\times(-1)\times(-2)\times...\times(-100)}{100\times98\times99\times97\times...\times1}$$

We know, any natural number $\times (0) = 0$

Hence, numerator $= 0 \Rightarrow \frac{0}{100!} = 0.$

29. (a) $x+\frac{1}{x-1}$

$$1+\sqrt{2}+\sqrt{3}+\frac{1}{1+\sqrt{2}+\sqrt{3}-1}$$

$$=1+\sqrt{2}+\sqrt{3}+\frac{1}{\sqrt{2}+\sqrt{3}}$$

$$=1+\sqrt{2}+\sqrt{3}+\sqrt{3}-\sqrt{2}=1+2\sqrt{3}$$

30. (a) Use Shortcut Approach-7

31. (a) Use Shortcut Approach-6

32. (a) Use Shortcut Approach-11

33. (a) $4x+\frac{1}{3x}=5 \Rightarrow 12x^2+1=15x$

Dividing both sides by 4x,

$$\frac{12x^2+1}{4x}=\frac{15x}{4x} \Rightarrow 3x+\frac{1}{4x}=\frac{15}{4}$$

Squaring both sides, $\left(3x+\frac{1}{4x}\right)^2=\left(\frac{15}{4}\right)^2$

$$\Rightarrow \quad 9x^2+\frac{1}{16x^2}+\frac{3}{2}=\frac{225}{16}$$

$$\Rightarrow \quad 9x^2+\frac{1}{16x^2}=\frac{225}{16}-\frac{3}{2}=\frac{225-24}{16}=\frac{201}{16}$$

34. (d) $a^4+a^2b^2+b^4=12 \Rightarrow a^2-ab+b^2=4$ ...(i)

We know,

$a^4+a^2b^2+b^4=(a^2-ab+b^2)(a^2+ab+b^2)$

$\Rightarrow$ $12=4\times(a^2+ab+b^2)$

$\Rightarrow$ $a^2+ab+b^2=3$ ....(ii)

Subtracting (ii) from (i)

$$2ab=-1 \quad \Rightarrow \quad ab=\frac{-1}{2}$$

Alternate solution :–

$a^4+a^2b^2+b^4=12$

$\Rightarrow$ $a^4+b^4=12-(a^2b^2)$ ...(i)

$a^2-ab+b^2=4 \Rightarrow a^2+b^2=4+ab$

Squaring both sides

$(a^2+b^2)^2=(4+ab)^2$

$\Rightarrow$ $a^4+b^4+2a^2b^2=16+a^2b^2+8ab$

$\because$ $a^4+b^4=12-(a^2b^2)$ [From eq. (i),]

$\Rightarrow$ $12-a^2b^2+2a^2b^2=16+a^2b^2+8ab$

$\Rightarrow \quad -4=8ab \Rightarrow ab=\frac{-4}{8}=\frac{-1}{2}$

35. (b) Let $a=\frac{p}{x}, b=\frac{q}{y}, c=\frac{r}{z}$

Then, $a+b+c=1$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

$$\Rightarrow \quad \frac{ab+bc+ca}{abc}=0 \Rightarrow ab+bc+ca=0$$

Now, $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow$ $1=a^2+b^2+c^2+0 \Rightarrow a^2+b^2+c^2=1.$

Hence $\frac{p^2}{x^2}+\frac{q^2}{y^2}+\frac{r^2}{z^2}=1$

36. (c) $a^2+a+1=0$

$(a-1)(a^2+a+1)=0$

$a^3-1=0 \Rightarrow a^3=1$

Now,

$a^2+1=-a$ and $a+1=-a^2$

So, $(a^2-a+1)(a+1-a^2)$

$=(-2a)(-2a^2)=4a^3$

Hence, $4\times1=4.$

37. (a)

$$\frac{1}{x+1}+\frac{2}{x+2}+\frac{3}{x+3}+.....+\frac{1007}{x+1007}=1249$$

$$\Rightarrow \quad \frac{1}{x+1}-1+\frac{2}{x+2}-1+\frac{3}{x+3}-1+....+\frac{x}{x+1007}-1$$

$$=1249-1007$$

$$\Rightarrow \quad \frac{-x}{x+1}+\frac{-x}{x+2}+\frac{-x}{x+3}+.....+\frac{-x}{x+1007}=242$$

$$\Rightarrow \quad \frac{x}{x+1}+\frac{x}{x+2}+\frac{x}{x+3}+.....+\frac{x}{x+1007}=-242.$$

38. (c) $x^5-12x^4+13x^3-12x^2+12x-5$

$=x^5-11x^4-x^4+11x^3+x^3-11x^2-x^2+11x+x-5+x^3$

$=x^4(x-11)-x^3(x-11)+x^2(x-11)-x(x-11)+x-5+x^3$

Substituting $x = 11$

$x^4(0)-x^3(0)+x^2(0)-x(0)+11-5+11^3$

Hence, $1331+6=1337.$

39. (c) $x^2+\frac{1}{x^2-2}+5=x^2-2+\frac{1}{x^2-2}+5+2$

Let $(x^2-2)=a$

We know minimum value of $a+\frac{1}{a}=2$

Hence, minimum $\left[\left(x^2-2\right)+\frac{1}{x^2-2}\right]=2$

Minimum value of the given expression $=2+7=9$.

40. (d) AM ≥ GM

$$\frac{2x+y}{2}\geq\sqrt{(2x).y}\Rightarrow 2x+y\geq 2\sqrt{2xy}$$

$\Rightarrow \quad 2x+y\geq 2\sqrt{2\times 8}$

Hence, minimum value of $2x+y=8$.

41. (a) $a+\frac{1}{a}=1$

$\Rightarrow \quad a^2+1=a \Rightarrow a^2-a+1=0$

Multiplying both sides by $(a+1)$

$(a+1)(a^2-a+1)=(a+1)(0)$

$\Rightarrow \quad a^3+1=0$

Now, $a^{100}+a^{99}+a^{98}+a^{97}+a^{96}+a^{95}$

$=a^{100}+a^{97}+a^{99}+a^{96}+a^{98}+a^{95}$

$=a^{97}(a^3+1)+a^{96}(a^3+1)+a^{95}(a^3+1)$

$=a^{97}(0)+a^{96}(0)+a^{95}(0)=0.$

42. (c) $ax^2+bx+c=a(x-p)^2$

$\Rightarrow \quad ax^2+bx+c=a(x^2+p^2-2xp)$

$\Rightarrow \quad ax^2+bx+c=ax^2+ap^2-2axp$

Comparing coefficients of x and constant term,

$$b=-2ap \quad \Rightarrow p=\frac{-b}{2a}$$

$$c=ap^2 \Rightarrow c=a\left(\frac{-b}{2a}\right)^2=\frac{b^2}{4a}\Rightarrow b^2=4ac$$

43. (b) $a^2+b^2=c^2$

$\Rightarrow \quad (a^2+b^2)^3=(c^2)^3$

$\Rightarrow \quad a^6+b^6+3(a^2+b^2)a^2b^2=c^6$

$\Rightarrow \quad a^6+b^6+3a^2b^2c^2=c^6 \qquad [\because a^2+b^2=c^2]$

$\Rightarrow \quad a^6+b^6-c^6=-3a^2b^2c^2$

So, $\frac{-3a^2b^2c^2}{a^2b^2c^2}=-3.$

44. (b) $(a+b)^2=21+c^2$

$\Rightarrow \quad (a+b)^2-c^2=21$

$\Rightarrow \quad [(a+b-c)(a+b+c)]=21 \qquad ....(i)$

Similarly,

$[(b+c+a)(b+c-a)]=32 \qquad ....(ii)$

and, $[(c+a+b)(c+a-b)]=28 \qquad ....(iii)$

Adding (i),(ii) and (iii)

$(a+b+c)[a+b-c+b+c-a+c+a-b]=81$

$\Rightarrow \quad (a+b+c)[a+b+c]=81\Rightarrow(a+b+c)^2=81$

$a+b+c=\pm 9$

45. (b) $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow \quad (6)^2=14+2(ab+bc+ca)$

$\Rightarrow \quad ab+bc+ca=\frac{36-14}{2}=11$

$a^3+b^3+c^3-3abc$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow \quad 36-3abc=6(14-11)$

$\Rightarrow \quad 3abc=18 \ =abc=\frac{18}{3}=6.$

46. (c) $x=2+\sqrt{3},\ \frac{1}{x}=\frac{1}{2+\sqrt{3}}=2-\sqrt{3}$

$$x+\frac{1}{x}=2+\sqrt{3}+2-\sqrt{3}=4$$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$=(4)^3-3.4=52$

So, $\frac{x^6+x^4+x^2+1}{x^3}=x^3+x+\frac{1}{x}+\frac{1}{x^3}$

$$x^3+\frac{1}{x^3}+x+\frac{1}{x}=52+4=56.$$

47. (d) $a^2+b^2+4c^2=2(a+b-2c)-3$

$\Rightarrow \quad a^2+b^2+4c^2-2a-2b+4c+3=0$

$\Rightarrow \quad a^2-2a+1+b^2-2b+1+4c^2+4c+1=0$

$\Rightarrow \quad (a-1)^2+(b-1)^2+(2c+1)^2=0$

Every term is a perfect square

So, $a-1=0, \quad a=1$

$b-1=0, b=1$

$$2c+1=0, c=-\frac{1}{2}$$

$$a^2+b^2+c^2=(1)^2+(1)^2+\left(-\frac{1}{2}\right)^2=2\frac{1}{4}.$$

48. (b) $a^2+a+1=0$

Multiplying both sides by $(a-1)$

$(a-1)(a^2+a+1)=(a-1)\times 0\Rightarrow a^3-1=0$

$\therefore \quad a^3=1$

Now, $a^5+a^4+1=a^3.a^2+a^3.a+1$

$=a^2+a+1 \qquad$ [Substituting $a^3=1$]

$=0.$

49. (c) Since, $a^4+b^4+a^2b^2=(a^2+b^2)^2-a^2b^2$

$=(a^2+b^2+ab)(a^2+b^2-ab)$

So, $\frac{a^4+b^4+a^2b^2}{a^2+b^2+ab}$

$=a^2+b^2-ab=(a-b)^2+ab$

GP_3386

Now put $a=\frac{6}{5}$ and $b=\frac{5}{6}$

Required value

$=\left(\frac{6}{5}-\frac{5}{6}\right)^2+1=\frac{121}{900}+1=1\frac{121}{900}$

50. (a) Required value $=\frac{\left(\frac{2}{3}\right)^4+3^4+\left(\frac{2}{3}\times 3\right)^2}{\left(\frac{2}{3}\right)^4+3^4-2}$

$=\left(\frac{2}{3}\right)^2+3^2+\left(\frac{2}{3}\times 3\right)=\left(3+\frac{2}{3}\right)^2$

$-\left(\frac{2}{3}\times 3\right)=\frac{103}{9}$

51. (c) Given data $=\sqrt{\frac{(2\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})^2}}$

$=\sqrt{\frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}}$

$=\sqrt{2}(\sqrt{3}-\sqrt{2})=\sqrt{6}-2$

52. (a) $x=\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{4+2\sqrt{3}}{2}=2+\sqrt{3};$

Likewise $y=2-\sqrt{3}$

So, $x^2+y^2=(2+\sqrt{3})^2+(2-\sqrt{3})^2=14$

53. (a) $\sqrt{12-\sqrt{140}}=\sqrt{5+7-2\sqrt{5}\sqrt{7}}=\sqrt{(\sqrt{7}-\sqrt{5})^2}$

$=\sqrt{7}-\sqrt{5}$

So, $\frac{1}{\sqrt{12-\sqrt{140}}}=\frac{1}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{\sqrt{7}+\sqrt{5}}{2}$

Likewise second term $=\frac{\sqrt{5}+\sqrt{3}}{2};$

and third term $=\frac{\sqrt{7}-\sqrt{3}}{2}$

So, required total $=0$

54. (c) Required solution

$=\sqrt{\frac{(0.75)^3+(1-0.75)(0.75+0.75^2+1)}{1-0.75}}$

$=\sqrt{\frac{(0.75)^3+1^3-(0.75)^3}{0.25}}=\sqrt{\frac{1}{0.25}}=\sqrt{4}=2$

55. (b) $\frac{4-1}{4}+\frac{9-4}{4\times 9}+\frac{16-9}{16\times 9}+.....+\frac{100-81}{81\times 100}$

$=\left(1-\frac{1}{4}+\frac{1}{4}-\frac{1}{9}+\frac{1}{9}-\frac{1}{16}+...+\frac{1}{81}-\frac{1}{100}\right)$

$=1-\frac{1}{100}=\frac{99}{100}$

56. (c) $\because a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Here $a=\frac{1}{3}, b=\frac{1}{4}, c=\frac{1}{5}$

So, $a+b+c=\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{47}{60}$

57. (a) $\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2.x.\frac{1}{x}$

$(2)^2=x^2+\frac{1}{x^2}+2$

Again

$\left(x^2+\frac{1}{x^2}\right)^3=x^6+\frac{1}{x^6}+3x^2.\frac{1}{x^2}\left(x^2+\frac{1}{x^2}\right)$

$(2)^3=x^6+\frac{1}{x^6}+3(2)\Rightarrow x^6+\frac{1}{x^6}=2$

58. (c) $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3x.\frac{1}{x}\left(x-\frac{1}{x}\right)$

$\left(x-\frac{1}{x}\right)^3=36-3\left(x-\frac{1}{x}\right)$

Let $\left(x-\frac{1}{x}\right)=P$

then $p^3=36-3P$

or $P^3+3P=36\Rightarrow P^3+3P-36=0$

Clearly P = 3 satisfies the equation.

$\therefore\quad x-\frac{1}{x}=3$

59. (c) Use Shortcut Approach-1

60. (b) Use Shortcut Approach-2

61. (b) Use Shortcut Approach-2

62. (b) Use Shortcut Approach-3

CHAPTER

6

Polynomials and Rational Expressions

(Including Factorisation of Polynomial, Factor Theorem & Remainder Theorem),

CONSTANT

A symbol having a fixed numerical value is called a **constant**.

For example : 5, –2, 3.14, $\frac{7}{9}$, $\sqrt{3}$, 0, π or $\frac{22}{7}$ etc. are constant.

VARIABLES

A symbol which may be assigned different numerical values is known as a **variable**.

For example :

In the formula of circumference of a circle, $C = 2\pi r$, 2 and π are constants and C and r are variables. Here, C is the circumference and r is the radius of the circle.

ALGEBRAIC EXPRESSIONS

A combination of variables and constants connected by some or all of the mathematical operations +, –, × and ÷ is known as an **algebraic expression**.

For example : $2x + 7, x^2 - 3x, 2y^3 - 7xy + 5$ etc.

Terms of an Algebraic Expression

The several parts of an algebraic expression separated by + or – sign are called the **terms of the expression**.

For example : $\frac{4}{3}x^4, -xy, 5y^2$ and 7 are the terms of the algebraic expression $\frac{4}{3}x^4 - xy + 5y^2 - 7$.

Coefficient of a Term

The numerical factor of a term is called the **coefficient** of the term.

For example : In the term $\frac{4}{3}x^4$, $\frac{4}{3}$ is the *coefficient* of the term $\frac{4}{3}x^4$. In the term $-xy$, -1 is the *coefficient* of the term $-xy$

Algebraic Expression	Term	Coefficient
$4x + 3y$	$4x$	4
	$3y$	3
$3xy^2 - 4x$	$3xy^2$	3
	$-4x$	–4
$3p^2q + 7pq - 8pq^2$	$3p^2q$	3
	$7pq$	7
	$-8pq^2$	–8

Some Important Types of Algebraic expressions on the Basis of its Number of Terms :

(i) **Monomial :** An algebraic expression containing one non-zero term is called a **monomial**.
For example : $2x^2, -5xy, 6x^3y^2$ etc.

(ii) **Binomial :** An algebraic expression containing two non-zero terms is called a **binomial**.
For example: $5x^4 - 6x, 2x^2 + 3, \sqrt{2}xy + 7y^2$ etc.

(iii) **Trinomial :** An algebraic expression containing three non-zero terms is called a **trinomial**.
For example: $6x^7 + 5x - 3, 8\sqrt{2}xy + 3x^2 + 7$ etc.

LIKE AND UNLIKE TERMS :

Two or more terms which have same variable(s) factors are called **like terms**. Two or more terms which have different variable(s) factors are called ***unlike terms.***

GP_3386

For example : $-5x^2y$, $\sqrt{3}x^2y$, $\frac{5}{7}yx^2$ are *like terms* because they have common variables factors x^2y (or yx^2). $7x^2$, $7x^3$ and $-5x^5$ are *unlike terms* because they have the different variable factor x^2, x^3 and x^5.

POLYNOMIALS

An algebraic expression in which the variables involved have only non-negative integral powers is called a **polynomial**.

For example : $2x^2+3x+4$, $\frac{2}{5}y^2+6$, $\sqrt{3}x^2+y^2$ are polynomials.

Note that $x+\frac{1}{x}$ is not a polynomial because it can be written as $x+x^{-1}$ and negative power is not allowed in a polynomial. $\sqrt{x}+6$ is also not a polynomial because it can be written as $x^{1/2}+6$ and fractional power is also not allowed in a polynomial. General form of the polynomial is

$a_0+a_1x+a_2x^2+a_3x^3+.....+a_nx^n$, where a_0, a_1, a_2, a_3, ... a_n are real numbers and n is a non-negative integer.

Degree of a Polynomial

A polynomial can have one variable or more than one variable in it.

(a) If a polynomial has one variable then the highest power of the variable is called **degree of the polynomial**.
For example : Degree of polynomial x^3-2x^2+4x+3 is 3 as the highest power of variable is 3.
Also, degree of $9y^4+5y^3+y^2-10$ is 4.

(b) If a polynomial has more than one variable, then degree of such polynomial is the highest value of the sum of powers of variables present in any term.
For example : $x+y+2xy$ has degree 2 because in third term (2xy), the sum of powers of variables x and y is 2 which is greater than the powers of variables in first and second term.
Degree of $15x^2y^3-10x^2y+7xy^2$ is 5.

Some Important Types of Polynomials on the Basis of their Degree:

Polynomials can be classified on the basis of their degree.

(i) Linear Polynomial : A polynomial of degree 1 is called a *linear polynomial*. (2x + 3) is a linear polynomial.

(ii) Quadratic Polynomial : A polynomial of degree 2 is called *quadratic polynomial*. x^2+x+4, $2x^2-3x+6$, ax^2+bx+c are examples of quadratic polynomials.

(iii) Cubic Polynomial : A polynomial of degree 3 is called a *cubic polynomial*. $3x^3-2x^2+4x-5$, $3x^3-2x+6$, ax^3+bx^2+cx+d etc. are examples of cubic polynomial.

(iv) Biquadratic Polynomial : A polynomial of degree 4 is called *Biquadratic polynomial*.
$x^4-7x^3+15x^2+7x-9$ is an example of biquadratic polynomial. A biquadratic polynomial can have maximum 5 terms.

Addition of Algebraic Expressions

In adding algebraic expressions, we collect different groups of like terms and find the sum of like terms in each group. The sum of several like terms is another like term whose coefficient is the sum of the coefficients of those like terms. Let us understand this throught he following illustration.

Illustration – 1

Add : $5x^3-2x^2+x+7$, $4x^2-3x+2$ and x^4+3x^2-x-3.

Solution : Required sum $=(5x^3-2x^2+x+7)+(4x^2-3x+2)+(x^4+3x^2-x-3)$.

$=x^4+5x^3+(-2x^2+4x^2+3x^2)+(x-3x-x)+(7+2-3)$

$=x^4+5x^3+(-2+4+3)x^2+(1-3-1)x+6$

$=x^4+5x^3+5x^2+(-3x)+6$

$=x^4+5x^3+5x^2-3x+6$

Subtraction of Algebraic Expressions

In order to subtract an algebraic expression from another, we change the signs (from '+' to '–' or from '–' to '+') of all the terms of the expression which to be subtracted and then the two expressions are added. Let us understand this through the following illustration.

Illustration – 2

Subtract: $\frac{3}{2}x^2y+\frac{4}{5}y-\frac{1}{3}x^2yz$ from

$$\frac{12}{5}x^2yz-\frac{3}{5}xyz+\frac{2}{3}x^2y.$$

Solution : We have,

$$\left(\frac{12}{5}x^2yz-\frac{3}{5}xyz+\frac{2}{3}x^2y\right)-\left(\frac{3}{2}x^2y+\frac{4}{5}y-\frac{1}{3}x^2yz\right)$$

$$=\frac{12}{5}x^2yz-\frac{3}{5}xyz+\frac{2}{3}x^2y-\frac{3}{2}x^2y-\frac{4}{5}y+\frac{1}{3}x^2yz$$

$$=\frac{12}{5}x^2yz+\frac{1}{3}x^2yz+\frac{2}{3}x^2y-\frac{3}{2}x^2y-\frac{3}{5}xyz-\frac{4}{5}y$$

$$=\frac{41}{15}x^2yz-\frac{5}{6}x^2y-\frac{3}{5}xyz-\frac{4}{5}y$$

Multiplication of Algebraic Expressions

In the multiplication of algebraic expressions, we are using the following rules :

(i) The product of two factors with like signs is positive and the product of two factors with unlike signs is negative

i.e. $(+)\times(+)=+$; $(+)\times(-)=-$

$(-)\times(+)=-$; $(-)\times(-)=+$

(ii) If a is any variable and m, n are positive integers, then $a^m \times a^n = a^{m+n}$

For example : $x^3 \times x^6 = x^{3+6} = x^9$

(iii) Distributive law of multiplication

i.e. $a(b+c) = ab + ac$

$(a+b)(c+d+e) = ac + ad + ae + bc + bd + be...$

Let us understand this through the following illustrations:

Illustration – 3

Multiply :

(i) $5x^2$, $17x^8$ **(ii)** $-3y^3$, y^2

(iii) $2p$, p^2q^3 **(iv)** $3x^4$, $-3x$, $2xy^3$

Solution :

(i) $5x^2 \times 17x^8 = (5\times 17)\times(x^2\times x^8)$

$= 85 \times x^{2+8} = 85x^{10}$

(ii) $-3y^3 \times y^2 = -3 \times y^{3+2} = -3y^5$

(iii) $2p \times p^2q^3 = 2\times(p\times p^2q^3) = 2\times(p^{1+2}q^3) = 2p^3q^3$

(iv) $3x^4\times(-3x)\times 2xy^3 = \{3\times(-3)\times 2\}\times(x^4\times x\times xy^3)$

$= -18\times(x^{4+1+1}y^3) = -18x^6y^3$

Illustration – 4

Multiply

(i) $(p+5)$ by $(p-3)$

(ii) $(3x+y)$ by $(3x-2y)$

(iii) $(x-y)$ by (x^2-y^2)

(iv) $(x+y^2+2xy)$ by $(2x^2-x-4)$

Solution : (i) $(p+5)(p-3) = p(p-3)+5(p-3)$

$= p\times p - 3p + 5p - 5\times 3 = p^2 + 2p - 15.$

(ii) $(3x+y)(3x-2y) = 3x(3x-2y) + y(3x-2y)$

$= 9x^2 - 6xy + 3xy - 2y^2$

$= 9x^2 - 3xy - 2y^2$

(iii) $(x-y)(x^2-y^2) = x(x^2-y^2) - y(x^2-y^2)$

$= x^3 - xy^2 - x^2y + y^3.$

(iv) $(x+y^2+2xy)(2x^2-x-4)$

$= x(2x^2-x-4) + y^2(2x^2-x-4) + 2xy(2x^2-x-4)$

$= 2x^3 - x^2 - 4x + 2x^2y^2 - xy^2 - 4y^2 + 4x^3y - 2x^2y - 8xy.$

Division of polynomial by Another polynomial :

If $p(x)$ and $q(x)$ are two polynomials such that degree of $p(x) >$ degree of $q(x)$ and $q(x) \neq 0$, then there exists unique polynomials $g(x)$ and $r(x)$ such that

$p(x) = g(x)\times q(x) + r(x)$,

where degree of $r(x)$ < degree of $g(x)$ or $r(x) = 0$. Then we say that on dividing $p(x)$ by $q(x)$, the quotient is $g(x)$ and remainder is $r(x)$. If remainder is zero (0), then $q(x)$ is a factor of $p(x)$.

Let us understand this through the following illustration.

Illustration – 5

Divide the polynomial $f(x) = 30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$. Also, find the quotient and remainder.

Solution : Using long divison method, we obtain

$$\begin{array}{r|l} 3x^2+2x-4 & 10x^2-3x-12 \\ \hline 30x^4+11x^3-82x^2-12x+48 & \\ 30x^4+20x^3-40x^2 & \\ \hline -\quad -\quad + & \\ -9x^3-42x^2-12x+48 & \\ -9x^3-6x^2+12x & \\ \hline +\quad +\quad - & \\ -36x^2-24x+48 & \\ -36x^2-24x+48 & \\ \hline +\quad +\quad - & \\ 0 & \end{array}$$

Clearly, quotient $q(x) = 10x^2 - 3x - 12$ and remainder $r(x) = 0$.

REMAINDER THEOREM

Let $f(x)$ be a polynomial of degree greater than or equal to 1. Then if $f(x)$ is divided by $(x-a)$, and a be any real number, then the remainder is $f(a)$.

Note that in $f(a)$, a is the value of x when $x - a = 0$.

Let us understand this through the following illustration.

Illustration – 6

Find the remainder when

(i) $x^3 + 2x^2 - 5x + 3$ is divided by $(x - 2)$.

(ii) $-x^4 - x^2 + 1$ is divided by $(x + 1)$.

(iii) $2x^3 - x^2 + 5x$ is divided by $(2x - 1)$.

Solution : (i) Let $p(x) = x^3 + 2x^2 - 5x + 3$

Remainder $= p(2) = 2^3 + 2 \times 2^2 - 5 \times 2 + 3$

$= 8 + 8 - 10 + 3 = 9.$

(ii) Let $p(x) = -x^4 - x^2 + 1$

Now $x + 1 = x - (-1)$

Hence remainder $= p(-1) = -(-1)^4 - (-1)^2 + 1 = 1$

(iii) Let $p(x) = 2x^3 - x^2 + 5x$

Put $2x - 1 = 0$, then $x = \frac{1}{2}$

$$\text{So, remainder} = p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right)$$

$$= 2 \times \frac{1}{8} - \frac{1}{4} + \frac{5}{2} = \frac{1}{4} - \frac{1}{4} + \frac{5}{2} = \frac{5}{2}$$

FACTOR THEOREM

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be any real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Conversly, if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$.

Let us understand this through the following illustration :

Illustration – 7

Is $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$?

Solution : Let $p(x) = x^3 + 3x^2 - 12x + 4$

Now $p(2) = (2)^3 + 3(2)^2 - 12 \times 2 + 4 = 8 + 12 - 24 + 4 = 0$.

Hence $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$.

Value of a Polynomial

A polynomial in any variable x is symbolically may be repersented by $p(x), f(x), \ldots$, (etc.)

The value of a polynomial $p(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $p(\alpha)$.

For example : Consider the polynomial

$p(x) = 5x^3 - 2x^2 + 3x - 2$

If we replace x by 1 everywhere in $p(x)$, we get

$p(1) = 5 \times (1)^3 - 2 \times (1)^2 + 3 \times (1) - 2$

$= 5 - 2 + 3 - 2 = 4$

So, we say that the value of $p(x)$ at $x = 1$ is 4.

ZERO OF A POLYNOMIAL

A number α is a zero of a polynomial $f(x)$, if $f(\alpha) = 0$.

Number of zero(s) of a polynomial = Degree of the polynomial.

Hence number of zero(s) of a linear polynomial, quadratic polynomial, cubic polynomial are 1, 2, 3 respectively.

Relation between Zero(s) and Coefficient of a Polynomial

(i) **Linear Polynomial ($ax + b$, $a \neq 0$) :**

If α be the zero of linear polynomial $ax + b$, then

$$\alpha = -\frac{b}{a}.$$

(ii) **Quadratic Polynomial ($ax^2 + bx + c$, $a \neq 0$):**

If α and β are zeros of quadratic equation $ax^2 + bx + c$ with $a \neq 0$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha \cdot \beta = \frac{c}{a}$$

(iii) **Cubic Polynomial ($ax^3 + bx^2 + cx + d$, $a \neq 0$):**

If α, β, γ are the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ with $a \neq 0$; then

$$\alpha + \beta + \gamma = -\frac{b}{a};$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

FACTORISATION OF POLYNOMIAL

The process of writing a given polynomial as the product of other polynomials of lower degree which does not express further as a product of other polynomials of more lower degree, is called the factorisation of polynomial.

Methods of Factorisation

METHOD-1 :

When each term of a polynomial has a common factor, we divide each term by this factor and take it out as a multiple.

For example: $54a^5b^2 - 12ab^6c^2 = 6ab^2 (9a^4 - 2b^4c^2)$

METHOD-2:
Sometimes in a given polynomial, it is not possible to take out a common factor directly. However the terms of the expression are grouped in such a manner that we may have a common factor. This can be factorised as discussed in the following illustration.

Illustration – 8

Factorise : $5xy^2 + 2x^3 - 15x^3y^3 - 6x^5y$.

Solution : $5xy^2 + 2x^3 - 15x^3y^3 - 6x^5y$
$= (5xy^2 + 2x^3) - (15x^3y^3 + 6x^5y)$
$= x(5y^2 + 2x^2) - 3x^3y(5y^2 + 2x^2)$
$= (5y^2 + 2x^2)(x - 3x^3y) = x(5y^2 + 2x^2)(1 - 3x^2y)$

METHOD-3 : Factorisation of Quadratic Trinomial
To factorise the quadratic trinomial $ax^2 + bx + c$, $a \neq 0$, split b in two numbers such that their sum is equal to b and their product is equal to the product of a and c. Then we follow as discussed in the following illustrations.

Illustration – 9

Factorise the following equation.
(i) $2x^2 - x - 3$ **(ii)** $5x^2 - 7x - 6$
(iii) $x^2 - 2x - 15$

Solution : (i) $2x^2 - x - 3 = 2x^2 + (-1)x + (-3)$
$= 2x^2 + (-3 + 2)x + (-3) = 2x^2 - 3x + 2x - 3$
$= x(2x - 3) + 1(2x - 3) = (2x - 3)(x + 1)$

(ii) $5x^2 - 7x - 6 = 5x^2 - 10x + 3x - 6$
$= 5x(x - 2) + 3(x - 2) = (x - 2)(5x + 3)$

Note that in short split –7 in two parts (i.e. –10 and 3) such that their sum is –7 and their product (i.e. -10×3) is equal to the product 5 and – 6.

(iii) $x^2 - 2x - 15 = x^2 - 5x + 3x - 15$
$= x(x - 5) + 3(x - 5)$
$= (x - 5)(x + 3)$

METHOD-4 : Factorisation of Polynomial using Algebraic Identities
You can also factorise the polynomial using the algebraic identities :

(i) $a^2 - b^2 = (a + b)(a - b)$

(ii) $a^2 + b^2 + 2ab = (a + b)^2$

(iii) $a^2 + b^2 - 2ab = (a - b)^2$

(iv) $a^3 + b^3 + 3a^2b + 3ab^2$
$= a^3 + b^3 + 3ab(a + b) = (a + b)^3$

(v) $a^3 - b^3 - 3a^2b + 3ab^2$
$= a^3 - b^3 - 3ab(a - b) = (a - b)^3$

(vi) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(vii) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(viii) $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2$

(ix) $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(x) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

This can be easily understand through the following illustrations :

Illustration – 10

Use a suitable identity to factorise each of the following :
(a) $4x^2 + 4xy + y^2$ **(b)** $25p^2 - 36q^2$
(c) $x^2 - y^2 + 2x + 1$ **(d)** $4a^2 - 4b^2 + 4a + 1$

Solution :

(a) Consider $4x^2 + 4xy + y^2 = (2x)^2 + 2(2x)(y) + (y)^2$
$= (2x + y)^2 = (2x + y)(2x + y)$
(Using Identity (ii))

(b) Consider $25p^2 - 36q^2 = (5p)^2 - (6q)^2$
$= (5p + 6q)(5p - 6q)$
(Using Identity (i))

(c) $x^2 - y^2 + 2x + 1 = x^2 + 2x + 1 - y^2$
$= (x + 1)^2 - (y)^2 = (x + 1 + y)(x + 1 - y)$
(Using Identities (i) and (ii))

(d) $4a^2 - 4b^2 + 4a + 1 = (4a^2 + 4a + 1) - 4b^2$
$= \{(2a)^2 + 2(2a)(1) + (1)^2\} - (2b)^2$
$= (2a + 1)^2 - (2b)^2$
$= (2a + 1 + 2b)(2a + 1 - 2b)$.
(Using Identities (i) and (ii))

Illustration – 11

Factorise : $(a + b - c)^3 + (a - b + c)^3 - 8a^3$

Solution : $(a + b - c)^3 + (a - b + c)^3 - 8a^3$
$= (a + b - c)^3 + (a - b + c)^3 + (-2a)^3$
$= 3(a + b - c)(a - b + c)(-2a)$
$[\because (a + b - c) + (a - b + c) + (-2a) = 0)]$
$= -6a(a + b - c)(a - b + c)$

HCF AND LCM

HCF (or GCD) of Two Polynomials

HCF of two polynomials $p(x)$ and $q(x)$ is that divisor which has the highest degree among all common divisors.

Note that the coefficient of the highest degree divisor is positive.

GP_3386

Illustration – 12

Find the HCF of following pair of polynomials:
$p(x) = (x^2 - 9)(x - 3)$; $q(x) = x^2 + 6x + 9$

Solution : $p(x) = (x^2 - 9)(x - 3)$

$= (x+3)(x-3)(x-3) = (x+3)(x-3)^2$

$q(x) = x^2 + 6x + 9$

$= x^2 + 3x + 3x + 9$

$= (x+3)(x+3) = (x+3)^2$

HCF of p (x) and q.(x) $= x + 3$

LCM of Two Polynomials

LCM of two polynomials $p(x)$ and $q(x)$ is a polynomial of lowest degree of which $p(x)$ and $q(x)$ both are multiples.

Illustration – 13

Find the LCM of $p(x) = (x+3)(x-2)^2$ and $q(x) = (x-2)(x-6)$.

Solution : $p(x) = (x+3)(x-2)^2$

$q(x) = (x-2)(x-6)$

HCF of p(x) and q(x) $= (x-2)$

$$\text{LCM of p(x) and q(x)} = \frac{p(x) \times q(x)}{\text{HCF}}$$

$$= \frac{(x+3)(x-2)^2 \times (x-2)(x-6)}{(x-2)} = (x+3)(x-2)^2(x-6)$$

Relation Between Two Polynomials and their HCF and LCM

(HCF) × (LCM) = (First Polynomial) × (Second Polynomial)

Illustration – 14

If HCF and LCM of two polynomials are $(x-1)(x+3)$ and $(x-1)^2(x+2)^3(x+3)$ and one of the two polynomials is $(x-1)(x+2)^3(x+3)$, then find the other polynomial?

Solution : (HCF) × (LCM) = (First polynomial) × (Second polynomial)

$$\therefore \text{Second polynomial} = \frac{(\text{HCF}) \times (\text{LCM})}{\text{First Polynomial}}$$

$$= \frac{(x-1)(x+3)(x-1)^2(x+2)^3(x+3)}{(x-1)(x+2)^3(x+3)}$$

$= (x+3)(x-1)^2$.

RATIONAL EXPRESSIONS

A rational expression is a ratio of two non-zero polynomials . Here are some examples of rational expressions

$$\frac{6}{x-1}, \frac{x^2-1}{x^2+5}, \frac{n^2+18n+1}{n^2-n-6}, \frac{4z^2+6z-10}{1}$$

The last example can be written as $4z^2 + 6z - 10$. Hence a polynomial is also a rational expression but every rational expression is not a polynomial.

Thus $\frac{p(x)}{q(x)}$ is a rational polynomial expression where $p(x)$ and $q(x)$ are both polynomial with a restriction that $q(x)$ is not a zero polynomial.

$\frac{6x^2 - 1}{2x + 1}$ is a rational expression and

$\frac{2x^2 + 5\sqrt{x} - 3}{3x - 1}$ is not a rational expression as numerator is not a polynomial.

Rational Expression in Lowest Terms: A rational expression $\frac{P(x)}{Q(x)}$ is said to be in its lowest terms if all the common factors of its numerator and denominator have been cancelled or in other words the HCF of numerator and denominator is1.

Illustration – 15

Reduce $\frac{x^2 - 25}{5x - x^2}$ to lowest term.

Solution :

$$\frac{x^2 - 25}{5x - x^2} = \frac{x^2 - 5^2}{5x - x^2} = \frac{(x-5)(x+5)}{-x(x-5)} = -\frac{x+5}{x}$$

We cannot further cancel any thing in $-\frac{x+5}{x}$.

So lowest form of $\frac{x^2 - 25}{5x - x^2}$ is $-\frac{x+5}{x}$.

OPERATIONS ON RATIONAL EXPRESSIONS

Like operations on rational numbers we can perform operations (Addition, Subtraction, Multiplication, Division) on rational expressions.

Addition and Subtraction of Rational Expressions

Let $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ are to be added. Here we see that both the rational expressions have different denominators.

$$\frac{p(x)}{q(x)}+\frac{r(x)}{s(x)}=\frac{p(x).s(x)+r(x).q(x)}{q(x).s(x).}.$$

If denominator of both rational expressions are equal then

$$\frac{p(x)}{q(x)}+\frac{r(x)}{q(x)}=\frac{p(x)+r(x)}{q(x)}$$

Difference of two rational expression can also be written as

$$\frac{p(x)}{q(x)}-\frac{r(x)}{s(x)}=\frac{p(x).s(x)-r(x).q(x)}{q(x).s(x).}$$

[with different denominators]

$$\frac{p(x)}{q(x)}-\frac{r(x)}{q(x)}=\frac{p(x)-r(x)}{q(x)}$$ [with equal denominators]

Illustration – 16

(a) **Add** $\frac{y}{xy+1}$ **and** $\frac{x-y}{xy+1}$

(b) **Subtract** $\frac{x-y}{xy+1}$ **from** $\frac{y}{xy+1}$

Solution :

(a) $\frac{y}{xy+1}+\frac{x-y}{xy+1}=\frac{y+x-y}{xy+1}=\frac{x}{xy+1}$

(b) $\frac{y}{xy+1}-\frac{x-y}{xy+1}=\frac{y-x+y}{xy+1}=\frac{2y-x}{xy+1}$

Illustration – 17

(a) **Subtract** $\frac{z-1}{z+2}$ **from** $\frac{2}{z+1}$

(b) **Add** $\frac{1}{x+3}$ **and** $\frac{2}{x-3}$

Solution : **(a)** $\frac{2}{z+1}-\frac{z-1}{z+2}$

$$=\frac{2(z+2)-(z-1)(z+1)}{(z+1)(z+2)}=\frac{2z+4-\left(z^2-1\right)}{z^2+3z+2}$$

$$=\frac{2z+4-z^2+1}{z^2+3z+2}=\frac{-z^2+2z+5}{z^2+3z+2}$$

(b) $\frac{1}{x+3}+\frac{2}{x-3}$

$$=\frac{(x-3)+2(x+3)}{(x+3)(x-3)}=\frac{x-3+2x+6}{x^2-9}$$

$$=\frac{3x+3}{x^2-9}=\frac{3(x+1)}{\left(x^2-9\right)}$$

Multiplication of Rational Expressions

Multiplication of rational expressions $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ is given by $\frac{p(x)}{q(x)}\times\frac{r(x)}{s(x)}=\frac{p(x).r(x)}{q(x).s(x)}$

Here we see that multiplication of rational expressions results in a rational expression.

Illustration – 18

Multiply $\frac{x+1}{x}$ **and** $\frac{x+2}{x^2-1}$.

Solution : $\left(\frac{x+1}{x}\right)\times\left(\frac{x+2}{x^2-1}\right)$

$$=\frac{(x+1)(x+2)}{x(x-1)(x+1)}=\frac{x+2}{x(x-1)}$$

Division of Rational Expressions:

If $\frac{p(x)}{q(x)}$ and $\frac{r(x)}{s(x)}$ are two non-zero rational expression

then

GP_3386

$$\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} \times \frac{s(x)}{r(x)}$$

Division of two non-zero rational expressions is also a rational expression.

Illustration – 19

Perform the indicated operations:

(a) $\dfrac{m^2 - 9}{m^2 + 5m + 6} \div \dfrac{3 - m}{m + 2}$ **(b)** $\dfrac{y^2 + 5y + 4}{\frac{y^2 - 1}{y+5}}$

Solution : **(a)** $\dfrac{m^2-9}{m^2+5m+6} \div \dfrac{3-m}{m+2}$

$$= \frac{(m-3)(m+3)}{(m+2)(m+3)} \div \frac{3-m}{m+2} = \frac{(m-3)}{(m+2)} \div \frac{3-m}{m+2}$$

$$= -\frac{(m-3)}{(m+2)} \times \frac{(m+2)}{(m-3)} = -1$$

(b) $\dfrac{y^2+5y+4}{\frac{y^2-1}{y+5}} = \dfrac{y^2+5y+4}{1} \times \dfrac{y+5}{y^2-1}$

$$= \frac{y^2 + 4y + y + 4}{1} \times \frac{y+5}{(y+1)(y-1)}$$

$$= \frac{(y+4)(y+1)}{1} \times \frac{y+5}{(y+1)(y-1)}$$

$$= \frac{(y+4)(y+5)}{y-1}$$

ADDITIVE INVERSE OF A RATIONAL EXPRESSION

A rational expression – $\dfrac{p(x)}{q(x)}$ is known as additive inverse of $\dfrac{p(x)}{q(x)}$. Sum of a rational expression and its additive inverse is zero i.e. $\dfrac{p(x)}{q(x)} + \left[-\dfrac{p(x)}{q(x)}\right] = 0$

MULTIPLICATIVE INVERSE OF A RATIONAL EXPRESSION

For every rational expression; $\dfrac{p(x)}{q(x)}$,

$q(x) \neq 0$; there exist a rational expression $\dfrac{r(x)}{s(x)}$ such that $\dfrac{p(x)}{q(x)} \times \dfrac{r(x)}{s(x)} = 1$, Such two rational expressions are called multiplicative inverse of each other.

LEVEL - 1

1. If $a + b + c = 0$, then $a^3 + b^3 + c^3 =$
(a) $2abc$ (b) $a^2(b^3 + c^3)$
(c) $a^2 + b^2 + c^2$ (d) $3abc$

2. Factorize : $(a-b)^3 + (b-c)^3 + (c-a)^3$
(a) $3(a+b)(a-b)(a+c)$
(b) $3(abc)$
(c) $3(a-b)(b-c)(c-a)$
(d) $3(a+b)(b+c)(c+a)$

3. Factors of polynomial $x^3 - 3x^2 - 10x + 24$ are
(a) $(x-2)(x+3)(x-4)$ (b) $(x+2)(x+3)(x+4)$
(c) $(x+2)(x-3)(x-4)$ (d) $(x-2)(x-3)(x-4)$

4. Which of the following is/are a polynomial?
(a) $x^3 - 4x^2 + 5\sqrt{x} + 1$ (b) $x^{-3} + 5x^2 + 2$
(c) $x^{-2} + 4$ (d) x

5. If $x - a$ is a factor of $x^3 - 3x^2a + 2a^2x + b$, then the value of 'b' is
(a) 1 (b) 0 (c) 2 (d) 4

6. The degree of a polynomial A is 7 and that of polynomial AB is 56, then find the degree of polynomial B.
(a) 49 (b) 42 (c) 50 (d) 48

7. One of the dimensions of the cuboid whose volume is $16x^2 - 26x + 10$ is
(a) 2 (b) $(8x-5)$
(c) $(x-1)$ (d) All of these

8. A .....polynomial has no zero.
(a) cubic (b) constant
(c) quadratic (d) linear

9. Which of the following is a biquadratic polynomial?
(a) $4x^3 + 3x^2 - 5x + 3$
(b) $\frac{1}{x^4} + 5x^2 - 2$
(c) $x^2y^2 + xy^3 + y^4 - 8xy + y^2 + 7$
(d) $4x^2 + 5x - 2$

10. Which of the following polynomials has $(x + 1)$ as a factor?
(a) $x^3 + x^2 + x + 1$
(b) $x^4 + x^3 + x^2 + x + 1$
(c) $x^4 + 3x^2 + 3x^2 + x + 1$
(d) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

11. The expression $2x^3 + 3px^2 - 4x + p$ has a remainder of 5 when divided by $x + 2$, then the value of p is
(a) 0 (b) 1 (c) 2 (d) –1

12. Find the value of a for which $(x + a)$ is a factor of the polynomial $f(x) = x^3 + ax^2 - 2x + a + 6$.
(a) -3 (b) -4
(c) 2 (d) None of these

13. Find the remainder when
$\lfloor 1 + \lfloor 2 + \lfloor 3 + \lfloor 4 + ... + \lfloor 100$ is divided by 6.
(a) 3 (b) 4 (c) 2 (d) 1

14. Find the remainder when 7^{99} is divided by 2400
(a) 1 (b) 49 (c) 343 (d) 7

15. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then remainder is :
(a) 135 (b) -135 (c) 63 (d) -63

16. Find the remainder when 5^{2450} is divided by 126
(a) 5 (b) 25 (c) 125 (d) 1

17. Find the remainder when $\left(32^{32}\right)^{32}$ is divided by 9
(a) 4 (b) 7 (c) 1 (d) 2

18. Find the remainder when $\left(33^{34}\right)^{35}$ is divided by 7
(a) 5 (b) 4 (c) 6 (d) 2

19. If $(x^3 - 5x^2 + 4p)$ is divisible by $(x + 2)$, then the value of p is
(a) 7 (b) -2 (c) 3 (d) -7

20. If $x^3 + 3x^2 + 4x + k$ contains $(x + 6)$ as a factor, the value of k is :
(a) 66 (b) 33 (c) 132 (d) 36

21. What will be the remainder when $(35)^{37}$ is divided by 9?
(a) 3 (b) 8 (c) 5 (d) 7

22. Find the remainder when 919293949596979899 is divided by 16
(a) 3 (b) 13 (c) 11 (d) 8

23. Find the remainder when 1234 ... 41 digits is divided by 8.
(a) 1 (b) 2 (c) 3 (d) 4

24. If $(x + 2)$ and $(x - 1)$ are the factors of $(x^3 + 10x^2 + mx + n)$, the values of m and n are :
(a) $m = 5, n = -3$ (b) $m = 17, n = -8$
(c) $m = 7, n = -18$ (d) $m = 23, n = -19$

25. If $x = 2$ and $x = 0$ are roots of the polynomials $f(x) = 2x^3 - 5x^2 + ax + b$. Then values of a and b respectively are
(a) 2, 0 (b) 1, 2 (c) $-1, 1$ (d) 0, 3

GP_3386

26. If two zeroes of the polynomial $p(x) = x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find the third zero.
(a) 1 (b) 0 (c) 3 (d) 4

27. Find the polynomial having $5 \pm \sqrt{3}$ as its zeroes.
(a) $x^2 - 10x + 22$ (b) $3x^2 + 4 + 5x$
(c) $6x^2 - 7x - 3$ (d) $x^2 - 10x + 75$

28. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$, find a and b.
(a) $1, \pm\sqrt{2}$ (b) $-1, \sqrt{2}$
(c) $\pm 1, \sqrt{2}$ (d) $\pm 1, -\sqrt{2}$

29. Which one of the following statements is correct?
(a) If $x^6 + 1$ is divided by $x + 1$, then the remainder is -2
(b) If $x^6 + 1$ is divided by $x - 1$, then the remainder is 2
(c) If $x^6 + 1$ is divided by $x + 1$, then the remainder is 1
(d) If $x^6 + 1$ is divided by $x - 1$, then the remainder is -1

30. If one zero of the polynomial $f(x) = 5x^2 + 13x + k$ is the reciprocal of the other, then the value of k is :
(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

31. The HCF of $45(2x^4 - x^3 - x^2)$ and $75(8x^5 + x^2)$ is
(a) $15x(2x + 1)$ (b) $15x^2(2x + 1)$
(c) $15x(2x + 1)^2$ (d) $15x^2(2x + 1)^2$

32. The LCM of $20x^2y(x^2 - y^2)$ and $35xy^2(x - y)$ is
(a) $140x^2y^2(x - y)$ (b) $140xy(x^2 - y^2)$
(c) $140x^2y^2(x^2 - y^2)$ (d) None

33. Find the remainder when $5p^3 - 13p^2 + 21p - 14$ is divided by $(3 - 2p + p^2)$.
(a) -5 (b) -15 (c) -10 (d) 5

34. The zeros of a quadratic polynomial $f(x) = x^2 - 7x + k$ are α and β such that $\alpha - \beta = 3$. Find the value of 'k'.
(a) 10 (b) 2 (c) 5 (d) 15

35. Which of the following is the quadratic polynomial whose zeros are $\frac{1}{3}$ and $\frac{-2}{5}$?
(a) $15x^2 + x - 2$ (b) $15x^2 + 5x - 6$
(c) $15x^2 - 5x + 6$ (d) $15x^2 - x + 2$

36. If the sum of the zeros of the polynomial $f(x) = (k^2 - 14)x^2 - 2x - 12$ is 1, which is one of the possible values of 'k'?
(a) $\sqrt{14}$ (b) -14 (c) 2 (d) ± 4

37. If $p(y) = 3y^4 - 5y^3 + y^2 + 8$, then $p(-1)$ will be
(a) 2 (b) 15 (c) 17 (d) -17

38. For what value of k is the polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$ exactly divisible by $(x + 2)$?
(a) 1 (b) -1 (c) 2 (d) -2

39. If α and β be the zeroes of the polynomial $p(x) = x^2 - 5x + 2$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 3\alpha\beta$.
(a) $\frac{5}{3}$ (b) $\frac{-7}{2}$ (c) $\frac{-5}{2}$ (d) $\frac{-4}{5}$

40. Twice the product of the zeroes of the polynomial $23x^2 - 26x + 161$ is 14p. Then p is
(a) 3 (b) 1 (c) $\frac{5}{2}$ (d) (-1)

41. If the sum of the square of zeroes of the quadratic polynomial $f(x) = x^2 - 8x + k$ is 40, find the value of k.
(a) 3 (b) 6 (c) 9 (d) 12

42. If α and β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that $\alpha - \beta = 1$, the value of K is-
(a) 12 (b) 6 (c) 4 (d) 1

43. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then the value of k is
(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

44. Perform the indicated operation and reduce the answer to lowest terms.

$$\frac{x^2 - 5x - 14}{x^2 - 3x + 2} \times \frac{x^2 - 4}{x^2 - 14x + 49}$$

(a) $\frac{(x+3)^2}{(x-1)(x-7)}$ (b) $\frac{(x+1)^2}{(x-1)(x-7)}$

(c) $\frac{(x+2)^2}{(x-1)(x-7)}$ (d) $\frac{(x-2)^2}{(x-1)(x+7)}$

45. Simplify: $\dfrac{y^2 + 5y + 4}{\dfrac{y^2 - 1}{y + 5}}$

(a) $\frac{(y+3)(y+5)}{y+1}$ (b) $\frac{(y+4)(y+5)}{y-1}$

(c) $\frac{(y-4)(y+5)}{(y+1)}$ (d) $\frac{(4+3)}{(y-1)}$

46. Simplify: $\frac{x+10}{(3x+8)^3}+\frac{x}{(3x+8)^2}$

(a) $\frac{3x^2+9x+10}{(3x+8)^3}$ (b) $\frac{2x^2+9x+10}{(3x+8)^2}$

(c) $\frac{3x^2+10}{(3x+8)^3}$ (d) None of these

47. Simplify: $\frac{1}{x^2-13x+42}+\frac{x+1}{x-6}-\frac{x^2}{x-7}$

(a) $\frac{-x^3+7x^2-6x-6}{(x-6)(x-7)}$ (b) $\frac{-x^3-7x^2+6x}{(x-6)(x-7)}$

(c) $\frac{+x^3-7x^2-6x}{(x-6)(x-7)}$ (d) None of these

LEVEL - 2

1. If $4x^4-3x^3-3x^2+x-7$ is divided by $1-2x$ then remainder will be –

(a) $\frac{57}{8}$ (b) $-\frac{59}{8}$ (c) $\frac{55}{8}$ (d) $-\frac{55}{8}$

2. If the polynomials $(2x^3+ax^2+3x-5)$ and (x^3+x^2-2x+a) leave the same remainder when divided by $(x-2)$, find the value of a.

(a) -8 (b) 4 (c) -3 (d) $\frac{6}{3}$

3. For what value of k is the polynomial $(2x^4+3x^3+2kx^2+3x+6)$ exactly divisible by $(x+2)$?

(a) -1 (b) 3 (c) 5 (d) -2

4. If $(x^{100}+2x^{99}+k)$ is divisible by $(x+1)$, then the value of k is

(a) 1 (b) 2 (c) -2 (d) -3

5. Let A and B are the remainders when the polynomial $y^3+2y^2-5ay-7$ and $y^3+ay^2-12y+6$ are divided by $y+1$ and $y-2$ respectively. If $2A+B=6$, find the value of a.

(a) -1 (b) 1 (c) -2 (d) 2

6. If $(x-a)$ and $(x-2)$ are two factors of $x^3-ax^2+14x+b$, the values of a and b are

(a) $a=6, b=7$ (b) $a=-8, b=-7$

(c) $a=7, b=-8$ (d) $a=7, b=8$

7. If $a+b+c=14$, $a^2+b^2+c^2=74$ and $a^3+b^3+c^3=434$, then the value of abc is

(a) 64 (b) 74 (c) 196 (d) 84

8. Let r_1 and r_2 be the remainders when the polynomials

$p(x)=x^3+x^2-5kx-7$

and $q(x)=x^3+kx^2-12x+6$

are divided by $x+1$ and $x-2$ respectively.

If $2r_1-2r_2=10$, the value of k is.

(a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{5}{7}$ (d) $\frac{7}{5}$

9. If two roots of equation $2x^2-7x+12=0$ are α & β, then find $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=?$

(a) $\frac{3}{25}$ (b) $\frac{7}{18}$ (c) $\frac{1}{6}$ (d) $\frac{1}{24}$

10. Find the remainder when 777777 ..... 363 times is divided by 11

(a) 0 (b) 7 (c) 1 (d) 3

11. 777777..... 129 times is divided by 37 the remainder will be?

(a) 0 (b) 1 (c) 2 (d) 3

12. Find the Remainder then 2^{591} is divided by 255.

(a) 225 (b) 128 (c) 127 (d) 64

13. Find the Remainder when 143^{321} is divided by 5

(a) 1 (b) 3 (c) 4 (d) 7

14. Find the remainder when $10^1+10^2+10^3+$ .....10^{100} is divided by 6

(a) 4 (b) 6 (c) 2 (d) 3

15. Other two zero(es) of $2x^4-3x^3-3x^2+6x-2$, if two of its zero(es) are $\sqrt{2}$ and $-\sqrt{2}$ are

(a) $\frac{1}{2}$ and 1 (b) 0 and 1 (c) 1 and 2 (d) $\frac{1}{2}$ and –1

16. If $x=\frac{4}{3}$ is a root of the polynomial $f(x)=6x^3-11x^2+kx-20$, then find the value of k.

(a) 10 (b) 19 (c) -5 (d) 3

17. Find α and β if $x+1$ and $x+2$ are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$

(a) 3, –1 (b) –1, 0 (c) 0, –3 (d) 5, 6

18. What must be subtracted from $x^3 - 6x^2 - 15x + 80$, so that the result is exactly divisible by $x^2 + x - 12$?

(a) $2x+3$ (b) $80-x$

(c) $4x-4$ (d) $3x^2+9x-8$

19. What must be added to the polynomial $f(x) = x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$?

(a) x^2+2 (b) x^3+1 (c) $x-2$ (d) $x+2$

20. Find the value of b for which the polynomial $2x^3 + 9x^2 - x - b$ is exactly divisible by $2x + 3$.

(a) 2 (b) –15 (c) 15 (d) –10

21. For what value of 'm' is one zero of the polynomial $(m^2+9)x^2 + 13x + 6m$, the reciprocal of the other?

(a) 9 (b) –3 (c) 6 (d) 3

22. Find a quadratic polynomial, whose sum and product of zeros are $\frac{-10}{\sqrt{3}}$ and 7 respectively

(a) $x^2 + \frac{10}{\sqrt{3}}x + \frac{7}{\sqrt{3}}$ (b) $\sqrt{3}x^2 - 10x + 7\sqrt{3}$

(c) $\sqrt{3}x^2 + 10x + 7\sqrt{3}$ (d) $x^2 + 10x + 7\sqrt{3}$

23. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$ respectively. Find g(x).

(a) x^2+2x+1 (b) x^2-2x+1

(c) $x + x + 1$ (d) x^2-x+1

24. The polynomials $(ax^3 + 3x^2 - 3)$ and $(2x^3 - 5x + a)$ when divided by $(x - 4)$ leave the same remainder. The value of a is

(a) 1 (b) –1 (c) 2 (d) –2

25. The value of k such that the quadratic polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of the zeroes as half of their product, is

(a) 2 (b) 3 (c) – 5 (d) 5

26. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, then the other zeroes are

(a) 5, 7 (b) –5, 7 (c) –5, –7 (d) 5, –7

27. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21 then find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

(a) 3, –2, –3 (b) 3, 2, 4 (c) 1, 2, –1 (d) –2, –1, –3

28. If one zero of $p(x) = 3x^2 - 8x + 2k + 1$ is seven times of the other, then find the zeroes and the value of k.

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$ (c) $-\frac{2}{3}$ (d) 7

29. If α, β, γ are zeroes of $p(x) = 6x^3 + 3x^2 - 5x + 1$, find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$.

(a) 5 (b) 4 (c) 7 (d) 11

30. If p, q and r are the zeroes of the polynomial $f(x) = ax^3 + bx^2 + cx + d$, then the value of $\frac{1}{p} + \frac{1}{q} + \frac{1}{r}$ is

(a) $\frac{-b}{a}$ (b) $\frac{c}{a}$ (c) $-\frac{c}{d}$ (d) $\frac{c}{d}$

31. Simplify:

$$\frac{8x^2 - 20^x - 48}{3x^2 - 27x + 60} \times \frac{9x^2 - 27x - 90}{16x^2 + 56x + 48}$$

(a) $\frac{3}{2}$ (b) $\frac{9}{8}$ (c) $\frac{4}{3}$ (d) $\frac{4}{9}$

32. Simplify: $\frac{3x^2 - 10x - 8}{6x^2 + x - 5} \times \frac{16x^2 + 24x + 9}{3x^2 + 13x - 10} \times \frac{6x^2 + 25x - 25}{4x^2 - 13x - 12}$

(a) $\frac{(3x-2)(4x+3)}{(x+1)(3x+2)}$ (b) $\frac{(3x-2)(x-4)}{(x+5)(3x+2)}$

(c) $\frac{(3x+2)(4x+3)}{(x+1)(3x-2)}$ (d) $\frac{(3x+2)(x-4)}{(x+5)(3x-2)}$

33. Simplify:

$$\left[\frac{x^3+y^3}{(x-y)^2+3xy}\right] \div \left[\frac{(x+y)^2 - 3xy}{x^3 - y^3}\right] \times \left[\frac{xy}{x^2 - y^2}\right]$$

(a) xy (b) $x+y$ (c) $x-y$ (d) $(x+y)^2$

34. Simplify:

$$\left[\frac{1}{1+a} + \frac{2a}{1-a^2}\right] \times \frac{(a^2+4a-5)}{(a^2+10a+25)}$$

(a) $\frac{-1}{a+1}$ (b) $\frac{1}{1-a}$ (c) $\frac{1}{a+5}$ (d) $\frac{-1}{a+5}$

Hints and Solutions

Level 1

1. (d) $\because \quad a^3 + b^3 + c^3 - 3abc$
$= (a+b+c) \times (a^2 + b^2 + c^2 - ab - bc - ca)$
$\because \quad a + b + c = 0$
$\therefore \quad a^3+b^3+c^3-3abc=0 \Rightarrow a^3+b^3+c^3=3abc$

2. (c) $\because \quad (a-b) + (b-c) + (c-a) = 0$
$\therefore \quad (a-b)^3 + (b-c)^3 + (c-a)^3$
$= 3(a-b)(b-c)(c-a)$

3. (a) $x^3 - 3x^2 - 10x + 24$
$= (x-2)(x^2 - x - 12) = (x-2)(x-4)(x+3)$

4. (d) Only x is a polynomial

5. (b) Since $(x-a)$ is a factor
$\therefore \quad (a)^3 - 3(a)^2 . a + 2a^2(a) + b = 0$
$\Rightarrow \quad a^3 - 3a^3 + 2a^3 + b = 0 \Rightarrow b = 0$

6. (a) The degree of polynomial $B = 56 - 7 = 49$

7. (d) $16x^2 - 26x + 10 = 2(8x^2 - 13x + 5)$
$= 2(8x^2 - 8x - 5x + 5) = 2(8x-5)(x-1)$

8. (b) Constant

9. (c) A biquadratic polynomial is a polynomial of the fourth degree.
Out of all the given options, option (c) is a polynomial of the fourth degree.

10. (a) $x^3 + x^2 + x + 1 = x^2(x+1) + 1(x+1)$
$= (x+1)(x^2+1)$

11. (b) $2(-2)^3 + 3p(-2)^2 - 4(-2) + p = 5$
$\Rightarrow \quad 13p = 13 \Rightarrow p = 1.$

12. (d) $(x+a)$ is a factor of
$f(x) = x^3 + ax^2 - 2x + a + 6.$
$\Leftrightarrow f(-a) = 0 \qquad [\because x + a = 0 \Leftrightarrow x = -a]$
$\Leftrightarrow (-a)^3 + a(-a)^2 - 2(-a) + a + 6 = 0$
$\Leftrightarrow 3a = -6 \Leftrightarrow a = -2$
Hence, the required value of a is -2.

13. (a) $1! + 2! + 3! + 4! + \ldots\ldots\ldots\ldots + 100!$

$$\text{Rem}\left[\frac{1!}{6}\right] = 1; \quad \text{Rem}\left[\frac{2!}{6}\right] = 2$$

$$\text{Rem}\left[\frac{3!}{6}\right] = 0; \quad \text{Rem}\left[\frac{4!}{6}\right] = 0$$

After 2!, every term is a multiple of 6.
Hence,

$$\text{remainder}\left[\frac{3! + 4! + 5! + 6! + \ldots\ldots\ldots + 100!}{6}\right] = 0$$

Required remainder $= 1 + 2 + 0 = 3.$

14. (c) $7^1 \longrightarrow 7; \; 7^2 \longrightarrow 49$
$7^3 \longrightarrow 343; \; 7^4 \longrightarrow 2401$

$$\text{Rem}\left[\frac{7^{99}}{2400}\right] = \text{Rem}\left[\frac{(7^4)^{24} \times 7^3}{2400}\right]$$

$$= \text{Rem}\left[\frac{1 \times 343}{2400}\right] = 343$$

15. (d) $5x^3 + 5x^2 - 6x + 9$
We know, when $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$.
$f(x) = 5x^3 + 5x^2 - 6x + 9$
So, remainder by $x + 3 = f(-3)$
$f(-3) = 5(-3)^3 + 5(-3)^2 - 6 \times (-3) + 9$
$= -135 + 45 + 18 + 9 = -63.$

16. (b) $5^3 = 125$

$$\text{Rem}\left[\frac{5^3}{126}\right] = -1 \text{ or } 125$$

$$\text{So, } \text{Rem}\left[\frac{5^6}{126}\right] = -1 \times -1 = 1$$

$$\text{Rem}\left[\frac{5^{2450}}{126}\right] = \text{Rem}\left[\frac{(5^6)^{408} \times 5^2}{126}\right]$$

$$= \text{Rem}\left[\frac{1 \times 25}{126}\right] = \text{Rem } 25.$$

17. (a) $\text{Rem}\left[\frac{32^{32}}{9}\right] = \text{Rem}\left[\frac{(36-4)}{9}\right]^{32}$

$$= \text{Rem}\left(\frac{4^{32}}{9}\right) = \text{Rem}\left[\frac{(4^3)^{10} \times 4^2}{9}\right]$$

$$= \text{Rem}\left[\frac{1 \times 16}{9}\right] = 7$$

$$\text{So, } \text{Rem}\left[\frac{(32^{32})^{32}}{9}\right] = \text{Rem}\left[\frac{7^{32}}{9}\right]$$

$$= \text{Rem}\left[\frac{(-2)^{32}}{9}\right] \quad \because \text{Rem}\frac{7}{9} = -2 \text{ or } 7$$

GP_3386

$$= \text{Rem}\left[\frac{2^{32}}{9}\right] = \text{Rem}\left[\frac{(2^3)^{10} \times 2^2}{9}\right]$$

$$= \text{Rem}\left[\frac{(-1)^{10} \times 2^2}{9}\right] = \text{Rem}\ 4.$$

18. (b) $\text{Rem}\left[\frac{(33^{34})^{35}}{7}\right] = \text{Rem}\left[\frac{(35-2)^{34}}{7}\right]$

$$= \text{Rem}\left[\frac{(-2)^{34}}{7}\right] = \text{Rem}\left[\frac{2^{34}}{7}\right]$$

$\because$ 34 is even.

$$= \text{Rem}\left[\frac{(2^3)^{11} \times 2}{7}\right] = \text{Rem}\left[\frac{1 \times 2}{7}\right] = 2$$

So, $\text{Rem}\left[\frac{(33^{34})^{35}}{7}\right] = \text{Rem}\left[\frac{2^{35}}{7}\right]$

$$= \text{Rem}\left[\frac{(2^3)^{11} \times 2^2}{7}\right] = \text{Rem}\left[\frac{1 \times 4}{7}\right] = 4$$

19. (a) Since $(x + 2)$ divides the polynomial $(x^3 - 5x^2 + 4p)$

Hence,

$f(-2) = 0.$

$f(x) = x^3 - 5x^2 + 4p$

$\Rightarrow$ $f(-2) = (-2)^3 - 5.(-2)^2 + 4p = 0$

$\Rightarrow$ $f(-2) = -8 - 20 + 4p = 0 \Rightarrow p = 27$

20. (c) $\because$ $x + 6$ is a factor of $x^3 + 3x^2 + 4x + k$. So, $x = -6$ divides the polynomial $x^3 + 3x^2 + 4x + k$.

$f(-6) = 0$

$(-6)^3 + 3(-6)^2 + 4 \times (-6) + k = 0$

$\Rightarrow$ $-216 + 108 - 24 + k = 0 \Rightarrow k = 132.$

21. $\text{Rem}\left[\frac{(35)^{37}}{9}\right] = \text{Rem}\left[\frac{(36-1)^{37}}{9}\right]$

$$= \text{Rem}\left[\frac{(-1)^{37}}{9}\right] = \text{Rem} -1 \text{ or } 8.$$

Hence $\text{Rem}\left[\frac{35^{37}}{9}\right] = 8.$

22. (c) For the divisibility of 16, the last four digit of the number should be divisible by 16.

So, $\text{Rem}\left[\frac{919293949596979899}{16}\right]$

$= \text{Rem}\left[\frac{9899}{16}\right]$

Hence, remainder 11.

23. (a) For the divisibility of 8, the last three digits of the number should be divisible by 8.

1234 ..... (41 digits)

$1 - 9 \rightarrow 9$ digits

$10 - 25 \rightarrow \frac{16 \times 2 \ \ 32 \text{ digits}}{\text{Total} \Rightarrow 32 + 9 = 41 \text{ digits}}$

Last '3' digits are 425.

Hence, remainder $= \frac{425}{8} = 1.$

24. (c) Since, $x + 2$ and $x - 1$ are factors of $x^3 + 10x^2 + mx + n$.

So, $x^3 + 10x^2 + mx + n$ is divisible by $x + 2$ and $x - 1$. $f(-2) = 0 \Rightarrow (-2)^3 + 10 \times (-2)^2 - 2m + n = 0$

$\Rightarrow$ $-2m + n = -32$ .... (i)

$f(1) = 0 \Rightarrow (1)^3 + 10.(1)^2 + m + n = 0$

$m + n = -11$ ... (ii)

From (i) and (ii) we get

$m = 7$ & $n = -18.$

25. (a) Let $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0 \Rightarrow 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow b = 0$

$\Rightarrow 2a = 4 \Rightarrow a = 2, b = 0$

26. (d) $\alpha + \beta + \gamma = -\left(\frac{-4}{1}\right)$

$\Rightarrow \sqrt{3} - \sqrt{3} + \gamma = 4$

$\Rightarrow \gamma = 4$ where, α, β and γ are three zeroes.

27. (a) Let $\alpha = 5 + \sqrt{3}$, $\beta = 5 - \sqrt{3}$

we know, p(x) having zeroes as α, β is given by $p(x) = (x - \alpha)(x - \beta)$

$\therefore$ $p(x) = [x - (5 + \sqrt{3})][x - (5 - \sqrt{3})]$

$= x^2 - (5 + \sqrt{3})x - (5 - \sqrt{3})x + (5 + \sqrt{3})(5 - \sqrt{3})$

$= x^2 - (5 + \sqrt{3} + 5 - \sqrt{3})x + (5^2 - 3)$

$\therefore$ $p(x) = x^2 - 10x + 22$

28. (a) Zeroes of polynomial $p(x) = x^3 - 3x^2 + x + 1$ are $a - b$, a and $a + b$

So, $a - b + a + a + b$

$= -\frac{(-3)}{1} \Rightarrow a = 1.$

$\alpha\beta\gamma = \frac{d}{a} \Rightarrow (a-b)a(a+b) = \frac{-1}{1} = -1$

$\Rightarrow (1-b) \times 1 \times (1+b) = -1$

$1 - b^2 = -1 \Rightarrow b^2 = 2 \Rightarrow b = \pm\sqrt{2}$

So, $a = 1,\ b = \pm\sqrt{2}$

29. (b) $f(x) = x^6 + 1 \Rightarrow f(1) = 1^6 + 1 = 2$

30. (b) Let the zeroes be α and $\frac{1}{\alpha}$. Then

Product of roots

$$\alpha \cdot \left(\frac{1}{\alpha}\right) = \frac{k}{5} \Rightarrow \frac{k}{5} = 1 \Rightarrow k = 5$$

31. (b) $2x^4 - x^3 - x^2 = x^2(2x^2 - x - 1)$

$= x^2(2x+1)(x-1)$

$8x^5 + x^2 = x^2(8x^3 + 1)$

$= x^2(2x+1)(4x^2 - 2x + 1)$

HCF of $(2x^4 - x^3 - x^2)$ & $(8x^5 + x^2)$

$= x^2(2x+1)$

HCF of 45 & 75 = 15

So, HCF $= 15x^2(2x+1)$

32. (c) $x^2y(x^2 - y^2) = x^2y(x+y)(x-y)$

$= x.\ x.\ y\,(x+y).\,(x-y)$

$xy^2(x-y) = xy^2(x-y) = x.\ y.\ y.\ (x-y)$

LCM of 20 & 35 = 140

So, LCM $= 140 \times x^2y^2(x^2 - y^2)$

33. (a)

$$p^2 - 2p + 3 \overline{)\ 5p^3 - 13p^2 + 21p - 14\ (\ 5p - 3}$$

$5p^3 - 10p^2 + 15p$

$-3p^2 + 6p - 14$

$-3p^2 + 6p - 9$

-5

Hence, the remainder is –5.

34. (a) If α and β are the zeros of the polynomial $f(x) = x^2 - 7x + k$,

$\alpha + \beta = 7$ and $\alpha\beta = k$

Given that $\alpha - \beta = 3$,

solving $\alpha + \beta = 7$ and $\alpha - \beta = 3$, we get $\alpha = 5$ and $\beta = 2$.

$\therefore\ \alpha\beta = 10$ and $k = 10$ $\quad[\because\ \alpha\beta = k]$

35. (a) Substituting the given zeros in $(x - a)(x - b)$, we get

$$\left(x - \frac{1}{3}\right)\left(x + \frac{2}{5}\right) = \frac{1}{15}\left[15x^2 + x - 2\right]$$

36. (d) $\alpha + \beta = \frac{2}{k^2 - 14} \Rightarrow \frac{2}{k^2 - 14} = 1$ (Given)

$\Rightarrow\ k = \pm 4$

37. (c) $p(-1) = 3(-1)^4 - 5(-1)^3 + (-1)^2 + 8$

$= 3 + 5 + 1 + 8 = 17$

38. (b) $f(-2) = 2 \times (-2)^4 + 3(-2)^3 + 2k \times (-2)^2 + 3 \times (-2) + 6$

$= 32 - 24 + 8k - 6 + 6 = 8k + 8$

$\therefore\ f(-2) = 0 \Rightarrow 8k + 8 = 0 \Rightarrow k = -1$

39. (b) $\alpha + \beta = 5,\ \alpha\beta = 2$

$$\frac{1}{\alpha} + \frac{1}{\beta} - 3\alpha\beta = \frac{\beta + \alpha}{\alpha\beta} - 3\alpha\beta = \frac{5}{2} - 3 \times 2$$

$$= \frac{5}{2} - 6 = -\frac{7}{2}$$

40. (b) Product of zeroes $= \frac{161}{23} = 7$

$\Rightarrow\ 2 \times$ product of zeroes $= 14p$

$\Rightarrow\ 2 \times 7 = 14p \Rightarrow p = 1$

41. (d) $\alpha^2 + \beta^2 = 40;\ \alpha + \beta = 8;\ \alpha\beta = k.$

Now use $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow 64 = 40 + 2k \Rightarrow k = 12$

42. (b) $\alpha + \beta = 5$ ...(i)

$\alpha\beta = k$ ...(ii)

$\alpha - \beta = 1$ ...(iii)

Solving (i) and (iii), we get $\alpha = 3$ and $\beta = 2$.

Putting the value of α and β in (ii), we get $k = 6$

43. (a) Since –3 is the zero of $(k-1)x^2 + kx + 1$,

$\therefore\ (k-1)(-3)^2 + k(-3) + 1 = 0$

$\Rightarrow\ 9k - 9 - 3k + 1 = 0$

$$\Rightarrow k = \frac{4}{3}$$

44. (c) $$\frac{x^2 - 5x - 14}{x^2 - 3x + 2} \times \frac{x^2 - 4}{x^2 - 14x + 49}$$

$$= \frac{(x-7)(x+2)}{(x-2)(x-1)} \times \frac{(x+2)(x-2)}{(x-7)(x-7)}$$

$$= \frac{(x+2)^2}{(x-1)(x-7)}$$

45. (b) $$\frac{y^2 + 5y + 4}{\frac{y^2 - 1}{y+5}} = \frac{(y^2 + 5y + 4)(y+5)}{(y+1)(y-1)}$$

$$= \frac{(y+4)(y+1)(y+5)}{(y+1)(y-1)} = \frac{(y+4)(y+5)}{y-1}$$

GP_3386

46. (a) $\Rightarrow \frac{x+10}{(3x+8)^3}+\frac{x(3x+8)}{(3x+8)^2(3x+8)}$

$\Rightarrow \frac{(x+10)+x(3x+8)}{(3x+8)^3} \Rightarrow \frac{3x^2+9x+10}{(3x+8)^3}$

47. (a) $\frac{1}{x^2-13x+42}+\frac{x+1}{x-6}-\frac{x^2}{x-7}$

$\Rightarrow \frac{1}{(x-6)(x-7)}+\frac{x+1}{x-6}-\frac{x^2}{x-7}$

$\Rightarrow \frac{1+(x+1)(x-7)-x^2(x-6)}{(x-6)(x-7)}$

$\Rightarrow \left[\frac{1+x^2-6x-7-x^3+6x^2}{(x-6)(x-7)}\right] \Rightarrow \frac{-x^3+7x^2-6x-6}{(x-6)(x-7)}$

Level 1

1. (b) Remainder

$=4\left(\frac{1}{2}\right)^4-3\left(\frac{1}{2}\right)^3-3\left(\frac{1}{2}\right)^2+\frac{1}{2}-7=\frac{-59}{8}$

2. (c) Let $f(x)=2x^3+ax^2+3x-5$ and $g(x)=x^3+x^2-2x+a$.
When f(x) is divided by (x – 2), remainder = f(2).
When g(x) is divided by (x – 2), remainder = g(2).
Now, $f(2)=(2\times 2^3+a\times 2^2+3\times 2-5)=(17+4a)$.
And, $g(2)=(2^3+2^2-2\times 2+a)=(8+a)$
$\therefore \quad 17+4a=8+a \Rightarrow 3a=-9 \Rightarrow a=-3$.

3. (a) Let $f(x)=2x^4+3x^3+2kx^2+3x+6$
Now, $x+2=0 \Rightarrow x=-2$
By the factor theorem, f(x) will be exactly divisible by (x + 2) if f(– 2) = 0.
Now, $f(-2)=2\times(-2)^4+3\times(-2)^3+2k\times(-2)^2+3\times(-2)+6$
$=32-24+8k-6+6=8k+8$.
$f(-2)=0 \Leftrightarrow 8k+8=0 \Leftrightarrow 8k=-8 \Leftrightarrow k=-1$
Hence, f(x) is exactly divisible by (x + 2), when k = – 1

4. (a) Let $p(x)=(x^{100}+2x^{99}+k)$ be divisible by (x + 1).
Then, p(– 1) = 0
$\Rightarrow (-1)^{100}+2(-1)^{99}+k=0$
$\Rightarrow \quad 1-2+k=0 \Rightarrow k-1=0 \Rightarrow k=1$.

5. (d) A is a remainder when $y^3+2y^2-5ay-7$ is divided by y + 1
$\therefore \quad (-1)^3+2(-1)^2-5a(-1)-7=A$
$\Rightarrow \quad -1+2+5a-7=A \Rightarrow 5a-6=A$...(i)
B is a remainder when $y^3+ay^2-12y+6$ is divided by y – 2
$\therefore \quad (2)^3+a(2)^2-12(2)+6=B$
$8+4a-24+6=B$
$4a-10=B$ ...(ii)
It is given that 2A + B = 6
$2(5a-6)+(4a-10)=6 \Rightarrow 14a-22=6$
$14a=28 \Rightarrow a=2$

6. (c) Let $f(x)=x^3-ax^2+14x+b$
Here, we will perform the synthetic division of f(x) by (x – 1) and (x – 2). Remainder will be zero as (x – 1) and (x – 2) are the factors of f(x).

1	1	–a	14	b
		1	1 – a	15 – a
	1	1 – a	15 – a	15 – a + b = 0

$\Rightarrow \quad a-b=15$ ...(i)

2	1	–a	14	b
		2	4 – 2a	36 – 4a
	1	2 – a	18 – 2a	36 – 4a + b = 0

$\Rightarrow \quad 4a-b=36$ ....(ii)
Solving (i) and (ii), we get a = 7, b = –8

7. (d) We have a + b + c = 14
$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$
$\Rightarrow \quad 2(ab+bc+ca)$
$=(a+b+c)^2-a^2-b^2-c^2$
$\Rightarrow \quad 2(ab+bc+ca)=(14)^2-74$
$=196-74=122$
$\Rightarrow \quad ab+bc+ca=122\div 2=61$
Now, $a^3+b^3+c^3-3abc$
$= \quad (a+b+c)\times\{a^2+b^2+c^2-(ab+bc+ca)\}$
$\Rightarrow 434-3abc=14(74-61)=14\times 13=182$
$\Rightarrow \quad 434-182=3abc \Rightarrow abc=252\div 3=84$

8. (a) Here, we will use remainder theorem to find r_1 and r_2. Remainder theorem states that if a polynomial f(x) is divided by (x – a), then the remainder is given by f(a).
Let us perform synthetic division of p(x) and q(x) by (x + 1) and (x – 2) respectively.

–1	1	1	–5k	–7
		–1	0	5k
	1	0	–5k	$5k-7=r_1$

and

2	1	k	–12	6
		2	2k + 4	4k – 16
	1	k + 2	2k – 8	$4k - 10 = r_2$

Given that $2r_1 - r_2 = 10$

$\Rightarrow 2(5k-7)-(4k-10)=10$

$\Rightarrow 10k-14-4k+10=10$

$\Rightarrow 6k=14$

$\therefore k=\frac{7}{3}$

9. (d) $2x^2-7x+12=0$

$a=2, b=-7, c=12$

$\alpha+\beta=\frac{-b}{a}=\frac{-(-7)}{2}=\frac{7}{2}$

$\alpha.\beta=\frac{c}{a}=\frac{12}{2}=6$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha.\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(\frac{7}{2}\right)^2-2\times6}{6}=\frac{49-48}{4\times6}=\frac{1}{24}$$

10. (b) Any digit repeated (P – 1) times is always divisible by P, where; P is a prime greater than 5. So 77777 ..... (7 written 10 times) is divisible by 11.

$$\Rightarrow \text{Rem}\left[\frac{7777.....(360\text{ times})}{11}\right]=0$$

$$\text{Rem}\left[\frac{777}{11}\right]=7$$

Alternate :

Any digit repeated 6 times is divisible by 11, 7, 13, 1001, 37.

So, 360 times written 7 is divisible by 11.

11. (a) If any is made by repeating a digit 6 times the number will be divisible by 7, 11, 13 and 37. So, 777777 ..... 126 times is divisible by 37 because 126 is the multiple of 6. So, the remaining three digits will be divided by 37 to get the remainder

$$\Rightarrow \frac{777777.....126\text{ Times},777}{37}$$

$37\overline{)777}\,(21 \quad \frac{777}{74}=0$

$\underline{74}$

37

$\underline{37}$

$\times$ Remainder = 0

Hence, the number is divisible by 37.

12. (b) $\frac{2^{591}}{255}$ $(\because 2^8=256)$

Now $\frac{\left(2^8\right)^{73}\times2^7}{255} \overset{+1}{\uparrow} = \frac{(256)^{73}\times128}{255}$

$$=\frac{(1)^{73}\times128}{255}=\frac{128}{255}$$

Remainder = 128

13. (b) When 143 is divided by 5 we get remainder 3 thus 143^{321} is divided by 5 then remainder 3^{321}. This remainder is very large divisor so again divided $\frac{(143)^{321}}{5}=\frac{(3)^{321}}{5}$

The remainder when 3^1 is divided by

$5=\frac{3^1}{5}=R=3$

$\frac{3^2}{5}=\frac{9}{5}=R=4$

$\frac{3^3}{5}=\frac{27}{5}=R=2$

$\frac{3^4}{5}=\frac{81}{5}=R=1$

$3^4=\frac{243}{5}=R=3$

$3^6=\frac{729}{7}=R=4$

$$\frac{3^{321}}{5}=\frac{(3)^{4\times80}\times3}{3}$$

$\overset{+1}{\uparrow}$ $\overset{+3}{\uparrow}$

$$=\frac{(3^4)^{80}\times3}{5}$$

$$=\frac{(+1)^{80}\times3}{5}=\frac{1\times3}{5}=3$$

Remainder = 3

14. (a) $\text{Rem}\left[\frac{10}{6}\right]=4 \Rightarrow \text{Rem}\left[\frac{10^2}{6}\right]=4$

$\Rightarrow \text{Rem}\left[\frac{10^3}{6}\right]=4$

$\Rightarrow \quad \text{Rem}\left[\frac{10^4}{6}\right] = 4$

So, $\frac{10^n}{6} \Rightarrow \text{Rem} = 4.$

$$\text{Rem}\left[\frac{10+10^2+10^3+.....+10^{100}}{6}\right]$$

$$= \text{Rem}\left[\frac{4\times 100}{6}\right] \Rightarrow \text{Rem} = 4.$$

15. (a) Two zeroes are $\sqrt{2}$ and $-\sqrt{2}$

Now, $(x-\sqrt{2}),(x+\sqrt{2}) = x^2-2$, is a factor of the given polynomial.

Now, we divide the given polynomial by x^2-2

$$\begin{array}{r|l} & 2x^2-3x+1 \\ \hline x^2-2 & 2x^4-3x^3-3x^2+6x-2 \\ & 2x^4 \quad -4x^2 \\ & - \quad + \\ \hline & -3x^3+x^2+6x-2 \\ & -3x^3 \quad +6x \\ & + \quad - \\ \hline & x^2 \quad -2 \\ & x^2 \quad -2 \\ & - \quad + \\ \hline & 0 \end{array}$$

So, $2x^4-3x^3-3x^2+6x-2$
$= (x^2-2)(2x^2-3x+1).$
Now, by splitting $-3x$, we factorise $2x^2-3x+1$ as $(2x-1)(x-1)$.
So, the remaining two zero(es) are given by

$x = \frac{1}{2}$ and $x = 1.$

16. (b) Let $f(x) = 6x^3-11x^2+kx-20$

$$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$$

$$\Rightarrow \quad 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$$

$$\Rightarrow \quad 128-176+12k-180 = 0$$

$$\Rightarrow \quad 12k+128-356 = 0 \Rightarrow k = 19$$

17. (b) Put $x+1=0$ or $x=-1$ and $x+2=0$ or $x=-2$ in $p(x)$

Then, $p(-1)=0$ and $p(-2)=0$

$\Rightarrow p(-1) = (-1)^3+3(-1)^2-2\alpha(-1)+\beta = 0$

$\Rightarrow \quad -1+3+2\alpha+\beta = 0 \Rightarrow \beta = -2\alpha-2 \quad ....(i)$

$p(-2) = (-2)^3+3(-2)^2-2\alpha(-2)+\beta = 0$

$\Rightarrow -8+12+4\alpha+\beta = 0 \Rightarrow \beta = -4\alpha-4 \quad ....(ii)$

By equalising both of the above equations, we get

$-2\alpha-2 = -4\alpha-4 \Rightarrow 2\alpha = -2 \Rightarrow \alpha = -1$

put $\alpha = -1$ in eq. (i)

$\Rightarrow \beta = -2(-1)-2 = 2-2 = 0$

Hence, $\alpha = -1, \ \beta = 0$

18. (c) In above problem, we see that if in $p(x)$ $= x^3-6x^2-15x+80$ we subtracted $ax+b$ so that is exactly divisible by x^2+x-12

$q(x) = x^3-6x^2-15x+80-(ax+b)$
$\quad = x^3-6x^2-(15+a)x+(80-b)$

Dividend = Divisor × quotient + remainder

But remainder will be zero.

$\therefore$ Dividend = Divisor × quotient

$q(x) = (x^2+x-12) \times$ quotient

$q(x) = x^3-6x^2-(15+a)x+(80-b)$
$\quad = x(x^2+x-12)-7(x^2+x-12)$
$\quad = x^3+x^2-7x^2-12x-7x+84$
$\quad = x^3-6x^2-19x+84$

Hence, $x^3-6x^2-19x+84$
$= x^3-6x^2-(15+a)x+(80-b)-15-a = -19$
$\Rightarrow a = +4$ and $80-b = 84 \Rightarrow b = -4$

So, if in $p(x)$ we subtracted $4x-4 = (ax+b)$, then it is exactly divisible by x^2+x-12

19. (c) The given polynomial is $f(x) = x^4+2x^3-2x^2+x-1$. To make this polynomial exactly divisible by x^2+2x-3, let us add p then $x^4+2x^3-2x^2+x-1+p$ is exactly divisible by x^2+2x-3.

$$\begin{array}{r|l} & x^2 \\ \hline x^2+2x-3 & x^4+2x^3-2x^2+x-1+p \\ & x^4+2x^3-3x^2 \\ & - \quad - \quad + \\ \hline & x^2+x-1+p \end{array}$$

For remainder to be zero,
$x^2+2x-3 = x^2+x-1+p$
So, $p = (x-2)$,
So, $x-2$ to be added to make the polynomial divisible by x^2+2x-3.

20. (c)

$$2x+3\overline{)\,2x^3+9x^2-x-b\,(\,x^2+3x-5}$$

$2x^3+3x^2$

$6x^2-x-b$

$6x^2+9x$

$-10x-b$

$-10x-15$

$-b+15$

For the dividend to be exactly divisible by the divisor, the remainder must be zero.

$\therefore \quad -b+15=0 \Rightarrow b=15$

21. (d) Given polynomial is $(m^2+9)x^2+13x+6m$.

Let one zero be α. Then, according to the problem, the other zero is $\frac{1}{\alpha}$.

$\therefore$ Product of zeros $= \alpha \times \frac{1}{\alpha} = 1$

But product of zeros $= \frac{6m}{m^2+9}$

$\Rightarrow \quad m^2-6m+9=0 \Rightarrow m-3=0 \Rightarrow m=3$

22. (c) Let α and β be the zeros of the required polynomial f(x).

Given, $\alpha+\beta=\frac{-10}{\sqrt{3}}, \alpha\beta=7$

$f(x)=x^2-(\alpha+\beta)x+\alpha\beta$

$= x^2-\left(\frac{-10}{\sqrt{3}}\right)x+7 = \sqrt{3}x^2+10x+7\sqrt{3}$

Hence, the quadratic polynomial is

$\sqrt{3}x^2+10x+7\sqrt{3}$

23. (d) Here, Dividend, $p(x)=x^3-3x^2+x+2$

Divisor = g(x), Quotient = (x – 2) and Remainder = (–2x + 4)

Since, (Quotient × Divisor) + Remainder = Dividend

$\therefore [(x-2)\times g(x)]+[(-2x+4)]=x^3-3x^2+x+2$

$\Rightarrow (x-2)\times g(x)=x^3-3x^2+x+2-(-2x+4)$

$=x^3-3x^2+x+2+2x-4$

$=x^3-3x^2+3x-2$

$\therefore \quad g(x)=\frac{x^3-3x^2+3x-2}{x-2}$

Now, dividing x^3-3x^2+3x-2 by $x-2$, we have

$$x-2\overline{)\,x^3-3x^2+3x-2\,(\,x^2-x+1}$$

x^3-2x^2

$-x^2+3x-2$

$-x^2+2x$

$x-2$

$x-2$

0

$\therefore \quad g(x)=\frac{x^3-3x^2+3x-2}{x-2}=x^2-x+1$

24. (a) Let $f(x)=ax^3+3x^2-3$ and $g(x)=2x^3-5x+a$

$\therefore f(4)=64a+45$ and $g(4)=108+a$

Since f(4) = g(4)

$\therefore \quad 64a+45=108+a \Rightarrow 63a=63 \Rightarrow a=1.$

25. (d) $\alpha+\beta=\frac{-\{-(k+6)\}}{1}=k+6$

$\alpha\beta=\frac{2(2k+1)}{1}=2(2k+1)$

But $\frac{\alpha\beta}{2}=\alpha+\beta \Rightarrow \frac{2(2k+1)}{2}=k+6 \Rightarrow k=5$

26. (b) Sum of the zeroes = 4 and product of the zeroes = 1

$\therefore x^2$ – (sum of the zeroes)x + product of zeroes

$\Rightarrow \quad x^2 - 4x + 1$ is a factor of the given polynomial.

On dividing the given polynomial by x^2-4x+1, we get quotient

$=x^2-2x-35=(x-7)(x+5)$

Hence, other zeros are 7 and –5.

27. (a) Let $p(x)=x^3+2x^2+kx+3$

Then, $p(3)=3^3+2\times 3^2+3k+3=21$

i.e., $3k=-27$ i.e., $k=-9$

Hence, the given polynomial will become x^3+2x^2-9x+3.

GP_3386

Now, $x-3\overline{)x^3+2x^2-9x+3}\,(x^2+5x+6$

x^3-3x^2

$5x^2-9x+3$

$5x^2-15x$

$6x+3$

$6x-18$

21

So, $x^3+2x^2-9x+3=(x^2+5x+6)(x-3)+21$

i.e., $x^3+2x^2-9x-18$

$=(x-3)(x^2+5x+6)=(x-3)(x+2)(x+3)$

So, the zeroes of $x^3+2x^2+kx-18$ are $3,-2,-3$.

28. (b) Let the zeroes be α and 7α

$\alpha+7\alpha=\frac{8}{3} \Rightarrow \alpha=\frac{1}{3}$ ....(1)

and $(\alpha)(7\alpha)=\frac{2k+1}{3} \Rightarrow 7\alpha^2=\frac{2k+1}{3}$ ....(2)

Using (1) and (2), we get,

$$7\left(\frac{1}{3}\right)^2=\frac{2k+1}{3} \Rightarrow \frac{7}{3}=2k+1 \Rightarrow k=\frac{2}{3}$$

29. (a) $\alpha+\beta+\gamma=\frac{-3}{6}, \frac{-1}{2}$

$=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{-5}{6}, \ \alpha\beta\gamma=\frac{-1}{6}$

$\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}$

$$=\left(\frac{\frac{-5}{6}}{\frac{-1}{6}}\right)=5$$

30. (c) $f(x)=ax^3+bx^2+cx+d$

Sum of the roots taken two at a time,

$pq+qr+pr=\frac{c}{a}$ ....(i)

Product of the roots, $pqr=\frac{-d}{a}$ ....(ii)

We have to find the value of $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$

$\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{pq+qr+pr}{pqr}$ ....(iii)

Dividing (i) by (ii), we get (iii)

So, $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{\frac{c}{a}}{\frac{-d}{a}}=\frac{-c}{d}$

31. (a) $\frac{8x^2-20x-48}{3x^2-27x+60}\times\frac{9x^2-27x-90}{16x^2+56x+48}$

$=\frac{4(x-4)(2x+3)}{3(x-4)(x-5)}\times\frac{9(x-5)(x+2)}{8(2x+3)(x+2)}=\frac{3}{2}$

32. (c)

$\frac{(3x^2+2)(x-4)}{(6x-5)(x+1)}\times\frac{(4x+3)(4x+3)}{(3x-2)(x+5)}\times\frac{(6x-5)(x+5)}{(4x+3)(x-4)}$

$=\frac{(3x+2)(4x+3)}{(x+1)(3x-2)}$

33. (a)

$=\left[\frac{x^3+y^3}{(x-y)^2+3xy}\right]\div\left[\frac{(x+y)^2-3xy}{x^3-y^3}\right]\times\left[\frac{xy}{x^2-y^2}\right]$

$=\left[\frac{(x+y)(x^2-xy+y^2)}{x^2-2xy+y^2+3xy}\right]\div\left[\frac{x^2+2xy+y^2-3xy}{(x-y)(x^2+xy+y^2)}\right]\times\left[\frac{xy}{x^2-y^2}\right]$

$=\left[\frac{(x+y)(x^2-xy+y^2)}{(x^2+xy+y^2)}\right]\div\left[\frac{x^2-xy+y^2}{(x-y)(x^2+xy+y^2)}\right]\times\left[\frac{xy}{x^2-y^2}\right]$

$=\left[\frac{(x+y)(x^2-xy+y^2)}{(x^2+xy+y^2)}\right]\times\left[\frac{(x-y)(x^2+xy+y^2)}{x^2-xy+y^2}\right]\times\frac{xy}{x^2-y^2}=xy$

34. (d) $\left[\frac{1}{1+a}+\frac{2a}{1-a^2}\right]\times\frac{(a^2+4a-5)}{(a^2+10a+25)}$

$=\left(\frac{1-a+2a}{1-a^2}\right)\times\frac{(a+5)(a-1)}{(a+5)^2}$

$=\frac{1+a}{(1+a)(1-a)}\times\frac{(a+5)(a-1)}{(a+5)^2}=\frac{-1}{a+5}$

CHAPTER

7

Linear Equations, Quadratic Equations & Inequalities (Including Simultaneous Linear Equations)

EQUATION

When an algebraic expression equated to any number or two algebraic expressions are connected by the sign of equality (=), they form an **equation**.

For example :

$2x^3 - 5x = 6$, $2p^2 + 3p = 3p^3 - 5$, etc.

LINEAR EQUATIONS

A polynomial of any variable of degree one is called a **linear polynomial.**

Standard form of a linear polynomial in one variable (say x) is $ax + b$, where a and b are some real numbers but $a \neq 0$.

Standard form of a linear equation in two variables (say x and y) is $ax + by + c$, where a, b, c are real numbers but $a, b \neq 0$.

When a linear polynomial is equated to zero (0), they form a linear equation.

Hence standard form of a linear equation in any one variable x is $ax + b = 0$ and standard form of linear equation in any two variable x and y is $ax + by + c = 0$.

For example :

$2x + 3 = 0$ (Linear equation in one variable),

$3x - 2y + 6 = 0$ (Linear equation in two variables) etc.

Solution of a Linear Equation of One Variable

The value of variable which satisfy a given equation is called **solution of the equation**. The solution is also known as **root** of the equation.

For example, for a linear equation: $2x - 8 = 0$, $x = 4$ is a solution or root, because by putting $x = 4$ in the equation, both sides of the equation become equal. In other words, $x = 2$ satisfies equation $2x - 8 = 0$.

Illustration – 1

Verify that $x = 3$ is a solution of equation $3x + 5 = 14$

Solution : If $x = 3$ is a solution of the given equation, then by putting $x = 3$ in the given equation will make both sides of equation equal

$\therefore$ Put $x = 3$ in the equation, we get

L. H. S. $= 3 \times 3 + 5 = 9 + 5 = 14 =$ R H S

Hence $x = 3$ is a solution of $3x + 5 = 14$

Illustration – 2

Verify that $x = 2$ is a solution of $2x + 3 = 5x - 6$.

Solution : By putting $x = 2$, we get

L. H. S. $= 2x + 3 = 2 \times 2 + 3 = 7$

R. H. S. $= 5x - 6 = 5 \times 2 - 6 = 4$

So $x = 2$ is not a solution of $2x + 3 = 5x - 6$

Rules for Solving a Linear Equation in One Variable

While solving a linear equation, following rules are to be remembered.

(a) Addition/ subtraction of same quantity to/from both sides of an equation does not change the equality

(b) Multiplication by a same number on both sides of an equation does not change the equality

(c) Division by a same non-zero number on both sides of an equation does not change the equality.

(d) Any term may be transposed from one side of equality to another side by changing the sign of the term (from + to –, or from –to +),

(e) A multiplier may be removed from one side of an equation by making it a divisor on the other side.

(f) A divisor may be removed from one side of an equation by making it a multiplier on the other side.

GP_3386

Illustration – 3

Solve: $\frac{1}{3}x-\frac{5}{2}=6$

Solution : We have, $\frac{1}{3}x-\frac{5}{2}=6$

$\Rightarrow \quad \frac{1}{3}x-\frac{5}{2}+\frac{5}{2}=6+\frac{5}{2}\left(\text{Adding } \frac{5}{2} \text{ on both sides}\right)$

$\Rightarrow \quad \frac{1}{3}x=6+\frac{5}{2} \quad \Rightarrow \frac{1}{3}x=\frac{12+5}{2}$

$\Rightarrow \quad \frac{1}{3}x=\frac{17}{2} \quad \Rightarrow 3\times\frac{1}{3}x=3\times\frac{17}{2}\Rightarrow x=\frac{51}{2}$

($\because$ Multiplying both sides by 3)

Thus, $x=\frac{51}{2}$ is the solution of the given equation.

Illustration – 4

Solve : $5x-\frac{1}{2}=5+\frac{1}{3}x$

Solution : $5x-\frac{1}{2}=5+\frac{1}{3}x$

$\Rightarrow 5x-\frac{1}{3}x=5+\frac{1}{2}=\frac{10+1}{2}$

$\Rightarrow\left(5-\frac{1}{3}\right)x=\frac{11}{2}\Rightarrow\left(\frac{15-1}{3}\right)x=\frac{11}{2}$

$\Rightarrow \frac{14}{3}x=\frac{11}{2}\Rightarrow x=\frac{11}{2}\times\frac{3}{14}=\frac{33}{28}$

To Find the Solution(s) or Root(s) of a Linear Equation in Two Variables

Standard form of a linear equation in two variables :

$ax + by + c = 0 \quad (a, b \neq 0)$

Express any one variable y in terms of other variable say x as

$$y=-\frac{ax+c}{d} \qquad \ldots(i)$$

Now by putting any real value of x in above equation (i), you will get the corresponding real value of y. The value of x and its corresponding value of y is one set of solution of the given linear equation in two variables. In the same way by putting another real value of x in the above equation (i), you will get another value of y. Thus you will get another set of solution. In this way, you will get infinite sets of solution.

Illustration – 5

Find any three sets of solution of $-x + 2y = 4$

Solution : Consider $-x + 2y = 4$

$\Rightarrow \quad x = 2y - 4 \qquad \ldots(i)$

Put $y = 0$ in equation (i), then $x = 2 \times 0 - 4 = -4$

Hence, one set of solution : $x = -4, y = 0$

Put $y = 1$ in equation (i), then $x = 2 \times 1 - 4 = 2 - 4 = -2$

Hence, second set of solution : $x = -2, y = 1$

Put $y = 2$ in equation (i), then $x = 2 \times 2 - 4 = 0$

Hence, third set of solution : $x = 0, y = 2$

Thus, three sets of solution are $(-4, 0)$, $(-2, 1)$ and $(0, 2)$

A Pair of Linear Equations in Two Variables

Standard form of a pair of linear equations in two variables (say x and y) is

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Here a_1, b_1, c_1, a_2, b_2 and c_2 are some real numbers.

We know that a linear equation is two variables has always infinite sets of solution. But number of solution of a pair of linear equations in two variables depends upon the different conditions. These are

Condition-I :

If $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, then there is a unique (i.e. one) solution.

Condition-II :

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, then there are infinite set of solution.

Condition-III :

If $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, then there is no solution.

There are three methods to solve a pair of linear equations in two variables.

(A) First Method — Substitution Method

Algorithm to find the solution :

A system of a pair of linear equations in two variables:

$a_1x + b_1y + c_1 = 0$ ...(1)

$a_2x + b_2y + c_2 = 0$ ...(2)

Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation (1) or (2), whichever is convenient. Consider this equation as equation (3).

Step 2 : Substitute this value of y in the other equation, which reduce it to an equation in one variable, i.e., in terms of x, which can be solved. Sometimes, on solving you can get an equation with no variable. If this statement is true, you can conclude that the pair of linear equations has infinite set of solutions. If the statement is false, then the pair of linear equations has no solution.

Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation (3) to obtain the value of the other variable.

Let us understand this method through the following illustration.

Illustration – 6

Find the solution of the following system by substitution method :

$4x + y = 11$ ...(1)

$x + 2y = 8$ ...(2)

Solution : It is easy to solve for either y from the first equation or x from the second equation. Let us solve for y from the equation (1)

The result is $y = 11 - 4x$ ... (3)

Now substitute the above value of y in the equation (2).

We get, $x + 2(11 - 4x) = 8$

$\Rightarrow x + 22 - 8x = 8 \Rightarrow -7x = 8 - 22$

$\Rightarrow -7x = -14 \Rightarrow x = 2$

Now substitute the value of x in equation (3).

We get $y = 11 - 4(2) = 11 - 8 = 3$

Hence, solution is $x = 2$ and $y = 3$.

(B) Second Method — Elimination Method

Algorithm to find the solution:

Step 1 : If coefficients of one variable (either x or y) in both the equations are not equal, then first multiply each of the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.

Step 2 : Add the two equations or subtract one equation from the other so that one variable gets eliminated. Now solve for the variable remains in the equation. If you get a solution for the variable, go to step 3.

If in this step we obtain a true equation involving no variable, then the original pair of equations has infinitely many solutions.

If in this step we obtain a false equation in volving no variable, then the original pair of equations has no solution, i.e., it is the given system is inconsistent.

Step 3 : Substitute this value of variable (x or y) in either of the original equations to get the value of the other variable. Let us understand this method through the following illustration.

Illustration – 7

Solve the following simultaneous equations by using the elimination method $2x + 3y = 15$; $4x - 3y = 3$

Solution : $2x + 3y = 15$ ..... (i)

$4x - 3y = 3$ ..... (ii)

Notice that $3y$ appears on the left-hand side of both equations with opposite sign

Add the equations (i) and (ii)

we get $2x + 3y + 4x - 3y = 15 + 3$

$6x = 18 \Rightarrow x = 3$

Substituting $x = 3$ in (i) gives :

$2 \times 3 + 3y = 15 \Rightarrow 6 + 3y = 15$

$\Rightarrow 3y = 15 - 6 \Rightarrow 3y = 9 \Rightarrow y = \frac{9}{3} = 3$

So, the solution is $x = 3$, $y = 3$

Illustration – 8

Solve the equations

$3x + 2y = 11$ ...(i)

$2x + 3y = 4$ ...(ii)

Solution : To eliminate y, we have to make the coefficients of y in (i) and (ii) equal.

Multiplying (i) by 3, we get

$9x + 6y = 33$ ...(iii)

Multiplying (ii) by 2, we get

$4x + 6y = 8$ ...(iv)

Subtracting (iv) from (iii), we have

$5x = 25$

$\therefore \quad x = 5$

Putting this value of x in equation (i), we get

$3 \times 5 + 2y = 11 \Rightarrow 15 + 2y = 11 \Rightarrow 2y = 11 - 15$

$\Rightarrow \quad 2y = -4 \Rightarrow y = -2$

$\therefore x = 5$; $y = -2$ is the solution.

GP_3386

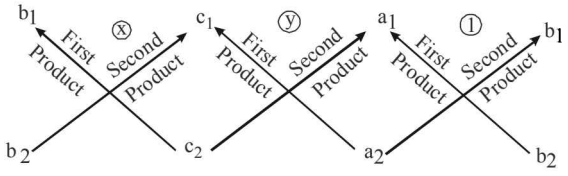
(C) Third Method — Cross-multiplication Method

Algorithm to find the solution:

Step 1 : Write the given equations in the standard form :

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

Step 2 : Taking the help of the diagram given below :



The arrows between the two numbers indicate that they are to be multiplied and the second product is to be subtracted from the first product.

Write equation as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Step 3 : $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$, provided $a_1b_2 - a_2b_1 \neq 0$

Illustration – 9

Solve the following equatins by cross-multiplication method :

$3x + 2y + 25 = 0, x + y + 15 = 0$

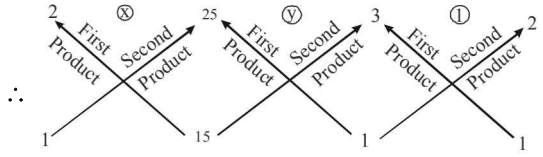
Solution : $3x + 2y + 25 = 0$ ..... (i)

$x + y + 15 = 0$ ..... (ii)

Here, $a_1 = 3, b_1 = 2, c_1 = 25$

$a_2 = 1, b_2 = 1, c_2 = 15$

$\therefore$



$$\frac{x}{2\times15 - 25\times1} = \frac{y}{25\times1 - 15\times13} = \frac{1}{3\times1 - 2\times1}$$

$$\Rightarrow \frac{x}{30 - 25} = \frac{y}{25 - 45} = \frac{1}{3 - 2}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-20} = \frac{1}{1} \Rightarrow \frac{x}{5} = 1, \frac{y}{-20} = 1 \Rightarrow x = 5,\ y = -20$$

QUADRATIC EQUATIONS

A polynomial of any variable of degree 2 is called a **quadratic polynomial**. Standard form of a quadratic polynomial of any variable (say x) is $ax^2 + bx + c$, where a, b, c are some real numbers but $a \neq 0$. When a quadratic polynomial equated to zero (0), they form a quadratic equation. Hence standard form of the quadratic equation of any variable x is

$ax^2 + bx + c = 0$.

Examples of quadratic equation are $2x^2 - 4x + 3 = 0$, $x^2 + 5x - 7 = 0$, $2x^2 + 7 = 0$, etc.

Solutions (or Roots) of a Quadratic Equation

The values of the variable present in the quadratic equation, which satisfy the quadratic equation, is called the **solution** of the quadratic equation.

For example: 3 and $-\frac{1}{2}$ are the solutions of the quadratic equation $2x^2 - 5x - 3 = 0$, because they satisfy the quadratic equation $2x^2 - 5x - 3 = 0$.

Solution of a quadratic equation has either -

(a) a pair of same real number.

(b) a pair of different real numbers or

(c) no real number.

The above type of solutions are called nature of the roots of the quadratic equation.

Nature of the roots of a given quadratic equation

$ax^2 + bx + c = 0$

is decided by the value of discriminant (denoted by D) and is equal to $b^2 - 4ac$.

Thus $D = b^2 - 4ac$.

Discriminant and Nature of The Roots of a Quadratic Equation

Quadratic Equation : $ax^2 + bx + c = 0$

Discriminant : $D = b^2 + 4ac$

Nature of the Roots :

(a) If $D = 0$, then roots are real and equal

(b) If $D > 0$, then roots are real and unequal

(c) If $D < 0$, then roots are not real.

Illustration – 10

Find the nature of the roots of the following quadratic equations.

(i) **$2x^2 - 3x + 5 = 0$**

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Solution : **(i)** $2x^2 - 3x + 5 = 0$

$a = 2, b = -3, c = 5$

Discriminant $D = b^2 - 4ac = 9 - 4 \times 2 \times 5$

$= 9 - 40 = -31 \Rightarrow D < 0$

Hence, no real root.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

$a = 3, b = -4\sqrt{3}, c = 4$

Discriminant

$D = b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0 \Rightarrow D = 0$

$\Rightarrow$ two equal real roots

(iii) The given equation is $2x^2 - 6x + 3 = 0$

Here, $a = 2, b = -6$ and $c = 3$

$\therefore D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12 > 0$

So, the given equation has two different real root.

Sum and Product of Roots

It α and β are the roots of quadratic equation $ax^2 + bx + c = 0$, then

(i) Sum of roots, $\alpha + \beta$

$$= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(ii) Product of roots, $\alpha\beta$

$$= \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

For example : In equation $3x^2 + 4x - 5 = 0$

Sum of roots = –4/3, Product of roots = –5/3

To Find the Roots (or Solutions)of a Quadratic Equation

METHOD-1 : Solution of Quadratic Equation by Factorisation

Factorise the polynomial $ax^2 + bx + c$ of the LHS of the quadratic equation $ax^2 + bx + c = 0$ by splitting the middle term bx (as discussed in the chapter 6, under the heading - Factorisation of quadratic polynomial).

$ax^2 + bx + c = 0$

$\Rightarrow a(x - \alpha)(x - \beta) = 0$

(After factorisation, supposed)

$\Rightarrow x - \alpha = 0, x - \beta = 0 \Rightarrow x = \alpha, x = \beta$

Illustration – 11

Solve the following quadratic equations by factorization:

(i) $x^2 + 6x + 5 = 0$

(ii) $9x^2 - 3x - 2 = 0$

Solution : (i) $x^2 + 6x + 5 = 0$

$\Rightarrow \quad x^2 + 5x + x + 5 = 0$

$\Rightarrow \quad x(x + 5) + 1(x + 5) = 0$

$\Rightarrow \quad (x + 1)(x + 5) = 0$

$\Rightarrow \quad x + 1 = 0$ or $x + 5 = 0 \Rightarrow x = -1$ or -5

(ii) $9x^2 - 3x - 2 = 0$

$\Rightarrow \quad 9x^2 - 6x + 3x - 2 = 0$

$\Rightarrow \quad 3x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow \quad (3x + 1)(3x - 2) = 0$

$\Rightarrow \quad 3x + 1 = 0$ or $3x - 2 = 0$

$\Rightarrow \quad x = \frac{-1}{3}$ or $\frac{2}{3}$

METHOD-2 : Roots (or solution) of Quadratic Equation by using Quadratic Formula

Quadratic equation : $ax^2 + bx + c = 0$

Roots, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$

[Quadratic Formula]

Illustration – 12

Find the roots of the equation $-2x^2 + 3x - 2 = 0$.

Solution : $-2x^2 + 3x - 2 = 0$

$a = -2, b = 3, c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(3)^2 - 4 \times (-2) \times (-2)}}{2 \times (-2)}$

GP_3386

$$= \frac{-3 \pm \sqrt{9+16}}{-4} = \frac{-3 \pm 5}{-4} = \frac{-3+5}{-4}, \frac{-3-5}{-4} = -\frac{1}{2}, 2$$

Formation of Quadratic Equation with Given Roots

- An equation whose roots are α and β can be written as $(x-\alpha)(x-\beta)=0$ or $x^2-(\alpha+\beta)x+\alpha\beta=0$ or $x^2-(\text{sum of the roots})\,x+$ product of the roots $=0$.
- Further if α and β are the roots of a quadratic equation $ax^2+bx+c=0$, then $ax^2+bx+c=a(x-\alpha)(x-\beta)$ is an identity.

Illustration – 13

Of the following quadratic equations, which is the one whose roots are 2 and – 15 ?

(a) $x^2-2x+15=0$ **(b)** $x^2+15x-2=0$

(c) $x^2+13x-30=0$ **(d)** $x^2-30=0$

Solution : (c) Sum of roots $= 2-15=-13$

Product of roots $= 2\times(-15)=-30$

Required equation

$= x^2 - x$ (sum of roots) + product of roots $=0$

$\Rightarrow x^2+13x-30=0$

Illustration – 14

If a and b are the roots of the equation $x^2-6x+6=0$, then the value of a^2+b^2 is :

(a) 36 **(b)** 24 **(c)** 17 **(d)** 6

Solution : (b) The sum of roots $= a+b=6$

Product of roots $= ab=6$

Now, $a^2+b^2=(a+b)^2-2ab=36-12=24$

Illustration – 15

If a, b are the two roots of a quadratic equation such that $a+b=24$ and $a-b=8$, then the quadratic equation having a and b as its roots is :

(a) $x^2+2x+8=0$ **(b)** $x^2-4x+8=0$

(c) $x^2-24x+128=0$ **(d)** $2x^2+8x+9=0$

Solution : (c) $a+b=24$ and $a-b=8$

$\Rightarrow a=16$ and $b=8 \Rightarrow ab=16\times8=128$

A quadratic equation with roots a and b is

$x^2-(a+b)x+ab=0$ or $x^2-24x+128=0$

Application of Linear and Quadratic Equation

Linear and quadratic equations are used to solve many types of questions related to word problems - Ages, boat and stream, two digits numbers, fractions, etc.

Illustration – 16

The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there ?

Solution : Let the ten's and the unit's digits in the first number be x and y, respectively.

So, the first number may be written as $10x+y$ in the expanded form (for example, $56=10(5)+6$).

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit. This number, in the expanded notation is $10y+x$ (for example, when 56 is reversed, we get $65=10(6)+5$).

According to the given condition.

$(10x+y)+(10y+x)=66$ i.e., $11(x+y)=66$

i.e., $x+y=6$ ..... (i)

We are also given that the digits differ by 2, therefore,

either $x-y=2$ ..... (ii)

or $y-x=2$ ..... (iii)

If $x-y=2$, then solving (i) and (ii) by elimination, we get $x=4$ and $y=2$.

In this case, we get the number 42.

If $y-x=2$, then solving (i) and (iii) by elimination, we get $x=2$ and $y=4$.

In this case, we get the number 24.

Thus, there are two such numbers 42 and 24.

Verification : Here, $42+24=66$ and $4-2=2$. Also $24+42=66$ and $4-2=2$.

Illustration – 17

A boat goes 12 km. upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Solution : Let the speed of the boat in still water be x km/hour and the speed of the stream be y km/hr. then speed of boat in downstream is $(x+y)$ km/hr. and the speed of boat upstream is $(x-y)$ km/hr.

In Ist case : Distance covered in upstream = 12 km

$\therefore$ time $= \frac{12}{x-y}$ hr.

Distance covered in downstream = 40 km

$\therefore$ time $= \frac{40}{x+y}$ hr.

Total time is 8 hrs. $\therefore \frac{12}{x-y} + \frac{40}{x+y} = 8$

In IInd case : Distance covered in upstream = 16 km

$\therefore$ time $= \frac{16}{x-y}$ hr., downstream = 32 km

$\therefore$ time $= \frac{32}{x+y}$ hr.

Total time taken = 8 hrs. $\therefore \frac{16}{x-y} + \frac{32}{x+y} = 8$

Solve them to get , x = Speed of boat = 6 km/hr,

y = speed of stream = 2 km/hr.

Illustration – 18

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Solution : Let the ages of the two friends be x years and $(20-x)$ years because the sum of their ages 20 years.

Four years ago, their ages were $(x-4)$ years and $(16-x)$ years.

We are given that $(x-4) \times (16-x) = 48$

$\Rightarrow \quad 16x - x^2 - 64 + 4x = 48$

$\Rightarrow \quad -x^2 + 20x - 64 - 48 = 0$

Illustration – 19

Out of a group of swans, $\frac{7}{2}$ times the square root of the number are playing on the shore of a tank. The two remaining ones are playing, with amorous fight, in the water. What is the total number of swans?

Solution : Let the number of swans be x.

There are two remaining swans.

Therefore, $x = \frac{7}{2}\sqrt{x} + 2$

or $(x-2)^2 = \left(\frac{7}{2}\right)^2 x$ or $4(x^2 - 4x + 4) = 49x$

or $4x^2 - 65x + 16 = 0$ or $4x^2 - 64x - x + 16 = 0$

or $4x(x-16) - 1(x-16) = 0$ or $(x-16)(4x-1) = 0$

This gives $x = 16$ or $x = \frac{1}{4}$

We reject $x = \frac{1}{4}$ ($\because$ Fractional swans cannot exist)

and take $x = 16$.

Hence, the total number of swans is 16.

Check : $\frac{7}{2} \times$ square root of 16 = 14.

i.e., 14 swans are playing on the shore of the tank. The remaining are 16 – 14 = 2, in accordance with the problem.

INEQUALITY

Two real numbers, two algebraic expressions or an algebraic expression and a real number related by the symbol >, <, ≥ or ≤ form an inequality.

'>' means 'greater than'. Hence $a > b$ read as a is greater than b.

'<' means 'less than'. Hence $a < b$ read as a is less than b.

'≥' means 'greater than or equal to'. Hence $a \geq b$ is read as a is greater than or equal to b.

'≤' means 'less than or equal to'. Hence $a \leq b$ is read as a is less than or equal to b.

Some Properties of Inequality

(i) If $a > b$, then evidently $b < a$ i.e. if the sides of an equality be transposed, the sign of equality must be reversed.

(ii) Sign of inequality does not change when equal numbers added to (or subtracted from) both sides of an inequality.

i.e. $a > b$

$\Rightarrow \quad a + 5 > b + 5$

and also $a - 4 > b - 4$

(iii) Sign of inequality does not change when both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied or divided by a negative number, then the sign of inequality is reversed.

GP_3386

i.e. $a \le b$

$\Rightarrow 3a \le 3b$

and also $\frac{a}{5} \le \frac{b}{5}$

But $-3a \ge -3b$

and also $\frac{a}{-5} \ge \frac{b}{-5}$

(iv) If $a > b$ and $b > c$, then $a > c$. Since $5 > 4$ and $4 > 2$, therefore $5 > 2$.

(v) If $a > b > 0$ then $\frac{1}{a} < \frac{1}{b}$

Since $6 > 2 > 0$, therefore $\frac{1}{6} < \frac{1}{2}$.

(vi) If $a > b > 0$ and $n > 0$ then $a^n > b^n$ and $(a)^{1/n} > (b)^{1/n}$

Since $3 > 2 > 0$ and $4 > 0$, therefore $(3)^4 > (2)^4$ and also $(3)^{1/4} > (2)^{1/4}$

(vii) If $x > y > 0$ and $a > 1$, then $a^x > a^y$

Since $5 > 3 > 0$ and $6 > 1$, therefore $(6)^5 > (6)^3$

(viii) If $x > y > 0$ and $0 < a < 1$ then $a^x < a^y$

Since $6 > 4 > 0$ and $0 < \frac{2}{3} < 1$, therefore $\left(\frac{2}{3}\right)^6 < \left(\frac{2}{3}\right)^4$.

Important Results

(i) Square of any real number is always equal or greater than 0. i.e. if a is a real number, then $a^2 \ge 0$.

(ii) For any real number a, $|a| \ge 0$

(iii) If a is a positive real number and $|x| \le a$, then $-a \le x \le a$

(iv) If a is a positive real number and $|x| \ge a$, then $x \le -a$ or $x \ge a$

(v) $|a + b| \le |a| + |b|$

In general

$|a_1 + a_2 + a_3 + ... + a_n| \le |a_1| + |a_2| + |a_3| + ... + |a_n|$

(vi) $|a - b| \ge |a| - |b|$

(vii) $a^2 + b^2 \ge 2ab$

Notation and Ranges

If a, b, c, d are four numbers such that $a < b < c < d$, then

(i) $x \in (a, b)$ means $a < x < b$

(ii) $x \in [a, b]$ means $a \le x \le b$

(iii) $x \in [a, b)$ means $a \le x < b$

(iv) $x \in (a, b]$ means $a < x \le b$

(v) $x \in (a, b) \cup (c, d)$ means $a < x < b$ or $c < x < d$

Solutions of Linear Inequalities in one Unknown

Inequalities of the form $ax + b > 0$, $ax + b \ge 0$, $ax + b < 0$ and $ax + b \le 0$ are called **linear inequalities**.

(i) $ax + b > 0 \Rightarrow x > -\frac{b}{a}$, if $a > 0$

and $x < -\frac{b}{a}$, if $a < 0$

(ii) $ax + b \ge 0 \Rightarrow x \ge -\frac{b}{a}$, if $a > 0$

and $x \le -\frac{b}{a}$, if $a < 0$

(iii) $ax + b < 0 \Rightarrow x < -\frac{b}{a}$, if $a > 0$

and $x > -\frac{b}{a}$, if $a < 0$

(iv) $ax + b \le 0 \Rightarrow x \le -\frac{b}{a}$, if $a > 0$

and $x \ge -\frac{b}{a}$, if $a < 0$

Illustration – 20

Solve : $2(x - 3) + 4 \ge 4 - x$

Solution : $2(x-3)+4 \ge 4-x$

$\Rightarrow 2x-6+4 \ge 4-x \Rightarrow 2x+x-2 \ge 4$

$\Rightarrow 3x \ge 4+2 \Rightarrow 3x \ge 6 \Rightarrow x \ge \frac{6}{3} \Rightarrow x \ge 2$

This solution can also be written as $x \in [2, \infty)$.

Illustration – 21

Solve : 3 (x + 4) + 1 < 2 (3x + 1) + 15

Solution : $3(x+4)+1<2(3x+1)+15$

$\Rightarrow \quad 3x+12+1<6x+2+15$

$\Rightarrow \quad 3x-6x<17-13$

$\Rightarrow \quad -3x<4 \Rightarrow x>\frac{4}{-3} \Rightarrow x>-\frac{4}{3}$

This solution can also be written as $x \in \left(-\frac{4}{3}, \infty\right)$.

Illustration – 22

Solve the following inequations:

$$\frac{2x+4}{x-1} \geq 5$$

Solution : We have, $\frac{2x+4}{x-1} \geq 5$

$\Rightarrow \frac{2x+4}{x-1}-5 \geq 0 \Rightarrow \frac{x-3}{x-1} \leq 0$

[Dividing both sides by 3]

$\Rightarrow \quad 1<x \leq 3 \Rightarrow x \in (1,3]$

Hence, the solution set of the given inequation is (1,3].

Illustration – 23

Solve: $-5 \leq \frac{2-3x}{4} \leq 9$.

Solution : We have, $-5 \leq \frac{2-3x}{4} \leq 9$

$\Rightarrow \quad \frac{22}{3} \geq x \geq \frac{-34}{3} \Rightarrow \frac{-34}{3} \leq x \leq \frac{22}{3}$

$\Rightarrow x \in [-34/3, 22/3]$

Hence, the interval [–34/3, 22/3] is the solution set of the given system of inequations.

Inequalities Containing a Modulus

(i)
- If $a > 0$, then $|x| \leq a \Rightarrow -a \leq x \leq a$
- If $a > 0$, then $|x| < a \Rightarrow -a < x < a$

(ii)
- If $a > 0$, then $|x| \geq a \Rightarrow x \leq -a$ and $x \geq a$
- If $a > 0$, then $|x| > a \Rightarrow x < -a$ and $x > a$
- If $a < 0$, then $|x| \geq a \Rightarrow x \leq a$ and $x \geq -a$
- If $a < 0$, then $|x| > a \Rightarrow x < a$ and $x > -a$

Illustration – 24

Solve: | x – 3 | ≥ 4

Solution : $|x-3| \geq 4$

$\Rightarrow \quad (x-3) \leq -4$ and $(x-3) \geq 4$

$\Rightarrow \quad x \leq -4+3$ and $x \geq 4+3 \Rightarrow x \leq -1$ and $x \geq 7$

i.e. $\quad x \in (-\infty, -1] \cup [7, \infty)$

Illustration – 25

Solve: | 5 – 4x | < – 2

Solution : $|5-4x| < -2$

$\Rightarrow \quad (5-4x) < -2$ and $(5-4x) > 2$

$\Rightarrow \quad -4x < -2-5$ and $-4x > 2-5$

$\Rightarrow \quad -4x < -7$ and $-4x > -3$

$\Rightarrow \quad 4x > 7$ and $4x < 3 \Rightarrow x > \frac{7}{4}$ and $x < \frac{3}{4}$

i.e. $\quad x \in \left(-\infty, \frac{3}{4}\right) \cup \left(\frac{7}{4}, \infty\right)$

Illustration – 26

If $0 < x < 5$ and $1 < y < 2$, then which of the following is true?

(a) $x + y < 0$ **(b)** $-3 < 2x - 3y < 4$

(c) $-6 < 2x - 3y < 7$ **(d)** $-3 < 3x - y < 2$

Solution : (c) $0 < x < 5$ ...(1)

$0 < 2x < 10$ ...(2) (multiply (1) by 2)

$1 < y < 2$ ...(3)

$-6 < -3y < -3$ ...(4) (multiply (3) by –3)

Subtracting (4) from (2)

$0 - 6 < 2x - 3y < 10 - 3$ *i.e.*, $-6 < 2x - 3y < 7$.

Illustration – 27

Which of the following is the solution set of | 2x – 3 | < 7 ?

(a) $\{x : -5 < x < 2\}$ **(b)** $\{x : -5 < x < 5\}$

(c) $\{x : -2 < x < 5\}$ **(d)** $\{x : x < -5 \text{ or } x > 2\}$

Solution : (c) If the expression between the absolute value bars is positive. It's less than +7 or, if the expression between the bars is negative, it's greater than –7. In other words,

$2x - 3$ is between –7 and +7

$-7 < 2x - 3 < 7$

$-4 < 2x < 10$

$-2 < x < 5$

GP_3386

Illustration – 28

$|x^2 - 4x| < 5$

(a) $-1 \le x \le 5$ (b) $1 \le x \le 5$

(c) $-1 \le x \le 1$ (d) $-1 < x < 5$

Solution : (d) At $x = 0$, inequality is satisfied, option (b) is rejected.

At $x = 2$, inequality is satisfied, option (c) is rejected.

At $x = 5$, LHS = RHS.

At $x = -1$, LHS = RHS.

Illustration – 29

If $1 \le x \le 3$ and $2 \le y \le 4$, what is the maximum value of $\left(\frac{x}{y}\right)$?

(a) $\frac{2}{3}$ (b) 4 (c) $\frac{3}{2}$ (d) 2

Solution : (c) $\text{Max}\left(\frac{x}{y}\right) = \frac{\text{Max}(x)}{\text{Min}(y)} = \frac{3}{2}$

Illustration – 30

The solution set of $\frac{1}{4} + \frac{1}{x} < \frac{1}{2} + \frac{1}{3}$, when $x \in R$, is:

(a) $\frac{1}{x} < \frac{7}{12}$ (b) $x \ge \frac{12}{7}$

(c) $x < \frac{14}{7}$ (d) $\frac{1}{x} > \frac{3}{2}$

Solution : (a) $\frac{1}{4} + \frac{1}{x} < \frac{5}{6} \Rightarrow \frac{1}{x} < \frac{5}{6} - \frac{1}{4}$

$\Rightarrow \frac{1}{x} < \frac{7}{12}$.

Illustration – 31

x > 2, y > -1, then which of the following holds good?

(a) $xy > -2$ (b) $xy < -2$

(c) $x > -\frac{2}{y}$ (d) None of these

Solution : (d) Given, $y > -1$ *i.e.,* if y is positive no. then product of x and y also positive.

But any option does not give xy is +ve.

By putting different values of x and y, we see that none of these three hold good.

Illustration – 32

If $x > 5$ and $y < -1$, then which of the following statements is true?

(a) $(x + 4y) > 1$ (b) $x > -4y$

(c) $-4x < 5y$ (d) None of these

Solution : (d) $x > 5$ and $y < -1 \Rightarrow 4y < -4$

(i) $x > 5$ and $4y < -4$ so $x + 4y < 1$

(ii) Let $x > -4y$ be true $\Rightarrow 4y < -4$ or $-4y > 4$

So, $x > 4$, which is not true as given $x > 5$.

So, $x > -4y$ is not necessarily true.

(iii) $x > 5 \Rightarrow -4x < -20$ and $5y < -5$

It is not necessary that $-4x < 5y$ as $-4x$ can be greater than $5y$, since $5y < -5$.

Hence, none of the options is true.

Illustration – 33

If a and b are negative, and c is positive, which of the following statements is/are true?

I. $a - b < a - c$ II. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$

III. $\frac{1}{b} < \frac{1}{c}$

(a) I only (b) II ouly

(c) III only (d) II and III only

Solution : (d) Let $a = x$, $b = -y$ and $c = z$

Statement I : $-x + y < -x - z$

So I is not true.

Statement II : $\frac{-x}{z} < \frac{-y}{z}$

Since $x < y$ so II is true

Statement III : $\frac{1}{-y} < \frac{1}{z}$

Since y is negative and z is positive. So III is true

Hence, statements II and III are true.

Illustration – 34

If $x + y > 5$ and $x - y > 3$, then which of the following gives all possible values of x?

(a) $x > 3$ (b) $x > 4$ (c) $x > 5$ (d) $x < 5$

Solution : (b) $x+y>5$ ...(1)

$x-y>3$ ...(2)

Adding inequations (1) and (2), we get

$2x>8$ i.e. $x>4$

Sign of Roots

Let α, β are real roots of the quadratic equation $ax^2+bx+c=0$ that is $D=b^2-4ac\geq 0$. Then

(i) Both the roots are positive if a and c have the same sign and the sign of b is opposite.

(ii) Both the roots are negative if a, b and c all have the same sign.

(iii) The roots have opposite sign if sign of a and c are opposite.

(iv) The roots are equal in magnitude and opposite in sign if $b=0$ (that is its roots α and $-\alpha$)

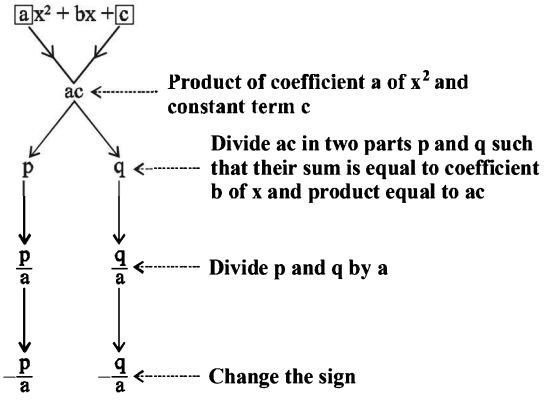
(v) The roots are reciprocal if $a=c$.

(that is the roots are α and $\frac{1}{\alpha}$)

SHORTCUTS

Shortcut Approach - 1

Shortcut Approach to solve quadratic equation $ax^2+bx+c=0$, if $b^2-4ac\geq 0$,



Here $\frac{-p}{a}$ and $\frac{-q}{a}$ are two roots or solutions of quadratic equation $ax^2+bx+c=0$

i.e. $x=-\frac{p}{a}$ or $-\frac{q}{a}$.

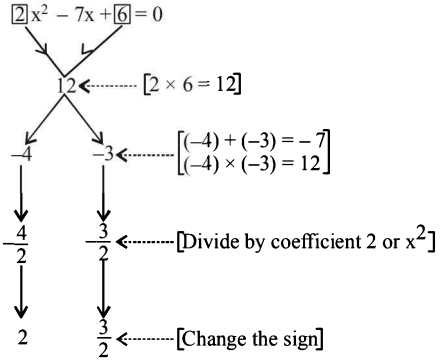
Illustration – 35

Solve : $2x^2+6=7x$

Shortcut Solution :

$2x^2+6=7x$

$\Rightarrow 2x^2-7x+6=0$



$\therefore$ Solutions or roots of given quadratic equation

$2x^2+6=7x$ are $\frac{3}{2}$ and 2.

Illustration – 36

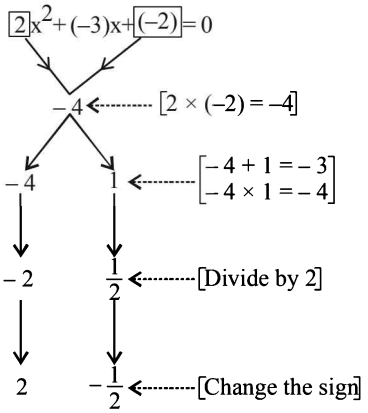
Solve $x-\frac{1}{x}=1\frac{1}{2}$

Shortcut Solution :

$x-\frac{1}{x}=1\frac{1}{2}$

$\Rightarrow \frac{x^2-1}{x}=\frac{3}{2}$ $\Rightarrow 2(x^2-1)=3x$

$\Rightarrow 2x^2-2=3x$ $\Rightarrow 2x^2-3x-2=0$



Either $2x+1=0$ or $x-2=0$

$\Rightarrow 2x=-1$ or $x=2$

GP_3386

$\Rightarrow \quad x = \frac{-1}{2}$ or $x = 2$

$\therefore \quad x = \frac{-1}{2}, 2$ are solutions.

Shortcut Approach - 1

If present age of the father is F times the age of his son. T years hence, the father's age become Z times the age of son then present age of his son is given by

$$\frac{(Z-1)T}{(F-Z)}$$

Illustration – 37

Present age of the father is 9 times the age of his son. One year later, father's age become 7 times the age of his son. What are the present ages of the father and his son.

Shortcut Solution : (a)

By the formula

$$\text{Son's age} = \frac{(7-1)}{(9-7)} \times 1 = \frac{6}{2} \times 1 = 3 \text{ years.}$$

So, father's age = 9 × son's age = 9 × 3 = 27 years.

Shortcut Approach - 1

If T_1 years earlier the age of the father was n times the age of his son, T_2 years hence, the age of the father becomes m times the age of his son then his son's age is given by

$$\textbf{Son's age} = \frac{T_1(n-1)+T_2(m-1)}{n-m}$$

Illustration – 38

10 years ago, Shakti's mother was 4 times older than her. After 10 years, the mother will be twice older than the daughter. What is the present age of Shakti?

Shortcut Solution :

By using formula,

$$\text{Shakti's age} = \frac{10(4-1)+10(2-1)}{4-2} = 20 \text{ years.}$$

Shortcut Approach - 1

Present age of Father : Son = a : b
After / Before T years = m : n

$$\textbf{Then son's age} = b \times \frac{T(m-n)}{an-bm}$$

$$\textbf{and father's age} = a \times \frac{T(m-n)}{an-bm}$$

Illustration – 39

The ratio of the ages of the father and the son at present is 3 : 1. Four years earlier, the ratio was 4 : 1. What are the present ages of the son and the father?

Shortcut Solution :

Ratio of present age of father and son = 3 : 1

4 years before = 4 : 1

$$\text{Son's age} = 1 \times \frac{4(4-1)}{4\times1-3\times1} = 12 \text{ years.}$$

$$\text{Father's age} = 3 \times \frac{4(4-1)}{4\times1-3\times1} = 36 \text{ years.}$$

LEVEL - 1

1. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers?
(a) 21 (b) 27
(c) 32 (d) Cannot be determined

2. The total age of A and B is 12 years more than the total age of B and C. C is how many years younger than A?
(a) 12 (b) 24
(c) C is elder than A (d) Data inadequate

3. If $(x-3)(2x+1)=0$, then the possible values of $2x+1$ are:
(a) 0 only (b) 0 and 3
(c) $-\frac{1}{2}$ and 3 (d) 0 and 7

4. When 24 is subtracted from a number, it reduces to its four-seventh. What is the sum of the digits of that number ?
(a) 1 (b) 9
(c) 11 (d) Data inadequate

5. Smita was asked to multiply a certain number by 36. She multiplied it by 63 instead and got an answer of 3834 more than the correct one. What was the number to be multiplied?
(a) 152 (b) 126 (c) 142 (d) 148

6. There are some parrots and some tigers in a forest. If the total number of animal heads in the forest is 858 and the total number of animal legs is 1,846, what is the number of parrots in the forest?
(a) 845 (b) 833 (c) 800 (d) 793

7. The sum of third, fourth and fifth part of a number exceeds half of the number by 34. Find the number.
(a) 60 (b) 120
(c) 30 (d) None of these

8. If the sum of two numbers is 42 and their product in 437, then find the absolute difference between the numbers.
(a) 4 (b) 7
(c) 9 (d) Cannot be determined

9. If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 was subtracted from the number and then it was divided by 5?
(a) $9\frac{2}{3}$ (b) 10
(c) $10\frac{1}{5}$ (d) $11\frac{1}{5}$

10. If three numbers are added in pairs, the sums equal 10, 19 and 21. The numbers are
(a) 4, 6, 10 (b) 6, 4, 15
(c) 3, 5, 10 (d) 2, 5, 15

11. If $ax^2+bx+c=0$ has real and different roots, then
(a) $b^2-4ac=0$ (b) $b^2-4ac>0$
(c) $b^2-4ac<0$ (d) $b^2-4ac\le 0$

12. If a and b are the roots of the equation $x^2-5x+6=0$, find the value of $2(a^2+b^2)$. **(SSC CGL Tier II 2016)**
(a) 4.6 (b) 4.2 (c) 6.5 (d) 6.6

13. The difference in the roots of the equation $2x^2-11x+5=0$ is **(CDS 2013)**
(a) 4.5 (b) 4 (c) 3.5 (d) 3

14. The roots of the equation $2\sqrt{x}+2x^{-\frac{1}{2}}=5$ can be found by solving
(a) $4x^2-25x+4=0$ (b) $4x^2+25x-4=0$
(c) $4x^2-17x+4=0$ (d) None of these

15. The roots of the equation $x^2+2\sqrt{3}x+3=0$ are
(a) real and equal
(b) rational and equal
(c) rational and unequal
(d) imaginary

16. Two numbers whose sum is 6 and the absolute value of whose difference is 8 are the roots of the equation
(a) $x^2-6x+7=0$ (b) $x^2-6x-7=0$
(c) $x^2+6x-8=0$ (d) $x^2-6x+8=0$

17. The roots of the equation $ax^2+bx+c=0$ will be reciprocal if
(a) $a=b$ (b) $a=bc$
(c) $c=a$ (d) $b=c$

18. If $\frac{b}{x-a}=\frac{x+a}{b}$ then the value of x in terms of a and b is
(a) $\pm\sqrt{a^2+b^2}$ (b) $+\sqrt{a^2+b^2}$
(c) $-\sqrt{a^2+b^2}$ (d) None of these

GP_3386

19. What is the value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5...\infty}}}}$?

(a) 5 (b) $\sqrt{5}$

(c) 1 (d) $(5)^{1/4}$

20. If r and s are roots of $x^2 + px + q = 0$, then what is the value of $\frac{1}{r^2}+\frac{1}{s^2}$?

(a) $p^2 - 4q$ (b) $\frac{p^2-4q}{2}$

(c) $\frac{p^2-4q}{q^2}$ (d) $\frac{p^2-2q}{q^2}$

21. By applying the quadratic formula to a quadratic equation $f(x) = ax^2 + bx + c = 0$, we get $c = \frac{b^2}{4a}$. Then the graph of $y = f(x)$ will certainly

(a) have a maximum
(b) have a minimum
(c) be a tangent to x-axis
(d) be a tangent to the y-axis

22. If $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$, then the roots of the quadratic equation $g[f(x)]$ will be

(a) $-1, -1$ (b) $2, -1$

(c) $-1+\sqrt{2}, -1-\sqrt{2}$ (d) $1, 2$

23. If α and β are the roots of the equation $x^2 + 6x + 1 = 0$, then what is $|\alpha - \beta|$ equal to?

(a) 6 (b) $3\sqrt{2}$ (c) $4\sqrt{2}$ (d) 12

24. If roots of an equation $ax^2 + bx + c = 0$ are positive, then which one of the following is correct?

(a) Signs of a and c should be like
(b) Signs of b and c should be like
(c) Signs of a and b should be like
(d) None of the above

25. For what value of c the quadratic equation $x^2 - (c+6)x + 2(2c-1) = 0$ has sum of the roots as half of their product?

(a) 5 (b) -4 (c) 7 (d) 3

26. Sum of the real roots of the equation $x^2 - 5|x| + 6 = 0$

(a) Equals to 5 (b) Equals to 10
(c) Equals to -5 (d) None of these

27. Determine k such that the quadratic equation $x^2 + 7(3 + 2k) - 2x(1 + 3k) = 0$ has equal roots :

(a) 2, 7 (b) 7, 5

(c) $2, \frac{-10}{9}$ (d) None of these

28. The nature of the roots of the equation $x^2 - 5x + 7 = 0$ is :

(a) no real roots
(b) 1 real root
(c) can't be determined
(d) None of these

29. The roots of $a^2x^2 + abx = b^2$, $a \neq 0$ are

(a) real and equal (b) non real
(c) real and unequal (d) None of these

30. The equation $x^2 - px + q = 0$, $p, q \in R$ has no real roots if :

(a) $p^2 > 4q$ (b) $p^2 < 4q$
(c) $p^2 = 4q$ (d) none of these

31. Determine the value of k for which the quadratic equation $4x^2 - 3kx + 1 = 0$ has equal roots :

(a) $\pm\left(\frac{2}{3}\right)$ (b) $\pm\left(\frac{4}{3}\right)$

(c) ± 4 (d) ± 6

32. Find the value of k so that the sum of the roots of the quadratic equation is equal to the product of the roots : $(k+1)x^2 + 2kx + 4 = 0$

(a) -2 (b) -4 (c) 6 (d) 8

33. If α and β are the roots of the equation $3x^2 + 7x + 3 = 0$. Find the value of $\alpha\beta$:

(a) 1 (b) 0 (c) 2 (d) 3

34. The inequality $|2x - 3| < 1$ is valid when x lies in the interval :

(a) $(-4, 3)$ (b) $(-1, 2)$
(c) $(1, 2)$ (d) $(3, 4)$

35. If α and β are the roots of the equation $x^2 - 2x - 1 = 0$. find the value of $\alpha^2\beta + \alpha\beta^2$:

(a) -8 (b) -5 (c) 6 (d) -2

Directions (36-40): In the given questions two equations numbered I and II are given. You have to solve both the equations and give answer

(a) If $x > y$ (b) If $x \geq y$
(c) If $x < y$ (d) If $x \leq y$

36. I. $721x^2 - 657x^2 = 256$

II. $\sqrt{256}\, y^3 - 14y^3 = 16$

37. I. $12x + 6y = (400)^{1/2}$

II. $3x + 2y = (324)^{1/2}$

38. I. $x^2 - 15x + 44 = 0$

II. $y^2 - 8y + 16 = 0$

39. I. $4x^3 - \sqrt{27}\, x^3 = 125$

II. $\sqrt{121}\, y^2 - 15y + \sqrt{16} = 0$

40. I. $3x^2 - 19x + 6 = 0$

II. $4y^2 - 20y = 16$

Directions (41-45): In each of these questions, two equations (I) and (II) are given. You have to solve both the equations and give answer

(a) If $x > y$
(b) If $x \geq y$
(c) If $x < y$
(d) If $x = y$ or relationship between x and y cannot be established.

41. I. $676x^2 - 1 = 0$ II. $y = \frac{1}{\sqrt[3]{15625}}$

42. I. $8x + 13y = 62$ II. $13x - 17y + 128 = 0$

43. I. $7x^2 + 2x = 120$ II. $y^2 + 11y + 30 = 0$

44. I. $x^2 = 7x$ II. $(y + 7)^2 = 0$

45. I. $2x^2 + 5x - 33 = 0$ II. $y^2 - y - 6 = 0$

Directions (Q.46-50): In each of these questions two equations are given. You have to solve these equations and give answer.

(a) If $x > y$
(b) If $x = y$, or relation cannot be established
(c) If $x \geq y$
(d) If $x \leq y$

46. I. $3x^2 + 13x + 14 = 0$ II. $3y^2 + 11y + 10 = 0$

47. I. $49x^2 - 84x + 36 = 0$ II. $25y^2 - 30y + 9 = 0$

48. I. $3x + 4y = 49$ II. $5x + 8y = 91$

49. I. $x + \frac{1}{4} = \frac{17}{4}$ II. $4y^2 + 4 + 17y = 0$

50. I. $x^2 - 9x + 18 = 0$ II. $2y^2 - 5y = 3$

LEVEL - 2

1. The least value of $ax^2 + bx + c$ $(a > 0)$ is

(a) $\frac{-b}{2a}$ (b) $\frac{4ac - b^2}{4a}$

(c) $\frac{c}{a}$ (d) cannot be determined

2. If one root of the equation $ax^2 + bx + c = 0$ is three times the other, then _____

(a) $b^2 = 16ac$ (b) $b^2 = ac$
(c) $3b^2 = 16ac$ (d) None of these

3. If $ab > 0$, then the minimum value of $(a + b)\left(\frac{1}{a} + \frac{1}{b}\right)$ is

(a) < 1 (b) > 4
(c) < 7 (d) can't say

4. If $\frac{13x^2 + 30x + 17}{3x^2 + 14x + 8} > 3$, then find the value of x, if $x \in R$ and $x \neq \frac{-2}{3}, -4$.

(a) $(-\infty, -4) \cup \left(\frac{-2}{3}, \frac{7}{2}\right) \cup \left(\frac{9}{2}, \infty\right)$

(b) $(-\infty, -4) \cup \left(\frac{-2}{3}, \frac{-1}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$

(c) $\left(-4, \frac{-2}{3}\right) \cup \left(\frac{-1}{2}, \frac{7}{2}\right)$

(d) None of these

5. For positive numbers a, b, c, the least value of $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is

(a) 3 (b) 9

(c) $\frac{27}{4}$ (d) None of these

6. The number of solutions of the equation $2x + y = 40$, where both x and y are positive integers and $x \leq y$, is :

(a) 7 (b) 13 (c) 14 (d) 18

7. If -4 is a root of the quadratic equation $x^2 - px - 4 = 0$ and the quadratic equation $x^2 - px + k = 0$ has equal roots, find the value of k :

(a) 9/4 (b) 1 (c) 2.5 (d) 3

8. If α and β are the roots of the equation $6x^2 + x - 2 = 0$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$:

(a) $-\frac{25}{12}$ (b) $-\frac{12}{75}$

(c) $-\frac{4}{9}$ (d) None of these

9. If $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + ...\infty}}}}$, then x is :

(a) 1 (b) 2
(c) 3 (d) None of these

10. If $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$; $a > 0$ have a common root, then a is equal to :

(a) 1 (b) 2 (c) 4 (d) 5

11. Find the quadratic equation whose roots are $\sqrt{3}$ and $2\sqrt{3}$:

(a) $x^2 + 3\sqrt{3}x - 6 = 0$ (b) $x^2 - 3\sqrt{3}x + 6 = 0$

(c) $x^2 + 3\sqrt{3}x + 5 = 0$ (d) None of these

GP_3386

12. The number of real solutions of the equation $2^{3x^2-7x+4} = 1$ is :
(a) 0 (b) 4
(c) 2 (d) infinitely many

13. The number of real roots of the equation $(x-1)^2+(x-2)^2+(x-3)^2=0$:
(a) 0 (b) 2 (c) 3 (d) 6

14. The equation $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$ has :
(a) no solution (b) one solution
(c) two solutions
(d) more than two solutions

15. The minimum value of the expression $a+\frac{1}{a}; a>0$ is
(a) 0 (b) 1 (c) 2 (d) 4

16. If p and q are the roots of $x^2+px+q=0$, then
(a) $p=1$ (b) $p=1$ or 0
(c) $p=-2$ (d) $p=-2$ or 0

17. If $x>0$, $xy=1$, minimum value of $x+y$ is :
(a) 1 (b) 0 (c) 2 (d) $\sqrt{2}$

18. The ratio of present ages of Sita and her son is 5 : 2 respectively. Seven years hence, the ratio will be 2 : 1. What was the age of Sita when her son was born? **[Bank PO 2008]**
(a) 28 years (b) 35 years
(c) 21 years (d) Data insufficient

19. Present age of Ram's father is 4 times Ram's age. 5 years from now, father's age will become 3 times of Ram's age, find present age of Ram's father. **[Bank Clerk 2015]**
(a) 40 years (b) 45 years
(c) 50 years (d) 60 years

20. The ratio of the ages of Ram and Sohan is 5 : 8. The ratio of their ages 10 years hence would be 7 : 10. What is the present age of Ram?
(a) 25 years (b) 35 years
(c) 40 years (d) 50 years

21. A two digit number is such that the product of its digits is 12. When 9 is added to the number, the digits interchange their places, find the number :
(a) 62 (b) 34 (c) 26 (d) 43

22. The sum of the squares of two consecutive positive odd numbers is 290. Find the sum of the numbers :
(a) 14 (b) 24
(c) 28 (d) None of these

23. A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more for the same amount, each book would have cost ₹ 1 less. How many books did he buy?
(a) 8 (b) 16 (c) 24 (d) 28

24. 5 years ago, the father's age was 3 times of his son's age. 5 years hence, the ratio between the ages of father and the son becomes 11 :5. Find the father's present age.
(a) 40 years (b) 45 years
(c) 50 years (d) 55 years

25. A father's age was five times his son's age five years ago and will be three times the son's age after two years. The ratio of the present age of the father to that of the son is **[SSC CGL Tier I 2013]**
(a) 33 : 5 (b) 11 : 5 (c) 3 : 1 (d) 10 : 3

Directions (26–30): In the following questions two equations numbered I and II are given. You have to solve both the equations and give answer If
(a) $x>y$
(b) $x\geq y$
(c) $x<y$
(d) $x=y$ or the relationship cannot be established

26. I. $\frac{45}{x^3}-\frac{15}{x^2}-\frac{10}{x^3}=0$ II. $\sqrt{5^3-5^2}=y^2$

27. I. $5.0002x+3.0002=33.541$
II. $2.912y-3.142=45.526$

28. I. $3375=x^3$ II. $y=12^2-5^3$

29. I. $4x+6y=18$ II. $8x-2y=22$

30. I. $x^2-\sqrt{36}x+\sqrt[3]{512}=0$
II. $y^2-\sqrt[3]{512}+4=0$

Directions (31-35): In the following questions, two equations numbered I and II are given. You have to solve both the equations and give answer if
(a) If $x>y$
(b) If $x\geq y$
(c) If $x\leq y$
(d) If $x=y$ or the relationship cannot be established.

31. I. $x^2-19x+84=0$ II. $y^2-25y+156=0$

32. I. $x^3-468=1729$ II. $y^2-1733+1564=0$

33. I. $x^2-20x+91=0$ II. $y^2=169$

34. I. $\sqrt{784}x+1234=1486$
II. $\sqrt{1089}y+2081=2345$

35. I. $3x^2-29x+18=0$ II. $2y^2-22y+56=0$

Directions (Qs.36-40) : In each of these questions, two equations are given. You have to solve these equations and give answer.
(a) If $x>y$
(b) If $x\geq y$
(c) If $x<y$
(d) If $x=y$ or relationship cannot be established

36. I. $3x^2-31x+78=0$ II. $5y^2-43y+92=0$

37. I. $9x^2+16x+7=0$ II. $27y^2-220y+217=0$

38. I. $x-\frac{752}{x}+31=0$ II. $y^2-31y-816=0$

39. I. $5x-19y=13$ II. $13x-17y=5$

40. I. $x^2-7x-1628=0$ II. $5y^2-176y-333=0$

Hints and Solutions

Level 1

1. (c) Let the four numbers be A, B, C and D.
Let $A+3=B-3=3C=D/3=x$ (let)
Then, $A=x-3, B=x+3, C=x/3$ and $D=3x$.
$A+B+C+D=64$
$\Rightarrow (x-3)+(x+3)+x/3+3x=64 \Rightarrow x=12$
Thus, the numbers are 9, 15, 4 and 36.
$\therefore$ Required difference $=(36-4)=32$.

2. (a) $(A+B)-(B+C)=12 \Rightarrow A-C=12$. C is 12 year younger than A.

3. (d) $(x-3)(2x+1)=0 \Rightarrow (x-3)=0$ or $(2x+1)=0$
when $x-3=0 \Rightarrow x=3$

when $2x+1=0 \Rightarrow x=-\frac{1}{2}$

When $x=3$, then $(2x+1)=7$ and when $x=-\frac{1}{2}$, then $2x+1=0$,

Possible values of $(2x+1)$ are 0 and 7.

4. (c) Let the number be x, Then,

$$x-24=\frac{4}{7}x \Leftrightarrow x-\frac{4}{7}x=24 \Leftrightarrow x=56$$

5. (c) Let the number be x.
$63x-36x=3834 \Rightarrow 27x=3834 \Rightarrow x=142$

6. (d) Let the number of parrots be p and the number of tigers be t. Then
$p+t=858$ ...(1)
$2p+4t=1846$ ...(2)
After rearranging equation (2), we get
$p+2t=923$ ...(3)
Solving (1) & (3) we get
$t=65$ & $p=793$

7. (b) Let X be the given number. Then
$X/3+X/4+X/5-X/2=34$.
Solving this, we get $X=120$.

8. (a) Let the numbers be x and y.
Then, $x+y=42$ and $xy=437$.

$$x-y=\sqrt{(x+y)^2-4xy}=\sqrt{(42)^2-4\times 437}$$

$=\sqrt{1764-1748}=\sqrt{16}=4$.
$\therefore$ Required difference = 4.

9. (b) Let the number be x. Then,

$$\frac{x-4}{6}=8 \Rightarrow x-4=48 \Rightarrow x=52$$

$$\therefore \frac{x-2}{5}=\frac{52-2}{5}=\frac{50}{5}=10.$$

10. (b) Let the numbers be x, y and z. Then,
$x+y=10$ ..... (1) $y+z=19$ ..... (2)
$x+z=21$ ..... (3)
Adding (1), (2) and (3), we get :
$2(x+y+z)=50$
or $x+y+z=25$.
Thus, $x=25-19=6$; $y=25-21=4$;
$z=25-10=15$.
Hence, the required numbers are 6, 4 and 15.

11. (b) If $ax^2+bx+c=0$ has real and different roots, then discriminant >0 i.e. $b^2-4ac>0$

12. (c) Use Shortcut Approach-1

13. (a) Use Shortcut Approach-1

14. (c) $2\sqrt{x}+\frac{2}{\sqrt{x}}=5$

$2x+2=5\sqrt{x}$
$\Rightarrow 4x^2+8x+4=25x \Rightarrow 4x^2-17x+4=0$

15. (a) $b^2-4ac=(2\sqrt{3})^2-4(1)(3)=0$. So the roots are real and equal.

16. (b) Let α and β are the roots
$\alpha+\beta=6$
$\alpha-\beta=8$
$2\alpha=14 \Rightarrow \alpha=7\ \beta=-1$
$\alpha+\beta=6, \alpha\beta=-7$
The quadratic equation is $x^2-6x-7=0$

17. (c) Since roots are reciprocal,

product of the roots $=1 \Rightarrow \frac{c}{a}=1$

$\Rightarrow c=a$.

18. (a) $\frac{b}{x-a}=\frac{x+a}{b}$

$x^2-a^2=b^2$ or $x^2=b^2+a^2$

$x=\pm\sqrt{a^2+b^2}$

19. (a) Let $x=\sqrt{5\sqrt{5\sqrt{5...\infty}}}$
$\Rightarrow x^2=5x \Rightarrow x=0, 5$

GP_3386

20. (d) $\frac{1}{r^2}+\frac{1}{s^2}=\frac{s^2+r^2}{(rs)^2}$

$=\frac{(s+r)^2-2sr}{(rs)^2}=\frac{p^2-2q}{q^2}$

21. (c) Since $c=\frac{b^2}{4a}$

$\Rightarrow b^2=4ac$ or $b^2-4ac=0$

This implies that $f(x)=0$ has equal roots, i.e., the curve touches the x-axis at exactly one point.

22. (a) $g(f(x))=5x^2+10x+5$

Roots are -1 and -1

23. (c) $\because$ α and β are the roots of the equation $x^2+6x+1=0$

$\therefore \alpha+\beta=-6$ and $\alpha\beta=1$

Now, $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$=(-6)^2-4=36-4=32$

$\Rightarrow |\alpha-\beta|=\sqrt{32}=4\sqrt{2}$

24. (a) If roots of an equation $ax^2+bx+c=0$ are positive, then signs of a and c should be like.

25. (c) $(c+6)=(1/2)\times 2(2c-1)\Rightarrow c+6=2c-1\Rightarrow c=7$

26. (d) The equation is:

$x^2+5x+6=0$ and $x^2-5x+6=0$

Sum of roots $=-5+5=0$.

27. (c) For equal roots $D=0$

i.e., $b^2-4ac=0$

$\Rightarrow$ $[-2(1+3k)]^2-4\times 1\times 7\times(3+2k)=0$

$k=2, -\frac{10}{9}$

28. (a) $D=b^2-4ac=25-4\times 1\times 7=-3$

Since $D<0$, therefore roots are not real i.e, roots will be imaginary.

29. (c) $a^2x^2+abx-b^2=0$

$D=b^2-4ac=(ab)^2-4\times a^2\times(-b^2)$

$=(ab)^2+4a^2b^2=(ab)^2[1+4]=(ab)^2(5)$

$\therefore$ The roots are real and unequal

30. (b) For non - real roots $D<0$

$\therefore b^2-4ac<0$

$\therefore (-p)^2-4\times 1\times q<0$

$\Rightarrow p^2-4q<0 \Rightarrow p^2<4q$

31. (b) $4x^2-3kx+1=0$

$D=b^2-4ac=0$

$\therefore 9k^2-4\times 1\times 4=0$

$\Rightarrow k^2=\frac{16}{9}\Rightarrow k=\pm\frac{4}{3}$

32. (a) $\alpha+\beta=\alpha\beta$

$\Rightarrow \frac{-b}{a}=\frac{c}{a}\Rightarrow -b=c$

$\therefore -2k=4 \Rightarrow k=-2$

33. (a) $\alpha\beta=\frac{c}{a}=\frac{3}{3}=1$

34. (c) We have $|2x-3|<1 \Leftrightarrow -1<2x-3<1$

$\Leftrightarrow 1<x<2 \Leftrightarrow x\in(1,2)$

35. (d) $\alpha+\beta=2$ and $\alpha\beta=-1$

$\therefore$ $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$

$=2(-1)=-2$

36. (d) I. $721x^2-657x^2=256$

$\Rightarrow 64x^2=256$

$\Rightarrow x^2=4$. So $x=\pm 2$

II. $\sqrt{256}\,y^3-14y^3=16$

$\Rightarrow 16y^3-14y^3=16$

$\Rightarrow 2y^3=16. \Rightarrow y=2 \Rightarrow$ Hence $x\le y$

37. (c) I. $12x+6y=(400)^{1/2}$

$\Rightarrow 12x+6y=20$ ...(I)

II. $3x+2y=(324)^{1/2}$

$\Rightarrow 3x+2y=18$ ...(II)

From (I) and (II)

We get $y=26$ and $x=-34/3$. So $y>x$

38. (b) I. $x^2-15x+44=0$

$\Rightarrow (x-4)(x-11)\Rightarrow x=4, 11$

II. $y^2-8y+16=0$

$\Rightarrow (y-4)(y-4)\Rightarrow y=4$, So $x\ge y$

39. (a) I. $4x^3-\sqrt{27}\,x^3=125$

$\Rightarrow 4x^3-3x^3=125$

$\Rightarrow x^3=125$ So $x=5$

II. $\sqrt{121}\,y^2-15y+\sqrt{16}=0$

$\Rightarrow 11y^2-15y+4=0$

$\Rightarrow (11y-4)(y-1)\Rightarrow y=4/11, 1$ So $x>y$

40. (c) I. $3x^2-19x+6$

$\Rightarrow (x-1)(3x-16)\Rightarrow x=1, 16/3$

II. $4y^2-20y=16$

$\Rightarrow (4y-4)(y-4)\Rightarrow y=4,1$ So $y>x$

41. (c) I. $676x^2-1=0$

$x=\pm\frac{1}{26}$;

II. $y=\frac{1}{\sqrt[3]{15625}}$

$y=\frac{1}{25}$. So $x<y$

42. (c) I. $8x+13y=62$ II. $13x-17y+128=0$
$x=-2; y=6;$ So $x<y$

43. (a) I. $7x^2+2x=120$
$\Rightarrow 7x^2+30x-28x-120$
$\Rightarrow (x-4)(7x+30)$
$x=\frac{-30}{7}, 4;$
II. $y^2+11y+30=0$
$\Rightarrow y^2+6y+5y+30$
$\Rightarrow (y+5)(y+6)\ y=-6,-5;$
So $x>y$

44. (a) I. $x^2=7x$ $x=7, 0;$
II. $(y+7)^2=0$ $y=-7;$ So $x>y$

45. (d) I. $2x^2+5x-33=0$
$\Rightarrow 2x^2+11x-6x-33$
$\Rightarrow (x-3)(2x+11) \Rightarrow x=\frac{-11}{2}, 3;$
II. $y^2-y-6=0$
$\Rightarrow y^2-3y+2y-6$
$\Rightarrow (y+2)(y-3)$
$y=3, -2;$
So no relation can be established.

46. (d) I. $3x^2+13x+14=0$
$\Rightarrow (3x+7)(x+2)$
$\Rightarrow x=-7/3, -2$
II. $3y^2+11y+10=0$
$\Rightarrow (y+2)(3y+5)$
$\Rightarrow y=-2, -5/3$ So $x \le y$

47. (a) I. $49x^2-84x+36=0$
$\Rightarrow (7x-6)(7x-6) \Rightarrow x=6/7, 6/7$
II. $25y^2-30y+9=0$
$\Rightarrow (5y-3)(5y-3) \Rightarrow y=3/5, 3/5$ So $x>y$

48. (d) I. $3x+4y=49$ ...(I)
II. $5x+8y=91$ ...(II)
From (I) and (II)
We get $x=7$ and $y=7$
So relation cannot be established.

49. (a) I. $x+\frac{1}{x}=\frac{17}{4}$
$\Rightarrow (x-4)(4x-1) \Rightarrow x=4, 1/4$
II. $4y^2+4+17y=0$
$\Rightarrow (y+4)(4y+1) \Rightarrow y=-4, -1/4$ So $x>y$

50. (c) I. $x^2-9x+18=0$
$\Rightarrow (x-6)(x-3) \Rightarrow x=6, 3$
II. $2y^2-5y=3$
$\Rightarrow (y-3)(2y+1) \Rightarrow y=3, -1/2$ So $x \ge y$

Level 2

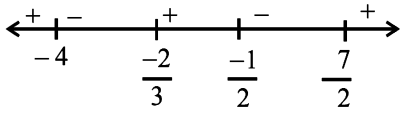
1. (b) If $a>0$ then, ax^2+bx+c has a minimum value at $x=\frac{-b}{2a}$ and is equal to $\frac{4ac-b^2}{4a}$.
If $a<0$, then ax^2+bx+c has a maximum value at $x=\frac{-b}{2a}$ and is equal to $\frac{4ac-b^2}{4a}$.

2. (c) Let $\alpha, 3\alpha$ are the roots.
$\alpha+3\alpha=\frac{-b}{a} \Rightarrow 4\alpha=\frac{-b}{a}$
$\Rightarrow \alpha=\frac{-b}{4a}$ ...(1)
$\alpha \times 3\alpha=\frac{c}{a} \Rightarrow 3\alpha^2=\frac{c}{a}$
$\frac{3b^2}{16a^2}=\frac{c}{a}$ [by (1)]
$3b^2=16ac.$

3. (b) $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{(a+b)^2}{ab}$
$=\frac{a^2}{ab}+\frac{b^2}{ab}+\frac{2ab}{ab}=\frac{a}{b}+\frac{b}{a}+2$
When $ab>0 \Rightarrow a$ and b have the same sign
$\therefore \frac{a}{b}+\frac{b}{a}$ will have a minimum value of 2
$\therefore \frac{a}{b}+\frac{b}{a}+2$ will have minimum of 4.

4. (b) $\frac{13x^2+30x+17}{3x^2+14x+8}-3>0$
$\frac{13x^2+30x+17-9x^2-42x-24}{3x^2+14x+8}>0$
$\frac{4x^2-12x-7}{3x^2+14x+8}>0$ or $\frac{(2x+1)(2x-7)}{(3x+2)(x+4)}>0$
Multiplying both the numerator and the denominator by $(3x+2)(x+4)$, we get
$(2x+1)(2x-7)(3x+2)(x+4)>0$
(The sign does not change as $(3x+2)^2$ $(x+4)^2$ is always positive).
Representing the above inequality on the number line we get,

GP_3386



The region where the inequality is satisfied is

$$(-\infty, -4) \cup \left(\frac{-2}{3}, \frac{-1}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$$

5. (b) Since a, b, c are three positive numbers.
As we know that AM ≥ GM

$$\Rightarrow \quad \frac{a+b+c}{3} \geq \sqrt[3]{abc} \qquad ...(1)$$

Their reciprocals are $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$

Again applying AM ≥ GM

$$\Rightarrow \quad \frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \geq \sqrt[3]{\frac{1}{abc}} \qquad ...(2)$$

Multiplying eqs. (1) and (2), we get

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$

6. (b) $2x+y=40;\ x \leq y\ ;x, y \in I^+$

This problem can be solved by putting various values for x and y. Starting from $x=1$. The above equation can be solved till $x=13$. At $x=13, y=14$ which is $>x$. But above this value of x, it becomes greater than y so the condition $x \leq y$ is violated.

7. (a) Since -4 is a root of $x^2-px-4=0$

$\therefore$ $(-4)^2-p(-4)-4=0 \Rightarrow p=-3$

$\therefore$ The equation becomes $x^2+3x+k=0$

Since both the roots of this equation are equal.

$\therefore$ Sum of the roots $(\alpha+\alpha)=2\alpha=-3$

$$\Rightarrow \quad \alpha=-\frac{3}{2}$$

$\therefore$ Product of the roots

$$(\alpha.\alpha)=\alpha^2=(-3/2)^2=\frac{9}{4}=k \ \therefore \ k=\frac{9}{4}$$

8. (a) $6x^2+x-2=0$

$\Rightarrow$ $6x^2-3x+4x-2=0$

$\Rightarrow$ $3x(2x-1)+2(2x-1)=0$

$$\Rightarrow \quad x=\frac{1}{2} \quad \text{or} \quad x=-\frac{2}{3}$$

$$\therefore \quad \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{1/2}{-2/3}+\frac{-2/3}{1/2}=-\frac{3}{4}-\frac{4}{3}=-\frac{25}{12}$$

9. (b) Let $x=\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+.....\infty}}}}$

$\therefore$ $x=\sqrt{2+x}$

$\Rightarrow$ $x^2=2+x$

$\Rightarrow$ $x^2-x-2=0$

$$\Rightarrow x=\frac{1\pm\sqrt{9}}{2}=\frac{1\pm3}{2}=2,-1$$

$\because$ $x>0, \therefore x=2$

10. (c) Let α be a common root of the two given equation, then

$\alpha^2-3a\alpha+35=0$ and $\alpha^2-a\alpha-21=0$

On subtracting we get $-2a\alpha+56=0$

$$\text{or } \alpha=\frac{28}{a}$$

as α is a root of $x^2-ax-21=0$

$$\therefore \quad \left(\frac{28}{a}\right)^2-a\left(\frac{28}{a}\right)-21=0$$

or $a^2=4^2$ or $a=\pm4$

11. (b) **Method – I.**

If α and β be the roots of a quadratic equation, then the equation will be

$(x-\alpha)(x-\beta)=0$

$$\therefore \quad (x-\sqrt{3})(x-2\sqrt{3})=0$$

$$\Rightarrow \quad x^2-3\sqrt{3}x+6=0$$

Method – II.

The required equation

$x^2-(\alpha+\beta)x+(\alpha.\beta)=0$

$$\Rightarrow \quad x^2-(\sqrt{3}+2\sqrt{3})x+(\sqrt{3}.2\sqrt{3})=0$$

$$\Rightarrow \quad x^2-3\sqrt{3}x+6=0$$

12. (c) $2^{3x^2-7x+4}=1=2^0$

$\Rightarrow$ $3x^2-7x+4=0$

$\Rightarrow$ $3x^2-3x-4x+4=0$

$\Rightarrow$ $3x(x-1)-4(x-1)=0$

$$\Rightarrow \quad x=1 \text{ or } x=\frac{4}{3} \qquad \therefore \quad x=1,\frac{4}{3}$$

13. (a) $(x-1)^2+(x-2)^2+(x-3)^2=0$ ...(i)

It is possible only when $x-1=0, x-2=0, x-3=0$, So $x=1, x=2, x=3$.

but $x = 1, 2, 3$ do not satisfy eq. (i).

14. (a) $\sqrt{x+1}-\sqrt{x-1}=\sqrt{4x-1}$

$\therefore$ $x+1 \geq 0, x-1 \geq 0, 4x-1 \geq 0$

All these inequalities are satisfied when $x \geq 1$. Squaring both the sides of the equation we get

$$(x+1)+(x-1)-2\sqrt{x^2-1}=4x-1$$

$\Rightarrow \quad 2\sqrt{x^2-1}=1-2x$

Squaring again, we get

$4(x^2-1)=1-4x+4x^2$

$\Rightarrow \quad x=\frac{5}{4}$

For $x=\frac{5}{4}$, $\text{LHS}=\sqrt{\frac{5}{4}+1}-\sqrt{\frac{5}{4}-1}=1$

and $\text{RHS}=\sqrt{4\left(\frac{5}{4}\right)-1}=\sqrt{4}=2$

but LHS $\neq$ RHS

$\therefore \quad x=\frac{5}{4}$ is not a root of the equation.

15. (c) Let $y=a+\frac{1}{a}$ ...(i)

Differentiating above eq. (i), we get

$$\frac{dy}{da}=1-\frac{1}{a^2} \qquad \text{....(ii)}$$

Equating eq. (ii) with zero, we get

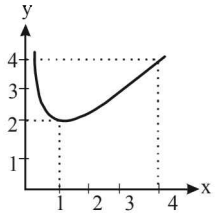
$$1-\frac{1}{a^2}=0\Rightarrow a^2=1\Rightarrow a=\pm 1$$

But since a > 0, hence only a = 1 is acceptable. Now substituting a = 1 in the original eq. (i) we get the required minimum value of the expression.

Hence, minimum of

$$y=1+\frac{1}{1}=2$$

Alternatively : See the following graph of $y=x+\frac{1}{x}$.



Here, $x=a$

$$y=x+\frac{1}{x}=a+\frac{1}{a}$$

a	$\frac{1}{a}$	$a+\frac{1}{a}$
1	1	2
2	1/2	2.5
3	1/3	3.33
4	1/4	4.25
5	1/5	5.20
1/2	2	2.5
1/3	3	3.33
1/4	4	4.25

16. (b) Since p and q are the roots of given equation

$x^2+px+q=0$, then $p+q=-p\Rightarrow q=-2p$

and $pq=q\Rightarrow p=1$

So, when p = 1, then q = –2

Again, when q = 0, then p = 0 hence,

p = 1, 0 and q = – 2, 0

Thus, option (b) is most appropriate.

17. (c) $(x+y)^2=x^2+y^2+2xy$

$(x+y)^2=x^2+y^2+2 \qquad (\because xy=1)$

For the minimum value of the expression

$x=y$

$\therefore x=1 \quad$ and $\quad y=1$

Hence, $(x+y)_{min}=\sqrt{x^2+y^2+2}=\sqrt{4}=2$

$(x+y)_{min}=2$ (Since $x>0$ and $xy>0\ \therefore y>0$)

Alternatively :

xy	x	y	$\sqrt{x^2+y^2+2}=(x+y)$
1	1/10	10	101/10
1	1/5	5	26/5
1	1/2	2	5/2
1	1	1	2
1	2	1/2	5/2
1	4	1/4	17/4
1	10	1/10	101/10

18. (c) Use Shortcut Approach-2

19. (a) Use Shortcut Approach-4

20. (a) Use Shortcut Approach-2

21. (b) Let the tens digit be x and unit digit be y.

Then the two digit number = 10 x + y

but $x\times y=12\Rightarrow y=\frac{12}{x}$

$\therefore$ the number is $\left(10x+\frac{12}{x}\right)$

Again $10x+\frac{12}{x}+9=10\times\left(\frac{12}{x}\right)+x$

$\Rightarrow 10x^2+12+9x=120+x^2$

$\Rightarrow 9x^2+9x-108=0 \Rightarrow x^2+x-12=0$

$\Rightarrow (x+4)(x-3)=0 \Rightarrow x=-4,3$

but x cannot be negative

$\therefore x=3$ only

$\therefore y=\frac{12}{x}=\frac{12}{3}=4$

$\therefore$ the number $=10x+y=10\times3+4=34$

22. (b) Let the two consecutive odd numbers be x and $x+2$

$\therefore x^2+(x+2)^2=290$

$\Rightarrow x^2+x^2+4+4x=290$

$\Rightarrow 2x^2+4x+4=290$

$\Rightarrow x^2+2x+2=145$

$\Rightarrow x^2+2x-143=0 \Rightarrow (x+13)(x-11)=0$

$\Rightarrow x=11$ or $x=-13$, but x is positive $\therefore x+2=13$

$\therefore$ Sum of the numbers $=11+13=24$

23. (b) Let the shopkeeper buys x books

$\therefore$ Rate of the book $=\frac{80}{x}$

the new number of books $=(x+4)$

$\therefore$ the new price $=\frac{80}{(x+4)}$

$\therefore \frac{80}{(x+4)}+1=\frac{80}{x} \Rightarrow \left(\frac{1}{x}-\frac{1}{(x+4)}\right)=\frac{1}{80}$

$\Rightarrow \frac{4}{x^2+4x}=\frac{1}{80}$

$\Rightarrow x^2+4x-320=0 \Rightarrow (x+20)(x-16)=0$

$\Rightarrow x=16,$ $(x=-20,$ is inadmissible)

$\therefore$ He bought 16 books.

24. (c) Use Shortcut Approach-3

25. (d) Use Shortcut Approach-3

26. (d) I. $\frac{45}{x^3}-\frac{15}{x^2}-\frac{10}{x^3}=0 \Rightarrow x=7/3$

II. $\sqrt{5^3-5^2}=y^2 \Rightarrow y\pm10$

So the relationship cannot be established.

27. (c) I. $5.0002x+3.0002=33.541$

$\Rightarrow 5.0002x=30.5408 \Rightarrow x=6.10$

II. $2.912y-3.142=45.526$

$\Rightarrow 2.912y=48.668 \Rightarrow y=16.71$ So $y>x$

28. (c) I. $3375=x^3 \Rightarrow x=15$

II. $y=12^2-5^3 \Rightarrow y=19$ So $y>x$

29. (a) I. $4x+6y=18$

II. $8x-2y=22 \Rightarrow x=3, y=1$ So $x>y$

30. (a) I. $x^2-\sqrt{36}x+\sqrt[3]{512}=0$

$\Rightarrow (x-2)(x-4) \Rightarrow x=2,4$

II. $y^2-\sqrt[3]{125}+4=0$

$\Rightarrow y=\pm1$ So $x>y$

31. (c) I. $x^2-19x+84=0$

$(x-7)(x-12)=0 \Rightarrow x=7,12$

II. $y^2-25y+156=0$

$(y-13)(y-12)=0 \Rightarrow y=12,13$ So $y\geq x$

32. (b) I. $x^3-468=1729 \Rightarrow x^3=2197\ x=13$

II. $y^2-1733+1564=0$

$\Rightarrow y^2=169 \Rightarrow y=\pm13$ So $x\geq y$

33. (d) I. $x^2-20x+91=0$

$(x-7)(x-13)=0 \Rightarrow x=7,13$

II. $y^2=169 \Rightarrow y=\pm13$

So relationship cannot be established.

34. (a) I. $\sqrt{784}\,x+1234=1486$

$\Rightarrow 28x=252 \Rightarrow x=9$

II. $\sqrt{1089}\,y+2081=2345$

$\Rightarrow 33y=264$

$\Rightarrow y=8 \Rightarrow x>y$

35. (d) I. $3x^2-29x+18=0$

$(x-9)(3x-2)=0 \Rightarrow x=9,2/3$

II. $2y^2-22y+56=0$

$(y-11)(y-2)=0 \Rightarrow y=11,2$

So relationship cannot be established.

36. (d) $x=13/3,6 \Rightarrow y=23/5,4$

No relationship can be established.

37. (c) $x=-7/9,-1 \Rightarrow y=31/27,7$, So $x<y$

38. (d) $x=-47,16 \Rightarrow y=48,-17$

No relationship can be established.

39. (a) $x=-7/9,$

$\Rightarrow y=-8/9,$ So $x>y$

40. (d) $x=44,-37$

$\Rightarrow y=37,-9/5$

No relationship can be established.

CHAPTER

8

Ratio, Proportion, Variation and Partnership

More At @Aj_ebooks

RATIO, PROPORTION & VARIATION

RATIO

Ratio is the comparison between two quantities in terms of their magnitudes. The ratio of two quantities is equivalent to a fraction that one quantity is of the other.

For example: Let Swati has 5 note books and Priya has 7 notebooks. Then the ratio of the number of books that have with Swati to the number of books that have with Priya is 5 is to 7. This ratio is expressed as 5 : 7 or $\frac{5}{7}$, which is a quotient of 5 and 7. Ratio of any two numbers a and b is expressed as $a : b$ or $\frac{a}{b}$. The numbers that form the ratio is called the **terms** of the ratio. The numerator of the ratio is called the **antecedent** and the denominator is called the **consequent** of the ratio.

Remember:

- In the ratio of two quantities the two quantities must be of the same kind and in same unit.
- The ratio is a pure number, i.e., without any unit of measurement.
- The ratio would stay unaltered even if both the numerator and the denominator are multiplied or divided by the same number.

Illustration – 1

Salaries of Rajesh and Sunil are in the ratio of 2 : 3. If the salary of each one is increased by ₹ 4000 the new ratio becomes 40 : 57. What is Sunil's present salary ?

(a) ₹ 17000 **(b) ₹ 20000**
(c) ₹ 25500 **(d) None of these**

Solution : (d) Let the salaries of Rajesh and Sunil be ₹ $2x$ and ₹ $3x$ respectively.

Then, $\frac{2x+4000}{3x+4000} = \frac{40}{57}$

or $114x + 228000 = 120x + 160000$

or $6x = 68000$

or $3x =$ ₹ 34000

Illustration – 2

The ratio between the present ages of P and Q is 5 : 8. After four years, the ratio between their ages will be 2 : 3. What is Q's age at present ?

(a) 36 years **(b) 20 years**
(c) 24 years **(d) None of these**

Solution : (d) $\frac{P}{Q} = \frac{5}{8}$ or $P = \frac{5Q}{8}$ ... (1)

$\frac{P+4}{Q+4} = \frac{2}{3}$

or $3P + 12 = 2Q + 8$

or $2Q - 3P = 4$ ... (2)

Putting value of P from eq. (1),

$2Q - 3 \times \frac{5}{8}Q = 4 \Rightarrow Q = 32$.

Properties of Ratios

(i) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \ldots = \frac{a_1 + a_2 + a_3 + \ldots}{b_1 + b_2 + b_3 + \ldots}$

This means that if two or more ratios are equal, then the ratio whose numerator is the sum of the numerators of all the ratios and denominator is the sum of the denominators of all the ratios is equal to each of the given ratio.

Since $\frac{35}{50} = \frac{7}{10}$

GP_3386

$\therefore \quad \frac{35}{50}=\frac{7}{10}=\frac{35+7}{50+10}=\frac{42}{60}$

(ii) If $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, ..., \frac{a_n}{b_n}$ are unequal ratios (or fractions), then $\frac{a_1+a_2+a_3+...+a_n}{b_1+b_2+b_3+...+b_n}$ lies between the lowest and the highest of these ratios.

(iii) If the ratio $\frac{a}{b}>1$ and k is a positive number, then $\frac{a+k}{b+k}<\frac{a}{b}$ and $\frac{a-k}{b-k}>\frac{a}{b}$

Similarly, if $\frac{a}{b}<1$, then

$\frac{a+k}{b+k}>\frac{a}{b}$ and $\frac{a-k}{b-k}<\frac{a}{b}$

(vi) If $\frac{c}{d}>\frac{a}{b}$, then $\frac{a+c}{b+d}>\frac{a}{b}$

and if $\frac{c}{d}<\frac{a}{b}$, then $\frac{a+c}{b+d}<\frac{a}{b}$

Comparison of Ratios

Method-I: Cross Multiplication Method

$\frac{a}{b}>\frac{c}{d}$, if $ad>bc$ & $\frac{a}{b}<\frac{c}{d}$, if $ad<bc$

For example: $\frac{6}{7}>\frac{3}{5}$ because $6\times5>7\times3$

and $\frac{4}{5}<\frac{7}{8}$ because $4\times8<5\times7$

Method-II: Denominator Equating Method

By making the denominator of each ratio equal to the LCM of the denominators of both ratios, we can compare the two ratios by checking their numerators. If two ratios have same denominator then the ratio with greater numerator is greater.

Illustration – 3

Which of the two ratios $\frac{5}{6}$ and $\frac{8}{9}$ is greater?

Solution : LCM of 6 and 9 = 18

$\frac{5}{6}=\frac{5\times3}{6\times3}=\frac{15}{18}$ and $\frac{8}{9}=\frac{8\times2}{9\times2}=\frac{16}{18}$

Since numerator of second ratio is greater than the numerator of first ratio, thus $\frac{16}{18}>\frac{15}{18}\Rightarrow\frac{8}{9}>\frac{5}{6}$

Types of Ratios

(i) Duplicate ratio
Duplicate ratio of $x : y$ is $x^2 : y^2$.

(ii) Triplicate ratio
Triplicate ratio of $x : y$ is $x^3 : y^3$.

(iii) Subduplicate ratio
Subduplicate ratio of $x : y$ is $\sqrt{x} : \sqrt{y}$.

(iv) Subtriplicate ratio
Subtriplicate ratio of $x : y$ is $\sqrt[3]{x} : \sqrt[3]{y}$.

(v) Reciprocal or Inverse ratio
Reciprocal ratio of $a : b$ is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

(vi) Compounded ratio
Compounded ratio of two or more ratios say $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, \ldots\ldots$ etc. is $\frac{ace.....}{bdf.....}$ i.e.

$$\frac{\text{Product of numerator}}{\text{Product of denominator}}$$

PROPORTION

When two ratios are equal, the four quantities composing them are said to be proportionals. Hence, if $\frac{a}{b}=\frac{c}{d}$, then a, b, c, d are in proportional and is written as $a : b :: c : d$

a : b :: c : d
1st Proportional, 2nd Proportional, 3rd Proportional, 4th Proportional

The terms a and d are also called **extremes** while the terms b and c are called the **means**.

$$a : b :: c : d \Rightarrow \frac{a}{b}=\frac{c}{d} \Rightarrow ad=bc$$

Hence product of extremes = Product of means

Illustration – 4

What must be added to each of the four numbers 10, 18, 22, 38 so that they become in proportion ?

Solution : Let the number to be added to each of the four numbers be x.
By the given condition, we get

$(10+x):(18+x) :: (22+x):(38+x)$

$\Rightarrow (10+x)(38+x)=(18+x)(22+x)$

$\Rightarrow 380+48x+x^2 = 396+40x+x^2$

$\Rightarrow 48x-40x = 396-380$

$\Rightarrow 8x=16 \Rightarrow x=\frac{16}{8}=2$

$\therefore$ 2 should be added to each of the given numbers.

Continued Proportion

(i) If $\frac{a}{b}=\frac{b}{c}$, then a, b, c, are said to be in continued proportion and vice-versa.
Here b is called *mean (mid) proportional* of a & c, and c is called *third proportional* of a and b.
Also $\frac{a}{b}=\frac{b}{c} \Rightarrow ac=b^2$

(ii) If a, b, c and d are in continued proportion, then $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$
Also if $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$ (let), a constant
Then $c=dk$; $b=ck=dk$ $k=dk^2$
& $a=bk=dk^2$ $k=dk^3$

Properties of Proportion

(i) **Invertendo :** If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$

(ii) **Alternando :** If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$

(iii) **Componendo :** If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$

(iv) **Dividendo:** If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$

(v) **Componendo and Dividendo :** If $\frac{a}{b}=\frac{c}{d}$, then

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

Illustration – 5

Find the value of $\frac{x+a}{x-a}+\frac{x+b}{x-b}$, if $x=\frac{2ab}{a+b}$

Solution : $x=\frac{2ab}{a+b} \Rightarrow \frac{x}{a}=\frac{2b}{a+b}$

By componendo – dividendo, $\frac{x+a}{x-a}=\frac{3b+a}{b-a}$

Similary, $\frac{x}{b}=\frac{2a}{a+b} \Rightarrow \frac{x+b}{x-b}=\frac{3a+b}{a-b}$

$\therefore \frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}=\frac{2a-2b}{a-b}=2.$

VARIATIONS

We come across many situations in our day to day life where we see change in one quantity bringing change in the other quantity. ***For example:***

(a) If the number of items purchased increases, its cost also increases.

(b) If the number of workers working to complete a job increases then days required to complete the job will decrease.

Here we observe that change in one quantity leads to change in other quantity. This is called **variation**.

Types of Variations

There are three types of variations: Direct variation, Indirect variation and Compound variation.

(i) Direct Variations

There is a direct variation in two quantities if they are related in such a way that an increase in one causes an increase in the other in the same ratio or a decrease in one causes a decrease in the other in the same ratio. This means that if one quantity becomes double then the other quantity also becomes double and if one quantity becomes half then the other quantity also becomes half etc.

Some Examples of Direct Variations

- More number of persons, more work
- More days, more work
- More working rate, more work
- More efficient worker, more work
- More cost, more number of articles
- More work, more wages
- More working hours of a machine, more fuel consumed
- More speed, more distance covered

(ii) Inverse Variations

There is an inverse variation in two quantities if they are so related that an increase in one causes a decrease in the other in the same ratio or vice-versa. This means that if one quantity becomes double then other quantity becomes half and if one quantity becomes one third then other quantity becomes thrice etc.

GP_3386

Rule of Three

The method of finding the 4th term of a proportion when the other three are given is called **Rule of Three**. In some problems, two variables are related to each other in such a way that change in one variable bring change in other variable either directly (Direct variation) or indirectly (Indirect variation). In such type of problems, there are two situations. Value of one of the variable (say A) is given in both situation and the value of other variable (say B) is given in any one situation.

To find the value of the variable B in other situation, we suppose the value of the variable B in other situation be x.

	Variable-A	Variable-B
Situation-I	A_1 (Given)	B_1 (Given)
Situation-II	A_2 (Given)	x (to be find out)

If variables A and B are in direct variation, then

$$A_1 : A_2 :: B_1 : x \text{ or } \frac{A_1}{A_2} = \frac{B_1}{x}$$

If variables A and B are in indirect variation, then

$$A_2 : A_1 :: B_1 : x \text{ or } \boxed{\frac{A_2}{A_1} = \frac{B_1}{x}}$$

To show the type of variation between variables A and B, we draw a downward ray along the values of variable B whose value in other situation is to be find out.

Now if variable A is in direct variation with variable B, then we draw a downward ray along the values of variable A and if variable A is in indirect variation with variable B, then draw an upward ray along the values of variable A as shown below :

(i) For direct variation between variables A and B :

	Variable - A	Variable - B
Situation - I	A_1 ↓	B_1 ↓
Situation - II	A_2 ↓	x

$$\frac{A_1}{A_2} = \frac{B_1}{x}$$

(ii) For indirect variation between variables A and B :

	Variable - A	Variable - B
Situation - I	A_1 ↑	B_1 ↓
Situation - II	A_2	x

$$\frac{A_2}{A_1} = \frac{B_1}{x}$$

Compound Variation (Repeated use of 'Rule of Three')

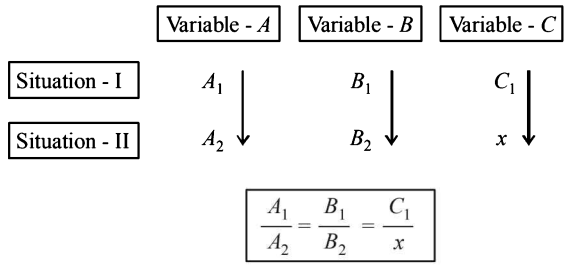
In some problems, more than two variables are related to each other in such a way that change in one variable bring change in other variables either directly or indirectly or directly in some variables and indirectly in remaining variables. For convenient, here we consider only three variables. In such type of problems, there are two situations Value of any two variables (say A and B) are given in both the situations and the value of third variable (say C) is given in any one situation. To find the value of the variable C in other situation, we suppose the value of the variable C in other situation be x.

	Variable - A	Variable - B	Variable - C
Situation - I	A_1 (Given)	B_1 (Given)	C (Given) ↓
Situation - II	A_2 (Given)	B_2 (Given)	x (To be find)

Draw a downward ray along the values of variable C whose value in other situation is to be find out.

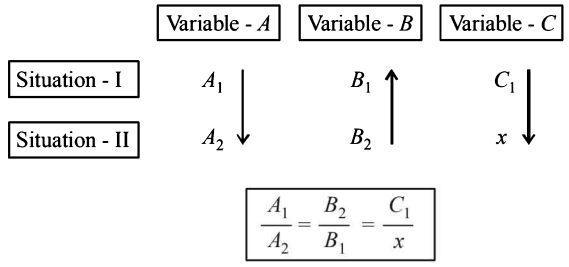
Now draw a downward ray along the values of variable A, if the variation between variables A and C is direct variation. Draw an upward ray along the value of the variable A, if the variation between variables A and C is indirect variation. In the same way draw a downward ray along the values of variable B, if variation between B and C is direct variation and upward ray along the values of variable B, if variation between B and C is indirect variation.

(a) For direct variation between A and C, direct variation between B and C :



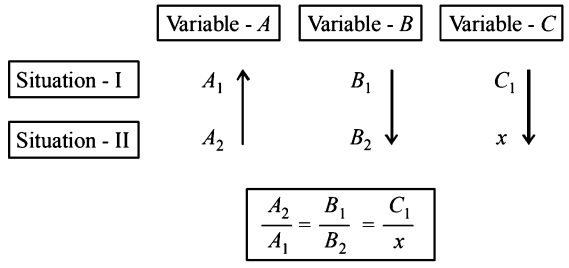
$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{x}$$

(b) For direct variation between *A* and *C*, indirect variation between *B* and *C*:



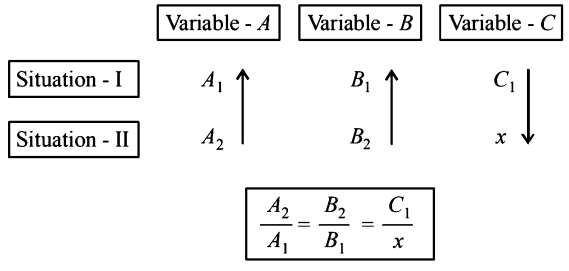
$$\frac{A_1}{A_2} = \frac{B_2}{B_1} = \frac{C_1}{x}$$

(c) For indirect variation between *A* and *C*, direct variation between *B* and *C*:



$$\frac{A_2}{A_1} = \frac{B_1}{B_2} = \frac{C_1}{x}$$

(d) For indirect variation between *A* and *C*, indirect variation betwen *B* and *C*:



$$\frac{A_2}{A_1} = \frac{B_2}{B_1} = \frac{C_1}{x}$$

SHORTCUTS

Shortcut Approach - 1

To divide a given quantity into a given ratio
Suppose any given quantity *a* is to be divided in the ratio *m* : *n*, then

$$\text{First part} = \frac{ma}{m+n}; \text{Second part} = \frac{na}{m+n}$$

Illustration – 6

Divide 80 in the ratio 2 : 3.

Shortcut Solution :

$$1^{st}\text{ part} = \frac{2\times 80}{2+3} = 32;\ 2^{nd}\text{ part} = \frac{3\times 80}{2+3} = 48.$$

Shortcut Approach - 2

(i) If $a : b = N_1 : D_1, b : c = N_2 : D_2$
Then $a : b : c = N_1N_2 : D_1N_2 : D_1D_2$.

(ii) If $a : b = N_1 : D_1, b : c = N_2 : D_2, c : d = N_3 : D_3$
Then $a : b : c : d = N_1N_2N_3 : D_1N_2N_3 : D_1D_2N_3 : D_1D_2D_3$

(iii) If $a : b = N_1 : D_1, b : c = N_2 : D_2, c : d = N_3 : D_3, d : e = N_4 : D_4$
Then $a : b : c : d : e = N_1N_2N_3N_4 : D_1N_2N_3N_4 : D_1D_2N_3N_4 : D_1D_2D_3N_4 : D_1D_2D_3D_4$.

Illustration – 7

Ratio of the age of *A* and *B* is 3 : 5 and ratio of the age of *B* and *C* is 4 : 7. Find the ratio of the age of *A* and *C*.

Shortcut Solution :

$A : B = 3 : 5 ; B : C = 4 : 7$
$\Rightarrow A : B : C = 3\times 4 : 5\times 4 : 5\times 7 = 12 : 20 : 35$
Here, A is correspond to the product of both numerators (3×4).
B is correspond to the product of first denominator and second numerator (5×4) and C is correspond to the product of both denominators (5×7)
Hence ratio of the age of A and $C = 12 : 35$

Illustration – 8

If $A : B = 4 : 5 ; B : C = 3 : 7 ; C : D = 6 : 7$ $D : E = 12 : 17$, then find the value of ratio $A : E$.

Shortcut Solution :

$A : B : C : D : E = (4\times 3\times 6\times 12) : (5\times 3\times 6\times 12) : (5\times 7\times 6\times 12) : (5\times 7\times 7\times 12) : (5\times 7\times 7\times 17)$
$\therefore A : E = (4\times 3\times 6\times 12) : (5\times 7\times 7\times 17) = 864 : 4165$

GP_3386

Note that here we have found the ratio of $A : E$ directly without finding the consolidate ratio $(A : B : C : D : E)$ of A, B, C, D and E.

Shortcut Approach - 1

If the ratio between two numbers is $a : b$ and if each number is increased by x, the ratio becomes $c : d$.

Then two numbers are given by $\frac{xa(c-d)}{ad-bc}$ and $\frac{xb(c-d)}{ad-bc}$

Illustration – 9

The ratio between two numbers is 3 : 4. If each number be increased by 2, the ratio becomes 7 : 9. Find the number.

Shortcut Solution :

Numbers are $\frac{2\times3\,(7-9)}{3\times9-4\times7}$ and $\frac{2\times4\,(7-9)}{3\times9-4\times7}$ or 12 and 16.

Shortcut Approach - 1

If two numbers are in the ratio of $a : b$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b}$ and $\frac{bx}{a+b}$.

Illustration – 10

The ratio of the rural and urban population of a country is 9 : 5. If the total population of the country is 560 millions, find the urban population of that country.

(a) 360 millions (b) 200 millions
(c) 190 millions (d) None of these

Shortcut Solution : (b)

Urban population $= \frac{bx}{a+b} = \frac{5\times560}{9+5} = 200$ millions

Shortcut Approach - 1

If two numbers are in the ratio $a : b$ and the difference between these numbers is x, then these numbers will be

(i) $\frac{ax}{a-b}$ **and** $\frac{bx}{a-b}$ **respectively (if $a > b$)**

(ii) $\frac{ax}{b-a}$ **and** $\frac{bx}{b-a}$ **respectively (if $a < b$)**

Illustration – 11

Two numbers are in the ratio of 4 : 5. If the larger number is 15 more than the smaller number, find the product of the numbers.

(a) 4050 (b) 4500 (c) 4550 (d) 4620

Shortcut Solution : (b)

Product of the numbers $= \frac{ax}{b-a}\times\frac{bx}{b-a}$ $(\because b > a)$

$= \frac{4\times15}{5-4}\times\frac{5\times15}{5-4} = 60\times75 = 4500$

Shortcut Approach - 2

If three numbers are in the ratio $a : b : c$ and the sum of these numbers is x, then these numbers will be $\frac{ax}{a+b+c}$, $\frac{bx}{a+b+c}$ and $\frac{cx}{a+b+c}$ respectively.

Illustration – 12

An amount of ₹ 3600 is distributed among A, B and C in the ratio of 1 : 2 : 3. Find the share of B.

(a) ₹ 2000 (b) ₹ 1600
(c) ₹ 1200 (d) ₹ 800

Shortcut Solution : (c)

Share of $B = \frac{bx}{a+b+c} = \frac{2\times3600}{1+2+3} =$ ₹ 1200.

Shortcut Approach - 2

If two numbers X and Y are in the ratio $x : y$. Then

(i) $\frac{X+Y}{X-Y} = \frac{x+y}{x-y}$, **if $x > y$. (ii)** $\frac{Y+X}{Y-X} = \frac{y+x}{y-x}$, **if $x < y$.**

Illustration – 13

A sum of money is divided between two persons in the ratio 5 : 2. If the share of one person is ₹ 15 less than that of the other, find the sum.

(a) 35 (b) 37 (c) 40 (d) 45

Shortcut Solution : (a)

$\frac{\text{Sum}}{15} = \frac{5+2}{5-2} \Rightarrow$ Sum = ₹ 35. $(\because a > b)$

Shortcut Approach - 2

The ratio between two numbers is $a : b$. If each number is increased by x, the ratio becomes $c : d$.
If $c - a = d - b$, then the numbers are given as

$\frac{ax}{c-a}$ **and** $\frac{bx}{c-a}$.

Illustration – 14

The ratio between two numbers is 7 : 11. If each number is increased by 6, the ratio becomes 5 : 7. Find the two numbers.

(a) 7, 15 (b) 14, 22 (c) 22, 35 (d) 24, 40

Shortcut Solution : (b)

Here $a : b = 7 : 11$ and $c : d = 5 : 7 = 10 : 14$

Since $c - a = 10 - 7 = 3$ and $d - b = 14 - 11 = 3$.

Here, $c - a = d - b$.

$\therefore$ the numbers are $\frac{7 \times 6}{3}$ and $\frac{11 \times 6}{3}$ i.e. 14 and 22.

Shortcut Approach - 12

If the sum of two numbers is A and their difference is a, then the ratio of numbers is given by $(A + a) : (A - a)$.

Illustration – 15

The sum of two numbers is 44 and their difference is 4. What is the ratio of the two numbers?

(a) 6 : 7 (b) 6 : 5 (c) 5 : 7 (d) 6 : 8

Shortcut Solution : (b)

Ratio of two numbers $= \frac{A + a}{A - a} = \frac{44 + 4}{44 - 4} = \frac{48}{40} = 6 : 5.$

Shortcut Approach - 10

A number which, when added to the terms of the ratio $a : b$ makes it equal to the ratio $c : d$ is $\frac{ad - bc}{c - d}$.

Illustration – 16

Find the number which when added to the terms of the ratio 17 : 31 makes it equal to the ratio 13 : 20.

(a) 6 (b) 7 (c) 8 (d) 9

Shortcut Solution : (d)

Reqd. number $= \frac{ad - bc}{c - d} = \frac{17 \times 20 - 31 \times 13}{13 - 20} = \frac{63}{7} = 9.$

Shortcut Approach - 10

A number which when subtracted from the terms of the ratio $a : b$ makes it equal to the ratio $c : d$ is $\frac{bc - ad}{c - d}$.

Illustration – 17

Find the number which when subtracted from the terms of the ratio 11 : 25 makes it equal to the ratio 4 : 11.

(a) 1 (b) 3 (c) 5 (d) 7

Shortcut Solution : (b)

Reqd. number $= \frac{bc - ad}{c - d} = \frac{25 \times 4 - 11 \times 11}{4 - 11} = \frac{21}{7} = 3.$

Shortcut Approach - 12

The ratio between two numbers is $a : b$, if each number is increased by x, the ratio becomes $c : d$. Then sum of two numbers $= \frac{x(a + b)(c - d)}{ad - bc}$.

Illustration – 18

The ratio between two numbers is 11 : 15. If each number be increased by 22, the ratio becomes 11 : 13. Find the sum of the numbers.

(a) 52 (b) 51 (c) 47 (d) 46

Shortcut Solution : (a)

Sum of the numbers $= \frac{x(a + b)(c - d)}{ad - bc}$

$= \frac{22(11 + 15)(11 - 13)}{11 \times 13 - 15 \times 11} = \frac{44 \times 26}{22} = 52.$

Shortcut Approach - 12

The ratio between the two numbers is $a : b$. If each number is increased by x, the ratio $c : d$. Then difference of the two numbers $= \left| \frac{x(a - b)(c - d)}{ad - bc} \right|$.

Illustration – 19

The ratio between two numbers is 9 : 7. If each number is increased by 6, the ratio becomes 21 : 17. Find the difference of the two numbers.

(a) 8 (b) 11 (c) 12 (d) 16

Shortcut Solution : (a)

Difference $= \left| \frac{x(a - b)(c - d)}{ad - bc} \right| = \left| \frac{6(9 - 7)(21 - 17)}{9 \times 17 - 7 \times 21} \right|$

$= \left| \frac{12 \times 4}{3(51 - 49)} \right| = \frac{48}{6} = 8.$

Shortcut Approach - 10

The ratio between two numbers is $a : b$. If each number be decreased by x, the ratio becomes $c : d$. Then, sum of the two numbers $= \frac{x(a + b)(d - c)}{ad - bc}$.

GP_3386

Illustration – 20

The ratio of two numbers is 7 : 8. If each number is decreased by 2, the ratio becomes 6 : 7. Find the sum of the two numbers.

(a) 35 (b) 30 (c) 29 (d) 26

Shortcut Solution : (b)

$$\text{Sum of two numbers} = \frac{x(a+b)(d-c)}{ad-bc}$$

$$= \frac{2(7+8)(7-6)}{7\times7-8\times6} = \frac{30}{1} = 30$$

Shortcut Approach - 12

The ratio between two numbers is $a : b$. If each number be decreased by x, the ratio becomes $c : d$. Then, the difference of the two numbers $= \left|\frac{x(a-b)(d-c)}{ad-bc}\right|$

Illustration – 21

The ratio of two numbers is 5 : 8. If each number is decreased by 6, the ratio becomes 1 : 2. Find the difference of the numbers.

(a) 6 (b) 9 (c) 10 (d) 15

Shortcut Solution : (b)

Difference of the two numbers

$$= \left|\frac{x(a-b)(d-c)}{ad-bc}\right| = \left|\frac{6(5-8)(2-1)}{5\times2-8\times1}\right| = \left|\frac{-18}{2}\right| = 9.$$

Shortcut Approach - 12

Income of two persons are in the ratio of $a : b$ and their expenditures are in the ratio $c : d$. If each of them saves ₹ x then their incomes are given by $\frac{xa(d-c)}{ad-bc}$ **and** $\frac{xb(d-c)}{ad-bc}$.

Illustration – 22

Income of A and B are in the ratio 7 : 5 and their expenditures are in the ratio 5 : 3. If each saves ₹ 1600, what are their incomes?

(a) ₹ 5600, ₹ 4000 (b) ₹ 5200, ₹ 4100
(c) ₹ 5150, ₹ 3900 (d) ₹ 5025, ₹ 4000

Shortcut Solution : (a)

Their incomes are $\frac{xa(d-c)}{ad-bc}$ and $\frac{xb(d-c)}{ad-bc}$

i.e. $\frac{1600\times7(3-5)}{7\times3-5\times5}$ and $\frac{1600\times5(3-5)}{7\times3-5\times5}$

= ₹ 5600 and ₹ 4000.

Shortcut Approach - 12

Income of two persons are in the ratio $a : b$ and their expenditure are in the ratio $c : d$. If each of them saves ₹ x, then their expenditures are given by $\frac{xc(b-a)}{ad-bc}$ **and** $\frac{xd(b-a)}{ad-bc}$.

Illustration – 23

Income of A and B are in the ratio 4 : 3 and their expenditures are in the ratio 5 : 2. If each saves ₹ 4900. What are their expenditure?

(a) ₹ 4200, ₹ 2500 (b) ₹ 3700, ₹ 2200
(c) ₹ 3500, ₹ 1200 (d) ₹ 3500, ₹ 1400

Shortcut Solution : (d)

Expenditures are $\frac{xc(b-a)}{ad-bc}$ and $\frac{xd(b-a)}{ad-bc}$

i.e. $\frac{4900\times5(3-4)}{4\times2-3\times5}$ and $\frac{4900\times2(3-4)}{4\times2-3\times5}$

$= \frac{4900\times5}{7}$ and $\frac{4900\times2}{7}$ = ₹ 3500, ₹ 1400.

Shortcut Approach - 12

Two candles of the same height are lighted at the same time. The first is consumed in T_1 hours and the second in T_2 hours. Assuming that each candle burns at a constant rate, the time after which the ratio of first candle to second candle becomes $x : y$ is given by

$$\frac{T_1T_2\left(\frac{x}{y}-1\right)}{\left(\frac{x}{y}\right)T_1-T_2} \text{ hours.}$$

Illustration – 24

Two candles of the same height are lighted at the same time. The first is consumed in 3 hours and the second in 1 hour. Assuming that each candle burns at a constant rate, in how much time after being lighted, the ratio between the first and second candles become 2 : 1.

(a) 32 minutes (b) 36 minutes
(c) 46 minutes (d) 1 hour 10 minutes

Shortcut Solution : (b)

$$\text{Required time} = \frac{T_1T_2\left(\frac{x}{y}-1\right)}{\left(\frac{x}{y}\right)T_1-T_2} = \frac{3\times1\left(\frac{2}{1}-1\right)}{\frac{2}{1}\times3-1} \text{ hour}$$

$= \frac{3}{5}$ hour $= \frac{3}{5} \times 60$ minutes $= 36$ minutes.

Shortcut Approach - 9

If on adding x to four numbers a, b, c and d; the resulting numbers become proportional, then

$$x = \frac{bc - ad}{(a+d)-(b+c)}.$$

Illustration – 25

The number to be added to each of the number 7, 16, 43, 79 so that the resulting number become proportional is

(a) 5 (b) 4 (c) 2 (d) 1

Shortcut Solution : (a)

$$\text{Required number} = \frac{bc - ad}{(a+d)-(b+c)}$$

$$= \frac{16 \times 43 - 7 \times 79}{(7+79)-(16+43)} = \frac{135}{27} = 5.$$

Shortcut Approach - 10

Two numbers are in the ratio $a : b$. If p is added to the first number and q is added to the second number respectively, then ratio of the resulting numbers become $m : n$. Then,

The original first number $= \frac{a(mq - np)}{an - bm}$ and the original second number $= \frac{b(mq - np)}{an - bm}$

Illustration – 26

Two numbers are in the ratio 3 : 5. If 2 is added to the first number and 5 is added to the second number, then ratio of the resulting numbers become 4 : 7. Find the smaller number.

(a) 10 (b) 16 (c) 17 (d) 18

Shortcut Solution : (d)

Smaller number = First number

$$= \frac{a(mq - np)}{an - bm} = \frac{3(4 \times 5 - 7 \times 2)}{3 \times 7 - 5 \times 4} = \frac{3 \times 6}{21 - 20} = 18.$$

PARTNERSHIP

A partnership is an association of two or more persons who invested their money/ resources in order to carry on a business. All these people, who have invested their resources are called **Partners**.

Partners are basically of two types :

(i) **Sleeping partner :** A partner who just invests his or her money/ resources but does not participates in the working and management of business is called as *sleeping partner*.

(ii) **Working partner :** A partner who is directly involved in the working and management of the business is called as *working partner*.

General Rules of Partnership

(i) A working partner gets either monthly payment or a share in the profit for his contribution in the working and management of the business. This payment is deducted from the total profit before its distribution.

(ii) If the partners invest different amounts for the same period of time, then the profits/ losses of all the partners are shared in the ratio of their investments.

(iii) If the partners invests same amount for the different time periods, then the profits/losses of all the partners are shared in the ratio of time periods for which their amounts were invested.

(iv) If the partners invest different amounts for different time periods, then their profits/losses are shared in the ratio of product of their investment and time period for which they invested.

Illustration – 1

Three partners Rahul, Puneet and Chandan invest ₹ 1600, ₹ 1800 and ₹ 2300 respectively in a business. How should they divide a profit of ₹ 399 ?

Solution : Profit is to be divided in the ratio 16 : 18 : 23

$$\text{Rahul's share of profit} = \frac{16}{16+18+23} \times 399 = ₹112$$

$$\text{Puneet's share of profit} = \frac{18}{57} \times 399 = ₹126$$

$$\text{Chandan's share of profit} = \frac{23}{57} \times 399 = ₹161$$

Illustration – 2

A, B and C enter into a partnership by investing 1500, 2500 and 3000 rupees, respectively. A as manager gets one-tenth of the total profit and the remaining profit is divided among the three in the ratio of their investment. If A's total share is ₹ 369, find the shares of B and C.

GP_3386

Solution : If total profit is x, then

A's share $= \frac{1}{10}x + \frac{15}{15+25+30}$ of the balance $\frac{9}{10}x$

$\Rightarrow \frac{1}{10}x + \frac{27x}{140} = 369 \Rightarrow 14x + 27x = 369 \times 140$

$\Rightarrow x = \frac{369 \times 140}{41} = 9 \times 140 = 1260$

B's share $= \frac{5}{14} \times \frac{9}{10} \times 1260 =$ ₹405

C's share $= \frac{6}{14} \times \frac{9}{10} \times 1260 =$ ₹486

Illustration – 3

A and B invested in the ratio 3 : 2 in a business. If 5% of the total profit goes to charity and A's share is ₹855, find the total profit.

Solution : Let the total profit be ₹100.

Then, ₹5 goes to charity.

Now, ₹95 is divided in the ratio 3 : 2.

$\therefore$ A's share $= \frac{95}{3+2} \times 3 =$ ₹57

But A's actual share is ₹855.

$\therefore$ Actual total profit $= 855\left(\frac{100}{57}\right) =$ ₹1500

Illustration – 4

A and B start a business. A invests ₹600 more than B for 4 months and B for 5 months. A's share is ₹48 more than that of B, out of a total profit of ₹528. Find the capital contributed by each.

Solution : B's profit $= \frac{528-48}{2} =$ ₹240

A's profit $= 528 - 240 =$ ₹288

$\therefore \frac{\text{A's capital} \times 4}{\text{B's capital} \times 5} = \frac{288}{240} = \frac{6}{5} \Rightarrow \frac{\text{A's capital}}{\text{B's capital}} = \frac{6}{5} \times \frac{5}{4} = \frac{3}{2}$

$\Rightarrow \frac{\text{B's capital} + 600}{\text{B's capital}} = \frac{3}{2}$

$\Rightarrow$ B's capital = ₹1200 and A's capital = ₹1800

Note: In a group of n persons invested different different amount for different period then their profit or loss ratio is

$At_1 : Bt_2 : Ct_3 : Dt_4 \ldots\ldots : Xt_n$

[Here first person invested amount A for t_1 period, second person invested amount B for t_2 period and so on.]

Illustration – 5

Three persons A, B, C rent the grazing of a park for ₹ 570. A puts in 126 oxen in the park for 3 months, B puts in 162 oxen for 5 months and C puts in 216 oxen for 4 months. What part of the rent should each person pay ?

Solution : Monthly equivalent rent of A $= 126 \times 3 = 378$

Monthly equivalent rent of B $= 162 \times 5 = 810$

Monthly equivalent rent of C $= 216 \times 4 = 864$

$\therefore$ Rent is to be divided in the ratio 378 : 810 : 864, i.e. 7 : 15 : 16

$\therefore$ A would have to pay $\frac{7}{7+15+16}$ of the rent

$= \frac{7}{38}$ of the rent $= \frac{7}{38} \times 570 =$ ₹105

$\therefore$ B would have to pay $\frac{15}{38}$ of the rent $\frac{15}{38} \times 570 =$ ₹225

and C would have to pay $\frac{16}{38}$, i.e. $\frac{8}{19}$ of the rent

$= \frac{8}{19} \times 570 =$ ₹240

Illustration – 6

Shekhar started a business investing ₹25,000 in 1999. In 2000, he invested an additional amount of ₹10,000 and Rajeev joined him with an amount of ₹35,000. In 2002, Shekhar invested another additional amount of ₹10,000 and Jatin joined them with an amount of ₹35,000. What will be Rajeev's share in the profit of ₹1,50,000 earned at the end of 3 years from the start of the business in 1999 ?

(a) ₹45,000 **(b)** ₹50,000

(c) ₹70,000 **(d)** ₹75,000

Solution : (b) Ratio of Shekhar, Rajeev and Jatin's investments = 25000 × 36 + 10000 × 24 + 10000 × 12 : 35000 × 24 : 35000 × 12,

$= 25 \times 36 + 10 \times 24 + 10 \times 12 : 35 \times 24 : 35 \times 12$

$= 25 \times 3 + 10 \times 2 + 10 \times 1 : 35 \times 2 : 35 \times 1$

$= 75 + 20 + 10 : 70 : 35 = 105 : 70 : 35$, i.e. $3 : 2 : 1$

$\therefore$ Rajeev's profit $= \frac{2}{6} \times 150000 = ₹ 50000$

SHORTCUTS

Shortcut Approach - 12

If A, B, C, ... enter into a partnership with capital investment ratio a : b : c :..... and ratio of timings of their investments $t_1 : t_2 : t_3$: .....; then the ratio of their profits is $at_1 : bt_2 : ct_3$: ..... .

Illustration – 7

A, B and C invested capitals in the ratio 7 : 6 : 5, the timings of theirs investments being in the ratio 4 : 7 : 9. In what ratio would their profit be distributed?

(a) 18 : 23 : 30 (b) 15 : 29 : 31
(c) 14 : 21 : 23 (d) 28 : 42 : 45

Shortcut Solution : (d)

Profit of A : Profit of B : Profit of C $= at_1 : bt_2 : ct_3$

$= 7 \times 4 : 6 \times 7 : 5 \times 9 = 28 : 42 : 45$

Shortcut Approach - 12

If A, B, C, ... enter into partnership and they invested a, b, c, ... respectively for the periods t_1, t_2, t_3,..... respectively, then their profit/loss ratio is $at_1 : bt_2 : ct_3$: ..... respectively.

Illustration – 8

A, B and C enter into a partnership. A invested ₹ 4000 for the whole year, B puts in ₹ 6000 at the first and increasing to ₹ 8000 at the end of 4 months, while C puts in at first ₹ 8000 but withdraw ₹ 2000 at the end of 9 months. Find the profit of A at the end of the year, if the total profit is ₹ 16950.

(a) ₹ 2800 (b) ₹ 3200 (c) ₹ 3600 (d) ₹ 4100

Shortcut Solution : (c)

Profit of A : Profit of B : Profit of C $= at_1 : bt_2 : bt_3$

$= (4000 \times 12) : (6000 \times 4 + 8000 \times 8) : (8000 \times 9 + 6000 \times 3)$

$= (2 \times 12) : (3 \times 4 + 4 \times 8) : (4 \times 9 + 3 \times 3) = 24 : 44 : 45$

Profit of A $= \frac{24}{24 + 44 + 45} \times 16950 = ₹ 3600$

Shortcut Approach - 12

If capital investment of A, B, C, ... are in the ratio of a : b : c : ..... and their profits ratio is $p_1 : p_2 : p_3$: .....; then the ratio of timings of their investments

$= \frac{p_1}{a} : \frac{p_2}{b} : \frac{p_3}{c} :$ .....

Illustration – 9

A, B and C invested capitals in the ratio 4 : 5 : 6. At the end of the business term, they received the profits in the ratio 2 : 3 : 4. Find the ratio of time for which they contributed their capitals.

(a) 10 : 9 : 11 (b) 15 : 9 : 10
(c) 12 : 13 : 15 (d) None of these

Shortcut Solution : (b)

Ratio of timing $= \frac{p_1}{a} : \frac{p_2}{b} : \frac{p_3}{c} = \frac{2}{4} : \frac{3}{5} : \frac{4}{6}$

$= 30 : 18 : 20 = 15 : 9 : 10$

Shortcut Approach - 10

If ratio of periods of investments of A, B, C, ... is $t_1 : t_2 : t_3$: .....; and ratio of their profits is $p_1 : p_2 : p_3$: .....; then ratio of their capital investments

$= \frac{p_1}{t_1} : \frac{p_2}{t_2} : \frac{p_3}{t_3} :$ .....

Illustration – 10

A, B and C invest their capitals in a business. If the ratio of their periods of investments is 2 : 3 : 4 and their profits are in the ratio of 4 : 3 : 2. Find the ratio in which the investments are made by A, B and C.

(a) 4 : 2 : 1 (b) 3 : 2 : 1 (c) 5 : 3 : 1 (d) 5 : 4 : 3

Shortcut Solution : (a)

Ratio of investments $= \frac{4}{2} : \frac{3}{3} : \frac{2}{4} = 2 : 1 : \frac{1}{2} = 4 : 2 : 1$

Shortcut Approach - 10

Two partners A and B invests in the ratio a : b in a business. If p% of the total profit goes to charity and A's share is ₹ X, then the total profit

$= ₹ X \left(\frac{100}{100 - p}\right)\left(1 + \frac{b}{a}\right)$

Illustration – 11

A and B invested in the ratio 5 : 4 in a business. If 4% of the total profit goes to charity and A's share is ₹ 480, find the total profit.

(a) ₹ 750 (b) ₹ 825 (c) ₹ 900 (d) ₹ 960

Shortcut Solution : (c)

Total profit $= X \left(\frac{100}{100 - p}\right)\left(1 + \frac{b}{a}\right)$

$= 480 \left(\frac{100}{100 - 4}\right)\left(1 + \frac{4}{5}\right) = ₹ 900$

GP_3386

LEVEL - 1

1. Two glasses A and B with their capacities in the ratio 2 : 3. If the total capacities of the glasses is 45 litres. Find the capacity of glass A. **[CDS 2015]**
(a) 5 (b) 6 (c) 7 (d) 18

2. Amit and Sudesh have invested in the ratio of 4 : 7. If both invested an amount of ₹ 49500, then find the investment of sudesh. **[Bank Clerk 2010]**
(a) 31500 (b) 1800 (c) 31000 (d) 18500

3. ₹ 1220 is divided, among A, B, C and D, such that B's share is $\frac{5}{9}^{th}$ of A's; C's share is $\frac{7}{10}^{th}$ of B's and D has $\frac{1}{3}$ as much as B and C together. Find A's share.
(a) ₹ 540 (b) ₹ 802 (c) ₹ 100 (d) ₹ 650

4. A and B entered into a partnership with investments of ₹ 15000 and ₹ 40000 respectively. After 3 months A left from the business, at the same time C joins with ₹ 30000. At the end of 9 months, they got ₹ 7800 as profit. Find the share of B.
(a) ₹ 4800 (b) ₹ 600 (c) ₹ 2400 (d) ₹ 1200

5. A and B entered into a partnership, investing ₹ 16000 and ₹ 12000 respectively. After 3 months, 'A' withdraw ₹ 5000, while B invested ₹ 5000 more. After 3 more months, C joins the business with a capital of ₹ 21000. After a year, they obtained a profit of ₹ 26,400. By what value does the share of B exceed the share of C ? **[SSC CHSL 2015]**
(a) ₹ 3600 (b) ₹ 2100 (c) ₹ 3000 (d) ₹ 2300

6. A certain sum of money was divided among A, B and C in a certain way. C got half as much as A and B together got. A got one third of what B and C together got. What is the ratio of A's share to that of C's share?
(a) 1 : 4 (b) 3 : 4 (c) 4 : 1 (d) 3 : 5

7. A B and C invested capitals in the ratio 2 : 3: 5; the timing of their investments being in the ratio 4 : 5: 6. In what ratio would their profit be distributed ?
(a) 7: 16 : 21 (b) 2 : 4: 5
(c) 5 : 9 : 14 (d) 8 : 15 : 30

8. Two numbers are in the ratio of 3 : 4. If 5 is subtracted from each, the resulting numbers are in the ratio 2 : 3. Find the numbers
(a) 12, 16 (b) 24, 32 (c) 60, 80 (d) 15, 20

9. Divide 1458 into two parts such that one may be to the other as 2 :7 **[Bank Clerk 2014]**
(a) 324, 1134 (b) 358, 1100
(c) 462, 996 (d) 458, 1000

10. Find the fourth proportional to $12X^3$, $9aX^2$, $8a^3X$.
(a) $4a^3$ (b) $6a^4$ (c) $5a$ (d) $7a^5$

11. Find $a : b : c$, if $6a = 9b = 10c$.
(a) 12 : 10 : 8 (b) 15 : 4 : 3
(c) 15 : 18 : 9 (d) 15 : 10 : 9

12. If $a : b = 2 : 3, b : c = 3 : 4, c : d = 4 : 5$, find $a : b : c : d$.
(a) 5 : 4 : 3 : 2 (b) 30 : 20 : 15 : 12
(c) 2 : 3 : 4 : 6 (d) 2 : 3 : 4 : 5

13. The ratio of A: B = 1 : 3, B : C = 2: 5 and C : D = 2 : 3. Find the value of A : B : C: D. **[SSC CGL 2010]**
(a) 3 : 9 : 12 : 15 (b) 4 : 12 : 30 : 45
(c) 5 : 10 : 15 : 20 (d) 4: 8 : 12 : 20

14. If ratio A : B = 1 : 2, B : C = 3 : 4 and C: D = 5: 6 find D : C : B : A = ? **[SSC CGL 2008]**
(a) 6 : 5 : 4 : 2 (b) 6 : 4 : 2 : 1
(c) 6 : 3 : 2: 1 (d) 48 : 40 : 30 : 15

15. Two numbers are in the ratio 2 : 3. If 3 be added to both of them, then their ratio becomes 3 : 4. Find the sum of the numbers. **(SSC CGL Tier 1 2015)**
(a) 10 (b) 15 (c) 20 (d) 25

16. The sum of three numbers is 98. If the ratio of the first to the second is 2 : 3 and that of the second to the third is 5 : 8, then the second number is:
(a) 20 (b) 30 (c) 38 (d) 48

17. Two numbers are in the ratio of 2: 3. If 15 is added to both the numbers, then the ratio between two numbers becomes $\frac{11}{14}$. Find the greater number. **[SSC CGL Tier II 2013]**
(a) 18 (b) 30 (c) 27 (d) 9

18. Tea worth ₹ 126 per kg and ₹ 135 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth ₹ 153 per kg, then the price of the third variety per kg is
(a) ₹ 169.50 (b) ₹ 170 (c) ₹ 175.50 (d) ₹ 180

19. Incomes of two companies A and B are in the ratio of 5 : 8. Had the income of company A been more by ₹ 25 lakh, the ratio of their incomes would have been 5 : 4. What is the income of company B?
(a) ₹ 80 lakh (b) ₹ 50 lakh
(c) ₹ 40 lakh (d) ₹ 60 lakh

20. There is a ratio of 5 :4 between two numbers. If 40 percent of the first number is 12 then what would be the 50 percent of the second number?
(a) 12 (b) 24
(c) 18 (d) None of these

21. A and B rent a pasture for 10 months; A puts in 80 cows for 7 months. How many can B put in for the remaining 3 months, if he pays half as much again as A?
(a) 120 (b) 180 (c) 200 (d) 280

22. A sum of ₹1300 is divided amongst P, Q, R and S such that

$$\frac{P\text{'s share}}{Q\text{'s share}}=\frac{Q\text{'s share}}{R\text{'s share}}=\frac{R\text{'s share}}{S\text{'s share}}=\frac{2}{3}.$$

Then, P's share is
(a) ₹ 140 (b) ₹ 160 (c) ₹ 240 (d) ₹ 320

23. If 10 persons can clean 10 floors by 10 mops in 10 days, in how many days can 8 persons clean 8 floors by 8 mops?
(a) $12\frac{1}{2}$ days (b) 8 days
(c) 10 days (d) $8\frac{1}{3}$ days

24. Zinc and copper are melted together in the ratio 9 : 11. What is the weight of melted mixture, if 28.8 kg of zinc has been consumed in it?
(a) 58 kg (b) 60 kg (c) 64 kg (d) 70 kg

25. ₹ 1104 is divided between 3 men, 4 women and 6 boys, so that the share of a man, a woman and a boy are in the proportion of 3 : 2 : 1. How much does each boy get?
(a) ₹ 48 (b) ₹ 64
(c) ₹ 96 (d) Cannot be determined

26. A man completes 5/8 of a job in 10 days. At this rate, how many more days will it take him to finish the job?
(a) 5 (b) 6 (c) 7 (d) $7\frac{1}{2}$

27. In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for whole time. Find A's share of the total profit of ₹ 2,300.
(a) ₹ 100 (b) ₹ 200 (c) ₹ 300 (d) ₹ 400

28. A, B and C invested capitals in the ratio 2 : 3: 4. At the end of the business term, they received the profits in the ratio 3 : 6: 10. Find the ratio of the periods for which they contributed their capitals.
(a) 1: 2: 3 (b) 3: 4: 5 (c) 4 : 5: 6 (d) 2 : 1: 3

29. In a partnership, A invested $\frac{1}{6}$ th of the capital for $\frac{1}{6}$ th of the time, B invested $\frac{1}{3}$ rd of the capital for $\frac{1}{3}$ rd of the time, and C invested the rest of the capital for whole period. At the end of the period, they earned a profit of ₹ 4600.Find the share of B.
(a) ₹ 1600 (b) ₹ 800 (c) ₹ 1000 (d) ₹ 1200

30. A photograph measuring $2\frac{1}{2}''\times 1\frac{7}{8}''$ is to be enlarged so that the length will be 4". How many inches will the enlarged breadth be?
(a) $1\frac{1}{2}$ (b) $2\frac{1}{8}$ (c) 3 (d) $3\frac{3}{8}$

31. Out of a total amount of ₹ 4,898, B receives 20 % more than A and 25% more than C. What is B's share?
(a) ₹ 930 (b) ₹ 1,860 (c) ₹ 1,400 (d) ₹ 1, 540

32. Two numbers are in the ratio 5 : 7. On diminishing each of them by 40, they become in the ratio 17 : 27. The difference of the numbers is :
(SSC CGL Tier I 2014)
(a) 18 (b) 52 (c) 137 (d) 50

33. Two numbers are in the ratio $1\frac{1}{2}:2\frac{2}{3}$. When each of these is increased by 15, they become in the ratio $1\frac{2}{3}:2\frac{1}{2}$. The greater of the numbers is :
(a) 27 (b) 36 (c) 48 (d) 64

34. The total emoluments of A and B are equal. However, A gets 65% of his basic salary as allowances and B gets 80% of his basic salary as allowances. What is the ratio of the basic salaries of A and B ?
(a) 16 : 13 (b) 5 : 9 (c) 12 : 11 (d) 7 : 8

GP_3386

35. If A : B : C = 2 : 3 : 4, then $\frac{A}{B}:\frac{B}{C}:\frac{C}{A}$ is equal to :
(a) 4 : 9 : 16 (b) 8 : 9 : 12
(c) 8 : 9 : 16 (d) 8 : 9 : 24

36. Given that 24 carat gold is pure gold, 18 carat gold is $\frac{3}{4}$ gold and 20 carat gold is $\frac{5}{6}$ gold, the ratio of the pure gold in 18 carat gold to the pure gold in 20 carat gold is :
(a) 3 : 8 (b) 9 : 10 (c) 15 : 24 (d) 8 : 5

37. A mixture contains spirit and water in the ratio 3: 2 If it contains 3 litres more spirit than water, then the quantity of spirit in the mixture is:
(SSC CGLTier I 2016)
(a) 10 litres (b) 12 litres
(c) 8 litres (d) 9 litres

38. If ₹ 900 is divided among A, B, C; the division is such that $\frac{1}{2}$ of A's money = $\frac{1}{3}$rd of B's money = $\frac{1}{4}$th of C's money. Find the amount (in ₹) received by A, B, C. **(SSC CGLTier I 2017)**
(a) 300, 400, 200 (b) 350, 450, 100
(c) 200, 300, 400 (d) 400, 150, 130

39. If 0.75 : x : : 5 : 8, then x is equal to:
(a) 1.12 (b) 1.20 (c) 1.25 (d) 1.30

40. The fourth proportional to 5, 8, 15 is:
(a) 18 (b) 24 (c) 19 (d) 20

41. What must be deducted from each of the numbers 7, 10, 12 and 18 so that the resultant numbers are in proportion ?
(a) 1 (b) 2 (c) 3 (d) 4

42. The income of A and B are in the ratio 3 : 2 and expenses are in the ratio 5 : 3. If both save ₹ 200, what is the income of A?
(a) ₹ 1000 (b) ₹ 1200 (c) ₹ 1500 (d) ₹ 1800

43. The incomes of Ram and Shyam are in the ratio 8 : 11 and their expenditure are in the ratio 7 :10. If each of them saves ₹ 500, what are their incomes and expenditures ?
(a) 4000, 5500, 3500, 5000
(b) 5000, 5000, 4000, 3500
(c) 4000, 5000, 3500, 4500
(d) None of these

44. Divide ₹ 671 among A, B, C such that if their shares be increased by ₹3, ₹7 and ₹9 respectively, the remainder shall be in the ratio 1 : 2 : 3.
(a) ₹ 112, ₹ 223, ₹ 336 (b) ₹ 114, ₹ 221, ₹ 336
(c) ₹ 112, ₹ 227, ₹ 332 (d) ₹ 114, ₹ 223, ₹ 334

45. Salaries of A, B and C were in the ratio 3 : 5 : 7, respectively. If their salaries were increased by 50%, 60% and 50% respectively, what will be the new ratio of the their respective new salaries ?
(a) 4 : 5 : 7 (b) 3 : 6 : 7
(c) 4 : 15 : 18 (d) 9 : 16 : 21

46. A sum of money is to be divided among A, B and C in the ratio 2 : 3 : 7. If the total share of A and B together is ₹ 1,500 less than C, What is A's share in it?
(a) ₹ 1,000 (b) ₹ 1,500
(c) ₹ 2,000 (d) Data insufficient

47. If 12 men can build a wall 100 m long, 3 m high, and 0.5 m thick in 25 days, in how many days will 20 men build a wall 60 m × 4 m × 0.25 m?
(a) 6 days (b) 8 days (c) 12 days(d) 15 days

48. In a dairy farm, 40 cows eat 40 bags of husk in 40 days. In how many days one cow will eat one bag of husk ?
(a) 1 (b) $\frac{1}{40}$ (c) 40 (d) 80

49. In a camp, there is a meal for 120 men or 200 children. If 150 children have taken the meal, how many men will be catered to with the remaining meal ?
(a) 20 (b) 30 (c) 40 (d) 50

50. 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day, complete the work ?
(a) 10 (b) 13 (c) 14 (d) 15

51. In 1 kg mixture of sand and iron, 20% is iron. How much sand should be added so that the proportion of iron becomes 10%?
(a) 1 kg (b) 200 gms (c) 800 gms(d)1.8 kg

52. When 30 percent of a number is added to another number the second number increases to its 140 per cent. What is the ratio between the first and the second number?
(a) 3 : 4 (b) 4 : 3
(c) 3 : 2 (d) None of these

53. The ratio of number of ladies to gents at a party was 1 : 2, but when 2 ladies and 2 gents left, the ratio became 1 : 3. How many people were originally present at the party?
(a) 6 (b) 9 (c) 12 (d) 10

54. The ratio of males and females in a city is 7 : 8 and the percentage of children among males and females is 25% and 20% respectively. If the number of adult females in the city is 156800 what is the total population?
(a) 245000 (b) 367500 (c) 196000 (d) 171500

55. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ and its perimeter is 104 cm. The length of the longest side is

(a) 52 cm (b) 48 cm (c) 32 cm (d) 26 cm

56. A bag contains an equal number of one rupee, 50 paise and 25 paise coins respectively. If the total value is ₹ 35, how many coins of each type are there?

(a) 20 coins (b) 30 coins
(c) 28 coins (d) 25 coins

57. If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, then each fraction is equal to

(a) $(a+b+c)^2$ (b) 1/2
(c) 1/4 (d) 0

58. If ₹ 126.50 is divided among A, B and C in the ratio of 2 : 5: 4, the share of B exceeds that of A by **(SSC CGL Tier II 2013)**

(a) ₹ 36.50 (b) ₹ 35.50 (c) ₹ 34.50 (d) ₹ 33.50

59. The speed of three buses are in the ratio 2 : 3 : 4. The ratio between the time taken by these buses to travel the same distance is

(a) 2 : 3 : 6 (b) 4 : 3 : 2 (c) 4 : 3 : 6 (d) 6 : 4 : 3

60. The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is :

(a) 21 : 19 (b) 11:9 (c) 22 : 9 (d) 11 : 18

61. The ratio between sum and difference of two numbers is 5 : 1. Find the ratio between the numbers.

(a) 5 : 2 (b) 3 : 2 (c) 2 : 3 (d) 2 : 5

62. The difference between the two positive numbers is 10 and the ratio between them is 5 : 3. Find the product of the two numbers.

(a) 375 (b) 325 (c) 275 (d) 125

63. If 30 oxen can plough 1/7th of a field in 4 hours, in how many hours will 18 oxen take to do the remaining work?

(a) 30 hours (b) 20 hours
(c) 15 hours (d) 40 hours

64. Four numbers in the ratio of 1 : 3 : 4 : 7 add up to give a sum of 75. Find the value of the biggest number.

(a) 42 (b) 35 (c) 49 (d) 63

65. According to the Boyle's law, at a constant temperature, pressure of a definite mass of gas is inversely proportional to the volume. If the pressure is reduced by 20% find the respective change in volume.

(a) −33.33% (b) +25%
(c) −25% (d) +33.33 %

66. The ratio of two numbers is 10:7 and their difference is 105. The sum of the numbers is: **(SSC CGL Tier II 2015)**

(a) 595 (b) 805 (c) 1190 (d) 1610

67. A certain sum of money is divided between A and B in the ratio 3:7. If B gets ₹ 800 more than A, find the total sum.

(a) ₹ 2400 (b) ₹ 3200 (c) ₹ 3000 (d) ₹ 2000

68. What number must be subtracted from both the numerator and the denominator 27 : 35, so that it becomes equal to 2 : 3?

(a) 8 (b) 12
(c) 6 (d) None of these

69. If p/q = 3/4, then find the value of the expression (5p − 3q)/(7p + 2q).

(a) 3/29 (b) 5/27 (c) 7/31 (d) 5/58

70. A and B invested ₹ 12,000 and ₹ 18,000 respectively in a business for the whole year. At the year-end, there was a total profit of ₹ 2,000. What is the share of A in the profit?

(a) ₹ 800 (b) ₹ 1, 200
(c) ₹ 1,600 (d) None of these

71. A and B invest ₹ 12,000 and ₹ 16,000 respectively in a business. At the year-end, they share the profit in the ratio of 3 : 1. If A has invested his capital for the whole year, for how many months B has invested his capital?

(a) 4 months (b) 3 months
(c) 6 months (d) 8 months

72. A and B are partners in a business. A contributes 1/4 of the capital for 15 months and B received 2/3 of the profit. Find how long B's money was used. **(SSC CGL Tier I 2012)**

(a) 6 months (b) 8 months
(c) 10 months (d) 12 months

73. A began business with ₹ 45000 and was joined afterwards by B with ₹ 54000. After how many months did B join if the profits at the end of the year were divided in the ratio 2 : 1 ? **(SSC CGL 2011)**

(a) 4 (b) 5 (c) 6 (d) 7

74. A and B invested in the ratio 3 : 2 in a business. If 5 % of the total profit goes to charity and A's share is ₹ 855, find the total profit.

(a) ₹ 1200 (b) ₹ 1500 (c) ₹ 1800 (d) ₹ 2100

75. The sum of the ages of the 4 members of Sinha family is 140 years. 5 years ago the ages of the 4 members Nishu, Vicky, Mrs Sinha nad Mr. Sinha were in the ratio of 2 : 3 : 7 : 8. After how many years would Nishu be as old as the present age of his mother?

(a) 10 yrs (b) 17 yrs (c) 30 yrs (d) 32 yrs

GP_3386

76. Two cogged wheels of which one has 32 cogs and other 54 cogs, work into each other. If the latter turns 80 times in three quarter of a minute, how often does the other turn in 8 second? (Assume equal size cogs and equis-spaced).
(a) 24 (b) 16 (c) 32 (d) 8

77. A bag contains ₹ 600 in the form of one rupee, 50 paise and 25 paise coins in the ratio of 3 : 4 : 12. Find the total number of 25 paise coins in the bag.
(a) 75 (b) 200 (c) 300 (d) 900

78. The prices of a scooter and a moped are in the ratio of 9 : 5. If a scooter costs ₹ 4200 more than a moped, find the total price of both vehicles.
(a) ₹14000 (b) ₹14700 (c) ₹ 15000 (d) ₹15200

79. What number should be subtracted from both terms of the ratio 15 : 19 in order to make it 3 :4 ?
(SSC CHSL 2016)
(a) 9 (b) 2 (c) 5 (d) 3

80. What must be added to each term of the ratio 7 : 11 so as to make it equal to 3:4 ?
(SSC CGL 2011)
(a) 1 (b) 2 (c) 3 (d) 5

LEVEL - 2

1. Three friends *A, B* and *C* started a business by investing a sum of money in the ratio of 5 : 7 : 6. After 6 months *C* withdraws half of his capital. If the sum invested by '*A*' is ₹ 40,000, out of a total annual profit of ₹ 33,000, *C*'s share will be
(a) ₹9,000 (b) ₹12,000 (c) ₹11,000 (d) ₹10,000

2. *A*, *B* and *C* are partners. *A* receives 9/10 of the profit and *B* and *C* share the remaining profit equally. *A*'s income is increased by ₹ 270 when the total profit rises from 12 to 15%. Find the capital invested by *B* and *C* each.
(a) ₹5000 (b) ₹1000 (c) ₹500 (d) ₹1500

3. *A* started a business with ₹ 21,000 and is joined afterwards by *B* with ₹ 36,000. After how many months did *B* join if the profits at the end of the year are divided equally?
(a) 3 (b) 4 (c) 5 (d) 6

4. Abhishek started a business investing ₹ 50,000. After one year he invested another ₹ 30,000 and Sudin also joined him with a capital of ₹ 70,000. If the profit earned in three years from the starting of business was ₹ 87,500, then find the share of Sudin in the profit.
(a) ₹ 37,500 (b) ₹ 35,000
(c) ₹ 38,281 (d) ₹ 52,500

5. *A* and *B* start a business with investments of ₹5000 and ₹4500 respectively. After 4 months, *A* takes out half of his capital. After two more months, *B* takes out one-third of his capital while *C* joins them with a capital of ₹ 7000. At the end of a year, they earn a profit of ₹ 5080. Find the share of each member in the profit.
(a) *A* – ₹ 1400, *B* – ₹ 1900, *C* – ₹ 1780
(b) *A* – ₹ 1600, *B* – ₹ 1800, *C* – ₹ 1680
(c) *A* – ₹ 1800, *B* – ₹ 1500, *C* – ₹ 1780
(d) *A* – ₹ 1680, *B* – ₹ 1600, *C* – ₹ 1800

6. *A*, *B* and *C* enter into a partnership. They invest ₹40,000, ₹80,000 and ₹1,20,000 respectively. At the end of the first year, *B* withdraws ₹ 40,000, while at the end of the second year, *C* withdraws ₹ 80,000. In what ratio will the profit be shared at the end of 3 years?
(a) 2 : 3 : 5 (b) 3 : 4 : 7
(c) 4 : 5 : 9 (d) None of these

7. In a class, the number of boys and girls is in the ratio of 4: 5. If 10 more boys join the class, the ratio of numbers of boys and girls becomes 6 : 5. How many girls are there in the class?
[Bank Clerk 2009]
(a) 20 (b) 30
(c) 25 (d) Couldn't be determined

8. Two candles of the same height are lighted at the same time. The first is consumed in 7 hours and the second is consumed in 4 hours. Assuming that each candle burns at a constant rate, in how many hours, after being lighted, was the first candle four times the height of the second ?
(a) 3.5 hrs (b) 2.5 hrs (c) 2 hrs (d) 3 hrs

9. The ratio of the prices of two houses *A* and *B* was 4 : 5 last year. This year, the price of *A* is increased by 25% and that of *B* by ₹ 50000. If their prices are now in the ratio 9 : 10, the price of *A* last year was :
(a) ₹3,60,000 (b) ₹4,50,000
(c) ₹4,80,000 (d) ₹5,00,000

10. In two alloys, the ratio of iron and copper is 4 : 3 and 6 : 1, respectively. If 14 kg of the first alloy and 42 kg of the second alloy is mixed together to form a new alloy, then what will be the ratio of iron to copper in the new alloy?

(a) 11 : 3 (b) 11 : 8
(c) 8 : 1 (d) None of these

11. A and B are two alloys of gold and copper prepared by mixing metals in the ratio 7 : 2 and 7 : 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the ratio of gold and copper in C will be:
(a) 5 : 7 (b) 5 : 9 (c) 7 : 5 (d) 9 : 5

12. A contractor employed 25 labourers on a job. He was paid ₹ 275 for the work. After retaining 20% of this sum, he distributed the remaining amount amongst the labourers. If the number of male to female labourers was in the ratio 2 : 3 and their wages in the ratio 5 : 4, what wages did a female labourer get ?
(a) ₹ 15 (b) ₹ 8 (c) ₹ 14 (d) ₹ 10

13. A man ordered 4 pairs of black socks and some pairs of brown socks. The price of a black pair is double that of a brown pair. While preparing the bill, the clerk interchanged the number of black and brown pairs by mistake which increased the bill by 50%. The ratio of the number of black and brown pairs of sock in the original order was :
(a) 4 : 1 (b) 2 : 1 (c) 1 : 4 (d) 1 : 2

14. The Binary Ice-cream Shopper sells two flavours : Vanilla and Chocolate. On Friday, the ratio of Vanilla cones sold to Chocolate cones sold was 2 : 3. If the store had sold 4 more Vanilla cones, then, the ratio of Vanilla cones sold to the Chocolate cones sold would have been 3 : 4. How many Vanilla cones did the store sell on Friday ?
(a) 32 (b) 35 (c) 42 (d) 48

15. At a start of a seminar, the ratio of the number of male participants to the number of female participants was 3 : 1. During the tea break 16 participants left and 6 more female participants registered. The ratio of the male to the female participants now became 2 : 1. What was the total number of participants at the start of the seminar ?
(a) 64 (b) 48 (c) 54 (d) 72

16. The monthly income of two persons are in the ratio of 4 : 5 and their monthly expenditures are in the ratio of 7 : 9. If each saves ₹ 50 a month, then what are their monthly incomes ?
(a) ₹ 100, ₹ 125 (b) ₹ 200, ₹ 250
(c) ₹ 300, ₹ 375 (d) ₹ 400, ₹ 500

17. The ratio of monthly incomes of A, B is 6 : 5 and their monthly expenditures are in the ratio 4 : 3. If each of them saves ₹ 400 per month, find the sum of their monthly incomes.
(SSC CGLTier I 2013)
(a) ₹ 2300 (b) ₹ 2400 (c) ₹ 2200 (d) ₹ 2500

18. The electricity bill of a certain establishment is partly fixed and partly varies as the number of units of electricity consumed. When in a certain month, 540 units are consumed, the bill is ₹ 1800. In another month, 620 units are consumed and the bill is ₹ 2040. In yet another month, 500 units are consumed. The bill for that month would be:
(a) ₹ 1560 (b) ₹ 1680 (c) ₹ 1840 (d) ₹ 1950

19. A certain number of persons can dig a trench 100 m long, 50 m broad and 10 m deep in 10 days. The same number of persons can dig another trench 20 m broad and 15 m deep in 30 days. The length of the second trench is :
(a) 400m (b) 500m (c) 800m (d) 900m

20. The numbers of students speaking English and Hindi are in the ratio of 4:5. If the number of students speaking English increased by 35% and that speaking Hindi increased by 20%, what would be the new respective ratio?
(a) 19 : 20 (b) 7 : 8 (c) 8 : 9 (d) 9 : 10

21. In Ramnagar Colony, the ratio of school going children to non-school going children is 5 : 4. If in the next year, the number of non-school going children is increased by 20%, making it 35,400, what is the new ratio of school going children to non-school going children?
(a) 4 : 5 (b) 3 : 2
(c) 25 : 24 (d) None of these

22. In a journey of 45 km performed by tonga, rickshaw and cycle in that order, the distance covered by the three ways in that order are in the ratio of 8 : 1 : 3 and charges per kilometre in that order are in the ratio of 8 : 1 : 4. If the tonga charges being 24 paise per kilometre, the total cost of the journey is
(a) ₹ 9.24 (b) ₹ 10
(c) ₹ 12 (d) None of these

23. *A* and *B* invest ₹ 3,000 and ₹ 4,000 in a business. *A* receives ₹ 10 per month out of the profit as a remuneration for running the business and the rest of profit is divided in proportion to the investments. If in a year '*A*' totally receives ₹ 390, what does *B* receive?
(a) ₹ 375 (b) ₹ 360 (c) ₹ 350 (d) ₹ 260

24. In a mixture of milk and water of volume 30 litres, the ratio of milk and water is 7 : 3. Find the quantity of water to be added to the mixture to make the ratio of milk and water 1 : 2.
(a) 30 (b) 32 (c) 33 (d) 35

GP_3386

25. A and B invested the same capital in a business. At the year-end they share the profit in the ratio of 3 : 2. If A has invested his capital for the whole year, for how many months B has invested his capital?
(a) 6 months (b) 8 months
(c) 9 months (d) none of these

26. King Dashrath decided to distribute gold coins to his three queens in the following way : 2nd queen would get 5/7th of what the 1st queen would get and the 3rd queen would get 3/5th of what the 2nd queen would get. 1st queen got 60 gold coins more than the 3rd queen. How many gold coins were distributed to the three queens?
(a) 120 (b) 175 (c) 225 (d) 250

27. In the prime minister office, one-third of the workers are women, half of the women are married and one-third of the married women have children. If three-fourth of the men are married and two-third of the married men have children, what part of the persons working at the PMO are without children?
(a) 5/18 (b) 4/9 (c) 11/18 (d) 17/36

28. Two-fifth of Anil's salary is equal of Bhuvan's salary and seven-ninth of Bhuvan's salary is equal to Chandra's salary. The sum of the salary of all of them is ₹ 770. Which of the following is the salary of each?
(a) 300, 225, 250 (b) 500, 425, 375
(c) 450, 180, 140 (d) 520, 610, 475

29. Inspector Vijay Shankar spends one-fourth of his salary on the house rent, one-third on food and one-sixth on travel. After spending one-tenth of the remaining amount on clothes, he is left with ₹ 13500. What is the difference between the amounts spent on the house rent and food?
(a) ₹ 7500 (b) ₹ 10000 (c) ₹ 4000 (d) ₹ 5000

30. Bidhan is planning to buy a bike worth ₹ 35000, provided his brother agrees to lend him 3/2 times the money, Bidhan contributes and the financer provides 3 times the brother's contribution. How much does Bidhan's brother contribute?
(a) 5000 (b) 7500 (c) 10000 (d) 7000

31. An alloy of manganese, tin and bronze contains 90% bronze, 7% manganese and 3% tin. A second alloy of bronze and tin is melted with the first and the mixture contains 85% of bronze, 5% of manganese and 10% of tin. What is the percentage of bronze in the second alloy?
(a) 67.5% (b) 72.5% (c) 77.5% (d) 82.5%

32. A certain sum of money was divided among Anand, Bidhan and Chandra in the following way: Chandra got half as much as Anand and Bidhan together got. Anand got one-third of what Bidhan and Chandra together got. What is the ratio of Anand's share to that of Chandra's share?
(a) 1 : 2 (b) 3 : 5
(c) 4 : 7 (d) None of these

33. Anil started a manufacturing unit with a certain amount of money. After a few months, Dheeraj became his partner, contributing three times of what Anil had contributed. At the end of the year, each was entitled to half the total profit. If Anil started the unit in January, then when did Dheeraj join as a patner?
(a) August (b) September
(c) July (d) October

34. The work done by a man, a woman and a child is in the ratio of 3 : 2 : 1. There are 20 men, 30 women and 36 children in a factory. Their weekly wages amount to ₹ 780, which is divided in the ratio of work done by the men, women and children. What will be the wages of 15 men, 21 women and 30 children for 2 weeks?
(a) ₹ 585 (b) ₹ 292.5 (c) ₹ 1170 (d) ₹ 900

Hints and Solutions

Level 1

1. **(d)** Use Shortcut Approach-4

2. **(a)** Use Shortcut Approach-4

3. **(a)** If A's share is 1, B's share $= \frac{5}{9} \times 1 = \frac{5}{9}$

C's share $= \frac{7}{10} \times \frac{5}{9} = \frac{7}{18}$;

D's share $= \frac{1}{3}\left(\frac{5}{9}+\frac{7}{18}\right) = \frac{17}{54}$

$\therefore A : B : C : D = 1 : \frac{5}{9} : \frac{7}{18} : \frac{17}{54}$

$= 54 : 30 : 21 : 17.$

$\therefore A$'s share $= \frac{54}{122} \times 1220 = ₹540.$

4. **(a)** Ratio of capitals of A, B and C
$= (15000 \times 3) : (40000 \times 9) : (30000 \times 6)$
$= 1 : 8 : 4$

B's share $= ₹\left(\frac{8}{13} \times 7800\right) = ₹\ 4800$

5. **(a)** Use Shortcut Approach-22

6. **(b)** Let us represent their shares by the corresponding letter of their names.
$A + B = 2C$ and $B + C = 3A$.
$\Rightarrow A + 3A - C = 2C$ (since $B = 3A - C$)
$4A = 3C \Rightarrow A : C = 3 : 4$

7. **(d)** Use Shortcut Approach-21

8. **(d)** Let $3X$ and $4X$ be the numbers

$\Rightarrow \frac{3X-5}{4X-5} = \frac{2}{3} \Rightarrow X = 5$

$\Rightarrow$ The required numbers are 15 and 20.

9. **(a)** Use Shortcut Approach-1

10. **(b)** Let r be the 4th proportional.

Then $\frac{12X^3}{9aX^2} = \frac{8a^3X}{r} \Rightarrow r = 6a^4$

11. **(d)** $\frac{a}{b} = \frac{9}{6} = 3 : 2 = 15 : 10$

$\frac{b}{c} = \frac{10}{9} = 10 : 9$

Hence, $a : b : c = 15 : 10 : 9$

12. **(d)** Obviously the ratio is 2 : 3 : 4 : 5

13. **(b)** Use Shortcut Approach-2

14. **(d)** Use Shortcut Approach-2

15. **(b)** Use Shortcut Approach-12

16. **(b)** Let A, B and C be the first, second and third nos. respectively.
Then, $A : B = 2 : 3$ and $B : C = 5 : 8$
Consider, $A : B = 2 : 3 = 2 \times 5 : 3 \times 5 = 10 : 15$
and $B : C = 5 : 8 = 5 \times 3 : 8 \times 3 = 15 : 24$
$\therefore$ $A : B : C = 10 : 15 : 24$
Let the reqd. number be $10x$, $15x$ and $24x$.
Given, sum of three numbers = 98
$\therefore$ $10x + 15x + 24x = 98$
$\Rightarrow 49x = 98 \Rightarrow x = 2$
$\Rightarrow$ Second number $= 15x = 15 \times 2 = 30$

17. **(c)** Use shortcut Approach-3

18. **(c)** Let the third type of tea is priced at ₹ x per kg. Also suppose that the three types of tea mixed together are 1, 1 and 2 kg,respectively.

Now, $\frac{126 \times 1 + 135 \times 1 + 2x}{1+1+2} = 153$

$\Rightarrow \frac{261+2x}{4} = 153 \Rightarrow 261 + 2x = 612$

$\Rightarrow x = \frac{351}{2} = ₹\ 175.5$ per kg.

19. **(c)** Let the incomes of two companies A and B be $5x$ and $8x$ respectively.
From the question,

$\frac{5x+25}{8x} = \frac{5}{4} \Rightarrow 20x + 100 = 40x \ \therefore\ x = 5$

$\therefore$ Income of company $B = 8x = ₹\ 40$ lakh

20. (a) $\frac{a}{b}=\frac{5}{4}, b=\frac{4}{5}a$ Given $(40\%\text{ of } a)=\frac{2}{5}a=12$

$\therefore \quad a=5\times 6$ and $b=\frac{4}{5}\times 5\times 6=24$

$\therefore \quad 50\%\text{ of } b=\frac{24}{2}=12$

21. (d) Let B puts $= x$ cows

then amount paid by

$B=\frac{3}{2}\times$ amount paid by A.

$\therefore \frac{80\times 7}{x\times 3}=\frac{\text{amount paid by } A}{3/2\times \text{ amount paid by } A}$

$\Rightarrow x=\frac{80\times 7\times 3}{3\times 2}=280$ cows

22. (b) Let $P = 2x$ and $Q = 3x$. Then, $\frac{Q}{R}=\frac{2}{3}$

$\Rightarrow R=\frac{3}{2}Q=\left(\frac{3}{2}\times 3x\right)=\frac{9x}{2}$.

Also, $\frac{R}{S}=\frac{2}{3}$

$\Rightarrow S=\frac{3}{2}R=\left(\frac{3}{2}\times\frac{9x}{2}\right)=\frac{27x}{4}$

$\therefore P=2x, Q=3x, R=\frac{9x}{2}$ and $S=\frac{27x}{4}$

Now, $P + Q + R + S = 1300$

$\Leftrightarrow \left(2x+3x+\frac{9x}{2}+\frac{27x}{4}\right)=1300$

$\Leftrightarrow 65x = 5200 \Leftrightarrow x=\frac{5200}{65}=80.$

$\therefore$ P's share = ₹(2×80) = ₹160.

23. (c) Do not get confused by the distractions given in the problem. 10 men and 10 days means 100 man-days are required to clean 10 floors.

That is, 1 floor requires 10 man-days to get cleaned. Hence, 8 floors will require 80 man-days to clean.

Therefore, 10 days are required to clean 8 floors.

24. (c) For 9 kg zinc, mixture melted = (9 + 11) kg.

For 28.8 kg zinc, mixture melted

$=\left(\frac{20}{9}\times 28.8\right)\text{kg}=64$ kg.

25. (a) Let each boy gets x, so the women gets $2x$ and a man gets $3x$.

Now, $(3 \times 3x) + (4 \times 2x) + (6 \times x) = 1104$

$\Rightarrow \quad 23x = 1104 \Rightarrow x = 48$

$\therefore$ Each boy gets ₹ 48.

26. (b) Work done $=\frac{5}{8}$.

Balance work $=\left(1-\frac{5}{8}\right)=\frac{3}{8}$.

Less work, Less days (Direct Proportion)

Let the required number of days be x. Then,

Work	days
5/8	10
3/8	x

Then, $\frac{5}{8}:\frac{3}{8}::10:x \Rightarrow \frac{5}{8}\times x=\frac{3}{8}\times 10$

$\Rightarrow x=\left(\frac{3}{8}\times 10\times\frac{8}{5}\right)=6.$

27. (a) Remaining capital $=1-\left(\frac{1}{6}+\frac{1}{3}\right)=\frac{1}{2}$

Ratio of their profit

$=\frac{1}{6}\times\left[\frac{1}{6}\times 12\right]:\frac{1}{3}\times\left[\frac{1}{3}\times 12\right]:\frac{1}{2}\times 12$

$=\frac{1}{3}:\frac{4}{3}:6=1:4:18$

$\therefore A$'s share $=\frac{1}{1+4+18}\times 2300=$ ₹ 100

28. (b) Use Shortcut Approach-23

29. (b) Use Shortcut Approach-22

30. (c) Let enlarged breadth be x inches. Then,

$\frac{5}{2}:4::\frac{15}{8}:x$

$\Rightarrow \frac{5}{2}x=4\times\frac{15}{8}\Rightarrow x=3$ inches

31. (b) Given $A+B+C=4898$ ..... (i)

Also $B=\frac{120}{100}A$ and $B=\frac{125}{100}C$

$\therefore$ By (i), $\frac{100}{120}B+B+\frac{100}{125}B=4898$

$\Rightarrow B=\frac{4898\times 30}{79}=$₹1,860

32. **(d)** Use Shortcut Approach-15

33. **(c)** Use Shortcut Approach-14

34. **(a)** Let the basic salary of A be ₹ x and that of B be ₹ y.

$$\text{Now, } x \times \frac{65}{100} = y \times \frac{80}{100} \Rightarrow x : y = 16 : 13$$

35. **(d)** Let A = 2x, B = 3x and C = 4x. Then,

$$\frac{A}{B} = \frac{2x}{3x} = \frac{2}{3}, \frac{B}{C} = \frac{3x}{4x} = \frac{3}{4} \text{ and } \frac{C}{A} = \frac{4x}{2x} = \frac{2}{1}$$

$$\Rightarrow \frac{A}{B} : \frac{B}{C} : \frac{C}{A} = \frac{2}{3} : \frac{3}{4} : \frac{2}{1} = 8 : 9 : 24.$$

36. **(b)** 18 carat gold

$$= \frac{3}{4} \text{ pure gold} = \frac{3}{4} \times 24 = 18 \text{ carat gold}$$

$$20 \text{ carat gold} = \frac{5}{6} \text{ pure gold} = \frac{5}{6} \times 24$$

= 20 carot gold

Required ratio = 18 : 20 = 9 : 10

37. **(d)** Use shortcut Approach-7

38. **(c)** Use shortcut Approach-6

39. **(b)** $(x \times 5) = (0.75 \times 8) \Rightarrow x = \frac{6}{5} = 1.20.$

40. **(b)** Let the fourth proportional to 5, 8, 15 be x.

$\therefore 5 : 8 :: 15 : x \Rightarrow 5x = (8 \times 15) \Rightarrow x = 24.$

41. **(b)** Use shortcut Approach-19

42. **(b)** Let income of A = ₹ 3x, income of B = ₹ 2x and expenditure of A = ₹ 5y,

expenditure of B = ₹ 3y

Now, saving = income – expenditure

$\therefore 3x - 5y = 2x - 3y = 200$

$\Rightarrow x = 2y$ and $y = 200 \Rightarrow x = 400$

$\therefore$ A's income = ₹ 1200

43. **(a)** Use shortcut Approach-16 and 17

44. **(a)** Let A's share be ₹ x,

B's share be ₹ y. Then,

C's share = ₹ [671 – (x + y)]

Now, $x + 3 : y + 7 : 671 - (x + y) + 9 = 1 : 2 : 3$

$$\Rightarrow x + 3 : y + 7 : 680 - (x + y) = 1 : 2 : 3$$

$$\therefore x + 3 = \frac{1}{6} \times 690 = 115 \Rightarrow x = ₹ 112$$

$$\text{Also } y + 7 = \frac{2}{6} \times 690 = 230 \Rightarrow y = ₹ 223$$

$\therefore$ C's share = ₹ [671 – (112 + 223)] = ₹ 336

45. **(d)** New ratio of their respective salaries

$$= 3 \times \frac{150}{100} : 5 \times \frac{160}{100} : 7 \times \frac{150}{100} = 9 : 16 : 21$$

46. **(b)** Let A's share = ₹ 2x, B's share = ₹ 3x and C's share = ₹ 7x

Now, $7x - (2x + 3x) = 1500 \Rightarrow x = 750$

$\therefore$ A's share = ₹ 2x = ₹ 1500

47. **(a)** Let the required days be x.

More men, less days (Indirect proportion)

More size, more days (Direct proportion)

$$\left.\begin{array}{ll} \text{Men} & 20 \quad\quad 12 \\ \text{size} & 100\times3\times0.5 \quad 60\times4\times0.25 \end{array}\right\} :: 25 : x$$

$\therefore 20 \times 100 \times 3 \times 0.5 \times x = 12 \times 60 \times 4 \times 0.25 \times 25$

$\Rightarrow x = 6$ days

48. **(c)** Let the required number of days be x.

Less cows, More days (Indirect proportion)

Less bags, Less days (Direct proportion)

$$\left.\begin{array}{ll} \text{Cows} & 1:40 \\ \text{Bags} & 40:1 \end{array}\right\} :: 40 : x$$

$\therefore 1 \times 40 \times x = 40 \times 1 \times 40 \Rightarrow x = 40.$

49. **(b)** There is a meal for 200 children. 150 children have taken the meal.

Remaining meal is to be catered to 50 children.

Now, 200 children $\equiv$ 120 men

$\therefore$ 50 children

$$\equiv \left(\frac{120}{200} \times 50\right) \text{ men} = 30 \text{ men.}$$

50. **(b)** Let the required number of days be x.

Less persons, More days (Indirect proportion)

More working hrs per day, Less days (Indirect Proportion)

$$\left.\begin{array}{ll} \text{Persons} & 30:39 \\ \text{Working hrs/day} & 6:5 \end{array}\right\} :: 12 : x$$

$$\therefore 30 \times 6 \times x = 39 \times 5 \times 12 \Rightarrow x = \frac{39 \times 5 \times 12}{30 \times 6}$$

$\Rightarrow x = 13.$

51. **(a)** In 1 kg mixture quantity of iron = 200 gm

Let x gm sand should be added, then

10% of $(1000 + x) = 200$

$\therefore \quad x = 1000$ gm $= 1$ kg

GP_3386

52. **(b)** Let the first and the second numbers be x and y respect then
$y + 30\%$ of $x = 140\%$ of y
or, $y + 0.3x = 1.4y$
or, $0.3x = 0.4y$
$\therefore \quad x : y = 0.4 : 0.3 = 4 : 3$

53. **(c)** Let number of ladies $= x$
then, number of gents $= 2x$
Now, $\frac{x-2}{2x-2} = \frac{1}{3} \Rightarrow 3x - 6 = 2x - 2$
$\Rightarrow x = 4$
$\therefore$ Total number of people originally present $= 4 + 8 = 12$

54. **(b)** Number of females $= 156800 \times \frac{100}{80} = 196000$
$\therefore$ Number of males $= \frac{7}{8} \times 196000 = 171500$
$\therefore$ Total population $= 196000 + 171500 = 367500$

55. **(b)** Ratio of sides $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$
Largest side $= \left(104 \times \frac{6}{13}\right)$ cm $= 48$ cm

56. **(a)** Let number of each type of coin $= x$. Then,
$1 \times x + .50 \times x + .25x = 35$
$\Rightarrow 1.75x = 35 \Rightarrow x = 20$ coins

57. **(b)** The given condition has a, b and c symmetrically placed. Thus, if we use $a = b = c = 2$ (say) we get each fraction as 1/2.

58. **(c)** Use Shortcut Approach-6

59. **(d)** Required ratio $= 1/2 : 1/3 : 1/4$
To find the simple ratio, we will multiply each of these by the LCM of 2, 3 and 4.
$= 12\,(1/2 : 1/3 : 1/4) = 6 : 4 : 3$

60. **(b)** Use shortcut Approach-9

61. **(b)** Use shortcut Approach-9

62. **(a)** Let the nos. be 5x and 3x.
then, $5x - 3x = 10 \Rightarrow 2x = 10 \quad \Rightarrow \quad x = 5$
Required product $= 5x \times 3x = 5 \times 5 \times 3 \times 5 = 375$

63. **(d)**

No. of oxen	Work	Time
30	1/7	4
18	$\frac{6}{7}$	x

Applying the variation, we get

$30 \times 4 \times 7 = \frac{18 \times x \times 7}{6} \quad \Rightarrow \quad x = 40$ hrs.

64. **(b)** Let the numbers be x, 3x, 4x and 7x, then,
$x + 3x + 4x + 7x = 75 \Rightarrow 15x = 75 \Rightarrow x = 5$
Required value $= 7 \times 5 = 35$

65. **(b)** p = pressure, v = volume

$p = \frac{k}{v} \Rightarrow k = pv$ (k = constant term)

p is reduced by 20% i.e., by $\frac{1}{5}$, then v is increased $\frac{1}{4}$ i.e., 25%

66. **(a)** Use Shortcut Approach-5

67. **(d)** Use Shortcut Approach-5

68. **(d)** $\frac{27-x}{35-x} = \frac{2}{3} \Rightarrow 81 - 3x = 70 - 2x \Rightarrow x = 11$

69. **(a)** $\frac{5p - 3q}{7p + 2q} = \frac{5.\frac{3}{4}q - 3q}{7.\frac{3}{4}q + 2q} = \frac{15q - 12q}{21q + 8q} = 3/29$

70. **(a)** Ratio of investments $= 12000 : 18000 = 2 : 3$
A's share in profit $= (2/5) \times 2000 = ₹\,800$

71. **(b)** Let B invested for n months
then $\frac{12000 \times 12}{16000 \times n} = \frac{3}{1} \Rightarrow n = 3$

72. **(c)** Use Shortcut Approach-23

73. **(d)** Use Shortcut Approach-22

74. **(b)** Use Shortcut Approach-25

75. **(c)** Let their ages 5 years ago be 2x, 3x, 7x and 8x.
Their ages now $2x + 5$, $3x + 5$, $7x + 5$, $8x + 5$.
Or, $20x + 20 = 140$
$\Rightarrow \quad 20x = 120 \Rightarrow x = 6$
Present age of Nishu $= 2 \times 6 + 5 = 17$ years
Present age of mother $= 7 \times 6 + 5 = 47$ years
Hence, required years $(47 - 17)$ years $= 30$ years

76. **(a)** No. of cogs remains inversely proportional to the number of turns.

No. of turns $= \frac{80}{45} \times \frac{54}{32} \times 8 = 24$

77. **(d)** Let the no. of coins denomination ₹ 1, 50 paise and 25 paise be 3x, 4x and 12x respectively.

Then, $3x.1+4x.\frac{1}{2}+12x.\frac{1}{4}=600$

$3x+2x+3x=600 \Rightarrow 8x=600 \Rightarrow x=75$

No. of 25 paise coins $=12\times75=900$

78. **(b)** Use shortcut Approach-7

79. **(d)** Use Shortcut Approach-11

80. **(d)** Use Shortcut Approach-10

Level 1

1. **(a)** Sum invested by A, B and C is

$5\times12:7\times12:6\times6+3\times6$

$\Rightarrow 60:84:54 \Rightarrow 10:14:9$

$\therefore$ Share of $C=\frac{9}{33}\times33,000=₹9,000$

2. **(c)** Let the profit $=x$

Profit of $A=\frac{9x}{10}$, Remaining profit $=\frac{x}{10}$

Profit of $B=\frac{x}{20}$

Profit of $C=\frac{x}{20}$

Ratio of profit $=\frac{9}{10}:\frac{1}{20}:\frac{1}{20}=18:1:1$

A's income is increased by ₹ 270 . When profit rises 3%

Investment of $A=\frac{270}{3}\times100=₹9000.$

If investments of A, B and $C=18x$, x and x

$18x=9000 \Rightarrow x=500$

B invested $=₹500$ & C invested $=₹500$.

3. **(c)** Suppose B joined after x months.

Then, $21000\times12=36000\times(12-x)$

$\Rightarrow 36x=180 \Rightarrow x=5.$

Hence, B joined after 5 months.

4. **(b)** Ratio of investments of Abhishek and Sudin

$=(50,000\times36)+(30,000\times24):(70,000\times24)$

$=(18,00,000+7,20,000):16,80,000=3:2$

Hence, share of Sudin in the profit earned from the business $=\frac{87,500}{(3+2)}\times2=₹35,000.$

5. **(b)** $A:B:C=(5000\times4+2500\times8):(4500\times6+3000\times6):(7000\times6)$

$=40000:45000:42000=40:45:42$

$\therefore A$'s share $=₹\left(5080\times\frac{40}{127}\right)=₹1600;$

B's share $=₹\left(5080\times\frac{45}{127}\right)=₹1800;$

C's share $=₹\left(5080\times\frac{42}{127}\right)=₹1680.$

6. **(b)** $A:B:C=(40000\times36):(80000\times12+40000\times24):(120000\times24+40000\times12)$

$=144:192:336=3:4:7$

7. **(c)** Use shortcut Approach-20

8. **(a)** Use Shortcut Approach-18

9. **(a)** Let the prices of two houses A and B be ₹ $4x$ and ₹ $5x$, respectively for the last year.

Then, the prices of A this year $=₹(1.25\times4x)$ and that of $B=₹(5x+50,000)$

This year, Ratio of their prices $=9:10$

$\therefore \frac{1.25\times4x}{5x+50,000}=\frac{9}{10} \Rightarrow x=90,000$

Hence, the price of A last year was $4x=₹3,60,000$

10. **(a)**

	Iron		*Copper*	
Alloy I	8	:	6	14 kg.
Alloy II	36	:	6	42 kg.
	44	:	12	56 kg

11. **(c)** Gold in C $=\left(\frac{7}{9}+\frac{7}{18}\right)\text{units}=\frac{7}{6}\text{units}.$

Copper in C $=\left(\frac{2}{9}+\frac{11}{18}\right)\text{units}=\frac{5}{6}\text{units}.$

$\therefore$ Gold : Copper $=\frac{7}{6}:\frac{5}{6}=7:5.$

12. **(b)** Number of males $=\frac{2}{5}\times25=10$

Number of females $=\frac{3}{5}\times25=15$

Amount distributed among males and females $=275\times80\%=₹220$

Let the wage paid to a male be ₹ $5x$ and that to a female be ₹ $4x$. Therefore,

GP_3386

$10 \times 5x + 15 \times 4x = 220 \Rightarrow 50x + 60x = 220$
$\Rightarrow x = 2$
Wage received by a female labourer $= 2 \times 4 = ₹ 8$

13. (c) Let x pairs of brown socks were ordered.
Let P be the price of a brown pair.
$\therefore$ Price of the black pair of sock $= 2P$
Now, $4P + 2Px = 1.5\,(Px + 8P)$

$$\Rightarrow 4P + 2Px = \frac{3}{2}(Px + 8P)$$

$$\Rightarrow 8P + 4Px = 3Px + 24P$$
$$\Rightarrow Px = 16P \Rightarrow x = 16$$

$$\therefore \text{ Required ratio} = \frac{4}{16} = 1:4$$

14. (a) $\frac{V}{C} = \frac{2}{3}$ and $\frac{V+4}{C} = \frac{3}{4}$ ...(i)

$$\therefore \quad C = \frac{3V}{2} \Rightarrow \frac{V+4}{3V/2} = \frac{3}{4} \quad \text{[From (i)]}$$

where V denoted for vanilla and C for chocolate.
$$\Rightarrow 4V + 16 = \frac{9V}{2} \Rightarrow 8V + 32 = 9V \Rightarrow V = 32$$

15. (a) Let the number of male and female participants be 3x and x respectively.
Therefore total no. of participants are 4x.
During the tea break, the number of male participants are

$$(4x - 16) \times \frac{3}{4} = 3x - 12 \quad \ldots\ldots (i)$$

and the number of female participants are

$$(4x - 16) \times \frac{1}{4} + 6 = x + 2 \quad \ldots\ldots (ii)$$

Now, $\frac{3x-12}{x+2} = \frac{2}{1} \Rightarrow x = 16.$
Therefore, the total number of participants are $= 4 \times 16 = 64$

16. (d) Let the income of two persons be ₹ 4x and ₹ 5x and their expenses be ₹ 7y and ₹ 9y.
Therefore, $4x - 7y = 50$ ..... (i)
and $5x - 9y = 50$ ..... (ii)
From (i) and (ii), we get
$x = 100$ and $y = 50$
The income of the two persons are ₹ 400 and ₹ 500, respectively.

17. (c) Use Shortcut Approach-16

18. (b) Let the fixed amount be ₹ x and the cost of each unit be ₹ y. Then,
$540y + x = 1800$ ..... (i) and
$620y + x = 2040$ ..... (ii)
On subtracting (i) from (ii), we get
$80y = 240 \Rightarrow y = 3.$
Putting $y = 3$ in (i), we get :
$x = (1800 - 1620) = 180.$
$\therefore$ Fixed charges = ₹ 180,
Charge per unit = ₹ 3.
Total charges for consuming 500 units
$= ₹ (180 + 500 \times 3) = ₹ 1680.$

19. (b) Let the required length be x metres.
More breadth, Less length (Indirect Proportion)
More depth, Less length (Indirect Proportion)
More days, More length (Direct Proportion)

Breadth	20:50	: : 100 : x
Depth	15:10	
Days	10:30	

$\therefore 20 \times 15 \times 10 \times x = 50 \times 10 \times 30 \times 100$

$$\Rightarrow x = \frac{50 \times 10 \times 30 \times 100}{20 \times 15 \times 10} \Rightarrow x = 500.$$

20. (d) Required ratio $= \frac{4 \times 135}{5 \times 120} = 9:10$

21. (c) $5:4 \rightarrow 5:4.8 \rightarrow 25:24$
Option (c) is correct.

22. (a) Total distances covered under each mode = 32, 4 and 12 km respectively.
Total charges $= 32 \times 24 + 4 \times 3 + 12 \times 12$
$= 924$ paise $= ₹ 9.24.$

23. (b) In a year, for *A*, total amount as a remuneration $= 10 \times 12 = ₹ 120$
$\therefore$ Amount of *A*'s profit $= 390 - 120 = ₹ 270$
Ratio of investment $= 3:4$
Let total profit $= ₹ x$
Then, *B*'s profit $= ₹ (x - 270)$

$$\therefore \frac{3}{3+4} \times x = 270 \Rightarrow x = 630$$

$\therefore$ *B*'s profit $= 630 - 270 = ₹ 360$

24. (c) Volume of milk in mixture $= \frac{7}{10} \times 30 = 21l$

Volume of water = 9 L

Let x litres of water is added

$$\frac{21}{9+x} = \frac{1}{2} \Rightarrow 42 = 9 + x \Rightarrow x = 33$$

25. (b) Suppose B invested for n months

Hence, $\frac{12 \times x}{n \times x} = \frac{3}{2}$

where x = their investment

$$\frac{12}{n} = \frac{3}{2} \Rightarrow n = 8$$

26. (c) Let x gold coins were given to 1st queen. Then no. of gold coins given to 2nd queen

$$= \frac{5}{7}x$$

And no. of gold coins given to 3rd queen

$$= \frac{3}{7}x$$

then, $x - \frac{3}{7}x = 60 \Rightarrow \frac{4x}{7} = 60$

$x = 105$

$$\text{Required no.} = 105 + \frac{5}{7} \times 105 + \frac{3}{7} \times 105 = 225$$

27. (c) Let x = total no. of workers

No. of women workers $= \frac{1}{3}x$

No. of men workers $= \frac{2}{3}x$

No. of married women $= \frac{1}{6}x$

No. of women having children $= \frac{1}{3} \cdot \frac{1}{6}x = \frac{1}{18}x$

No. of married men $= \frac{3}{4} \cdot \frac{2}{3}x = \frac{1}{2}x$

No. of men having children = x / 3

No. of men without children $= \left(\frac{2}{3}x - \frac{1}{3}x\right) = \frac{x}{3}$

$$\text{Required fraction} = \frac{\frac{5x}{18} + x/3}{x} = \frac{\frac{5x+6x}{18}}{4} = \frac{11}{18}$$

28. (c) Let salary of Anil = ₹ x

Bhuvan's salary $= \frac{2}{5}x$

Chandra's salary $= \frac{7}{9} \cdot \frac{2}{5}x = \frac{14x}{45}$

$$x + \frac{2}{5}x + \frac{14x}{45} = 770$$

$$\Rightarrow \frac{77x}{45} = 770 \Rightarrow x = 450$$

29. (d) Let his salary be ₹ x

Money spent on house rent $= \frac{1}{4}x$

Money spent on food $= \frac{1}{3}x$

Money spent on travel $= \frac{1}{6}x$

Money spent $= \frac{1}{4}x + \frac{1}{3}x + \frac{1}{6}x = \frac{3}{4}x$

Money left $= \frac{x}{4}$

Money spent on clothes $= \frac{1}{10} \cdot \frac{x}{4} = \frac{x}{40}$

Money left $= \frac{x}{4} - \frac{x}{40} = \frac{10x - x}{40} = \frac{9x}{40}$

i.e., $\frac{9x}{40} = 13500 \Rightarrow x = 60000$

Required amount = 5000

30. (b) Let ₹ x be Bidhan's contribution. Then his brother's contribution $= \frac{3}{2}x$

Financers contribution $= 3 \cdot \frac{3}{2}x = \frac{9x}{2}$

then, $x + \frac{3}{2}x + \frac{9x}{2} = 35000 \Rightarrow x = 5000$

His brother's contribution = ₹ 7500

GP_3386

31. (b) Let x and y be the mass of 1st alloy and 2nd alloy respectively.

$$\text{Bronze in 1st alloy} = \frac{9}{10}x$$

$$\text{Manganese in 1st alloy} = \frac{7}{100}x$$

$$\text{Tin in 1st alloy} = \frac{3}{100}x$$

For manganese, we have

$$\frac{\frac{7}{100}x}{x+y} \times 100 = 5 \Rightarrow 7x = 5x + 5y \Rightarrow x/y = 5/2$$

Applying alligation

t = % of bronze in 2nd Alloy

$$\frac{y}{x} = \frac{5}{85-t} \Rightarrow \frac{2}{5} = \frac{5}{85-t} \Rightarrow t = 72.5$$

32. (d) Let Anand, Bidhan and Chandra got ₹ x, y and z respectively then, Chandra's share

$$= \frac{1}{2}(x+y) \text{ and Anand's share} = \frac{1}{3}(y+z)$$

$$\Rightarrow z = \frac{1}{2}(x+y) \text{ and } x = \frac{1}{3}(y+z)$$

$$\Rightarrow x + y - 2z = 0 \quad \text{.....(i)}$$

and $3x - y - z = 0$ .....(ii)

Adding (i) and (ii), we have

$$x + 3x - z - 2z = 0 \Rightarrow 4x = 3z \Rightarrow \frac{x}{z} = \frac{3}{4}$$

33. (b) Let Anil's invested ₹ x then Dheeraj's investment' = ₹ 3x

Let Dheeraj join him for n months

then, $\frac{x \times 12}{3x \times n} = 1 \Rightarrow x \times 12 = 3x \times n \Rightarrow n = 4$

Hence, Dheeraj joined him in September.

34. (c) Let the wages of a man, a woman and a child be 3x, 2x and x respectively,

then, $20 \times 3x + 30 \times 2x + x \times 36 = 780$

$60x + 60x + 36x = 780 \Rightarrow 156x = 780 \Rightarrow x = 5$

Reqd. amount $= 2(15 \times 3 \times 5 + 21 \times 2 \times 5 + 30 \times 5)$

$= 2\,(225 + 210 + 150) = ₹\,1170$

CHAPTER

9

Average
(Including Mean, Median and Mode)

AVERAGE

An *average* is a simple concept of mathematics but its uses are very common in day-to-day life.

An *average* of a group of numbers is a number that is the best representative of the group of numbers because it tells a lot about the entire numbers of the group.

In other words an *average* is a measure of central tendency called **arithmetic mean** of a group of numbers,

The formula for finding the average is

$$\text{Average} = \frac{\text{Sum of all numbers}}{\text{Number of numbers}}$$

Thus if A_v be the average of n numbers $x_1, x_2, x_3, \ldots, x_n$, then

$$A_v = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

$$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_n = n \,.\, A_v$$

$\Rightarrow$ Sum of n numbers = (Number of numbers) × (Average)

Illustration – 1

The average of the first nine prime numbers is:

(a) 9 (b) 11 (c) $11\frac{1}{9}$ (d) $11\frac{2}{9}$

Solution : (c) Average

$$= \frac{2+3+5+7+11+13+17+19+23}{9}$$

$$= \frac{100}{9} = 11\frac{1}{9}$$

Illustration – 2

The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers?

(a) 2 (b) 5
(c) 8 (d) Cannot be determined

Solution : (c) Let the numbers be $x, x+2, x+4, x+6$ and $x+8$.

Then, $\frac{x+(x+2)+(x+4)+(x+6)(x+8)}{5} = 61$

or $5x + 20 = 305$ or $x = 57$.

So, required difference = (57 + 8) – 57 = 8.

Weighted Average

If we have two or more groups of numbers whose individual averages are known, then combined average of all the numbers of all the groups is known as **weighted average**. Thus if there are k groups having number of numbers $n_1, n_2, n_3, \ldots, n_k$ with averages $A_1, A_2, A_3, \ldots, A_k$ respectively; then weighted average, A_w

$$= \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots + n_kA_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

Illustration – 3

The average score of a cricketer in two matches is 27 and in three other matches is 32. Then find the average score in all the five matches.

Solution : Average in 5 matches

$$= \frac{2\times 27 + 3\times 32}{2+3} = \frac{54+96}{5} = 30.$$

GP_3386

Illustration – 4

The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is

(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 3 : 5

Solution : (b) Let the number of boys in a class be x.

Let the number of girls in a class be y.

$\therefore$ Sum of the ages of the boys $= 16.4x$

Sum of the ages of the girls $= 15.4y$

$\therefore 15.8(x+y) = 16.4x + 15.4y \Rightarrow 0.6x = 0.4y$

$\Rightarrow \frac{x}{y} = \frac{2}{3}$

$\therefore$ Required ratio $= 2 : 3$

Illustration – 5

The average age of 30 boys of a class is equal to 14 yrs. When the age of the class teacher is included the average becomes 15 yrs. Find the age of the class teacher.

Solution : Total ages of 30 boys $= 14 \times 30$
$= 420$ yrs.

Total ages when class teacher is included
$= 15 \times 31 = 465$ yrs.

$\therefore$ Age of class teacher $= 465 - 420 = 45$ yrs.

By direct formula,

Age of new entrant
= (No. of old members) × (Increase in average) + (New average)

$= 30(15 - 14) + 15 = 45$ yrs.

Illustration – 6

The average weight of 45 students in a class is 52 kg. 5 of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class?

(a) 52.6 **(b)** $52\frac{2}{3}$

(c) $52\frac{1}{3}$ **(d) None of these**

Solution : (b) Weight of 45 students
$= 45 \times 52 = 2340$ kg

Weight of 5 students who left $= 5 \times 48 = 240$ kg

Weight of 5 students who join $= 5 \times 54 = 270$ kg

Therefore, new total weight of 45 students
$= 2340 - 240 + 270 = 2370$

$\Rightarrow$ New average weight $= \frac{2370}{45} = 52\frac{2}{3}$ kg

Ages and Averages

If the average age of a group of persons is x years today then after n years their average age will be $(x + n)$ years because for a group of people, 1 year is added to each person's age every year.

Similarly, n years ago their average age would have been
$(x - n)$ years, because 1 year is subtracted from each person's age before every year.

Illustration – 7

The average age of a family of 6 members is 22 yrs. If the age of the youngest member be 7 yrs, then what was the average age of the family at the birth of the youngest member?

Solution : Total ages of all members
$= 6 \times 22 = 132$ yrs.

7 yrs. ago, total sum of ages $= 132 - (6 \times 7) = 90$ yrs.

But at that time there were 5 members in the family.

$\therefore$ Average at that time $= 90 \div 5 = 18$ yrs.

Illustration – 8

The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?

Solution : Let the number of passed candidates be x.

Then total marks $= 120 \times 35 = 39x + (120 - x) \times 15$

or, $4200 = 39x + 1800 - 15x$ or $24x = 2400 \Rightarrow x = 100$

$\therefore$ number of passed candidates $= 100$.

Illustration – 9

A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must he make in his next innings so as to raise his average to 24?

Solution : Total of 10 innings $= 21.5 \times 10 = 215$

Suppose he needs a score of x in 11th innings; then

average in 11 innings $= \dfrac{215+x}{11} = 24$

or, $x = 264 - 215 = 49.$

More About Average

If average of some numbers is x, and

(i) a is added to each number, then new average $= x + a$

(ii) a is subtracted from each number, then new average $= x - a$

(iii) a is multiplied with each number, then new average $= ax$, $a \neq 0$

(iv) each number is divided by a, then new average

$= \dfrac{x}{a}$, $a \neq 0$

Remember

★ Average of first n natural numbers $= \dfrac{(n+1)}{2}$

★ Average of consecutive numbers

$= \dfrac{\text{First number} + \text{Last number}}{2}$

★ Average of 1 to n odd numbers

$= \dfrac{\text{Last odd number} + 1}{2}$

★ Average of 1 to n even numbers

$= \dfrac{\text{Last even number} + 2}{2}$

★ Average of squares of first n natural numbers

$= \dfrac{(n+1)(2n+1)}{6}$

★ Average of the cubes of first n natural numbers

$= \dfrac{n(n+1)^2}{4}$

★ Average of n multiples of any number

$= \dfrac{\text{Number} \times (n+1)}{2}$

★ If n is odd: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.

★ If n is even: The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.

★ The average of squares of first n consecutive even number is $\dfrac{2(n+1)(2n+1)}{3}$.

★ The average of squares of consecutive even numbers till n is $\dfrac{(n+1)(n+2)}{3}$.

★ The average of square of consecutive odd numbers till n is $\dfrac{n(n+2)}{3}$.

★ If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n-1)$.

Illustration – 10

Find average of 12, 14, 16, 18, 20 (an A.P)

Solution : Number of terms is 5, an odd number

$\Rightarrow$ Average $= \left(\dfrac{5+1}{2}\right)^{th}$ or 3rd term $= 16$

Illustration – 11

Find average of 4, 6, 8, 10. (an A.P.)

Solution : Number of terms is 4, an even number.

$\Rightarrow$ Average = Average of two mid-terms i.e., Average of $\left(\dfrac{4}{2}\right)^{th}$ and $\left(\dfrac{4}{2}+1\right)^{th}$ term

= Average of 2nd and 3rd term i.e., $\dfrac{6+8}{2} = 7$

SHORTCUTS

Shortcut Approach - 1

If average of n consecutive odd numbers x, then the difference between the smallest and the largest numbers is given by $2(n-1)$.

Note that the above formula is independent of x. Therefore, this formula always holds good irrespective of the value of x.

Illustration – 12

If average of 6 consecutive odd numbers is 48, what is the difference between the smallest and the largest numbers?

(a) 8 (b) 10 (c) 11 (d) 12

GP_3386

Shortcut Solution : (b)

Difference = $2(n-1) = 2(6-1) = 10$.

Shortcut Approach - 1

If the average of n numbers is A and on rechecking, it is noticed that some of the numbers n (i.e. $x_1, x_2, x_3, \ldots, x_n$) are wrongly taken as $(x'_1, x'_2, x'_3, \ldots, x'_n)$, then their correct average

$$= A + \frac{(x_1 + x_2 + x_3 + \ldots + x_n) - (x'_1 + x'_2 + x'_3 + \ldots + x'_n)}{n}$$

Illustration – 13

The average weight of a group of 9 boys was calculated to be 74.5 kg and it was later discovered that one weight was misread as 56 kg instead of the correct one of 65 kg. The correct average weight is

(a) 72 kg (b) 73.4 kg (c) 74.5 kg (d) 75.5 kg

Shortcut Solution : (d)

$$\text{Correct average} = A + \frac{x_1 - x'_1}{n} = 74.5 + \frac{65-56}{9} = 75.5$$

Shortcut Approach - 1

Let the average weight of n balls is a units. If the weight of a bag is included, the average weight increases by b units, then weight of the bag = $a + b(n+1)$.

Illustration – 14

The average weight of 40 balls is 4 gm. If the weight of the bag be included the average weight increases by 0.04 gm. What is the weight of the bag?

(a) 5.64 gm (b) 5.46 gm (c) 5.45 gm (d) 6.64 gm

Shortcut Solution : (a)

Weight of the bag = $a + b(n+1) = 4 + 0.04(40+1)$
$= 4 + 1.64 = 5.64$ gm

Shortcut Approach - 1

Let there were n students in a hostel. If the number of students increases by a, the expenses of mess increase by ₹ b per day while the average expenditure per head diminishes by ₹ c, then the original expenditure of the mess $= n\left[\frac{c(n+a)+b}{a}\right]$

Illustration – 15

There were 36 students in a hostel. If the number of students increases by 4, the expenses of the mess increase by ₹ 32 per day while the average expenditure per head diminishes by ₹ 1. Find the original expenditure of the mess.

(a) ₹ 648 (b) ₹ 648.50 (c) ₹ 650 (d) ₹ 662

Shortcut Solution : (a)

$$\text{Original expenditure} = n\left[\frac{c(n+a)+b}{a}\right]$$

$$= 36\left[\frac{1(36+4)+32}{4}\right] = 36 \times 18 = ₹\ 648.$$

Shortcut Approach - 1

If the average of $(a+b+1)$ result is x, that of the first a is y and that of the last b is z, then value of the $(n+1)$th result = Total of $(a+b+1)$ results – (Total of first a results + Total of last b results)
$= (a+b+1)x - (ay + bz)$

Illustration – 16

The average of 11 numbers is 37.15, the average of the first 3 is 36.41 and of the last 7 is 37.51. The fourth number is found to be wrong. Find the value of the rest.

(a) 35.15 (b) 36.85 (c) 37.24 (d) 38.16

Shortcut Solution : (b)

Value of the number = $(a+b+1)x - (ay + bz)$
$= (3+7+1)37.15 - (3 \times 36.41 + 7 \times 37.51)$
$= 408.65 - 371.80 = 36.85$.

Shortcut Approach - 1

If the average age of m boys is x and the average age of n boys out of m boys is y, then the average age of the rest of the boys $= \frac{mx - ny}{m-n}$

Illustration – 17

The average age of 30 students in a class is 12 years. The average of a group of 5 of the students is 12 years. Find the average age of the remaining students.

(a) 10 (b) 12 (c) 13 (d) 14

Shortcut Solution : (b)

$$\text{Average age} = \frac{mx-ny}{m-n} = \frac{30 \times 12 - 5 \times 12}{30-5} = \frac{300}{25} = 12$$

Shortcut Approach - 1

If the average of n quantities is equal to x. When a quantity is removed, the average becomes y. Then the value of the removed quantity = $n(x-y)+y$.

Illustration – 18

The average age of 30 students and the class teacher is 20 years. If the class teacher's age is excluded, the average reduces by 1 year. What is the age of the class teacher?

(a) 48 yrs (b) 50 yrs (c) 51 yrs (d) 52 yrs

Shortcut Solution : (b)

Age of the class teacher = $n(x-y)+y$

Here $n = 30+1 = 31$, $x = 20$ and $y = 20-1 = 19$

$\therefore$ Age of the class teacher = $31(20-19)+19$

$= 31 \times 1 + 19 = 50$ years.

Shortcut Approach - 9

If the average of the first and the second of three numbers is x more or less than the average of the second and the third of these numbers, then the difference between the first and the third of these three numbers is given by $2x$.

Note that, here only 2 numbers (i.e. first and second or second and third) are involved in calculating average, therefore, we multiply x by 2. If n numbers are involved, for getting answer, we multiply x by n.

Illustration – 19

The average marks scored by Ganesh in English, Science, Mathematics and History is less than 15 from that scored by him in English, History, Geography and Mathematics. What is the difference of marks in Science and Geography scored by him?

(a) 55 (b) 60 (c) 62 (d) 63

Shortcut Solution : (b)

Here four subjects are involved.

$\therefore$ Difference in marks in Science and Geography

$= 4x = 4 \times 15 = 60$.

Shortcut Approach - 10

If a batsman in his nth innings makes a score of x and thereby increases his average by y, then the average after n innings = $x - y(n-1)$.

Illustration – 20

A batsman in his 20th innings makes a score of 110 and thereby increases his average by 4. What is his average after 20 innings?

(a) 34 (b) 35 (c) 38 (d) 41

Shortcut Solution : (a)

Average = $x - y(n-1) = 110 - 4(20-1) = 110 - 76 = 34$.

Shortcut Approach - 10

If a cricketer has completed n innings and his average is x runs. The number of runs, he must make in his next iunings so as to raise his average to $y = n(y-x)+y$.

Illustration – 21

A cricketer has completed 14 innings and his average is 30 runs. How many runs must be make in his next innings so as to raise his average to 32?

(a) 57 (b) 59 (c) 60 (d) 65

Shortcut Solution : (c)

Required runs = $n(y-x)+y$

$= 14(32-30)+32 = 14 \times 2 + 32 = 60$.

Shortcut Approach - 11

The average of marks obtained by n candidates in a certain examination is T. If the average marks of passed candidates is P and that of the failed candidates is F. Then the number of candidates who passed the examination

$$= \frac{\text{Total candidates (Total Average – Failed Average)}}{\text{Passed Average – Failed Average}}$$

$$= \frac{n(T-F)}{P-F}$$

Illustration – 22

The average of marks obtained by 80 candidates in a certain examination is 32. If the average marks of passed candidates is 34 and that of the failed candidates is 18. What is the number of candidates who passed the examination?

(a) 60 (b) 62 (c) 70 (d) 75

Shortcut Solution : (c)

No. of passed candidates

$$= \frac{n(T-F)}{P-F} = \frac{80(32-18)}{34-18} = \frac{80 \times 14}{16} = 70.$$

Illustration – 23

The average of marks obtained by 110 candidates in a certain examination is 15. If the average marks of passed candidates is 25 and that of the failed candidates is 14. What is the number of candidates who failed the examination?

(a) 89 (b) 97 (c) 100 (d) 102

Shortcut Solution : (c)

No. of failed candidates

$$= \text{Total candidates} - \frac{n(T-F)}{P-F}$$

$$= 110 - \frac{110(15-14)}{25-14} = 100$$

Shortcut Approach - 1

The average weight of n persons is increased by x kg. when some of them whose weight [$y_1 + y_2 + \ldots\ldots$ where, $y_1 + y_2 + \ldots\ldots = y$ kg] are replaced by the same number of persons. Then the weight of the new persons

= Weight of removed person (s) + (No. of persons) × (Increase in average)

$= y + nx$.

GP_3386

Illustration – 24

The average weight of 15 men is increased by 2 kg. When one of the men who weighs 48 kg is replaced by a new man. Find the weight of the new man.

(a) 78 kg (b) 80 kg (c) 81.5 kg (d) 82 kg

Shortcut Solution : (a)

Weight of new man $= y + nx = 48 + 15 \times 2 = 78$ kg.

Shortcut Approach - 10

The average age of n persons is decreased by x years. When some of them aged [$y_1 + y_2 +$ ..... where, $y_1 + y_2 +$ .... $= y$ years] are replaced by the same number of persons. Then the total age of the new persons = Total age of removed persons – (No. of persons) × (Decrease in average) = $y - nx$.

Illustration – 25

In a class there are 18 students whose average age is decreased by 2 months, when 3 students aged 12, 13 and 14 years respectively are replaced by the same number of students. Find the average age of the new students.

(a) 12 yrs (b) 13 yrs (c) 15 yrs (d) 16 yrs

Shortcut Solution : (a)

Average age of new students

$$= \frac{\text{Total age of new persons}}{\text{No. of new persons}} = \frac{y - nx}{3}$$

$$= \frac{(12+13+14) - 18 \times \frac{2}{12}}{3} = \frac{39-3}{3} = \frac{36}{3} = 12 \text{ years.}$$

Shortcut Approach - 10

If the average of n results (where n is an odd number) is a and the average of first $\frac{n+1}{2}$ results is b and that of last $\frac{n+1}{2}$ is c. Then $\left(\frac{n+1}{2}\right)$th result $= \frac{n+1}{2}(b+c) - na$. or (the middle value)

Illustration – 26

The average of 17 numbers is 45. The average of first 9 of these numbers is 51 and the last 9 of these numbers is 36. What is the 9th number?

(a) 15 (b) 17 (c) 18 (d) 19

Shortcut Solution : (c)

9th number $= \frac{n+1}{2}(b+c) - na$

$$= \frac{17+1}{2}(51+36) - 17 \times 45$$

$$= 9 \times 87 - 765 = 783 - 765 = 18.$$

Shortcut Approach - 10

If the average value of all the members of group is x, the average value of the first part of member is y, the average value of the remaining part of members is z and the number of the first part of members is n, then the number of the other part of members is $\frac{n(x-y)}{z-x}$.

or

$$= \frac{\text{No. of 1st part members (Total average of whole group} - \text{Average of 1st part)}}{\text{Average of other part} - \text{Total average of whole group}}$$

Illustration – 27

The average salary of the entire staff in a office is ₹ 220 per month. The average salary of officer is ₹ 650 and that of non-officers is ₹ 170. If the number of officers is 25, then find the number of non-officers in the office.

(a) 200 (b) 205 (c) 212 (d) 215

Shortcut Solution : (d)

No. of non-officer in the office

$$= \frac{n(x-y)}{z-x} = \frac{25(220-650)}{170-220} = \frac{25 \times (-430)}{-50} = 215.$$

Illustration – 28

The average age of all the students of a class is 22 years. The average age of boys of the class is 24 years and that of the girls is 20 years. If the number of girls in the class is 30, then find the number of boys in the class

(a) 26 (b) 27 (c) 29 (d) 30

Shortcut Solution : (d)

Let the number of boys in the class be x

$$\therefore \frac{n(x-y)}{z-x} = 30 \Rightarrow \frac{x(22-24)}{20-22} = 30$$

$$\Rightarrow x \times \frac{-2}{-2} = 30 \Rightarrow x = 30$$

LEVEL - 1

1. The average age of 24 students and the class teacher is 16 years. If the class teacher's age is excluded, the average reduces by one year. What is the age of the class teacher?
(a) 50 years (b) 45 years
(c) 40 years (d) Data inadequate

2. The average age of 36 students in a group is 14 years. When teacher's age is included in it, the average increases by one. What is the teacher's age in years?
(a) 31 (b) 36
(c) 51 (d) cannot be determined

3. The average of 11 numbers is 10.9. If the average of the first six numbers is 10.5 and that of the last six numbers is 11.4, then the middle number is :
(a) 11.5 (b) 11.4 (c) 11.3 (d) 11.0

4. The average run scored by a batsman in 20 innings is 32. After 21st innings, the runs average becomes 34. How much runs does the batsman score in his 21st innings? **[CDS 2011]**
(a) 70 (b) 72 (c) 74 (d) 76

5. The average of 9 numbers is 30. The average of first 5 numbers is 25 and that of the last 3 numbers is 35. What is the 6th number? **[SSC CGL 2008]**
(a) 20 (b) 30 (c) 40 (d) 50

6. The average of 11 numbers is 10.8. If the average of the first six numbers be 10.4 and that of the last six is 11.5, the middle (6th) number is: **[SSC CGL Tier I 2015]**
(a) 10.3 (b) 12.6 (c) 13.5 (d) 15.5

7. The average temperature for the first four days of a week is 40.2°C and that of the last four days is 41.3°C. If the average temperature for the whole week is 40.6°C, then the temperature on the fourth day is
(a) 40.8°C (b) 38.5°C (c) 41.3°C (d) 41.8°C

8. A person covers half his journey by train at 60 kmph, the remaining half by bus at 30 kmph and the rest by cycle at 10 kmph. Find his average speed during the entire journey.
(a) 36 kmph (b) 24 kmph
(c) 48 kmph (d) None of these

9. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. The corrected (new) mean is: **[SSC CGL 2009]**
(a) 35. 2 (b) 36.1 (c) 36. 5 (d) 39.1

10. The average of 10 numbers is 40.2. Later it is found that two numbers have been wrongly copied. The first is 18 greater than the actual number and the second number added is 13 instead of 31. Find the correct average.
(a) 40.2 (b) 40.4 (c) 40.6 (d) 40.8

11. The arithmetic mean of 100 numbers was computed as 89.05. It was later found that two numbers 92 and 83 have been misread as 192 and 33 respectively, what is the correct arithmetic mean of the numbers? **[CDS 2010]**
(a) 88.55 (b) 87.55
(c) 89.55 (d) Data Insufficient

12. Of the three numbers, the first is twice the second and the second is twice the third. The average of the reciprocal of the numbers is $\frac{7}{72}$. The numbers are
(a) 16, 8, 4 (b) 20, 10, 5
(c) 24, 12, 6 (d) 36, 18, 9

13. In a bag, there are 150 coins of ₹ 1·50 p and 25 p denominations. If the total value of coins is ₹ 150, then find how many rupees can be constituted by 50 coins.
(a) ₹ 16 (b) ₹ 20
(c) ₹ 28 (d) None of these

14. The average of two numbers is XY. If one number is X, then the other number is
(a) Y (b) $\frac{Y}{2}$
(c) $2XY - X$ (d) $X(Y-1)$

15. If A, B, C and D are four consecutive odd numbers and their average is 52, what is the difference between A and D? **[SSC CGL 2010]**
(a) 6 (b) 7 (c) 8 (d) 9

16. If a, b, c, d, e are five consecutive odd numbers, then the average in terms of 'a' will be –
(a) $a+2$ (b) $a+3$ (c) $a+4$ (d) a

GP_3386

17. There are 50 boys in a class. Their average weight is 45 kg. When one boy leaves the class, the average reduces by 100 g. Find the weight of the boy who left the class.

(a) 40.9 kg (b) 42.9 kg (c) 49.9 kg (d) 39.9 kg

18. The average age of a family of 6 members is 25 years. The average age of the family after the demise of a 40 years old member will be

[SSC CHSL 2010]

(a) 20 yrs (b) 21 yrs (c) 22 yrs (d) 23 yrs

19. The average monthly income of a family of 4 earning members was ₹7350. One of the earning members died. Thereby the average came down to ₹6500. The monthly income of the deceased was: **[SSC CGL 2006]**

(a) ₹8500 (b) ₹8200 (c) ₹9900 (d) ₹10750

20. Rahul Ghosh walks from A to B at 8 km/h and comes back from B to A at 12 km/h. What is his average speed for the entire journey?

(a) 8.8 km/h (b) 9.6 km/h

(c) 10.2 km/h (d) 11.4 km/h

21. In a class of 60 students, the average height of 30 students is x cm and that of the remaining students is y cm. Find the average height of the whole class.

(a) (x + y) cm (b) 30 cm

(c) $\frac{x+y}{2}$ cm (d) None of these

22. The average weight of seven members of a family is 18 kg. If the head of the family is not considered, the average weight of the others would be 5 kg less. The weight of the head of the family is

(a) 42 kg (b) 48 kg (c) 52 kg (d) 36 kg

23. The average weight of 21 boys was recorded as 64 kg. If the weight of the teacher was added, the average increased by one kg. what was the teacher's weight?

[Bank Clerk 2011]

(a) 86 kg (b) 64 kg (c) 72 kg (d) 98 kg

24. A beggar has 50 coins with an amount of ₹ 15 in the denominations of 25 paise and 50 paise. How many 25 paise coins are there with the beggar?

(a) 36 (b) 40 (c) 30 (d) 25

25. The average salary of A, B and C is ₹ 10,000 and average expenditure of A is ₹ 6000, then the average saving of B and C is :

(a) ₹ 5500 (b) ₹ 4500

(c) ₹ 4000 (d) can't be determined

26. The average salary of A, B is ₹ 6000 and that of C, D and E is ₹ 8000. The average salary of all the 5 people is:

(a) ₹ 7200 (b) ₹ 7000

(c) ₹ 7500 (d) can't be determined

27. The average of 9 numbers is 11. If each of these 9 numbers is multiplied by 5 and then 5 is added to each of these resultant numbers, then the new average is :

(a) 20 (b) 30 (c) 60 (d) 50

28. The average rainfall in the months of January and February is 6 cm and in the months of March to June is 5 cm and July to October is 10 cm and in the November and December, it is 6 cm. The average rainfall for the whole year is :

(a) 7 (b) 5.5

(c) 7.5 (d) None of these

29. Average rainfall on Monday, Tuesday, Wednesday and Thursday is 420.5 cm and average on Tuesday, Wednesday, Thursday and Friday is 440.5 cm. If the ratio of rainfall for monday and Friday is 20:21, find the rainfall in cm on Monday and Friday.

[SSC CGL Tier II 2012]

(a) 1700, 1470 (b) 1682,1762

(c) 1800, 1890 (d) 1600, 1680

30. The mean temperature from Monday to Wednesday was 37°C and from Tuesday to Thursday was 34°C. If the temperature on Thursday was $4/5^{th}$ that of Monday, the temperature on Thursday was:

[SSC CGL Tier I 2013]

(a) 34°C (b) 35°C (c) 36°C (d) 37°C

31. The average of 7, 14, 21, 28.....77 is :
(a) 7 (b) 11 (c) 42 (d) 66

32. The average age of 3 children of Arihant Singh is 12 years and their ratio of ages is 3 : 4 : 5. The average age of the youngest and eldest child is if he had only 3 children :
(a) 12 (b) 21 (c) 8 (d) 9

33. The average of any 5 consecutive odd numbers a, b, c, d, e is:
(a) $\frac{(abcde)}{5}$ (b) $\frac{bd}{3}$
(c) $\left(\frac{a+c+e}{5}\right)$ (d) None of these

34. Three years ago, the average age of a family of 8 members was 30 years. If one child is also included in the family, the present average age of the family remained the same as three years ago. The present age of the child is
(a) 6 yrs (b) 1 year (c) 3 yrs (d) 4 yrs

35. In a class of 40 students, the average score is 35 marks. If 10 students, whose average is 50 marks, leave the class, then find the new average of the class?
(a) 30 marks (b) 32 marks
(c) 34.5 marks (d) 31.5 marks

36. Of three numbers, the second is thrice the first and the third number is three-fourth of the first. If the average of the three numbers is 114, the largest number is .
(a) 72 (b) 216 (c) 354 (d) 726

37. The average of 3 numbers is 154. The first number is twice the second and the second number is twice the third. The first number is-
(a) 264 (b) 132 (c) 88 (d) 66

38. The average age of 32 students is 10 years. If the teacher's age is also included, the average age increases by one year. What is the age of the teacher (in yr)?
(a) 43 (b) 50 (c) 34 (d) 42

39. Of the three numbers, the first number is twice of the second and the second is thrice of the third number. If the average of these 3 numbers is 20, then the sum of the largest and smallest numbers is
(a) 24 (b) 42 (c) 54 (d) 60

40. A student's mark was wrongly entered as 83 instead of 63. Due to that the average marks for the class got increased by half (1/2). What is the number of students in the class?
(a) 40 (b) 44
(c) 48 (d) None of these

41. The average age of 30 girls is 13 years. The average of first 18 girls is 15 years. Find out the average age of remaining 12 girls.
[Hotel mgmt. 2009]
(a) 12 yrs (b) 10 yrs (c) 16 yrs (d) 10.5 yrs

42. The sum of five numbers is 555. The average of first two numbers is 75 and the third number is 115. What is the average of last two numbers?
[Bank PO 2007]
(a) 145 (b) 290 (c) 265 (d) 150

43. The average of six numbers is x and the average of three of these is y. If the average of the remaining three is z, then
(a) $2x = y + z$ (b) $x = 2y + 2z$
(c) $x = y + z$ (d) $z + x = 5y$

44. The average of 11 results is 50. If the average of first six results is 49 and that of last six is 52, find the sixth result?
(a) 45 (b) 46 (c) 55 (d) 56

45. A bus started its journey from Ramgarh and reached Devgarh in 44 minutes at its average speed of 50 km/hr. If the average speed of the bus is increased by 5km/hour, how much time will it take to cover the same distance?
(a) 40 minutes (b) 38minutes
(c) 36 minutes (d) 31minutes

46. The average speed of a train is $1\frac{3}{7}$ of the average speed of a car. The car covers a distance of 588 km in 6 hours. How much distance will the train cover in 13 hours?
(a) 1750 km (b) 1760 km
(c) 1720 km (d) None of these

47. The average marks of 40 students in an English exam is 72. Later it is found that three marks 64, 62 and 84 were wrongly entered as 68, 65 and 73. The average after mistakes are rectified as–
(a) 70 (b) 72 (c) 71.9 (d) 72.1

GP_3386

LEVEL - 2

1. The average of 5 consecutive numbers is n. If the next two numbers are also included the average will
(a) remain the same (b) increase by 1
(c) increase by 1.4 (d) increase by 2

2. The average age of Mr. and Mrs Sinha at the time of their marriage in 1972 was 23 years. On the occasion of their anniversary in 1976, they observed that the average age of their family had come down by 4 years compared to their average age at the time of their marriage. This was due to the fact that their son Vicky was born during that period. What was the age of Vicky in 1980?
(a) 6 (b) 7 (c) 8 (d) 5

3. The average salary of all the workers in a workshop is ₹8,000. The average salary of 7 technicians is ₹12,000 and the average salary of the rest is ₹ 6,000. The total number of workers in the workshop is :
(a) 21 (b) 20 (c) 23 (d) 22

4. The average weight of 47 balls is 4 gm. If the weight of the bag (in which the balls are kept) be included, the calculated average weight increases by 0.3 gm. What is the weight of the bag?
(a) 14.8 gm (b) 15.0 gm
(c) 18.6 gm (d) None of these

5. A train travels with a speed of 20 m/s in the first 10 minutes, goes 8.5 km in the next 10 minutes, 11 km in the next 10 minutes, 8.5 km in the next 10 minutes and 6 km in the next 10 minutes. What is the average speed of the train in kilometer per hour for the journey described?
(a) 42 kmph (b) 35.8 kmph
(c) 55.2 kmph (d) 46 kmph

6. Find the average increase rate if increase in the population in the first year is 30% and that in the second year is 40%.
(a) 41 (b) 56 (c) 40 (d) 38

7. Find the average weight of four containers, if it is known that the weight of the first container is 100 kg and the total of the second, third and fourth containers' weight is defined by $f(x) = x^2 - 3/4\,(x^2)$ where $x = 100$
(a) 650 kg (b) 900 kg (c) 750 kg (d) 450 kg

Directions (8–10) : Eight years ago there were 5 members in the Arthur's family and then the average age of the family was 36 years. Mean while Arthur got married and gave birth to a child. Still the average age of his family is same now.

8. The present age of his wife is
(a) 25 years (b) 26 years
(c) 32 years (d) Data insufficient

9. The age of his wife at the time of his child's birth was. If the difference between the age of her child and herself was 26 years
(a) 25 years (b) 26 years
(c) 20 years (d) Can't be determined

10. The age of Arthur at the time of his marriage was
(a) 22 years (b) 23 years
(c) 26 years (d) Can't be determined

11. The average age of a group of 14 persons is 27 years and 9 months. Two persons, each 42 years old, left the group. What will be the average age of the remaining persons in the group?
(a) 26.875 years (b) 26.25 years
(c) 25.375 years (d) 25 years

12. There were 35 students in a hostel. Due to the admission of 7 new students, the expenses of mess were increased by ₹ 42 per day while the average expenditure per head diminished by ₹ 1. What was the original expenditure of the mess?
(a) ₹400 (b) ₹420 (c) ₹445 (d) ₹465

13. There were 52 students in a hostel. If the number of students increases by 13, the daily expenses of the mess increases by ₹65, while average expenses per student decreases by ₹1. Find the original expenditure of the mess.
[SSC CGL Tier-II 2015]
(a) ₹480 (b) ₹500 (c) ₹520 (d) ₹545

14. There were 35 students in a hostel. If the number of students increases by 7, the daily expenses of the mess increases by ₹42, while average expenses per student decreases by ₹1. Find the original expenditure of the mess.
[SSC CGL Tier I 2016]
(a) ₹420 (b) ₹500 (c) ₹520 (d) ₹545

15. The average attendance in a school for the first 4 days of the week is 30 and for the first 5 days of the week is 32. The attendance on the fifth day is
(a) 32 (b) 40 (c) 38 (d) 36

16. The average temperature for Monday, Tuesday and Wednesday was 55°, the average for Tuesday, Wednesday and Thursday was 60°, that for Thursday being 56°, what was the temperature on Monday ?
(a) 39° (b) 41°
(c) 45° (d) None of these

17. A family consists of grandparents, parents and three grandchildren. The average age of the grandparents is 67 years, that of the parents is 35 years and that of the grandchildren is 6 years. What is the average age of the family?
(a) $28\frac{4}{7}$ years (b) $31\frac{5}{7}$ years
(c) $32\frac{1}{7}$ years (d) None of these

18. A batsman in his 12th innings makes a score of 65 and thereby increases his average by 2 runs. What is his average after the 12th innings if he had never been 'not out'?
(a) 42 (b) 43 (c) 44 (d) 45

19. The average age of 7 members of a family is 25. If one of the members whose age is 28 years, is excluded, then others are in the ratio 1 : 2 : 3 : 4 : 5 : 6. Find the age of the eldest member of the family.
(a) 28 (b) 35 (c) 14 (d) 42

20. The average of seven consecutive even numbers is 10. If the next three even numbers are included, then what is the new average?
(a) 10 (b) 12 (c) 13 (d) 14

21. Average weight of three students is 40 kg. If the first student is twice the second and the second student is thrice the weight of the third student, then what is the weight of the student with lightest weight?
(a) 18 (b) 20
(c) 14 (d) None of these

22. The average age of 8 persons is increased by 2 years, when one of them, whose age is 24 years, is replaced by a new person. The age of the new person is: **[SSC CHSL 2015]**
(a) 42 yrs (b) 40 yrs (c) 38 yrs (d) 45 yrs

23. A shrewed milkman mixes water and milk in the ratio of 2 : 3. What part of this mixture should be removed and replaced with water so that the solution contains water and milk in the ratio 1 : 1?
(a) 1/6 (b) 1/4 (c) 1/3 (d) 5/6

24. Out of 10 teachers of a school, one teacher retires and in his place, a new teacher of age 25 years joins. As a result, average age of teachers is reduced by 3 years. The age (in years of the retired teacher is:
[SSC CGL Tier -I 2013]
(a) 50 (b) 58 (c) 60 (d) 55

25. The average age of 8 men is increased by 2 years when two of them whose ages are 21 and 23 years are replaced by two new men. The average age of the two new men is:
[SSC CGL 2011]
(a) 22 yrs (b) 24 yrs (c) 28 yrs (d) 30 yrs

26. There are 125 middle level employees in Due North Inc. The average monthly salary of the middle level employees is ₹ 5,500 and that of the senior level employees is ₹ 14,000. If the average monthly salary of all these employees is ₹ 8,687.5, find the total number of employees in the company, if middle and senior level employees of Due North Inc. form 80% of their total employees.
(a) 175 (b) 200 (c) 220 (d) 250

27. I went to a hotel along with 12 friends. I paid ₹ 145 and all the others paid an equal amount. In the end, we did some calculations and found that the average sum paid by all of us was ₹ 5 more than what was originally paid by each of my friends. How much money did each of my friends pay?
(a) ₹ 120 (b) ₹ 100 (c) ₹ 80 (d) ₹ 70

28. Brian Lara, the famous batsman, scored 6,000 runs in certain number of innings. In the next five innings he was out of form and hence, could make only a total of 90 runs, as a result of which his average fell by 2 runs. How many innings did he play in all, if he gets out in all the innings?
(a) 105 (b) 95 (c) 115 (d) 104

29. First ten multiples of 1, 2, ...10 are taken. What is the average of all these 100 numbers?
(a) 25.5 (b) 50.5
(c) 75.5 (d) None of these

30. A milkman dilutes 36 litres of pure milk with water. The percentage of milk in the solution is now 75%. How many litres of water did he add?
(a) 6 litres (b) 8 litres
(c) 12 litres (d) 16 litres

GP_3386

31. Mr. Patel walked 6 km to reach the station from his house, then he boarded a train whose average speed was 60 km/hr and thus he reached his destination. In thus way he took total of 3 hours. If the average speed of the entire journey was 32 km/hr then the average speed of walking is:
(a) 3 km/hr (b) 4.5 km/hr
(c) 4 km/hr (d) None of these

32. The average score of Sehwag in 10 innings was 77 runs. In the 11^{th} innings he had scored zero runs. The overall average score of Sehwag in all the 11 innings was :
(a) 77 (b) 7.7
(c) 11 (d) None of these

33. The average age of P,Q and R is 5 yrs more than R's age. If the total ages of P and Q together is 39 yrs, then R's age is
(a) 16 yrs (b) 14 yrs
(c) 12 yrs (d) 24 yrs

34. Sachin Tendulkar has a certain average for 11 innings. In the 12th innings, he score 120 runs and there by increases his average by 5 runs. His new average is
[SSC CGL Tier II 2013]
(a) 60 (b) 62 (c) 65 (d) 66

35. A cricket player after playing 15 tests scored 124 runs in the 16^{th} test. As a result, the average of his runs is increased by 4. The present average of runs is
(a) 55 (b) 64 (c) 60 (d) 68

36. A cricketer had a certain average of runs for his 64 innings. In his 65th innings, he is bowled out for no score on his part, This brings down his average by 2 runs his new average of runs is: **[SSC CGL 2008]**
(a) 130 (b) 128 (c) 70 (d) 68

37. A cricketer has completed 10 innings and his average is 21.5 runs. How many runs must be made in his next innings so as to raise his average to 24? **[SSC CHSL 2014]**
(a) 47 (b) 48 (c) 49 (d) 50

38. The respective ratio between the present ages of A ,B and C is 6 : 8 : 10 . If the average of their present ages is 28 years then what would be the sum of the ages of A and B together after 5 years ?
(a) 45 yrs (b) 55yrs (c) 52 yrs (d) 59yrs

39. Find the average of all 2-digit natural numbers which when divided by 9 leaves a remainder of 1 in each case?
(a) 90 (b) 50.5 (c) 45.5 (d) 101

40. A car covers 1/5 of the distance from A to B at the speed of 8 km/hour,1/10 of the distance at 25 km per hour and the remaining at the speed of 20km per hour. Find the average speed of the whole journey-
(a) 12.625 km/hr (b) 13.625 km/hr
(c) 14.625 km/hr (d) 15.625 km/hr

41. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. Find out the average age of the team.
(a) 23 years (b) 26 years
(c) 50 years (d) None of these

42. The average speed of a train is 3 times the average speed of a car. The car covers a distance of 520 kms in 8 hours. How much distance will the train cover in 13 hours?
(a) 2553 kms (b) 2585 kms
(c) 2355 kms (d) 2535 kms

43. Three years ago, the average age of a family of 10 members was 40 years. If one child is also included in the family, the present average age of the family remained the same as three years ago. The present age of the child is
(a) 10 years (b) 11 year
(c) 13 years (d) 4 years

44. The average of marks obtained by 120 candidates in a certain examination is 35. If the average mark of passed candidates is 39 and that of the failed candidates is 15, what is the number of candidates who passed the examination?
(a) 90 (b) 85 (c) 100 (d) 120

45. The average marks fetched by 140 students is 35. The average marks of passed students is 40 and that of failed students is 20, then what is the number of failed students?
[SSC CGL 2005]
(a) 32 (b) 35 (c) 28 (d) 41

46. The average of 8 numbers is a and the average of 2 numbers among them is b. If the average of the remaining 6 numbers is c, then
(a) $4a - c = 3b$ (b) $2c - b = 2a$
(c) $3a + b = 4c$ (d) $4a - b = 3c$

47. The average salary of all the staff in an office of a corporate house is ₹ 5,000. The average salary of the officers is ₹ 14,000 and that of the rest is ₹ 4,000. If the total number of staff is 500, the number of officers is–
(a) 10 (b) 15 (c) 25 (d) 50

48. The batting average for 40 innings of a cricketer is 50 runs. His highest score exceeds his lowest score by 172 runs. If these two innings are excluded, the average of the remaining 38 innings is 48 runs. The highest score of the player is-
(a) 165 (b) 170 (c) 172 (d) 174

49. Average cost of 8 apples and 5 oranges is ₹ 25. While the average cost of 4 apples and 7 oranges is ₹ 21. What is the total cost of 36 apples and 36 oranges?
(a) ₹ 1586 (b) ₹ 1668 (c) ₹ 1112 (d) ₹ 1724

50. Mahesh travels at the speed of 80 km/hr for 3 hours and at the speed of 120 km/hr for 4.5 hours. In this way he covers 3/5 of the total distance. At what average speed should he travel to cover the remaining distance in 4 hours?
(a) 160 km/hr (b) 170 km/hr
(c) 125 km/hr (d) 130 km.hr

51. The average salary of the entire staff in an office is ₹ 240 per month. The average salary of officers is ₹ 900 and that of non-officers is ₹ 120. If the number of officers is 30, then find the number of non-officers in the office.
(a) 150 (b) 165 (c) 220 (d) 520

52. The average salary of the entire staff in a office is ₹ 120 per month. The average salary of officers is ₹ 460 and that of non-officers is ₹ 110. If the number of officers is 15, then find the number of non-officers in the office.
[SSC CGL Tier II 2016]
(a) 510 (b) 520 (c) 530 (d) 540

53. A passenger train covers the distance between stations X and Y 50 minutes faster than a goods train. Find this distance if the average speeds of the passenger train is 180 kmph and that of goods train 60 kmph.
(a) 75km (b) 64 km (c) 55km (d) 80km

54. Total age of Ramesh and Sohan is 30 years more than that of Sohan and Mahesh. While the average age of Ramesh, Sohan and Mahesh is equal to that of Ramesh and Sohan. Then what is the difference between the ages of Ramesh and Sohan?
(a) 65 (b) 60 (c) 40 (d) 80

GP_3386

Hints and Solutions

Level 1

1. (c) Age of the class teacher = 25 × 16 – 24 × 15
= 400 – 360 = 40 years.

2. (c) Age of the teacher = (37 × 15 – 36 × 14) years = 51 years.

3. (a) The middle number = Sum of the first six + Sum of the the last six – Sum of all the 11
= 6 × 10.5 + 6 × 11.4 – 11 × 10.9
= 63 + 68.4 – 119.9
= 11.5

4. (c) Use Shortcut Approach-3

5. (c) Use shortcut Approach-5

6. (b) Use Shortcut Approach-14

7. (d) Temperature on the fourth day
= 40.2 × 4 + 41.3 × 4 – 40.6 × 7
= 160.8 + 165.2 – 284.2 = 41.8°C

8. (b) Recognise that the journey by bus and that by cycle are of equal distance. Hence, we can use the short cut illustrated earlier to solve this part of the problem.
Using the process explained above, we get average speed of the second half of the journey as
10 + 1 × 5 = 15 kmph
Then we employ the same technique for the first part and get 15 + 1 × 9 = 24 kmph

9. (c) Use Shortcut Approach-2

10. (a) Sum of 10 numbers = 402
Corrected sum of 10 numbers
= 402 – 13 + 31 – 18 = 402

Hence, correct average $= \frac{402}{10} = 40.2$

11. (a) Use Shortcut Approach-2

12. (c) Let the third number be x. Then, second number = $2x$. First number = $4x$.

$$\therefore \quad \frac{1}{x}+\frac{1}{2x}+\frac{1}{4x}=\left(\frac{7}{72}\times 3\right) \text{ or } \frac{7}{4x}=\frac{7}{24}$$

or $4x = 24$ or $x = 6$
So, the numbers are 24, 12 and 6.

13. (d) For 150 coins to be of a value of ₹150, using only 25 paise, 50 paise and ₹ 1 coins, we cannot have any coins lower than the value of ₹ 1. Thus, the number of 50 paise coins would be 0. Option (d) is correct.

14. (c) Let the other number be N.

Then, $\frac{X+N}{2} = XY \Rightarrow N = 2XY - X$

15. (a) Use Shortcut Approach-1

16. (c) If you assume the first number to be a, naturally next number would be 2 more than a and so on
The numbers are : a, a + 2, a + 4, a + 6, a + 8.
Hence, their average = Sum/5 = (a + 4)

17. (c) Here one boy is excluded and final average of the group decreases.
∴ change in average is (–)ve = – 0.1 kg.
Using the formula
Sum of the quantities excluded

$$= \begin{pmatrix}\text{Change in no. of quantities}\\ \times \\ \text{Original average}\end{pmatrix} + \begin{pmatrix}\text{Change in average}\\ \times \\ \text{Final no. of quantities}\end{pmatrix}$$

⇒ weight of the boy who left
= (1 × 45) – (– 0.1 × 49) = 49.9 kg
∴ weight of the boy who left the class is 49.9 kg.

18. (c) Use Shortcut Approach-7

19. (c) Use Shortcut Approach-7

20. (b) Average speed $= \frac{2\times 8\times 12}{8+12} = \frac{192}{20}$
= 9.6 km/h

21. (c) $\frac{30x+30y}{60} = \frac{x+y}{2}$

22. (b) Weight of the whole family
= 7 × 18 = 126
Weight of family excluding head of the family = 13 × 6 = 78 kg
Weight of head of the family = 48 kg

23. (a) Use Shortcut Approach-3

24. (b) x + y = 50

$$\frac{1}{4}x+\frac{1}{2}y=15$$

Solving both the equations, we get x = 40 and y = 10

25. (d) Total income = Total expenditure + Total savings
We can not find the average saving of B and C, since data is insufficient.

26. **(a)** Required average salary

$$= \frac{6000\times 2+8000\times 3}{(2+3)} = \frac{36000}{5} = ₹7200$$

27. **(c)** Hence required average $= 11\times 5+5 = 60$

28. **(a)** Average rainfall

$$= \frac{2\times 6+4\times 5+4\times 10+2\times 6}{12} = 7$$

29. **(d)** Use Shortcut Approach-8

30. **(c)** Use Shortcut Approach-8

31. **(c)** $\frac{7+14+21+....77}{11} = \frac{7(1+2+.....11)}{11}$

$$= \frac{7\times 11\times 12}{11\times 2} = 42$$

$$\left[\because 1+2+3.....n = \frac{n(n+1)}{2}\right]$$

Alternatively : Since all the numbers are in A.P Further there are odd number of numbers (i.e., 11) in the sequence. Thus the middle most term is the average of the sequence, which is 42.

32. **(a)** $\frac{3x+4x+5x}{3} = 12 \Rightarrow x = 3$

$\therefore$ The ages are 9, 12 and 15 years

and the required average age $= \frac{9+15}{2} = 12$

33. **(d)** The correct answer can be $\frac{a+b+c+d+e}{5}$

or $\frac{a+e}{2} = \frac{b+d}{2} = c$

where a, b, c, d, e are consecutive odd numbers.

34. **(a)** The present average age of 8 members will be 33 years.
Let the present age of child be 'x'. So,

$\frac{8\times 33+x}{8+1} = 30 \Rightarrow$ x = 6 years.

35. **(a)** New average

$$= \frac{35\times 40-50\times 10}{30} = \frac{900}{30} = 30$$

36. **(b)** Let the first number be x.
$\therefore$ Second number = 3x and Third number = 3x/4
According to question,
$\therefore$ x + 3x + 3x/4 = 3×114
$\Rightarrow$ 19x/4 = 342 $\Rightarrow$ x = 342×4/19 = 72
$\therefore$ The largest number = 3×72 = 216

37. **(a)** Let the first number be x.
$\therefore$ Second number = x/2 and Third number = x/4
$\because$ x + x/2 + x/4 = 3 × 154
$\Rightarrow$ 7x/4 = 462 $\Rightarrow$ x = 462 × 4/7 =264

38. **(a)** The total age of 32 students = 320 yrs.
The total age of 32 students and the teacher = 33 × 11 = 363 yrs.
$\therefore$ Age of the teacher = 363 – 320 = 43yrs.

39. **(b)** Let the third number be x.
$\therefore$ Second number = 3x
and first number = 6x
$\therefore$ 6x + 3x + x = 20 × 3 $\Rightarrow$10x = 60 $\Rightarrow$ x = 6
$\therefore$ Required sum = 6x +x = 7x = 7 × 6 = 42.

40. **(a)** Let the total number of students = x
The average marks increased by 1/2 due to an increase of 83 – 63 = 20 marks.
But total increase in the marks = 1/2×x = x/2
Hence we can write as x/2=20
$\Rightarrow$ x = 20×2= 40

41. **(b)** Use shortcut Approach-6

42. **(a)** Use shortcut Approach - 6

43. **(a)** Clearly, we have = x =(3y+3z/6)
or 2x = y + z

44. **(d)** The total of 11 results = 11 × 50 = 550
The total of first 6 results = 6 × 49 = 294
The total of last 6 results = 6 × 52 = 312
The 6th result is common to both;
Sixth result = 294 + 312 – 550 = 56

45. **(a)** Distance = (44/60)×50 = (x/60) ×55
$\therefore$ x= 40 minutes

46. **(d)** Speed of the car = 588/6 = 98 km/hr
Speed of train = (10/7) × 98 = 140 km/hr
Distance covered by the train in 13 hours = 140 × 13 = 1820 km

47. **(d)** Correct average
= (40×72 + (64 + 62 +84) – 68 – 65 – 73)/40
= (2880 + 210 – 206)/40 = 2884/40 = 72.1

Level 2

1. **(b)** Check as follows,

$$\frac{1+2+3+4+5}{5} = 3$$

$$\frac{1+2+3+4+5+6+7}{7} = 4$$

2. **(b)** Sum of ages of Mr. and Mrs. Sinha in 1972 = 46 years
Sum of age of their family in 1976 = 19 × 3 = 57 years
Sum of ages of Mr. and Mrs. Sinha in 1976 = (46 + 8) years = 54 years
$\therefore$ Age of Vicky in 1980

GP_3386

$= 57 - 54 + 4 = 7$ years.

3. (a) Let the total no. of workers be x.
Now, $8000x = 7 \times 12000 + (x-7) \times 6000$

$$\Rightarrow \quad x = \frac{42000}{2000} = 21$$

4. (d) As average weight increases by 0.3 gm after including weight of the bag.
So, weight of the bag

$$= (47 \times 0.3) + 4.3 = 14.1 + 4.3 = 18.4\ gm$$

5. (c) Find the total distance covered in each segment of 10 minutes. You will get total distance = 46 kilometers in 50 mins.

$$\therefore \text{ Average speed} = \frac{46 \times 60}{50} = 55.2 \text{ kmph}$$

6. (a) $100 \rightarrow 130 \rightarrow 182$. Hence, $82/2 = 41$.

7. (a) Put x = 100 to get the weight of the containers. Use these weights of find average weight as $2600/4 = 650$.

8. (d) From the above explanation we have no clue about his wife's age.

9. (b) Since we know that the difference between the age of any two persons remains always constant, while the ratio of their ages gets changed as the time changes.
So, if the age of his child be x (presently)
Then the age of wife be $x + 26$ (presently)
Thus, the total age $= x + (x + 26) = 32$
$\Rightarrow \quad x = 3 \qquad [\because\ 252 - 220 = 32]$
Therefore the age of her child is 3 years and herself is 29 years. Hence her age at the time of the birth of her child was 26 years.
Alternatively: As we have mentioned above that the age difference remains always constant. Therefore her age at the time of her child's birth was 26 years.

10. (d) Since there is no clue. So we can't determine.

11. (c) $(14 \times 27.75 - 2 \times 42)/12 = 25.375$

12. (b) Let the original average expenditure be ₹ x.
Then, $42(x-1) - 35x = 42 \Leftrightarrow 7x = 84 \Rightarrow x = 12$.
$\therefore$ Original expenditure = ₹ (35×12) = ₹ 420.

13. (c) Use Shortcut Approach-4

14. (a) Use Shortcut Approach-4

15. (b) Attendance on the fifth day
$= 32 \times 5 - 30 \times 4 = 160 - 120 = 40$

16. (b) Sum of temp. for Monday, Tuesday and Wednesday $= 55 \times 3 = 165°$
Sum of temp. for Tuesday, Wednesday, an Thursday $= 60 \times 3 = 180°$
Since temp. on Thursday = 56°
$\Rightarrow$ Temp for Tue and Wed $= 180 - 56 = 124°$
$\Rightarrow$ Temp of Monday $= 165 - 124 = 41°$

17. (b) Required average

$$= \left(\frac{67 \times 2 + 35 \times 2 + 6 \times 3}{2 + 2 + 3}\right)$$

$$= \left(\frac{134 + 70 + 18}{7}\right) = \frac{222}{7} = 31\frac{5}{7} \text{ years.}$$

18. (b) Let 'x' be the average score after 12th innings
$\Rightarrow 12x = 11 \times (x-2) + 65 \Rightarrow x = 43.$

19. (d) $25 \times 7 - 28 = 21x \Rightarrow x = 7$
Age of eldest member $= 6 \times 7 = 42$ years

20. (c) $7x + 42 = 70 \Rightarrow 7x = 28 \Rightarrow x = 4$

$$\text{Required average} = \frac{10x + 90}{10} = x + 9 = 13$$

21. (d) $6x + 3x + x = 120 \Rightarrow x = 12$

22. (b) Use Shortcut Approach-12

23. (d) Let x part of mixture is removed

$$\text{then, } \frac{2 - \frac{2}{5}x + x}{3 - \frac{3}{5}x} = 1 \Rightarrow x = \frac{5}{6}$$

24. (d) Use Shortcut Approach-13

25. (d) Use Shortcut Approach-12

26. (d) Let x = no. of senior level employees

$$\therefore \frac{125 \times 5500 + x \times 14000}{125 + x} = 8687.5 \Rightarrow x = 75$$

$\frac{4}{5}$ of total employees $= (125 + 75) = 200$

$$\text{Hence no. of total employees} = \frac{5}{4} \times 200 = 250$$

27. (c) $\frac{145 + 12x}{13} = x + 5 \Rightarrow x = 80$

28. (a) He played 100 matches previously,

$$\text{His old average} = \frac{6000}{100} = 60$$

His new average = 6090/105 = 58
Option (a) is satisfied.

29. (d) Average of 1st 10 multiple of 1

$$= \frac{1 + 2 + 3 + \ldots\ldots + 10}{10} = 5.5$$

Average of 1st 10 multiple of 2

$$= \frac{2+4+.....+20}{10} = 11$$

Average of 1st 10 multiple of 3

$$= \frac{2+6+.....+30}{10} = 16.5$$

Hence, required average

$$= \frac{5.5+11+16.5+.....\text{to 10 term}}{10}$$

$$= \frac{\frac{10}{2}[2\times 5.5+9\times 5.5]}{10}$$

$$= \frac{1}{2}\times 60.5 = 30.25$$

30. (c) Let x = volume of water he added

$$\frac{36}{36+x} = \frac{3}{4} \Rightarrow 144 = 108+3x \Rightarrow x = 12$$

31. (c) $\frac{6}{4} = 1.5$ hour, when he was walking

Therefore time of journey by train = 3 – 1.5 = 1.5

Now, the distance travelled by train = 1.5 × 60 = 96 – 6 = 90

Alternatively : Total distance = 32 × 3 = 6 + 60 × x

$\Rightarrow$ x = 1.5 hours

Thus, the speed of walking $= \frac{6}{1.5}$

$= 4$ km / hr

32. (d) Average $= \frac{770}{11} = 70$

33. (c) Let the age of P = x yrs,

Age of Q = y yrs and age of R = z yrs

Now, $\frac{x+y+z}{3} = z+5 \Rightarrow x + y - 2z = 15$

And x + y = 39

On solving Eqs. (i) and (ii),

we get z = 12 yrs

34. (c) Use Shortcut Approach-9

35. (b) Let the original average be x. Then

$$\frac{15x+124}{16} = x+4 \Rightarrow x = 60$$

Present average = x + 4 = 64

36. (b) Use shortcut Approach-9

37. (c) Use shortcut Approach-10

38. (d) Let the ages of A, B and C are 6a, 8a and 10a.

$$\text{Average age} = \frac{6a+8a+10a}{3} = 28$$

$\Rightarrow$ a = 3.5

Sum of the ages of A and B together after 5 years

= 6a + 5 + 8a + 5 = 14a + 10 = 59 years

39. (b) The two digit numbers which leave a remainder of 1 when divided by 9 will form a series 10, 19, 28,……., 91. Since, there are 10 numbers and the difference between the numbers is constant, then the average of the entire series is same as the average of first and last numbers. So, the average of the series is $\frac{10+91}{2} = 50.5$.

40. (d) If the whole journey be x km. The total time taken

$$= \left(\frac{\frac{x}{5}}{8} + \frac{\frac{x}{10}}{25} + \frac{\frac{7x}{10}}{20}\right) \text{hrs}$$

= (x/40 + x/250 + 7x/200) hrs

= (25x + 4x + 35x)/1000

= 64x/1000 hrs

$\therefore$ Average speed $= \frac{x}{\frac{64x}{1000}}$

= 15.625 km/hr

41. (a) Number of members in the team = 11

Let the average age of the team = x

$\Rightarrow$ (Sum of the ages of all the 11 members of the team) / (11) = x

$\Rightarrow$ Sum of the ages of all the 11 members of the team = 11x

Age of the captain = 26

Age of the wicket keeper = 26 + 3 = 29

Sum of the ages of 9 members of the team excluding captain and wicket keeper

= 11x – 26 – 29 = 11x – 55

Average age of 9 members of the team excluding captain and wicket keeper

= (11x – 55)/9

Given that (11x – 55)/9 = (x – 1)

$\Rightarrow$ 11x – 55 = 9(x – 1) $\Rightarrow$ x = 46/ 2 = 23 years

42. (d) Avg. Speed of car = 520 ÷ 8 = 65 km/hr
Avg. speed of train = 65 × 3 = 195 km/hr
So Train cover distance in 13 hr = 195 × 13 = 2535 km

43. (a) The present average age of 10 members will be 43 years. Let the present age of child be 'x'. So,

$$\frac{10\times 43+x}{10+1}=40 \Rightarrow x = 10 \text{ years.}$$

44. (c) Let the number of passed candidates be x.
Then total marks = 120 × 35
= 39x + (120 – x) × 15
⇒ 4200 = 39x + 1800 – 15x
⇒ x = 100

45. (b) Use Shortcut Approach-11

46. (d) Sum of 8 number = a × 8 = 8a
Sum of 2 numbers = b × 2 = 2b
Sum of remaining 6 number = c × 6 = 6c
∴ 2b + 6c = 8a ⇒ b + 3c = 4a ⇒ 4a – b = 3c

47. (d) Let the number of officers be x.
∵ 5000 × 500 = 14000x + 4000(500 – x)
∴ 2500000 = 14000x + 2000000 – 4000x
∴ x = 500000/10000 = 50

48. (d) Let the max. number of runs be x.
∴ The lowest score = (x–172)
∵ 40 × 50 = 38 × 48 + x + (x –172)
⇒ 2000 = 1824 + 2x – 172 ⇒ x = 174

49. (b) Let cost of 1 apple = x
Cost of 1 oranges = y
8x+5y = 25×13 = 325
4x+7y = 21×11 = 231
∴ 12x+12y = 556
∴ Total cost of 36 apples and 36 oranges = 556 × 6/2 = 1668

50. (d) 3/5 of the total distance
= 80 × 3 + 120 × 4.5 = 240 + 540 = 780 km
So total distance = 780/3 ×5 = 1300 km
Remaining distance = 1300 – 780 = 520 km
∴ Speed = 520/4 = 130 km/hr

51. (b) Let the required number of non-officers = x
Then, 120x + 900 ×30 = 240 (30 + x)
Or, 240 x – 120 x = 900× 30 – 240×30
⇒ 120x = (27000 – 7200) ⇒ x = 165

52. (a) Use Shortcut Approach-15

53. (a) Distance = (180 × 60)/(180 – 60) × 50/60
= 75 km

54. (b) R + S = S + M + 30
Or R – M = 30 .....(i)
And (R + S + M)/3 = (R + S)/2
⇒ M = (R + S)/2 .....(ii)
Putting (ii) in (i)
R – (R + S)/2 = 30 ⇒ R – S = 60

CHAPTER

10 Percentage

BASIC DEFINITION OF PERCENTAGE

The word per cent means per hundred or for every hundred. The symbol '%' is used for the term percent. Thus, 20 per cent is written as 20% and it means 20 out of 100.

This is written in ratio from as $\frac{20}{100}$.

Thus percentage value of the ratio $\frac{3}{5}$ will be

$\frac{3}{5} \times 100\% = 60\%$

Illustration – 1

A person saves ₹ 5,000 per month from his monthly salary ₹ 30,000. Find the percentage monthly saving of the person.

Solution : Out of monthly salary ₹ 30,000, saving is ₹ 5,000

$\Rightarrow$ Out of monthly salary ₹1, saving is ₹ $\frac{5,000}{30,000}$

$\Rightarrow$ Out of monthly salary ₹100, saving is

$$₹ \frac{5,000}{30,000} \times 100 = ₹ \frac{50}{3} = ₹ 16.67 \text{ (approx.)}$$

To convert the ratio to percentage value, multiply the ratio by 100 and put % sign.

Hence percentage monthly saving = 16.67% (approx.)

$x\%$ of any value $y = \frac{x}{100} \times y$

For example: 20% of 600 kg sugar

$= \frac{20}{100} \times 600 =$ 120 kg sugar

Illustration – 2

250 students of *ABC* school and 350 students of *XYZ* school appeared in secondary board examination conducted by CBSE in 2013. 20 students of *ABC* school and 25 students of *XYZ* school did not pass in this board examination. Students of which of the two schools *ABC* and *XYZ* have shown poor performance?

Solution : We cannot compare the performance of the students of the two schools in secondary board examination by just looking the number of students 20 of *ABC* school and 25 of *XYZ* school who did not pass in secondary board examination. To compare the performance, you have to find the percentage of the students who did not pass the secondary board examination of each school out of those students of each school who appeared in the secondary board examination.

Percentage of the students of *ABC* school who did not pass

$= \frac{20}{250} \times 100 = 8\%$

Percentage of the student of *XYZ* school who did not pass

$= \frac{25}{350} \times 100\% = 7.1\%$ (approximately)

Hence students of the *ABC* school have shown poor performance.

Illustration – 3

In a survey, voters of a national party *A* are increase by 2.5 lakhs and voters of national party *B* are increase by 4 lakhs in 2012. Which party *A* or *B* has grown more in 2012 ?

Solution : In first shot the answer to the question seems to be national party *B*. But actually the question can not be answered, because we don't know the just previous year's voters of each of the national party *A* and *B*.

GP_3386

If we had further information that in 2011, voters of national party A were 5 lakhs and voters of national party B were 10 lakhs, then we can compare growth rates of two national parties by finding their percentage growth.

Percentage growth rate of national party A in 2012

$$= \frac{250000}{500000} \times 100\% = 50\%$$

Percentage growth rate of national party B in 2012

$$= \frac{400000}{1000000} \times 100\% = 40\%$$

Hence, national party A has higher growth rate in 2012. Thus national party A has grown more than B in 2012.

Illustration – 4

Express the following as fraction

(a) 25% **(b)** $33\frac{1}{3}\%$

Solution :

(a) $25\% = \frac{25}{100}\left(\text{Since \% means } \frac{1}{100}\right) = \frac{1}{4}$

(b) $33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{3 \times 100} = \frac{1}{3}$

Illustration – 5

25% of a number is 80. What is number ?

Solution : Let the number be X. According to the given condition

$$\frac{25}{100} \times X = 80 \Rightarrow X = \frac{80 \times 100}{25} = 320.$$

Illustration – 6

Express $\frac{1}{8}$ as a percentage.

Solution : $\frac{1}{8} = \frac{1}{8} \times 100\%$

$$= \frac{100}{8}\% = \frac{25}{2}\% = 12\frac{1}{2}\%$$

Illustration – 7

Two third of three fifth of one eighth of a certain number is 268.50. What is 30% of the number?

(a) 1611 **(b) 1616**
(c) 1343 **(d) 594.60**

Solution : (a) Let the number be x.

According to the question $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{1}{8} \times x = 268.50$

$$\Rightarrow \frac{2}{3} \times \frac{3}{5} \times \frac{1}{8} \times x = 268.50$$

$$x = \frac{268.50 \times 3 \times 5 \times 8}{2 \times 3} = 5370$$

$$30\% \text{ of } x = \frac{30}{100} \times 5370 = 1611.00$$

Illustration – 8

4598 is 95% of ?

(a) 4800 **(b) 4840** **(c) 4850** **(d) 4880**

Solution : (b) Let 95% of $x = 4598$.

Then, $\frac{95}{100} \times x = 4598 \Rightarrow x = \left(4598 \times \frac{100}{95}\right) = 4840.$

Decimal to Percentage Conversion

To convert a decimal number to percentage, multiply the decimal number by 100 or simply move the decimal point to the right by two digits and put % sign.

For example: $2.381 = 2.381 \times 100\% = 238.1\%$

$0.53 = 0.53 \times 100\% = 53.0\% = 53\%$

$0.0436 = 0.0436 \times 100\% = 4.36\%$

Percentage to Decimal Conversion

To convert a percentage value to decimal, divide the percentage value by 100
or simply move the decimal point to the left by two digits and remove % sign.

For example: $526\% = 5.26$

$23.82\% = 0.2382$

$200\% = 2$

Percentage Increase, Percentage Decrease and Percentage Change

Percentage increase

$$= \frac{\text{Increase}}{\text{Initial value (i.e., Base value)}} \times 100$$

Percentage decrease

$$= \frac{\text{Decrease}}{\text{Initial value (i.e., Base value)}} \times 100$$

Percentage change

$$= \frac{\text{Change}}{\text{Initial value (i.e., Base value)}} \times 100$$

Let income of a family in the years 2010, 2011 and 2012 are ₹ 50000, ₹ 80000 and ₹ 60000 respectively.

Here income of the family increases in 2011 but decreases in 2012.

Increase in family income in 2011 from 2010

= (Higher Income) – (Lower Income)

= (Income in 2011) – (Income in 2010)

= ₹ 80000 – ₹ 50000 = ₹ 30000

Decrease in family income in 2012 from 2011

= (Higher Income) – (Lower Income)

= (Income in 2011) – (Income in 2012)

= ₹ 80000 – ₹ 60000 = ₹ 20000

Percentage increase in family income in 2011 from 2010

$$= \frac{\left(\begin{matrix}\text{Increase in income}\\ \text{in 2011 from 2010}\end{matrix}\right)}{(\text{Income in 2010})} \times 100$$

$$= \frac{30000}{50000} \times 100 = 60\%$$

Percentage decrease in family income in 2012 from 2011

$$= \frac{\left(\begin{matrix}\text{Decrease in income}\\ \text{in 2012 from 2011}\end{matrix}\right)}{(\text{Income in 2011})} \times 100$$

$$= \frac{20000}{80000} \times 100 = 25\%$$

Illustration – 9

Rent of the house is increased from ₹ 7000 to ₹ 7700. Express the increase in price as a percentage of the original rent.

Solution : Increase value = ₹ 7700 – ₹ 7000 = ₹ 700

$$\text{Increase \%} = \frac{\text{Increase value}}{\text{Base value}} \times 100 = \frac{700}{7000} \times 100 = 10$$

∴ Percentage rise = 10 %.

Illustration – 10

The cost of a bike last year was ₹ 19000. Its cost this year is ₹ 17000. Find the percent decrease in its cost.

Solution : (b) $\%\ \text{decrease} = \frac{19000 - 17000}{19000} \times 100$

$$= \frac{2000}{19000} \times 100 = 10.5.$$

∴ Percent decrease = 10.5 %.

NOTE : If the value of any thing increases, then percentage change is the percentage increase and if the value of any thing decreases, then percentage change is the percentage decrease. Thus,
Percentage change = Percentage increase, if value of the thing increases
and Percentage change = Percentage decrease, if value of the thing decreases.

Changing a Quantity by a Given Percent

(i) If any quantity x increased by $a\%$, then final value of $x = \left(\frac{100+a}{100}\right)x$

(ii) If any quantity x decreased by $a\%$, then final value of $x = \left(\frac{100-a}{100}\right)x$

Illustration – 11

Price of a car is ₹ 5, 00, 000. If its price increased by 15%, then the new price of the car is

(a) ₹ 5,25,000 **(b) ₹ 5,50,000**
(c) ₹ 5,75,000 **(d) ₹ 5,85,000**

Solution : (c) New price of the car

$$= ₹\left(\frac{100+15}{100}\right) \times 5,00,000 = ₹\,115 \times 5,000 = ₹\,5,75,000$$

Illustration – 12

Price of a flat is ₹ 20, 00, 000. If its price is decreased by 30%, then the new price of the flat is

(a) 12, 00, 000 **(b) 12, 25, 000**
(c) 13, 00, 000 **(d) 14, 00, 000**

Solution : (d) New price of the flat

$$= ₹\left(\frac{100-30}{100}\right) \times 20,00,000$$

$$= ₹\,70 \times 20,000$$

$$= ₹\,14,00,000$$

Successive change in a quantity by different given per cent

If any quantity x increased by $a\%$, then increased by $b\%$, then decreased by $c\%$, then increased by $d\%$, ..., then final value of x ...

$$= x\left(\frac{100+a}{100}\right) \times \left(\frac{100+b}{100}\right) \times \left(\frac{100-c}{100}\right) \times \left(\frac{100+d}{100}\right) \times \ldots$$

Illustration – 13

Let the price of a flat in the year 2012 is ₹ 50,00,000. If its price increased by 10% in the year 2013, then increased 20% in the year 2014, then decreased 25% in the year 2015, then increase by 15% the year 2016. Find the value of the flat in the year 2017.

(a) ₹ 56,92,500 **(b) ₹ 56,90,000**

(c) ₹ 56,00,000 **(d) ₹ 55,80,000**

Solution : (a) Value of the flat in the year 2017

$$= 50,00,000\left(\frac{100+10}{100}\right)\left(\frac{100+20}{100}\right)\left(\frac{100-25}{100}\right)\left(\frac{100+15}{100}\right)$$

$$= 5\times110\times120\times75\times\frac{115}{100} = 5\times11\times12\times75\times115$$

= ₹ 56,92,500

SHORTCUTS

Shortcut Approach - 3

If a is x% of b, then $x=\frac{a}{b}\times100\%$

Illustration – 14

30 is what per cent of 75?

(a) 36% (b) 40% (c) 42% (d) 45%

Shortcut Solution : (b)

$$x=\frac{a}{b}\times100\%$$

Hence, required percent

$$=\frac{30}{75}\times100\% = 10\times4\% = 40\%$$

Shortcut Approach - 3

If the price of a commodity increases by r%, then the reduction in consumption so as not to increase the expenditure, is $\left(\frac{r}{100+r}\times100\right)\%$

Illustration – 15

If the price of a commodity be raised by 15%, find how much per cent must a householder reduce his consumption of that commodity, so as not to increase his expenditure.

(a) 14% **(b) $12\frac{1}{23}\%$** **(c) $23\frac{1}{13}\%$** **(d) $13\frac{1}{23}\%$**

Shortcut Solution : (d)

Required percentage

$$=\left(\frac{r}{100+r}\times100\right)\% = \left[\frac{15}{100+15}\times100\right]\%$$

$$=\frac{1500}{115}\% = 13\frac{1}{23}\%$$

Shortcut Approach - 3

If the price of a commodity decreases by r%, then, increase in consumption, so as not to decrease expenditure on this item is $\left[\frac{r}{(100-r)}\times100\right]\%$,

Illustration – 16

If the price of rice falls down by 5%, by how much per cent must a householder increase its consumption, so as not to decrease expenditure in this item?

(a) 5% **(b) 20%** **(c) $5\frac{5}{19}\%$** **(d) $5\frac{3}{19}\%$**

Shortcut Solution : (c)

$$\text{Required percentage} = \left[\frac{r}{(100-r)}\times100\right]\%$$

$$=\left[\frac{5}{(100-5)}\times100\right]\% = \frac{500}{95} = 5\frac{5}{19}\%$$

Shortcut Approach - 3

If the original price of a commodity is ₹ x and new price of the commodity is ₹ y.

Case I: When original price > new price

Increase in consumption so as not to increase or decrease the expenditure.

$$=\left[\frac{x-y}{x}\times100\right]\%$$

$$=\left(\frac{\textit{Original Price}-\textit{New Price}}{\textit{Original Price}}\times100\right)\%$$

Case II: When new price > original price

Decrease in consumption so as not to increase or decrease the expenditure

$$=\left[\frac{y-x}{y}\times100\right]\%$$

$$=\left(\frac{\textit{New Price}-\textit{Original Price}}{\textit{New Price}}\times100\right)\%$$

GP_3386

Illustration – 17

Sugar is now being sold at ₹ 18 per kg. During last month its cost was ₹ 25 per kg. Find by how much per cent a family should increase its consumption, so as to keep the expenditure the same

(a) 30 % (b) 29 % (c) 28 % (d) 25 %

Shortcut Solution : (c)

$$\% \text{ Increase in expenditure} = \left[\frac{x-y}{x}\right]\times 100$$

$$= \frac{25-18}{25}\times 100\% = 28\%$$

Shortcut Approach - 3

If the price of a commodity is changed by x% and its consumption changed by y%, then

$$\frac{\text{New expenditure}}{\text{Initial expenditure}} = \frac{(100-x)(100+y)}{(100)^2}$$

Put x as $(+x)$ and y as $(+y)$ in the case of 'increase' and x as $(-x)$ and y as $(-y)$ in the case of 'decrease'.

Illustration – 18

The price of wheat is decreased by 25% and its consumption increases by 25%. Find the ratio of new expenditure to initial expenditure.

(a) 3 : 4 (b) 5 : 4

(c) 16 : 15 (d) 15 : 16

Shortcut Solution : (d)

Ratio of new expenditure to initial expenditure

$$= \frac{(100-x)(100+y)}{(100)^2} = \frac{(100-25)(100+25)}{(100)^2}$$

$$= \frac{9375}{10000} = \frac{15}{16}$$

Shortcut Approach - 3

If the price of an item is increased by x% and a housewife reduced the consumption of that item by x%, then her expenditure on that item decreases by

$$\left(\frac{x}{10}\right)^2 \%.$$

Illustration – 19

The price of groundnut oil is increased by 30% and a housewife reduced her consumption of groundnut oil by 30% and hence her expenditure on groundnut oil,

(a) remains unchanged (b) increases by 9%

(c) decreases by 6% (d) decreases by 9%

Shortcut Solution : (d)

$$\text{Expenditure on groundnut oil} = \left(\frac{x}{10}\right)^2 \% = \left(\frac{30}{10}\right)^2 \% = 9\%$$

Hence, expenditure on ground nut oil decreases by 9%.

Shortcut Approach - 3

If a number is successively increased by x%, y%, and z%, then single equivalent increase in that number will be

$$\left[(x+y+z)+\left(\frac{xy+yz+zx}{100}\right)+\frac{xyz}{100^2}\right]\%.$$

Illustration – 20

Find a single equivalent increase, if a number is successively increased by 20%, 25% and 30%.

(a) 90% (b) 75% (c) 95% (d) 85%

Shortcut Solution : (c) A single equivalent

$$\text{increase} = \left[(x+y+z)+\left(\frac{xy+yz+zx}{100}\right)+\frac{xyz}{(100)^2}\right]\%$$

$$= \left[(20+25+30)+\left(\frac{20\times 25+25\times 30+30\times 20}{100}\right)+\left(\frac{20\times 25\times 30}{(100)^2}\right)\right]\%$$

$$= (75+18.5+1.5)\% = 95\%$$

Shortcut Approach - 3

If three successive discounts of x%, y% and z% are allowed on an amount then a single discount that equivalent to the three successive discounts will be

$$\left[x+y+z-\frac{xy+yz+zx}{100}+\frac{xyz}{100^2}\right]\%.$$

or

Finding single discount rate equivalent to the series of two discounts

= 1st discount + 2nd discount

$$- \frac{\text{1st discount} \times \text{2nd discount}}{100}$$

Illustration – 21

Find a single discount equivalent to a discount series of 10%, 20% and 25%.

(a) 46% (b) 56% (c) 55% (d) 45%

GP_3386

Shortcut Solution : (a)

Required equivalent single discount

$$=\left[(x+y+z)-\left[\frac{xy+yz+zx}{100}\right]+\frac{xyz}{(100)^2}\right]\%$$

$$=\left[(20+25+30)-\left[\frac{20\times25+25\times30+30\times20}{100}\right]+\frac{20\times25\times30}{(100)^2}\right]\%$$

$=55-9.5+0.5=46\%$

Shortcut Approach - 3

If the value of a number is first increased by x% and later decreased by x%, then net change is always a decrease which is equal to x% of x or $\frac{x^2}{100}$.

Illustration – 22

The salary of a worker is first increased by 5% and thereafter it was reduced by 5%. What was the change in his salary?

(a) Increase in his salary, increase % is 0.25
(b) Decrease in his salary, decrease % is 0.25
(c) Increase in his salary, increase % is 4%
(d) Decrease in his salary, decrease % is 0.5

Shortcut Solution : (b)

Change in his salary

$$=\left(\frac{x^2}{100}\right)=\left(\frac{(5)^2}{100}\right)=\frac{25}{100}=0.25\%$$

Hence decrease in his salary, decrease by 0.25%.

Shortcut Approach - 10

If the value is increased successively by x% and y% then the final increase is given by $\left[x+y+\frac{xy}{100}\right]$%,

Illustration – 23

A shopkeeper marks the prices at 20% higher than the original price. Due to increase in demand, he further increases the price by 15%. How much % profit will he get?

(a) 38% **(b) 40%** **(c) 38.75%** **(d) 35%**

Shortcut Solution : (a)

Actual profit %

$$=\left[x+y+\frac{xy}{100}\right]\%=\left[20+15+\frac{20\times15}{100}\right]\%$$

$=[35+3]=38\%$

Shortcut Approach - 11

If the value is decreased successively by x% and y% then the final decrease is given by $\left[x+y-\frac{xy}{100}\right]$%,

Illustration – 24

The population of a town is decreased by 8% and 5% in two successive years. What per cent population is decreased after two years?

(a) 13% **(b) 12.6%** **(c) 12.5%** **(d) 13%**

Shortcut Solution : (b)

Population is decreased by

$$=\left[x+y-\frac{xy}{100}\right]\%=\left[8+5-\frac{8\times5}{100}\right]\%$$

$=[13-0.40]=12.6\%$

Shortcut Approach - 12

If the value is first increased by x% and then decreased by y%, then there is $\left(x-y-\frac{xy}{100}\right)$% increase or decrease, according to the +ve or –ve sign respectively.

Illustration – 25

The salary of a worker was first increased by 10% and thereafter, decreased by 15%. What was the change in his salary?

(a) increase, 6.5% **(b) decrease, 6.5%**
(c) increase, 5.5% **(d) decrease, 5.5%**

Shortcut Solution : (b)

Change in his salary

$$=\left[x-y-\frac{xy}{100}\right]\%=\left[10-15-\frac{10\times15}{100}\right]$$

$=-5-1.5=-6.5\%$

So, negative sign shows the decrease in salary by 6.5%

Shortcut Approach - 12

A man spends x% of his income. His income is increased by y% and his expenditure also increases by z%, then the percentage increase or decrease in his savings is given by $\left[\frac{100y-xz}{100-x}\right]$%, according to the +ve or –ve sign.

Illustration – 26

A man spends 50% of his income. His income increases by 30% and his expenditure also increases by 20%. Find the percentage increase in his savings.

(a) 25% **(b) 50%** **(c) 60%** **(d) 40%**

Shortcut Solution : (d)

$$\text{Percentage} = \left[\frac{100y - xz}{100 - x}\right]\%$$

$$= \left[\frac{100\times 30 - 50\times 20}{100 - 50}\right]\% = 40\%$$

Shortcut Approach - 12

The pass marks in an examination is x%. If a candidate who secures y marks fails by z marks, then the maximum marks, is given by $\left[\frac{100(y+z)}{x}\right]$.

Illustration – 27

A student has to secure 15% marks to get through. If he gets 80 marks and fails by 70 marks, find the maximum marks set for the examination.

(a) 100 (b) 1000 (c) 1500 (d) 900

Shortcut Solution : (b)

$$\text{Maximum marks} = \left[\frac{100(y+z)}{x}\right]$$

$$= \left[\frac{100(80+70)}{15}\right] = \frac{100\times 150}{15} = 1000.$$

Shortcut Approach - 10

A candidate scoring x% in an examination fails by 'a' marks, while another candidate who scores y% marks gets 'b' marks more than the minimum required pass marks. Then the maximum marks for that examination are $M = \frac{100(a+b)}{y-x}$

Illustration – 28

A candidate scores 26% and fails by 49 marks, while another candidate who scores 36% marks, gets 36 marks more than the minimum required marks to pass the examination. Find the maximum marks for the examination.

(a) 850 (b) 750 (c) 600 (d) 800

Shortcut Solution : (a)

$$\text{Maximum marks} = \left[\frac{100(a+b)}{y-x}\right]$$

$$= \left[\frac{100\times(49+36)}{36-26}\right] = \frac{100\times 85}{10} = 850$$

Shortcut Approach - 10

In an examination x% failed in English and y% failed in Maths. If z% of students failed in both the subjects, the percentage of students who passed in both the subjects is $100 - (x + y - z)$

Also, percentage of students who failed in either subject = $(x + y - z)$%

Illustration – 29

In an examination, 33% of the students failed in Maths, 24% failed in English and 17% failed in both. Find the percentage of students who passed in both the subject.

(a) 55% (b) 60% (c) 65% (d) 70%

Shortcut Solution : (b)

Percentage of student who passed in both subjects = 100 – (x + y – z) = (100 – (33 + 24 – 17) = 60%

Shortcut Approach - 3

(i) If the one factor is decreased by x% and the other factor is increased by y%, then effect on the product = $y - x - \frac{yx}{100}$

(ii) or, if the one factor is increased by x% and the other factor is decreased by y% then the effect on the product = $x - y - \frac{xy}{100}$ the value is increased or decreased according to the +ve or –ve sign obtained.

Illustration – 30

If the price is increased by 5% and the sale is decreased by 16%, then what will be effect on income?

(a) income increase by 11%

(b) income decrease by 10.2%

(c) income decreases by 11.8%

(d) income increases by 11.2%

Shortcut Solution : (c)

$$\text{Effect on income} = \left[x - y - \frac{yx}{100}\right]$$

$$= \left[5 - 16 - \frac{5\times 16}{100}\right] = -11.8\%$$

Here, negative sign shows the decrease in income by 11.8%

GP_3386

Shortcut Approach - 18

If one factor is decreased by x% and the other factor also decreases by y%, then the effect on the product is given by $\left[x+y-\frac{xy}{100}\right]$% decrease.

Illustration – 31

The land holding of a person is decreased by 15% due to late monsoon, the production decreases by 12%. Then what is the effect on the revenue?

(a) 27% (b) 12.18% (c) 26.8% (d) 25.2%

Shortcut Solution : (d)

Effect on the revenue $=\left[x+y-\frac{xy}{100}\right]$

$=\left[15+12-\frac{15\times12}{100}\right]=25.2\%$

Shortcut Approach - 12

Theorem : If one factor is increased by x% and the other increases by y% then the effect on the product is given by $\left[x+y+\frac{xy}{100}\right]$% increase.

Illustration – 32

The number of seats in a cinema hall is increased by 25%. The price on a ticket is also increased by 20%. What is the effect on the revenue collected?

(a) 45 % increase (b) 49% increase
(c) 50% increase (d) 52% increase

Shortcut Solution : (c)

Effect on the revenue

$=\left[x+y+\frac{xy}{100}\right]=\left[25+20+\frac{25\times20}{100}\right]=50\%$

Shortcut Approach - 10

If one of the sides of a rectangle is increased by x% and the other is decreased by y%, then the increase or decrease per cent in area is given by $\left(x-y-\frac{xy}{100}\right)$%, according to the +ve or –ve sign.

Illustration – 33

If the length is increased by 20% and the breadth is decreased by 25%, then what will be the effect on the area?

(a) 10% decrease (b) 12% decrease
(c) 12% increase (d) 10% increase

Shortcut Solution : (a) Effect on the area

$=\left[x-y-\frac{xy}{100}\right]=\left[20-25-\frac{20\times25}{100}\right]=-5-5=-10\%$

Here, negative sign shows the decrease in area by 10%

Shortcut Approach - 10

If one of the sides of a rectangle is increased by x% and the other is increased by y% then the per cent value by which area changes is given by $\left[x+y+\frac{xy}{100}\right]$% increase.

Illustration – 34

If one of the sides of a rectangle is increased by 25% and the other is increased by 15%, find the per cent value by which the area changes.

(a) 43.75% (b) 40.75% (c) 40% (d) 43.25%

Shortcut Solution : (a)

Percentage value by which area changes

$=\left[x+y+\frac{xy}{100}\right]\%=\left[25+15+\frac{25\times15}{100}\right]$

$=[40+3.75]=43.75.$

Shortcut Approach - 10

If one of the sides of a rectangle is decreased by x% and the other is decreased by y% then the per cent value by which area changes is given by $\left[x+y-\frac{xy}{100}\right]$% decrease.

Illustration – 35

In measuring sides of a rectangle, one side is taken 10% in deficit and the other 5% in deficit. Find the percentage decrease in the area of the rectangle.

(a) 14.5% (b) 15%
(c) 15.5% (d) 14%

Shortcut Solution : (a)

Percentage value by which area changes

$=\left[x+y-\frac{xy}{100}\right]=\left[10+5-\frac{10\times5}{100}\right]$

$=15-0.5=14.5\%$

LEVEL - 1

1. 50 kg is what percent of 250 kg?
(a) 26% (b) 15% (c) 20% (d) 18%

2. The price of sugar is increased by 25%. If a family wants to keep its expenses on sugar unaltered, then the family will have to reduce the consumption of sugar by **[SSC CGL 2010]**
(a) 20% (b) 21% (c) 22% (d) 25%

3. If the price of sugar is increased by 7%, then by how much per cent should a housewife reduce her consumption of sugar, to have no extra expenditure?
(a) 7 over 107% (b) 107 over 100%
(c) 100 over 107% (d) 7%

4. When the price of a radio was reduced by 20%, its sale increased by 80%. What was the net effect on the sale?
(a) 44% increase (b) 44% decrease
(c) 66% increase (d) 75% increase

5. *A* invested 10% more than *B*. *B* invested 10% less than *C*. If the total sum of their investment is ₹ 14450. how much did *C* get?
(a) ₹ 5000 (b) ₹ 4800
(c) ₹ 5100 (d) None of these

6. If the price of tea falls down by 6% by how much percent must a householder increase its consumption, so as not to decrease expenditure?
(a) $5\frac{16}{47}\%$ (b) $4\frac{18}{67}\%$ (c) $6\frac{18}{47}\%$ (d) $6\frac{17}{47}\%$

7. The price of consumer goods increased by 50% and its consumption decreased by 25%. Find the ratio of new expenditure to initial expenditure.
(a) 8 : 9 (b) 9 : 8 (c) 7 : 8 (d) 11 : 8

8. A batsman scored 110 runs which included 3 boundaries and 8 sixes. What percent of his total score did he make by running between the wickets ?
(a) 45% (b) $45\frac{5}{11}\%$ (c) $54\frac{6}{11}\%$ (d) 55%

9. How many litres of a 30% alcohol solution should be added to 40 litres of a 60% alcohol solution to prepare a 50% solution ?
(a) 30 (b) 20 (c) 24 (d) 32

10. A number is increased by 20% and then again by 20%. By what percent should the increased number be reduced so as to get back the original number ?
(a) $19\frac{11}{31}\%$ (b) $30\frac{5}{9}\%$ (c) 40% (d) 44%

11. Successive discount of 50% and 50% is equivalent to **[SSC CHSL 2013]**
(a) 100% (b) 75% (c) 50% (d) 25%

12. A salesman is allowed $5\frac{1}{2}\%$ discount on the total sales made by him plus a bonus of $\frac{1}{2}\%$ on the sales over ₹ 10,000. If his total earnings were ₹ 1990, then his total sales (in ₹) were:
(a) 30,000 (b) 32,000 (c) 34,000 (d) 35,000

13. Wheat is now being sold at ₹ 27 per kg. During last month its cost was ₹ 24 per kg. Find by how much per cent a family reduces its consumption so as to keep the expenditure fixed.
(a) 10.2% (b) 12.1% (c) 12.3% (d) 11.1%

14. A number is increased by 20% and then it is decreased by 10%. Find the net increase or decrease per cent. **[SSC CGL Tier - I 2012]**
(a) 10 % increase (b) 10% decrease
(c) 8% increase (d) 8% decrease

15. If price of a book is first decreased by 25% and then increased by 20%, the net change in the price of the book will be : **[SSC CGL Tier- I 2013]**
(a) 10 % decrease (b) 5% decrease
(c) No change (d) 5% increase

16. A scooter costs ₹ 25, 000 when it is brand new. At the end of each year, its value is only 80% of what it was at the beginning of the year. What is the value of the scooter at the end of 3 years?
(a) ₹ 10,000 (b) ₹ 12,500
(c) ₹ 12,800 (d) ₹ 12,000

17. The single discount which is equivalent to successive discount of 20%, 15% and 10% is.
(a) 32.7% (b) 34.2% (c) 36.2% (d) 38.8%

GP_3386

18. The length of a rectangular plot is increased by 25%. To keep its area unchanged, the width of the plot should be :
(a) kept unchanged (b) increased by 25%
(c) increased by 20% (d) reduced by 20%

19. p is six times as large as q. The percent that q is less than p, is :
(a) $16\frac{2}{3}$ (b) 60 (c) $83\frac{1}{3}$ (d) 90

20. In an examination a candidate must secure 40% marks to pass. A candidate who gets 220 marks, fails by 20 marks. What are the maximum marks for the examination? **[SSC CGL Tier I 2012]**
(a) 1200 (b) 800 (c) 600 (d) 450

21. In an examination, 60% of the candidates passed in English, 70% in Maths and 40% in both subjects. How many students failed in both subjects?
(a) 10% (b) 20% (c) 30% (d) 40%

22. If the length is increased by 20% and the breadth is decreased by 25%, then what will be the effect on the area?
(a) 10% decrease (b) 10% increase
(c) 20% increase (d) 20% decrease

23. The sum of two numbers is $\frac{28}{25}$ of the first number. The second number is what percent of the first?
(a) 12% (b) 14% (c) 16% (d) 18%

24. $A = 10\%$ of x, $B = 10\%$ of y, $C = 10\%$ of $x + 10\%$ of y. On the basis of the above equalities, what is true in the following?
(a) A is equal to B
(b) A is greater than B
(c) B is greater than A
(d) Relation cannot be established between A and B

25. If 90% of A = 30% of B and B = $2x$% of A, then the value of x.
(a) 450 (b) 400 (c) 220 (d) 150

26. If A's salary is 25% higher than B's salary, then how much per cent is B's salary lower than A's ?
(a) $16\frac{1}{3}\%$ (b) 20% (c) 25% (d) $33\frac{1}{3}\%$

27. In measuring the side of a square, an error of 5% in excess is made. The error % in the calculated area is,
(a) $10\frac{1}{4}\%$ (b) $10\frac{3}{4}\%$ (c) $1\frac{3}{4}\%$ (d) 25%

28. Ravi's salary is 150% of Amit's salary. Amit's salary is 80% of Ram's salary. What is the ratio of Ram's salary to Ravi's salary ?
(a) 1 to 2 (b) 2 to 3 (c) 5 to 6 (d) 6 to 5

29. In a housing society, 30 per cent of the residents are men over the age of 18 and 40 per cent are women over the age of 18. If there are 24 children living in the housing society, then how many total residents live ?
(a) 32 (b) 80 (c) 94 (d) 112

30. A number is increased by 10% and then reduced by 10%. After these operations, the number :
(a) does not change (b) decreases by 1%
(c) increases by 1% (d) increases by 0.1%

31. Which number will fit in place of ?
$\frac{3}{5}$ of $1450 + ? = 200\%$ of 450.
(a) 40 (b) 30
(c) 28 (d) none of these

32. A number is increased by 20% and then again by 20%. By what per cent should the increased number be reduced so as to get back the original number? **[SSC CGL Tier - I 2015]**
(a) $30\frac{5}{9}\%$ (b) $19\frac{11}{13}\%$ (c) 40% (d) 44%

33. Two-third of one-seventh of a number is 87.5% of 240. What is the number ?
(a) 2670 (b) 2450 (c) 2205 (d) 1470

34. A positive number is by mistake divided by 6 instead of being multiplied by 6. What is the % error on the basis of correct answer?
(a) 3 (b) 97 (c) 17 (d) 83

35. If 10 % of an electricity bill is deducted, ₹ 45 is still to be paid. How much was the bill?
(a) ₹ 50 (b) ₹ 60
(c) ₹ 55 (d) None of these

36. A store raised the price of an item by exactly 10 per cent. Which of the following could not be the resulting price of the item ?
(a) ₹ 5.50 (b) ₹ 7.60 (c) ₹ 11.00 (d) ₹ 12.10

37. An inspector rejects 0.08% of the metres as defective. How many metres will he examine to reject 2 metres?

(a) 200m (b) 250m (c) 2500m (d) 3000m

38. 10 litres of water is added to 50 litres of a solution containing 20% of alcohol in water. What is the concentration of alcohol in the solution now?

(a) 20% (b) $16\frac{2}{3}\%$ (c) $12\frac{1}{2}\%$ (d) $33\frac{1}{3}\%$

39. After spending 20% on clothes, 10% on books, 9% on purchasing gift for girl friend and 7% on others, Chandra has a balance of ₹ 2,700. How much money was there with him initially?

(a) ₹ 5,000 (b) ₹ 6,000

(c) ₹ 7,000 (d) None of these

40. In a maternity centre, 5% of all the childbirth cases result in twins. What is the approximate percentage of twins out of total children born?

(a) 5% (b) 7.6% (c) 9.5% (d) 10.4%

41. What is the percentage change in the result when we add 50 to a certain number x, instead of subtracting 50 from the same number x ?

(a) 50% (b) 100%

(c) 300% (d) Can't be determined

42. In a class, the no. of boys is more than the no. of girls by 12% of the total strength. The ratio of boys to girls is :

(a) 15 : 11 (b) 11 : 14 (c) 14 : 11 (d) 8 : 11

43. 220% of a number 'X' is 44. What is 44% of 'X' ?

(a) 88 (b) 8.8

(c) 66 (d) Data insufficient

44. In an examination a candidate got 30% marks and failed by 30 marks. If the passing marks are 60% of the total marks, then the maximum marks will be :

(a) 450 (b) 600 (c) 300 (d) 100

45. The population of a village is 5000 and it increases at the rate of 2% every year. After 2 years, the population will be :

(a) 5116 (b) 5202 (c) 5200 (d) 5204

LEVEL - 2

1. In an examination, 5% of the applicants were found ineligible and 85% of the eligible candidates belonged to the general category. If 4275 eligible candidates belonged to other categories, then how many candidates applied for the examination ?

(a) 30, 000 (b) 35, 000

(c) 37, 000 (d) None of these

2. A salesman's terms were changed from a flat commission of 5% on all his sales to a fixed salary of ₹ 1,000 plus 2.5% commission on all sales exceeding ₹ 4,000. If his remuneration as per the new scheme was ₹ 600 more than by the first scheme, what were his sales worth?

(a) ₹ 10,000 (b) ₹ 11,000

(c) ₹ 12,000 (d) ₹ 14,000

3. A sum of ₹ 4558 is divided among A, B and C such that A receives 20% more than C, and C receives 25% less than B. What is A's share in the amount ?

(a) ₹ 1548 (b) ₹ 1720 (c) ₹ 1290 (d) ₹ 1345

4. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's saving to B's saving.

(a) 5 : 1 (b) 1 : 5 (c) 3.5 : 1 (d) 2 : 7

5. A city had a population of 30,00,000 in the beginning of 1999. Its average growth rate is 4% per year, but due to a massive earthquake in 2001, its population is reduced by 8% in that year. But it again maintained the same growth rate of 4% in following years. What will be the approx. population of the city at the end of 2003?

(a) 32,06,460 (b) 34,68,420

(c) 31,52,360 (d) 32,28,810

6. A spider climbed $62\frac{1}{2}\%$ of the height of the pole in one hour and in the next hour it

covered $12\frac{1}{2}$ % of the remaining height. If the height of the pole is 192 m, then distance climbed in second hour is:

(a) 3 m (b) 5 m (c) 7 m (d) 9 m

7. A man buys a house for ₹ 100000 and rents it. He puts 12.5% of each month's rent aside for upkeep & repairs, pays ₹ 325 per year as taxes and realizes 5.5% annually on his investment. Find the monthly rent.

(a) 550 (b) 554.76 (c) 654.76 (d) 1620.45

8. In some quantity of ghee, 60% is pure ghee and 40% is vanaspati. If 10 kg of pure ghee is added, then the strength of vanaspati ghee becomes 20%. The original quantity was :

(a) 10 kg (b) 15 kg (c) 20 kg (d) 25 kg

9. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's savings to B's savings.

(a) 5 : 1 (b) 1 : 5 (c) 3.5 : 1 (d) 2 : 7

10. The price of cooking oil is increased by 25%. By what percent a family should reduce the consumption of cooking oil so as not to increase the expenditure on this account?

[SSC CGL Tier I-2012]

(a) 20 (b) 25 (c) 50 (d) 40

11. The price of commodity rises from ₹6 per kg to ₹7.50 per kg. If the expenditure cannot increase, then percentage of reduction in consumption is :

[SSC CGL Tier - I 2016]

(a) 15 (b) 20 (c) 25 (d) 30

12. A shopkeeper marks the price of his good 12% higher than its original price. After that he allows a discount of 12%. What is his percentage profit or loss?

(a) 1.44% (b) 1.14% (c) 14.4% (d) 4.44%

13. The digit at unit place of a two-digit number is increased by 100% and the digit at ten places of the same number is increased by 50%. The new number thus formed is 19 more than the original number. What is the original number?

(a) 22 (b) 63

(c) 24 (d) None of these

14. The price of oil is increased by 25%. If the expenditure is not allowed to increase, the ratio between the reduction in consumption and the original consumption is

(a) 1 : 3 (b) 1 : 4 (c) 1 : 5 (d) 1 : 6

15. By reduction of 20% in the price of oranges, one can purchase 5 oranges more for ₹ 2.50. Find the reduced price of the oranges per dozen. Find also the original price.

(a) 120 paise, 140 paise (b) ₹ 0.8, ₹ 1.5

(c) ₹ 1.0, ₹ 1.5 (d) ₹ 1.2., ₹ 1.5

16. A speaks truth in 75% and B in 80% cases. In what percentage of cases are they likely to contradict each other when narrating the same incident?

(a) 35% (b) 30% (c) 25% (d) 20%

17. Which of the following successive discounts is better to a customer? **SSC CGL Tier -I 2013]**

(i) 20%, 15%, 10% (ii) 25%, 12%, 8%

(a) (i) is better (b) (ii) is better

(c) Both are same (d) None of these

18. The population of a city is decreased by 20% in a year. In the next year, the population is again decreased by 30%. Find per cent decrease in population of the city from the original population.

(a) 44% (b) 50% (c) 55% (d) 56%

19. The number of seats in a cinema hall is increased by 25%. The cost of a ticket is also increased by 10%. The overall percentage increase in the revenue is : **[SSC CGL Tier - I 2015, 2012]**

(a) 10.5% (b) 27.5% (c) 37.5% (d) 40.5%

20. A man spends 75% of his income. His income is inereased by 20% and he inereases his expenditure by 10%. His savings are increased by :

[SSC CGL Tier- I 2014]

(a) 10% (b) 25% (c) 37% (d) 50%

21. The monthly income of a person was ₹13500 and his monthly expenditure was ₹9000. Next year his income increased by 14% and his expenditure increased by 7%. The percentage of increase in his savings is :

[SSC CGL Tier -II 2016]

(a) 7% (b) 21% (c) 28% (d) 35%

22. Of the 1000 inhabitants of a town, 60% are males of whom 20% are literate. If, of all the inhabitants, 25% are literate, then what percent of the females of the town are literate ?

(a) 22.5% (b) 27.5% (c) 32.5% (d) 37.5%

23. In a market survey, 20% opted for product *A* whereas 60% opted for product *B*. The remaining individuals were not certain. If the difference between those who opted for product *B* and those who were uncertain was 720, how many individuals were covered in the survey ?

(a) 1440 (b) 1800

(c) 3600 (d) Data inadequate

24. Due to an increase of 30% in the price of eggs, 3 eggs less are availabe for ₹ 7.80. The present rate of eggs per dozen is :

(a) ₹8.64 (b) ₹8.88 (c) ₹9.36 (d) ₹10.40

25. A screw driver and a hammer currently have the same price. If the price of a screw driver rises by 5% and the price of hammer goes up by 3%, then how much more will it cost to buy 3 screw drivers and 3 hammers ?

(a) 3% (b) 4% (c) 5% (d) 8%

26. In an examination, 65% students passed in Civics and 60% in Histroy, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students?

(a) 600 (b) 650 (c) 700 (d) 750

27. In a certain school, 20% of students are below 8 years of age. The number of students above 8 years of age is $\frac{2}{3}$ of the number of students of 8 years age which is 48. What is the total number of students in the school?

(a) 72 (b) 80

(c) 120 (d) None of these

28. Selling price of a commodity is increased by 20%. As a result of it, quantity sold is decreased by 30%. What is effect of this on total sales?

(a) 16% decrease (b) 10 % increase

(c) 10% decrease (d) 50% increase

29. An empty fuel tank of a car was filled with A type petrol. When the tank was half-empty, it was filled with B type petrol. Again when the tank was half-empty, it was filled with A type petrol. When the tank was half-empty again, it was filled with B type petrol. What is the percentage of A type petrol at present in the tank?

(a) 33.5% (b) 37.5% (c) 40% (d) 50%

30. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants.

(a) 5000 (b) 6000 (c) 7000 (d) 8000

31. A candidate secured 30% marks in an examination and failed by 6 marks. Another secured 40% marks and got 6 marks more than the base minimum to pass. The maximum marks are : **[SSC CHSL 2013]**

(a) 150 (b) 120 (c) 100 (d) 180

32. In an examination, 80% candidates passed in English and 85% candidates passed in Maths. If 73% candidates passed in both these subjects, what percent of students failed in both the subjects : **[SSC CGL Tier - 1 2014]**

(a) 8% (b) 19% (c) 27% (d) 35%

33. If the sides of a square are increased by 30%, find the per cent increase in its area.

(a) 70% (b) 68% (c) 69% (d) 71%

34. A student X passes his examination with 515 marks, having scored 3% above the minimum.If Y had obtained 710 marks, what % would have been above the minimum?

(a) 40% (b) 42%

(c) 50% (d) Cannot be determined

GP_3386

35. In a shipment of 120 machine parts, 5 per cent were defective. In an another shipment of 80 machine parts, 10 per cent were also defective. For the two shipments combined, what per cent of the machine parts were defective ?

(a) 6.5% (b) 7.0% (c) 7.5% (d) 8.0%

36. The ratio of salary of a worker in July to that in June was $2\frac{1}{2} : 2\frac{1}{4}$, by what % the salary of July more than salary of June. Also find by what %, salary of June was less than that of July.

(a) $11\frac{1}{9}\%$ and 10% (b) 10% and $11\frac{1}{9}\%$

(c) Both 10% (d) Both $11\frac{1}{9}\%$

37. In an examination 80% of student passed in English 85% in mathematics and 75% in both English and mathematics. If 40 student failed in both the subject find total number of students.

(a) 350 (b) 400 (c) 450 (d) 600

38. Anthony got 30% of the maximum marks in an examination and failed by 10 marks. However, Amar who took the same examination, got 40% of the total marks and got 15 more than the passing marks in the examination. What were the passing marks in the examination ?

(a) 35 (b) 250 (c) 75 (d) 85

39. In an examination 35% of total student failed in Hindi 45% failed in English and 20% in both. Find the percentage of those who passed in both the subjects.

(a) 40% (b) 60% (c) 50% (d) 30%

40. Find the least number of integral year in which a sum of money invested at 20% compound interest per annum will become more than double itself.

(a) 1 (b) 4 (c) 2 (d) 3

41. An amount is lent at x% p.a. simple interest for two years. Instead, had it been lent at 2x% p.a. simple interest for y more years, then the interest would have been five times the earlier interest. Find the value of y.

(a) 2 (b) 3 (c) 4 (d) 5

42. If the numerator of a fraction is increased by 20% and its denominator decreased by 10% the fraction becomes 3/2. What is the value of the original fraction?

(a) $\frac{2}{3}$ (b) $\frac{7}{8}$ (c) $\frac{9}{8}$ (d) $\frac{6}{7}$

43. Everyday a fruit seller sells 50% of his stock. 10% of the stock gets spoiled during the night. If a total of 1983 fruits got spoiled in the three days period, how many fruits did he start with?

(a) 20,000 (b) 25,000
(c) 24,000 (d) none of these

44. A person has to pay 20% tax on first ₹ 4, 500 earned, for amount earned between ₹ 4, 500 and ₹ 7,500 the tax is 25% and for the amount earned more than ₹ 7,500, the tax is 30%. If he paid 23.33% of his income as tax, then what is his income?

(a) ₹ 9, 000 (b) ₹ 8, 500
(c) ₹ 10, 000 (d) ₹ 10, 500

45. In a ΔABC measurement of $\angle ABC = 15°$. What will be the value of $\angle ABC$ when seen through a magnifying glass which can magnify any object by 20%?

(a) Value reduce by 20%
(b) Same
(c) Value increased by 20%
(d) Value decreased by 16.66%

46. Due to a price hike of 20%, 4 kg less sugar are available for ₹ 120. What is the initial price per kg of sugar?

(a) ₹ 5/kg (b) ₹ 4/kg (c) ₹ 6/kg (d) ₹5.5/kg

47. Three friends P, Q and R donate 8%, 7% and 9% of their salary to a charitable hospital in the given order. Salary of P and Q is same and the difference of their donations is ₹ 74. The total donation by the P and Q is ₹ 525 more than R's donation. What is the percentage of the total salary of the three donations?

(a) 7.95% (b) 8.3%
(c) 6.34% (d) None of these

48. The entry fee in an exhibition was ₹ 10. Later this was reduced by 25%, which increased the sale of tickets by 20%. Find the percentage increase in the number of visitors.

(a) 54 (b) 57 (c) 60 (d) 66

49. There were only two contestants A and B in the recent assembly election in Bihar at Raghopur constituency. B got 50% of the votes that A got. Had A got 200 votes less, there would have been a tie. What is the total number of votes polled?
(a) 1,000 (b) 800
(c) 1,200 (d) cannot be determined

50. In the New York Stock Exchange there are 45% female employees and thus the number of male employees is exceeded by 72. Hence the total no. of employees in the New York Stock Exchange is :
(a) 540 (b) 720 (c) 7200 (d) 550

51. The monthly salary of Shahid and Kareena together is $ 28, 000. The salary of Shahid and Kareena is increased by 25% and 12.5% respectively then the new salary of Kareena becomes 120% of the new salary of Shahid. The new (or increased) salary of Shahid is :
(a) $ 15,000 (b) $ 18,000
(c) $ 14,000 (d) $ 16,000

52. The charges per hour of internet surfing is increased by 25% then find the percentage decrease in the time period of surfing of a user (a net savy) who can afford only a 10% increase in expenditure :
(a) 22% (b) 12% (c) 15% (d) 9.09%

53. A book consists of 30 pages, 25 lines on each page and 35 characters on each line. If this content is written in another notebook consisting of 30 lines and 28 characters per line, then the required no. of pages will how much per cent greater than the previous pages?
(a) 4.16% (b) 5%
(c) 6.66% (d) None of these

Hints and Solutions

Level 1

1. **(c)** Use Shortcut Approach-1

2. **(a)** Use Shortcut Approach-2

3. **(a)** % reduction in consumption

$$= \frac{\%\text{ change in price}}{100 + \%\text{ change in price}} \times 100$$

$$= \frac{7}{100+7}\% = \frac{7}{107}\%$$

4. **(a)** Let the original price be x and sale be of y units.
Then, the revenue collected initially $= x \times y$
Now, new price $= 0.8x$, new sale $= 1.8y$
Then, new revenue collected $= 1.44xy$

$$\%\text{ increase in revenue} = \frac{0.44xy}{xy} \times 100$$

$$= 44\%\text{ increase}$$

5. **(a)** Let the investment of C = ₹ 100
Then B's investment = ₹ 90 and A's investment = ₹ 99
Sum of investment = ₹ (100 + 90 + 99) = ₹ 289
Hence, C's actual investment

$$= ₹\left(\frac{14450 \times 100}{289}\right) = ₹\ 5000$$

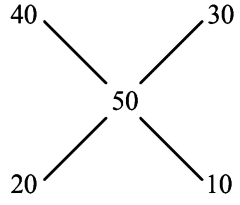
6. **(c)** Use Shortcut Approach-3

7. **(b)** Use Shortcut Approach-5

8. **(b)** Number of runs made by running
= 110 – (3 × 4 + 8 × 6) = 50.
∴ Required percentage

$$= \left(\frac{50}{110} \times 100\right)\% = 45\frac{5}{11}\%$$

9. **(b)** Required litres of solution



$$= \frac{40}{2} = 20 \text{ litres}$$

10. **(b)** Let original number = 100
New number = 120% of 120% of 100

$$= \left(\frac{120}{100} \times \frac{120}{100} \times 100\right) = 144.$$

Decrease on 144 = 44. Decrease on 100

$$= \left(\frac{44}{144} \times 100\right)\% = 30\frac{5}{9}\%$$

11. **(b)** Use Shortcut Approach-8

12. **(c)** Let the total sales be ₹ x. Then, $5\frac{1}{2}\%$ of

$$x + \frac{1}{2}\% \text{ of } (x - 10000) = 1990$$

$$\Leftrightarrow \frac{11}{2} \times \frac{1}{100} \times x + \frac{1}{2} \times \frac{1}{100} \times (x - 10000) = 1990$$

$$\Leftrightarrow 12x - 10000 = 398000 \Leftrightarrow 12x = 408000$$

$$\Leftrightarrow x = 34000$$

13. **(d)** Let the family consumes 1 kg wheat
To keep expenditure at ₹ 24, its new consumption should be $\frac{24}{27} = \frac{8}{9}$ kg

∴ Percentage decrease in consumption

$$= \frac{\left(1 - \frac{8}{9}\right)}{1} \times 100 = 11.1\%$$

Alternative method :

$$\text{Required \%} = \frac{27-24}{27} \times 100 = 11.1\%$$

14. **(c)** Use Shortcut Approach-12

15. **(a)** Use Shortcut Approach-12

16. **(c)** After first year, the value of the scooter = ₹ 20,000
After second year, the value of scooter = ₹ 16,000
After third year, the value of scooter = ₹ 12,800

17. **(d)** Single discount of successive discount 20% and 15%

$$= 20 + 15 - \frac{26 \times 15}{100} = 35 - 3 = 32$$

Now, single discount of successive discount 32% and 10%

$$= \quad 32 + 10 - \frac{32 \times 10}{100} = \quad 42 - 3.2 = 38.8\%$$

18. **(d)** Let the original length and breadth be both 10 cm each.
Then original area = 100 cm^2
New length = $10 \times 1.25 = 12.5$ cm
Let new breadth be x. Then, $12.5x = 100$

$$\Rightarrow \quad x = \frac{100}{12.5} = 8\text{cm}$$

Hence, % reduction in breadth

$$= \frac{2}{10} \times 100 = 20\%$$

19. **(c)** $p = 6q$. So, q is less than p by $5q$.
$\therefore$ Required percentage

$$= \left(\frac{5q}{p} \times 100 \right)\%$$

$$= \left(\frac{5q}{6q} \times 100 \right)\% = 83\frac{1}{3}\%$$

20. **(c)** Use Shortcut Approach-14
21. **(a)** Use Shortcut Approach-16
22. **(a)** Use Shortcut Approach-20
23. **(a)** Let the numbers be x and y. Then,

$$x + y = \frac{28}{25}x \Rightarrow y = \frac{28}{25}x - x \Rightarrow y = \frac{3}{25}x$$

$$\Rightarrow \frac{y}{x} = \left(\frac{3}{25} \times 100 \right)\% = 12\%.$$

24. **(d)** The given information gives no indication regarding the comparison of x and y.

25. **(d)** $\frac{A \times 90}{100} = \frac{B \times 30}{100}$

$$\Rightarrow \quad 3A = B$$

$$\Rightarrow \quad 3A = A \times \frac{2x}{100}$$

$$\Rightarrow \quad x = 150$$

26. **(b)** Let B's salary be ₹ 100,
then A's salary = ₹ 125

$$\% \text{ lesser} = \frac{125 - 100}{125} \times 100 = \frac{25}{125} \times 100$$

$$= \frac{1}{5} \times 100 = 20\%$$

27. **(a)** If side is increased by a%, area increased by $\left(2a + \frac{a^2}{100} \right)\% = 2 \times 5 + \frac{5^2}{100} = 10\frac{1}{4}\%$

28. **(c)** Let the salary of Ram be ₹ 100.
Then, salary of Amit = ₹ 80
and salary of Ravi = ₹ 120
Ratio of Ram's salary to Ravi's salary = 100 : 120 = 5 : 6

29. **(b)** 30% of the residents are children.
$\therefore$ 30% of the total residents = 24
$\therefore$ Total number of residents in the society

$$= \frac{24}{30} \times 100 = 80$$

30. **(b)** Let the original number be 100.
Then, the new number = $100 \times 1.1 \times 0.9 = 99$
i.e. the number decreases by 1%.

31. **(b)** $\frac{3}{5}$ of 1450 + ? = 200% of 450

$$\Rightarrow ? = \frac{200}{100} \times 450 - \frac{3}{5} \times 1450 = 900 - 870 = 30$$

32. **(a)** Use Shortcut Approach-10
33. **(c)** Let number be x then,

$$\frac{2}{3} \times \frac{1}{7} \times x = \frac{240 \times 87.5}{100}$$

or $\frac{2x}{21} = 87.5 \times 2.4$
or $x = 2205$

34. **(b)** Let the number be x. Then,
% error

$$= \frac{6x - x/6}{6x} \times 100 = \frac{35}{36} \times 100 = 97.2\%$$

35. **(a)** Let the bill be ₹ x. Then 90% of x = 45

$$\Rightarrow \quad x = \frac{45 \times 100}{90} = ₹ 50$$

36. **(b)** $5 + 10\% = 5.50$
$10 + 10\% = 11$
$11 + 10\% = 12.10$

37. **(c)** Let the inspector examined x metres,
then 0.08% of x = 2

$$\Rightarrow \quad \frac{x \times 0.08}{100} = 2 \text{ or } \quad x = \frac{200}{0.08} = 2500 \text{ metres}$$

38. **(b)** Alcohol in solution = 20% of $50l = 10l$
Concentration of alcohol in new solution

$= \frac{10}{60} \times 100 = 16\frac{2}{3}\%$

39. (a) Let initial amount = ₹ 100
Money left = 100 – (20 + 10 + 9 + 7)
= ₹ (100 – 46) = ₹ 54

Money he had $= \frac{100}{54} \times 2700 =$ ₹ 5000

40. (c) No. of twins cases out of 100 = 5
No. of twins out of 100 children = 10

Percentage of twins $= \frac{10}{105} \times 100$

$= \frac{200}{21} = 9.5\%$

41. (d) Use some different values for x then verify.
Let x = 150 then % error

$= \frac{200-100}{100} \times 100 = 100\%$

Again if x = 100 then % error
$= \frac{150-50}{50} \times 100 = 200\%$
Hence, we cannot determine.

42. (c)

Boys	Girls	Total
14	11	25
56	44	100

(12)

Since out of 100 students number of boys are greater than the numbers of girls by 12 i.e., 12%.
Alternatively:
Let the number of boy and girls be x and y respectively then
$(x - y)$

$= \frac{12 \times (x+y)}{100} \Rightarrow \frac{x}{y} = \frac{14}{11}$

43. (b) $\frac{x \times 220}{100} = 44$
$\Rightarrow x = 20$

So $\frac{20 \times 44}{100} = 8.8$

44. (d) Passing marks are 0.6x
So 0.3x + 30 = 0.6 x
$\Rightarrow$ x = 100

45. (b) Population after 2 years $= 5000\left(1 + \frac{2}{100}\right)^2$

$5000 \times \frac{51}{50} \times \frac{51}{50} = 5202$

Level-2

1. (a) Let the total number of applicants be x.
Number of eligible candidates = 95% of x.
Eligible candidates of other categories = 15% of (95% of x)

$= \left(\frac{15}{100} \times \frac{95}{100}\right) \times x = \frac{57}{400}x$

$\therefore \frac{57}{400}x = 4275 \Leftrightarrow x = \left(\frac{4275 \times 400}{57}\right) = 30000$

2. (c) Let his sales be worth ₹ x. Then,
$1000 + 2.5\%$ of $(x - 4000) = 5\%$ of $x + 600$

$\Rightarrow \frac{5x}{100} - \frac{2.5(x-4000)}{100} = 1000 - 600$
$\Rightarrow 2.5x + 10000 = 40{,}000$

$\Rightarrow x = \frac{30{,}000}{2.5} = 12{,}000$

3. (a) Let B get ₹ x. Then C gets = 75% of $x = \frac{3x}{4}$

and A gets = 120% of $\frac{3x}{4} = \frac{120}{100} \times \frac{3x}{4} = \frac{9x}{10}$

Now, $\frac{9x}{10} + \frac{3x}{4} + x = 4558$

$\Rightarrow \frac{53x}{20} = 4558 \Rightarrow x = \frac{4558 \times 20}{53} = 1720$

Hence, A's share $= \frac{9x}{10} =$ ₹ $\frac{9 \times 1720}{10} =$ ₹ 1548

4. (b) Let 100 units be B's income and X units be B's expenditure
$\Rightarrow$ A's income = 60 units.
A's expenditure = 70X/100 units.
But 60 = (75/100) X $\Rightarrow$ X = 80.
i.e., B's saving = (100 – 80) units = 20 units.

Hence A's saving $= 60 - \frac{70}{100} \times 80 = 4$ units.
i.e., A's saving : B's saving = 4 : 20 = 1 : 5.

5. (d) Population after 2000 = 3244800
Population after 2001 = 2985216
Population at the end of 2003 = 3228810

6. (d) Height climbed in second hour

$= 12\frac{1}{2}\%$ of $\left(100 - 62\frac{1}{2}\right)\%$ of 192 m

$= \left(\frac{25}{2} \times \frac{1}{100} \times \frac{75}{2} \times \frac{1}{100} \times 192\right)$ m = 9 m.

7. **(b)** We have 5.5% of 100000
= Rent – 12.5% of Rent – 325.

$$\Rightarrow \frac{5500}{12} = \text{Rent} - \frac{\text{Rent}}{8} - \frac{325}{12}$$

$$\Rightarrow \frac{5500}{12} + \frac{325}{12} = \frac{7}{8} \times \text{Rent}.$$

$$\Rightarrow \text{Rent} = \frac{5500}{12} \times \frac{8}{7} = ₹ 554.76 \text{ per month.}$$

8. **(a)** Let the original quantity be x kg. Vanaspati ghee in x kg $= \left(\frac{40}{100}x\right)$ kg $= \left(\frac{2x}{5}\right)$ kg.

Now, $\frac{\frac{2x}{5}}{x+10} = \frac{20}{100} \Leftrightarrow \frac{2x}{5x+50} = \frac{1}{5}$

$\Leftrightarrow 5x = 50 \Leftrightarrow x = 10$

9. **(b)** Let B's Income = ₹ x

A's Income = ₹ $\frac{3}{5}x$

And B's expenditure = ₹ y

A's expenditure = ₹ $\frac{7}{10}$y

Also, $\frac{3}{5}x = \frac{3}{4} \cdot \frac{7}{10}y$

$$\frac{\text{A's savings}}{\text{B's savings}} = \frac{x-y}{\frac{3}{5}x - \frac{7}{10}y} = \frac{\frac{7}{8}y - y}{\frac{3}{5} \cdot \frac{7}{8}y - \frac{7}{10}y}$$

$$= \frac{-y/8}{\frac{21y}{40} - \frac{7}{10}y} = \frac{5}{25} = 1:5$$

10. **(a)** Use Shortcut Approach-2

11. **(b)** Use Shortcut Approach-2

12. **(a)** Use Shortcut Approach-9
In this case, there will be loss

$$\therefore \text{Loss \%} = \frac{(12)^2}{100}\% = 1.44\%$$

13. **(d)**

	Original number	New number	Difference
(a)	22	34	12
(b)	63	96	33
(c)	24	38	14

Obviously, (d) is the correct option.

14. **(c)** Let original consumption be 1 unit costing ₹ 100
New cost = ₹125. New consumption

$$= \left(\frac{1}{125} \times 100\right) = \frac{4}{5} \text{ unit.}$$

$$\therefore \frac{\text{Reduction in consumption}}{\text{Original consumption}} = \frac{\left(1 - \frac{4}{5}\right)}{1} = \frac{1}{5}$$

i.e., 1 : 5.

15. **(d)** Let original price be ₹ x per orange. Then,
Reduced rate = $(1 - 0.2)x$ = ₹ $0.8x$

$$\therefore \frac{2.50}{0.8x} - \frac{2.50}{x} = 5$$

$$\Rightarrow \frac{25}{8x} - \frac{2.5}{x} = 5 \Rightarrow x = \frac{1}{8}$$

$\therefore$ Original price of oranges per dozen

$$= \frac{1}{8} \times 12 = ₹ 1.5$$

and Reduced price = ₹ (0.8)(1.5) = ₹ 1.2

16. **(a)** Let the truth spoken by A and B be p_1 and p_2 respectively, *i.e.*, $p_1 = \frac{3}{4}$ and $p_2 = \frac{4}{5}$

They will contradict each other only when one speaks truth and the other is lying.

i.e., $\frac{3}{4} \times \frac{1}{5} + \frac{4}{5} \times \frac{1}{4} = \frac{3}{20} + \frac{4}{20} = \frac{7}{20} = \frac{35}{100}$

i.e., 35%

17. **(b)** Use Shortcut Approach-8

18. **(a)** Use Shortcut Approach-11

19. **(c)** Use Shortcut Approach-19

20. **(d)** Use Shortcut Approach-13

21. **(c)** Use Shortcut Approach-13

22. **(c)** Number of males = 60% of 1000 = 600.
Number of females = (1000 – 600) = 400.
Number of literates = 25% of 1000 = 250.
Number of literate males = 20% of 600 = 120
Number of literate females = (250 – 120) = 130
$\therefore$ Required percentage

$$= \left(\frac{130}{400} \times 100\right)\% = 32.5\%$$

23. **(b)** Percentage of uncertain individuals
= [100 – (20 + 60)] % = 20%
$\therefore$ 60% of x – 20% of x = 720 $\Leftrightarrow$ 40% of x = 720

$$\Leftrightarrow \frac{40}{100}x = 720 \Leftrightarrow x = \left(\frac{720 \times 100}{40}\right)$$

= 1800.

GP_3386

24. (c) Let the original price per egg be ₹ x.

Then, increased price $= ₹\left(\frac{130}{100}x\right)$

$$\therefore \quad \frac{7.80}{x}-\frac{7.80}{\frac{130}{100}x}=3$$

$$\Leftrightarrow \quad \frac{7.80}{x}-\frac{780}{130x}=3$$

$$\Leftrightarrow \quad 1014-780=3\times 130x \Leftrightarrow 390x=234$$

$$\Leftrightarrow \quad x=0.6$$

So, present price per dozen

$$= ₹\left(12\times\frac{130}{100}\times 0.6\right) = ₹\ 9.36.$$

25. (b) Let the original price of a screw driver and a hammer be ₹ 100 each.

Then, price of 3 screw drivers and 3 hammers = ₹ 600

Now, after increase of 5%, the price of 3 screw drivers = ₹ 315

And after 3% increase the price of 3 hammers = ₹ 309

Increased price of 3 hammers and 3 screw drivers = ₹ 624

Therefore, % increase in price

$$=\frac{24}{600}\times 100=4\%$$

26. (a) Let the total number of students be x.

Number passed in one or both is given by :

$$n(A\cup B)=n(A)+n(B)-n(A\cap B)$$

$$= 65\%\text{ of }x+60\%\text{ of }x-40\%\text{ of }x$$

$$=\left(\frac{65}{100}x+\frac{60}{100}x-\frac{40}{100}x\right)=\frac{85}{100}x=\frac{17}{20}x.$$

$$\text{Failed in both} =\left(x-\frac{17}{20}x\right)=\frac{3x}{20}.$$

$$\therefore \frac{3x}{20}=90\Rightarrow x=\left(\frac{90\times 20}{3}\right)=600.$$

27. (d) Let the number of students be x. Then,

Total number of students of 8 years and above 8 years

$$=(100-20)\%\text{ of }x=80\%\text{ of }x.$$

$$\therefore\ 80\%\text{ of }x=48+2/3\text{ of }48\Rightarrow\frac{80}{100}x=80$$

$$\Rightarrow x=100.$$

28. (a) Use Shortcut Approach-17

29. (b) Let the capacity of the tank be 100 litres. Then,

Initially : A type petrol = 100 litres.

After first operation :

$$\text{A type petrol} =\left(\frac{100}{2}\right)=50\ \text{litres};$$

B type petrol = 50 litres.

After second operation :

$$\text{A type petrol} =\left(\frac{50}{2}+50\right)=75\ \text{litres};$$

B type petrol = (50/2) = 25 litres

After third operation :

$$\text{A type petrol} =\left(\frac{75}{2}\right)=37.5\ \text{liters};$$

$$\text{B type petrol} =\left(\frac{25}{2}+50\right)=62.5\ \text{litres}.$$

$\therefore$ Required percentage = 37.5%.

30. (b) Let the total number of original inhabitants be x. Then, (100 – 25)% of (100 – 10)% of x = 4050

$$\Rightarrow\left(\frac{75}{100}\times\frac{90}{100}\times x\right)=4050\Rightarrow\frac{27}{40}x=4050$$

$$\Rightarrow x=\left(\frac{4050\times 40}{27}\right)=6000.$$

$\therefore$ Number of original inhabitants = 6000.

31. (b) Use Shortcut Approach-15

$$\text{Maximum marks} = =\frac{100(6+6)}{40-30}=120$$

32. (a) Use Shortcut Approach-16

Candidates failed in English = (100 – 80)% = 20 %

Candidates failed in Maths = (100 – 85) = 15%

Let x% of students failed in both subjects.

$\therefore 73=100-(20+15-x)$

$\Rightarrow 73=100-35+x\Rightarrow 8\%=x$

33. (c) Use Shortcut Approach-21

$$\%\text{ inerease in area} =\left(30+30+\frac{30\times 30}{100}\right)\%$$

$$=(60+9)\%=69\%$$

34. (b)

$$\because\ \frac{\text{Marks of y}}{\text{Marks of x}}=\frac{100+\%\text{ above minimum of y}}{100+\%\text{ above minimum of x}}$$

$$\Rightarrow\frac{710}{515}=\frac{100+y}{103}\Rightarrow 100+y=\frac{710\times 103}{515}=142$$

$$\Rightarrow y=42\%$$

35. (b) Total no. of machine parts in both the

shipments = (120 + 80) = 200
Total defective machine parts in both the shipments = 120 × 5% + 80 × 10% = 6 + 8 = 14

Therefore, required % = $\frac{14}{200} \times 100 = 7\%$

36. (a) Let the salary of July be ₹ $\frac{5}{2}$x

and the salary of June be ₹ $\frac{9}{4}$x.

Required percentages

$$= \frac{\frac{5}{2}x - \frac{9}{4}x}{\frac{9}{4}x} \times 100 \text{ and } \frac{\frac{5}{2}x - \frac{9}{4}x}{\frac{5}{2}x} \times 100$$

$$= \frac{100}{9}\% \text{ and } \frac{100}{10}\% = 11\frac{1}{9}\% \text{ and } 10\%$$

37. (b) Let total no. of student = x
Let A and B represent the sets of students who passed in English and mathematics respectively

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 80\% \text{ of } x + 85\% \text{ of } x - 75\% \text{ of } x$$
$$= \frac{80}{100}x + \frac{85}{100}x - \frac{75x}{100}$$
$$= \frac{90x}{100} = \frac{9x}{10}$$

$\therefore$ Students failed in both subjects

$$= x - \frac{9x}{10} = \frac{x}{10}$$

So, $\frac{x}{10} = 40 \Rightarrow x = 400$

38. (d) Let the maximum marks be x.
Then, x × 30% + 10 = x × 40% − 15
$\Rightarrow$ x × 10% = 25 or x = 250
Therefore, passing marks

$$= 250 \times \frac{30}{100} + 10 = 85$$

39. (a) Let A and B be the sets of student who failed in
Hindi and English respectively
Then $n(A) = 35$
$n(B) = 45$
$n(A \cap B) = 20$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 35 + 45 - 20 = 60$$

% of student failed in Hindi or English or both = 60%
% percentage passed = 100 − 60 = 40%

40. (b) $A = P\left(1 + \frac{20}{100}\right)^n = P\left(\frac{6}{5}\right)^n$

When n = 4, only then condition given is satisfied.

41. (b) $5\left(\frac{P \times x \times 2}{100}\right) = \frac{P \times 2x \times (y+2)}{100}$
y = 3

42. (c) Required fraction

$$= \frac{3 - \frac{1}{2}}{\frac{18+2}{9}} = \frac{\frac{5}{2}}{\frac{20}{9}} = \frac{5}{2} \cdot \frac{9}{20} = \frac{9}{8}$$

43. (c) The condition will be summarized as

Day	Initial Amount	Sales	Remaining over night	Rotten
1	x	0.5x	0.5x	0.05x
2	0.45x	0.225x	0.225x	0.2025x
3	0.2025x	0.10125x	0.10125x	0.010125x

Total rotten fruits = $0.05x + 0.0225x + 0.010125x$
$\Rightarrow 0.082625x = 1983$
$\Rightarrow x = 24000$

44. (a) Tax paid on first ₹4500 = ₹4500 × $\frac{20}{100}$ = ₹900
Now, tax paid on amount between ₹ 4500 and

₹ 7500 = ₹ 300 × $\frac{25}{100}$ = ₹ 750

Let income = x (> ₹ 7500)

Then, tax paid = ₹ $[900 + 750 + (x - 7500) \times \frac{30}{100}]$

So, $x \times \frac{23.33}{100} = 1650 + (x - 7500) \times \frac{3}{10}$

$\Rightarrow x \times \frac{70}{3 \times 100} = 1650 + (x - 7500) \times \frac{3}{10}$
$\Rightarrow x =$ ₹9000

45. (b) There will be no change in the measurement of angles. It's the size that will increase, and not the slope or angle.

GP_3386

46. (a) Price Quantity

$\uparrow \frac{1}{5}$ $\frac{1}{6} \downarrow$

Or, $\frac{1}{6}x = 4$

$x = 24$

Initially price per kg $= \frac{120}{24} = ₹5/kg$

47. (a) Salary of P and Q = x and salary of R = y then

$\frac{8x}{100} - \frac{7x}{100} = 74$ and $x = 7400$

then, donation by R = ₹ 585

then $\frac{9}{100}y = 585$

$y = \frac{585 \times 100}{9} = ₹\ 6500$

Required percentage $= \frac{1695}{21300} = 7.95\%$

48. (c) $10 \times 100 = 1000$, 100 = No. of visitors

Now, 7.5 × No. of visitors = 1200

No. of visitors = 160

49. (d) $x - 200 = \frac{x}{2} \Rightarrow x = 400$

A got 400 votes and B got 200 votes. But information is not given regarding invalid votes.

50. (b) Male Female

55x 45x

$\Rightarrow 10x = 72$

$x = 7.2$

$100x = 7.2 \times 100 = 720$

51. (a)

S K

$\begin{pmatrix} 15,000 & 18,000 \\ 12,000 & 16,000 \end{pmatrix}$ going in the reverse direction.

Please option (a) is correct

Alternatively :

Shahid Kareena

25% $\begin{pmatrix} 4x & \frac{48x}{9} \\ 5x & 6x \end{pmatrix}$

$\xrightarrow{+120\%}$

$\Rightarrow$ Initially $S : K = 4x : \frac{48x}{9} = 3 : 4$

$\Rightarrow$ Shahid's initial salary = $ 12, 000

Shahid's changed salary = $ 15000

52. (b) Time × Rate = Total charges

$1 \times 1 = 1$

$x \times 1.25 = 1.1$

$x = \frac{1.1}{1.25} \times 100 = 88\%$

Thus decrease in time = 12%

53. (c) $30 \times 25 \times 35 = x \times 30 \times 28$

$\Rightarrow x = 31.25$

It means 32 pages.

So, the percentages increase in the no. of pages $= \frac{2}{30} \times 100 = 6.66\%$

CHAPTER

11

Profit, Loss and Discount

More At @Aj_ebooks

PROFIT, LOSS AND DISCOUNT

Cost Price

The amount paid to purchase an article or the price at which an article is made, is known as its *cost price*.
The cost price is abbreviated as C.P.

Selling Price

The price at which an article is sold, is known as its *selling price*. The selling price is abbreviated as S.P.

Profit

If the selling price (S.P.) of an article is greater than the cost price (C.P.), then the difference between the selling price and cost price is called *profit*.
Thus, If S.P. > C.P., then

Profit = S.P. – C.P.

$\Rightarrow$ S.P. = C.P. + Profit

$\Rightarrow$ C.P. = S.P. – Profit.

Profit

If the selling price (S.P.) of an article is less than the cost price (C.P.), then the difference between the cost price (C.P.) and the selling price (S.P.) is called *loss*.
Thus, if S.P. < C.P., then

Loss = C.P. – S.P.

$\Rightarrow$ C.P. = S.P. + Loss

$\Rightarrow$ S.P. = C.P. – Loss

Illustration – 1

An article was bought for ₹ 2000 and sold for ₹ 2200. Find the gain or loss.

Solution : C.P. of the article = ₹ 2000
S.P. of the article = ₹ 2200
Since S.P. > C.P. So there is gain.

Gain (profit) = S.P. – C.P. = ₹ 2200 – ₹ 2000 = ₹ 200

Profit and Loss percentage

The profit per cent is the profit that would be obtained for a C.P. of ₹ 100.
Similarly, the loss per cent is the loss that would be made for a C.P. of ₹ 100.

$$\text{Profit per cent} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Loss per cent} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

REMEMBER

★ $\text{Profit} = \frac{\text{C.P.} \times \text{Profit \%}}{100}$

★ $\text{Loss} = \frac{\text{C.P.} \times \text{Loss \%}}{100}$

★ $\text{S.P.} = \left(\frac{100 + \text{Profit\%}}{100}\right) \times \text{C.P.}$

★ $\text{S.P.} = \left(\frac{100 - \text{Loss\%}}{100}\right) \times \text{C.P.}$

★ $\text{C.P.} = \frac{100 \times \text{S.P.}}{100 + \text{Profit \%}}$

★ $\text{C.P.} = \frac{100 \times \text{S.P.}}{100 - \text{Loss \%}}$

NOTE : (i) If an article is sold at a certain gain (say 45%), then SP = 145% of CP
(ii) If an article is sold at certain loss (say 25%), then SP = 75% of CP.

GP_3386

Illustration – 2

A cycle was purchased for ₹ 1600 and sold for ₹ 1400. Find the loss and loss %.

Solution : C.P. of the cycle = ₹ 1600

S.P. of the cycle = ₹ 1400

Since S.P < C.P, so there is a loss.

Loss = C.P. – S.P. = ₹ 1600 – ₹ 1400 = ₹ 200.

$$\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{200}{1600} \times 100 = 12\frac{1}{2}\%$$

Illustration – 3

By selling a table for ₹ 330, a trader gains 10%. Find the cost price of the table.

Solution : S.P. = ₹ 330, Gain = 10%

$$\therefore \text{C.P.} = \left(\frac{100}{100 + \text{Gain \%}}\right) \times \text{S.P}.$$

$$= ₹\ \frac{100}{100+10} \times 330 = ₹\ 300.$$

Illustration – 4

A mobile phone is sold for ₹ 5060 at a gain of 10%. What would have been the gain or loss per cent if it had been sold for ₹ 4370 ?

Solution : S.P. = ₹ 5060, gain = 10%

$$\therefore \text{C.P.} = \frac{5060 \times 100}{100+10} = ₹\ 4600.$$

2nd S.P. = ₹ 4370

Since, S.P. < C.P., so there is loss.

$$\therefore \text{Loss \%} = \frac{(4600-4370) \times 100}{4600} = 5\%$$

Cost Price

The expenses incurred on transportation, maintenance, packaging, repairing and the like are included under the general heading of Overheads. These overheads and the profit when added to the cost price determine the Selling Price. If the overheads are not separately mentioned in the problem, we assume it to be zero or else these have been included in the Cost Price itself

Dishonest Dealing

A dishonest dealer sells his goods at cost price or at a loss but he uses false weight, meter scale or any other measuring instrument to get some profit in reality.

Profit % of dishonest dealer

$$= \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\% \text{ or}$$

$$= \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100\%$$

$$\text{Also } \frac{\text{True Scale}}{\text{False Scale}} = \frac{100 + \text{gain\%}}{100 - \text{loss\%}}$$

Illustration – 5

A cloth merchant says that due to slump in the market, he sells the cloth at 10% loss, but he uses a false metre-scale and actually gain 15%. Find the actual length of the scale.

(a) 78 cm **(b) 78.25 cm**

(c) 78.5 cm **(d) 78.75 cm**

Solution : (b) $\frac{\text{True scale}}{\text{False scale}} = \frac{100+\text{gain\%}}{100-\text{loss\%}}$

$$\frac{100}{\text{False scale}} = \frac{100+15}{100-10} \Rightarrow \text{False scale} = 78.26\,\text{cm}$$

Illustration – 6

A dishonest dealer professes to sell his goods at cost price, but he uses a weight of 960 g for the kg weight. Find his gain per cent.

Solution : Error = 1 kg – 960 g = 40 g.

$$\therefore \quad \text{Gain \%} = \frac{40}{1000-40} \times 100 = \frac{40}{960} \times 100 = 4\frac{1}{6}\%$$

Real Profit/Loss Percentage

If the profit or loss is calculated on S.P., then it is not actual profit or loss.

$$\text{Real profit \%} = \frac{\text{\% profit on S.P.}}{100 - \text{\% profit on S.P.}} \times 100$$

$$\text{Real loss\%} = \frac{\text{\% loss on S.P.}}{100 + \text{\% loss on S.P.}} \times 100$$

Marked Price, List Price, Discount and Successive Discounts

List Price

When a manufacturer decides the retail prices of its different products, then these retail prices are either printed on the products or a list of retail prices of different products is sent to all its retail shopkeepers. Since the list price is decided by the manufacturer

and not by its retail shopkeeper, therefore it is the same at all retail shops.

Marked Price (MP)

In big shops and departmental stores, every article is tagged with a card and its price is written on the card. This is called the **marked price** of the article.

Discount

In order to increase the sale or clear the old stock, sometimes the shopkeepers offer a certain percentage of rebate on the marked price or list price. This rebate is known as **discount.** Since discount is always given on marked price or list price. Hence

S.P = (M.P. or List price) – (Discount)

Illustration – 7

After allowing a discount of $\frac{15}{2}$% on marked price, an article is sold for ₹ 555. Find its M.P.

Solution : Let M.P.=₹100

Discount $=\frac{15}{2}$% and S.P. = 100 – 7.50 = ₹92.50

If SP is ₹ 92.50, then M.P. = ₹ 100

If SP is ₹ 555, then M.P. $=\frac{100}{92.50}\times 555$ = ₹ 600.

Illustration – 8

A garment dealer allows his customers 10% discount on marked price of the goods and still makes a profit of 25%. Find the cost price of a shirt if it is marked at ₹ 1250.

Solution : Marked Price (MP) = ₹ 1250,

Discount = 10%, Profit = 25%

Let Cost Price (C.P.)= ₹ 'x'

Selling Price (S.P.)=M.P. – Discount

SP = 1250 – 10 % of 1250

= 1250 – 125 = ₹1125

Now, % Profit $=\frac{SP-CP}{CP}\times 100$

$\Rightarrow \quad 25=\left(\frac{1125-x}{x}\right)\times 100$

$\Rightarrow \quad x=1125\times 4-4x$

$\Rightarrow \quad x=\frac{1125\times 4}{5}$ = ₹900

$\therefore$ C.P. = ₹900.

Illustration – 9

What price should Neha mark on a sari which cost her ₹ 3000, so as to gain 20% after allowing a discount of 10%?

Solution : Let Marked Price (MP) of sari = ₹ x,

Discount =10%

C.P. = ₹ 3000, % gain = 20%

SP =MP – Discount = x – 10% of $x = x-\frac{10x}{100}=\frac{9x}{10}$

% gain $=\frac{SP-CP}{CP}\times 100 \Rightarrow 20=\frac{\frac{9x}{10}-3000}{3000}\times 100$

$\Rightarrow 20\times 30=\frac{9x}{10}-3000=\frac{3600\times 10}{9}$ = ₹ 4000

Successive Discounts

If two or more discounts are allowed one after the other then such discounts are known as *successive discounts* or *discounts in series.*

Suppose a discount of 15% is given, then on the reduced price a discount of 10% is also given. In such a case, we say that the successive discounts of 15% and 10% are given.

Illustration – 10

A shopkeeper offers 5% discount on all his goods to all his customers. He offers a further discount of 2% on the reduced price to those customers who pay cash. What will you actually have to pay for an article in cash if its M.P. is ₹ 4800?

Solution : M.P. = ₹4800

First discount = 5% of M.P. $=\frac{5}{100}\times 4800$ = ₹240

Net price after discount = ₹ 4800 – ₹ 240 = ₹ 4560

2nd discount = 2% of ₹ 4560 = $\frac{2}{100}\times 4560$ = ₹ 91.20

Net price after discount = ₹ 4560 – ₹ 91.20 = ₹ 4468.80

SHORTCUTS

Shortcut Approach - 1

If an item is bought at the rate of X items for a rupee, then the number of items sold for a rupee in order to

GP_3386

gain $x\%$ is $\left[X\left(\frac{100}{100+x}\right)\right]$.

Illustration – 11

If oranges are bought at the rate of 27 for a rupee, how many must be sold for a rupee in order to gain 35%.?

(a) 26 (b) 25 (c) 20 (d) 24

Shortcut Solution : (c)

Number of oranges

$$=\left[X\left(\frac{100}{100+x}\right)\right]=\left[27\left(\frac{100}{100+35}\right)\right]=20.$$

Shortcut Approach - 1

If a man purchases 'a' items for ₹ 'b' and sells 'c' items for ₹ 'd', then the gain or loss [depending upon the respective (+ve) or (–ve) sign in the final result] made by him is $\left[\frac{ad-bc}{bc}\times 100\right]\%$

Illustration – 12

A person buys 12 eggs for ₹ 15 and sells them at 10 for ₹ 14. What does he gain or loss per cent?

(a) 10% loss (b) 12% profit

(c) 12% loss (d) 15% profit

Shortcut Solution : (b)

$$\therefore \text{Profit} = \left[\frac{ad-bc}{bc}\times 100\right]\%$$

$$=\left[\frac{(12\times 14-15\times 10)}{15\times 10}\times 100\right]\% = 12\% \text{ (Profit)}$$

Shortcut Approach - 1

If a hike of x% in the price of an article forces a person to buy *n* kg less then the new and the original prices per kg of the article are $\left[\frac{Ax}{100n}\text{ per kg}\right]$ **and** $\left[\frac{Ax}{(100+x)n}\text{ per kg}\right]$ **respectively.**

Illustration – 13

A 20% hike in the price of wheat forces a person to purchase 1 kg less for ₹ 20. Find the original price of the wheat.

(a) ₹ 3 per kg (b) ₹ $3\frac{1}{3}$ per kg

(c) ₹ $3\frac{2}{3}$ per kg (d) Data inadequate

Shortcut Solution : (b)

$$\therefore \text{Original price} = \left[\frac{Ax}{(100+x)n}\text{ per kg}\right]$$

$$=\left[\frac{20\times 20}{(100+20)1}\right]=\frac{400}{120}=\frac{10}{3}=₹3\frac{1}{3}\text{ per kg}$$

Shortcut Approach - 1

If a reduction of x% in the price of an article enables a person to buy n kg more for ₹ A. then the reduced and the original prices per kg of the article are $\left(\frac{Ax}{100n}\right)$ **per kg and** $\left(\frac{Ax}{(100-x)n}\right)$ **per kg respectively.**

Illustration – 14

A reduction of 25% in the price of rice enables a person to buy 4 kg more for ₹ 40. Find the reduced and the original price per kg of rice respectively.

(a) ₹ $\frac{10}{3}$, ₹ 2.5 **(b)** ₹ 2.5, ₹ $\frac{10}{3}$

(c) ₹ 3.5, ₹ $\frac{11}{3}$ **(d)** Can't be determined

Shortcut Solution : (b)

$$\therefore \text{Reduced price} = \left[\frac{Ax}{100n}\right]=\left[\frac{40\times 25}{100\times 4}\right]=₹2.5.$$

$$\therefore \text{Original price} = \left[\frac{Ax}{(100-x)n}\right]=\left[\frac{40\times 25}{(100-25)\times 4}\right]$$

$$= ₹\ \frac{10}{3}$$

Shortcut Approach - 1

When there are two successive profits of x% and y %, then the resultant profit per cent is given by $\left(x+y+\frac{xy}{100}\right)$.

Illustration – 15

A sells a bicycle to B at a profit of 15% and B sells it to C at a profit of 20%. Find the resultant profit per cent.

(a) 38% (b) 35% (c) 36% (d) 34%

Shortcut Solution : (a)

$\therefore$ Resultant profit $= \left[x + y + \frac{xy}{100}\right]$

$= \left[15 + 20 + \frac{15 \times 20}{100}\right] = \left[35 + \frac{15 \times 20}{100}\right] = 38\%$

Shortcut Approach - 1

When there is a profit of x% and loss of y% in a transaction, then the resultant profit or loss per cent is given by $\left(x - y - \frac{xy}{100}\right)$ according to the +ve and the –ve signs respectively.

Illustration – 16

A sells a bicycle to B at a profit of 20% and B sells it to C at a loss of 15%. Find the resultant profit or loss.

(a) 2% loss (b) 2% gain

(c) 8% gain (d) 8% loss

Shortcut Solution : (b)

Resultant profit $= \left(x - y - \frac{xy}{100}\right)$

$= \left(20 - 15 - \frac{20 \times 15}{100}\right) = 2\%\,(\text{gain})$

Shortcut Approach - 1

When there are two successive loss of x% and y%, then the resultant loss per cent is given by $\left[x + y - \frac{xy}{100}\right]$.

Illustration – 17

A sells a good to B at a loss of 15% and B sells it to C at a loss of 20%. Find the resultant loss per cent.

(a) 32% (b) 38% (c) 35% (d) 32.5%

Shortcut Solution : (a)

$\therefore$ Resultant loss $= \left[x + y - \frac{xy}{100}\right]$

$= \left[15 + 20 - \frac{15 \times 20}{100}\right] = [35 - 3] = 32\%$

Shortcut Approach - 1

If a dealer wants to earn x% profit on an article after offering y% discount to the customer. To arrive at label price, the marked price should be increased by $\left[\frac{x + y}{100 - y}\right] \times 100$ per cent.

Illustration – 18

If a dealer wants to earn 20% profit on an article after offering 25% discount to the customer, by what percentage should he increase his marked price to arrive at the label price?

(a) 60% (b) 50% (c) 45% (d) 55%

Shortcut Solution : (a)

$\therefore$ Marked price should be increased by

$= \left[\frac{x + y}{100 - y}\right] \times 100\% = \left[\frac{20 + 25}{100 - 25}\right] \times 100\% = 60\%$

Shortcut Approach - 1

If a dishonest trader professes to sell his items at CP but uses false measurement, then

$$\%\ \text{gain} = \frac{\text{True measurement} - \text{False measurement}}{\text{False measurement}} \times 100$$

Illustration – 19

A cloth dealer professes to sell poplin at cost price, but uses a metre having a length of 92 cm only and charges for the metre. Find his gain per cent.

(a) $8\frac{16}{23}\%$ **(b)** $9\frac{16}{23}\%$

(c) $7\frac{16}{23}\%$ **(d) Data inadequat**

GP_3386

Shortcut Solution : (a)

∴ Gain percentage

$$= \frac{\text{True length} - \text{False length}}{\text{False length}} \times 100$$

$$= \frac{100-92}{92} \times 100 = \frac{800}{92} = \frac{200}{23} = 8\frac{16}{23}\%$$

Shortcut Approach - 10

If a shopkeeper sells his goods at x% profit and uses a measurement which is y% less, then
Total percentage profit

$$= \frac{\%\text{ profit} + \%\text{ less in wt}}{100 - \%\text{ less in wt}} \times 100$$

Illustration – 20

A grocer sells rice at a profit of 5% and uses a weight which is 20% less. Find his total percentage gain.

(a) 31.25% **(b) I.5%**
(c) 31% **(d) Data inadequate**

Shortcut Solution : (a)

∴ Total percentage profit

$$= \left[\frac{\%\text{ profit} + \%\text{ less in wt}}{100 - \%\text{ less in wt}}\right] \times 100$$

$$= \left[\frac{5+20}{100-20}\right] \times 100 = \frac{2500}{80} = 31.25\%$$

Shortcut Approach - 12

If the shopkeeper sells his goods at x% loss on cost price but uses y gm instead of z gm, then his % profit or loss is $\left[100 - x\right]\frac{z}{y} - 100$ according as the sign is +ve or –ve.

Illustration – 21

A merchant professes to lose 10% on a certain tea, but he uses a weight equal to 900 g instead of I kg. Find his real loss or gain per cent.

(a) 10% gain **(b) 10% loss**
(c) Neither gain nor loss **(d) Data inadequate**

Shortcut Solution : (c)

∴ Profit or loss $= \left[100 - x\right]\frac{z}{y} - 100$

$$= [100-10]\frac{1000}{900} - 100 = 90 \times \frac{10}{9} - 100 = 0$$

(It means their is neither gain nor loss).

Shortcut Approach - 12

A dishonest dealer sells the goods at x% loss on cost price but uses y% less weight, then his percentage profit or loss is $\left[\frac{y-x}{100-y} \times 100\right]$ according as the sign is +ve or –ve.

Illustration – 22

A dishonest dealer sells the goods at 20% loss on cost price but uses 25% less weight. What is his percentage profit or loss?

(a) $6\frac{2}{3}\%$ **gain** **(b)** $6\frac{1}{3}\%$ **gain**

(c) 6% gain **(d)** $6\frac{2}{3}\%$ **loss**

Shortcut Solution : (a)

∴ Profit or loss

$$= \left[\frac{y-x}{100-y} \times 100\right] = \left[\frac{25-20}{100-25} \times 100\right] = 6\frac{2}{3}\%\,(\text{gain})$$

Shortcut Approach - 12

If a seller uses 'X' gm in place of one kg (1000 gm) to sell his goods and gains a profit of x% on cost price, then his actual gain or loss percentage is $\left[(100+x)\left[\frac{1000}{X}\right] - 100\right]$ according as the sign is +ve or –ve.

Illustration – 23

A seller uses 990 gm in place of one kg to sell his goods. Find his actual % profit or loss, when he sells his article on 10% gain on cost price.

(a) $11\frac{1}{9}\%$ **gain** **(b)** $9\frac{1}{11}\%$ **gain**

(c) $11\frac{1}{9}\%$ **loss** **(d)** $9\frac{1}{11}\%$ **loss**

Shortcut Solution : (a)

∴ Actual gain or loss

$$=\left[(100+x)\left[\frac{1000}{x}\right]-100\right]$$

$$=\left[(100+10)\left[\frac{1000}{990}\right]-100\right]$$

$$=\left[\frac{11000-9900}{99}\right]=\frac{1100}{99}=11\frac{1}{9}\%\ \text{(gain)}$$

Shortcut Approach - 12

A man purchases a certain number of articles at x a rupee and the same number at y a rupee. He mixes them together and sells them at z a rupee. then his gain or loss % $=\left[\frac{2xy}{z(x+y)}-1\right]\times 100$ according as the sign is $+ve$ or $-ve$.

Illustration – 24

A man purchases a certain number of toffees at 20 a rupee and the same number at 15 a rupee. He mixes them together and sells them at 40 for 2 rupees. What does he gain or loss per cent in the transaction?

(a) $14\frac{2}{7}\%$ **gain** **(b)** $14\frac{2}{7}\%$ **loss**

(c) $14\frac{3}{7}\%$ **gain** **(d)** $14\frac{4}{7}\%$ **loss**

Shortcut Solution : (b)

∴ Profit or loss

$$=\left[\frac{2xy}{z(x+y)}-1\right]\times 100$$

$$=\left[\frac{2\times 20\times 15}{2(20+15)}-1\right]\times 100=14\frac{2}{7}\%\ \text{loss}$$

Shortcut Approach - 10

If a tradesman marks his goods at x% above his cost price and allows purchasers a discount of y% for cash, then there is $\left(x-y-\frac{xy}{100}\right)\%$ profit or loss according to +ve or –ve sign respectively.

Note : When x = y, then formula becomes $-\frac{x^2}{100}$

– ve sign indicates that there will be always loss.

Illustration – 25

A tradesman's charges are 20 percent over cost price. If he allows his customers 10 per cent off their bills for cash payment, what is his gain per cent?

(a) 4% **(b)** 8%

(c) 10% **(d)** 12%

Shortcut Solution : (b)

$$\therefore \text{ Profit or loss } \% =\left[x-y-\frac{xy}{100}\right]\%$$

$$=\left[20-10-\frac{20\times 10}{100}\right]\%=\left[\frac{1000-200}{100}\right]=8\%$$

Shortcut Approach - 12

If a man buys two items A and B for ₹ P, and sells one item A so as to lose x% and the other item B so as to gain y%, and on the whole he neither gains nor loses, then

(i) the cost of the item A is $\left(\frac{Py}{x+y}\right)$ and

(ii) the cost of the item B is $\left(\frac{Px}{x+y}\right)$.

Illustration – 26

A man buys two tables for ₹ 1750. He sells one so as to lose 10% and the other so as to gain 15%. On the whole he neither gains nor loses. What does each table cost?

(a) ₹ 1000, ₹ 750 **(b)** ₹ 1050, ₹ 700

(c) ₹ 950, ₹ 800 **(d)** Can't be determined

Shortcut Solution : (b)

∴ Cost of 1st table

$$=\left[\frac{Py}{x+y}\right]=\left[\frac{1750\times 15}{10+15}\right]=₹\,1050$$

$$\therefore \text{ Cost of 2nd table } =\left[\frac{Px}{x+y}\right]$$

$$=\frac{1750\times 10}{25}=₹\,700.$$

GP_3386

Shortcut Approach - 12

By selling a certain item at the rate of 'X' items a rupee, a man loses x%. If he wants to gain y%, then the number of items should be sold for a rupee is

$$\left[\left(\frac{100-x}{100+y}\right)X\right]$$

Illustration – 27

By selling oranges at 35 a rupee, a man loses 25%. How many for a rupee should he sell in order to gain 25%?

(a) 35 **(b) 21**
(c) 25 **(d) 30**

Shortcut Solution : (b)

$\therefore$ The number of oranges

$$=\left[\left(\frac{100-x}{100+y}\right)X\right]$$

$$=\left[\left(\frac{100-25}{100+25}\right)\times 35\right]=\frac{75}{125}\times 35=21$$

Shortcut Approach - 18

When each of the two commodities is sold at the same price ₹ A, and a profit of P % is made on the first and a loss of L% is made on the second, then the percentage gain or loss is $\frac{100(P-L)-2PL}{(100+P)+(100-L)}$ according to the +ve or –ve sign.

Illustration – 28

A person sells two horses at ₹ 24000 each and by doing so he gains 27% on one horse and loses 27% on the other. What is the % loss or gain in this deal?

(a) 7.29% gain **(b) 7.29% loss**
(c) 8.29% loss **(d) 6.29% loss**

Shortcut Solution : (b)

$\therefore$ Profit or loss %

$$=\frac{100(P-L)-2PL}{(100+P)+(100-L)}$$

$$=\frac{100(27-27)-2\times 27\times 27}{(100+27)+(100-27)}=\frac{0-2\times 27\times 27}{200}$$

$=-7.29\%$

So, negative sign shows the loss of 7.29%.

Shortcut Approach - 19

If a dealer sells an item for ₹ A, making a profit of x%, and he sells another item at a loss of y%, and on the whole he makes neither profit nor loss, then the cost of the second table is ₹ $\left[A\left(\frac{100}{100+x}\right)\frac{x}{y}\right]$.

Illustration – 29

A dealer sells a goat for ₹ 2600, making a profit of 30% . He sells another goat at a loss of 20%, and on the whole he makes neither profit nor loss. What did the second goat cost him ?

(a) ₹ 3000 **(b) ₹ 3600**
(c) ₹ 3500 **(d) ₹ 2900**

Shortcut Solution : (a)

$$\therefore \text{Cost}=\left[A\left(\frac{100}{100+x}\right)\frac{x}{y}\right]$$

$$=\left[2600\left(\frac{100}{100+30}\right)\times\frac{30}{20}\right]=₹3000$$

Shortcut Approach - 10

If a person buys an article with x per cent discount on the marked price and sells the article with y per cent profit on the marked price, then his profit per cent on the price he buys the article is given by $\left(\frac{x+y}{100-x}\right)\times 100$ per cent.

Illustration – 30

Vinay bought an article with 20 per cent discount on the labelled price. He sold the article with 12 per cent profit on the labelled price. What was his per cent profit of the price he bought?

(a) 35% **(b) 25%**
(c) 20% **(d) 40%**

Shortcut Solution : (d)

$$\therefore \text{Profit \%}=\left[\frac{x+y}{100-x}\right]\times 100$$

$$=\left[\frac{20+12}{100-20}\right]\times 100=40\%$$

Shortcut Approach - 12

A certain company declares x per cent discount for wholesale buyers. If a person buys articles from the company for ₹ A after getting discount. He fixed up the selling price of the article in such a way that he earned a profit y% on original company price. Then the total selling price is given by ₹ $A\left(\frac{100+y}{100-x}\right)$.

Illustration – 31

A garment company declared 14% discount for wholesale buyers. Mr Sujeet bought garments from the company for ₹ 860 after getting discount. He fixed up the selling price of garments in such a way that he earned a profit of 6% on original company price. What is the approximate total selling price?

(a) ₹ 1060 (b) ₹ 1160
(c) ₹ 960 (d) Can't be determined

Shortcut Solution : (a)

$$\therefore \text{Total S.P.} = A\left(\frac{100+y}{100-x}\right) = 860\left(\frac{100+6}{100-14}\right) = ₹1060.$$

Shortcut Approach - 10

A businessman marks an article at ₹ A and allows x% discount (on the marked price). He gains y %. If the cost price of the article is ₹ B, then the selling price of the article can be calculated from the equation given below

$$\text{Selling price} = \frac{A(100-x)}{100} = \frac{B(100+y)}{100}$$

$$\text{i.e.} \frac{\text{M.P}(100-\text{Dis\%})}{100} = \frac{\text{CP}(100+\text{Profit\%})}{100}$$

Note : Remember discount is given on marked price, and gain is calculated on the cost price.

Illustration – 32

How much per cent above the cost price should a shopkeeper mark his goods so as to earn a profit of 36% after allowing a discount of 15% on the marked price?

(a) 60% (b) 50% (c) 40% (d) 65%

Shortcut Solution : (a)

Let C.P = ₹ 100

$$\text{Now, } \frac{\text{M.P.}(100-\text{Dis\%})}{100} = \frac{\text{C.P.}(100+\text{P\%})}{100}$$

$$\Rightarrow \text{MP} = \frac{\text{CP}(100+\text{P\%})}{(100-\text{Dis\%})} = 100\left(\frac{100+36}{100-15}\right) = ₹160$$

$\therefore$ A shopkeeper should mark his goods 60% above C.P.

Shortcut Approach - 10

A person sells articles at ₹ A each after giving x% discount on marked price. Had he not given the discount, he would have earned a profit of y% on the cost price. Then the cost price of each article is given by ₹ $\left[\frac{100^2 A}{(100-x)(100+y)}\right]$.

Illustration – 33

A shopkeeper sold books at ₹ 166 each after giving 17% discount on labelled price. Had he not given the discount, he would have earned a profit of 25% on the cost price. What was the cost price of each book?

(a) ₹ 165 (b) ₹ 155 (c) ₹ 160 (d) ₹ 164

Shortcut Solution : (c)

$$\text{C.P.} = \frac{(100)^2 \text{A}}{(100-\text{x})(100+\text{y})}$$

$$= \frac{(100)^2 \times 166}{(100-17)(100+25)} = ₹160$$

Shortcut Approach - 10

A shopkeeper sold an article for ₹ A after giving x% discount on the labelled price and made y% profit on the cost price. Had he not given the discount, the percentage profit would have been $\left[\frac{x+y}{100-x} \times 100\right]$ per cent.

Illustration – 34

A shopkeeper sold an article for ₹ 880 after giving 12% discount on the labelled price and made 32% profit on the cost price. What would have been the percentage profit, had he not given the discount?

(a) 25% (b) 35% (c) 40% (d) 50%

Shortcut Solution : (d)

$\therefore$ Profit

$$= \left[\frac{x+y}{100-x} \times 100\right] = \left[\frac{12+32}{100-12} \times 100\right] = 50\%.$$

LEVEL - 1

I. A man bought toffees at the rate of 15 for a rupee and sold them at the rate of 12 for a rupee. His gain percent is
(a) 20% (b) 25%
(c) 33.33% (d) None of these

2. The cost price of 20 articles is equal to the selling price of 25 articles. The loss percent in the transaction is
(a) 5 (b) 20 (c) 25 (d) 30

3. A property dealer sells a house for ₹ 6,30,000 and in the bargain makes a profit of 5%. Had he sold it for ₹ 5,00,000, then what percentage of loss or gain he would have made?
(a) $2\frac{1}{4}\%$ gain (b) 10% loss
(c) $12\frac{1}{2}\%$ loss (d) $16\frac{2}{3}\%$ loss

4. A fruit seller buys lemons at 2 for a rupee and sells them at 5 for three rupees. What is his gain percent? **[CDS 2011]**
(a) 10% (b) 15% (c) 20% (d) 25%

5. A person purchased 11 articles for ₹ 10 and at 10 articles for ₹11. Find the gain percentage. **[SSC CGL Tier-I 2012]**
(a) 22 (b) 20 (c) 1 (d) 21

6. The sale price of an article including the sales tax is ₹ 616. The rate of sales tax is 10%. If the shopkeeper has made a profit of 12%, then the cost price of the article is :
(a) ₹500 (b) ₹515 (c) ₹550 (d) ₹600

7. A sells a cycle to B at 20% profit. Then B sells it to C at 25% profit. If C pays ₹ 225, then what is the cost price of cycle for A?
(a) ₹100 (b) ₹125 (c) ₹150 (d) ₹175

8. An article is listed at ₹ 65. A customer bought this article for ₹ 56.16 and got two successive discounts of which the first one is 10%. The other rate of discount of this scheme that was allowed by the shopkeeper was :
(a) 3% (b) 4% (c) 6% (d) 2%

9. Two motor cars were sold for ₹ 9,900 each, gaining 10% on one and losing 10% on the other. The gain or loss per cent in the whole transaction is :
(a) Neither loss no gain
(b) $\frac{1}{99}\%$ gain
(c) $\frac{100}{99}\%$ profit
(d) 1% loss

10. A dishonest dealer sells his goods at 10% loss on cost price and uses 30% less weight. What is his profit or loss percent?
(a) $28\frac{4}{7}\%$profit (b) $28\frac{4}{7}\%$profit
(c) $26\frac{4}{7}\%$profit (d) $26\frac{4}{7}\%$loss

II. Three successive discounts of 10%, 12% and 15% amount to a single discount of:
(a) 36.28% (b) 34.68%
(c) 37% (d) 32.68%

12. Dhiraj purchased 150 kg of rice. He sold $\frac{1}{3}$rd of it at 10% loss. At what per cent of profit must he sell the remaining rice so that he can make 10% profit on the whole?
(a) 20% (b) 15%
(c) 10% (d) None of these

13. A shopkeeper purchased 50 dozens cups for ₹ 3000 out of these 60 cups were broken and could not be sold. At what rate per dozen should be sell the remaining cup to make a profit of 20% ?
(a) ₹60 (b) ₹75 (c) ₹80 (d) ₹90

14. $\frac{2}{3}$ of a consignment was sold at 6 % profit and the rest at a loss of 3 %. If there was an overall profit of ₹ 540, find the value of the consignment.
(a) ₹15,000 (b) ₹18000 (c) ₹35000 (d) ₹45000

15. A trader mixes three varieties of groundnuts costing ₹ 50, ₹ 20 and ₹ 30 per kg in the ratio 2 : 4 : 3 in terms of weight, and sells the mixture of ₹ 33 per kg. What percentage of profit does he make?
(a) 8% (b) 9%
(c) 10% (d) None of these

16. If the cost price is 96% of the selling price, then what is the profit percent?
(a) 4.5% (b) 4.2% (c) 4% (d) 3.8%

17. A dealer sold a mixer for ₹420 at a loss of 12.5%. At what price should he have sold it to gain 12.5%.
(a) ₹620 (b) ₹540 (c) ₹650 (d) ₹750

18. A man sold 10 eggs for 5 rupees and gained 20%.How many eggs did he buy for 5 rupees?
(a) 10 eggs (b) 12 eggs (c) 14 eggs (d) 16 eggs

19. If 15 oranges are bought for a rupee, how many must be sold for a rupee to gain 25%?
(a) 12 (b) 10 (c) 20 (d) 18

20. A dishonest dealer sells his goods at the cost price but still earns a profit of 25% by underweighing. What weight does he use for a kg?
(a) 750 g (b) 800 g (c) 825 g (d) 850 g

21. A retailer profess to sell his goods at cost price. If using a false weight, he still gains 25%, find the weight he uses in place of 1 kg.
(a) 200 g (b) 600 g (c) 700 g (d) 800 g

22. A merchant professes to sell goods at the loss of 5% but uses weight of 900 grams in place of one kilogram, what is his profit percent?
(a) 5% (b) $5\frac{5}{19}\%$
(c) 6% (d) 10%

23. A shopkeeper marks up his goods to gain 35%. But he allows 10% discount for cash payment. His profit on the cash transaction therefore, in percentage, is
(a) $13\frac{1}{2}$ (b) 25 (c) $21\frac{1}{2}$ (d) $31\frac{1}{2}$

24. A trader sells wheat at 20% profit and uses weight 20% less than the actual measure. His gain percent is
(a) 50% (b) 25% (c) 10% (d) 15%

25. A merchant professes to sell goods at 20% profit but uses weight of 800 grams in place of a kilogram. What is his actual profit percent?
(a) 20% (b) 25% (c) 50% (d) 33.3%

26. A man sold two steel chairs for ₹500 each. On one he gains 20% and on other, he loses 12%. How much does he gain or lose in the whole transaction?
(a) 1.5% gain (b) 2% gain
(c) 1.5% loss (d) 2% loss

27. A man purchases two watches at ₹560. He sells one at 15% profit and other at 10% loss. Then he neither gains nor loss. Find the cost price of each watch.
(a) ₹224, ₹300 (b) ₹200, ₹300
(c) ₹224, ₹336 (d) ₹200, ₹336

28. A man purchased two cows for ₹500. He sells the first at 12% loss and the second at 8% gain. In this bargain he neither gains nor loses. Find the selling price of each cow.
(a) ₹176, ₹324 (b) ₹324, ₹350
(c) ₹150, ₹300 (d) ₹184, ₹276

29. A man bought a horse and a carriage for ₹3000. He sold the horse at a gain of 20% and the carriage at a loss 10%, thereby gaining 2% on the whole. Find the cost of the horse.
(a) ₹1000 (b) ₹1200 (c) ₹1500 (d) ₹1700

30. A man purchases a certain number of oranges at 4 a rupee and the same number of oranges at 5 a rupee. He mixes them together and sells them at 4 a rupee. What is his gain or loss percent?
(a) $10\frac{1}{9}\%$ loss (b) $10\frac{1}{9}\%$ gain
(c) $11\frac{1}{9}\%$ gain (d) $11\frac{1}{9}\%$ loss

31. Two electronic musical instruments were purchased for ₹8000. The first was sold at a profit of 40% and the second at loss of 40%. If the sale price was the same in both the cases, what was the cost price of two electronic musical instruments?
(a) ₹2000, ₹5000 (b) ₹2200, ₹5500
(c) ₹2400, ₹5000 (d) ₹2400, ₹5600

32. A dealer sells a table for ₹400, making a profit of 25%. He sells another table at a loss of 10%, and on the whole he makes neither profit nor loss. What did the second table cost him?
(a) ₹600 (b) ₹800
(c) ₹700 (d) ₹900

33. A man sells an article at a gain 15%. If he had bought it at 10% less and sold it for ₹4 less, he would have gained 25%. Find the cost price of the article.
(a) ₹150 (b) ₹160 (c) ₹170 (d) ₹180

34. A dealer sold two T.V. sets for ₹7400 each. On one he gained 10% and on the other he lost 10%. The dealers loss or gain in the transaction is **[SSC CGL Tier I 2015]**
(a) no profit, no loss (b) 1% gain
(c) 0.1% loss (d) 1% loss

35. A single discount equal to a discount series of 10% and 20% is
(a) 25% (b) 28% (c) 30% (d) 35%

GP_3386

36. A man sold his book for ₹ 891, thereby gaining $\frac{1}{10}$ of its cost price. Find his cost price.
(a) ₹850 (b) ₹810 (c) ₹851 (d) ₹840

37. Kaberi buys a radio three-fourths of its value and sells it for 20% more than its value. What will be its profit percentage?
(a) 50% (b) 40% (c) 60% (d) 70%

38. A person sells 36 oranges per rupee and suffers a loss of 4%. Find how many oranges per rupee to be sold to have a gain of 8%?
(a) 30 (b) 31 (c) 32 (d) 33

39. A man buys milk at ₹ 6 per litre and adds one third of water to it and sells mixture at ₹ 7.20 per litre. The gain is
(a) 40% (b) 80% (c) 60% (d) 25%

40. A man sells an article at 5% profit. If he had ought it at 5% less and sold it for ₹ 1 less, he would have gained 10%. The cost price of the article is :
(a) ₹200 (b) ₹150 (c) ₹240 (d) ₹280

41. The list price of a watch is ₹ 160. A retailer bought the same watch ₹ 122.40. He got two successive discounts one at 10% and the other at a rate which was not legible. What is the second discount rate?
(a) 12% (b) 14% (c) 15% (d) 18%

42. The difference between the selling price of a clock at a profit of 8% and 10% is ₹ 6. Find the cost price of the clock.
(a) ₹1200 (b) ₹600 (c) ₹400 (d) ₹300

43. A shopkeeper buys 50 dozen eggs at ₹ 4 per dozen. Out of them, 40 eggs were found broken. At what rate should he sell the remaining eggs per dozen so as to gain 5% on the whole?
(a) ₹4 (b) ₹4.25 (c) ₹4.50 (d) ₹5.25

44. John sold a fan at a loss of 7%. If he had sold it for ₹ 48 more, he would have gained 5%. Find the cost price of the fan.
(a) ₹350 (b) ₹480 (c) ₹240 (d) ₹400

45. Goods are sold so that when a discount of 4 percent is given on the sale price, a profit of 20 percent is made. How much percent, is the sale price higher than the cost price?
(a) 20% (b) 24% (c) 25% (d) 27%

46. If a merchant estimates his profit as 20% of the selling price, what is his real profit per cent?
(a) 18% (b) 20% (c) 25% (d) 28%

47. A wholesaler sells 30 pens at the price of 27 pens to a retailer. The retailer sells the pens at their market price. The profit for the retailer is
(a) 11% (b) 10%
(c) $11\frac{1}{9}\%$ (d) $9\frac{1}{11}\%$

48. A trader marks his goods at such a price that he can deduct 15% for cash and yet make 20% profit. Find the marked price of an item which costs him ₹ 90 :
(a) ₹ $135\frac{11}{13}$ (b) ₹ $105\frac{3}{21}$
(c) ₹ $127\frac{1}{17}$ (d) ₹ $95\frac{1}{21}$

49. A tradesman is marketing his goods 20% above the cost price of the goods. He gives 10% discount on cash payment, find his gain percent.
(a) 12% (b) 8% (c) 15% (d) 18%

50. A business man sells lot of shirts at a profit of $12\frac{1}{2}\%$ and invests the proceeds to buy lot of pants, which he sells at a profit of 20%. If he makes a net profit of ₹ 700, the cost of the shirts was
(a) ₹2450 (b) ₹2225 (c) ₹2000 (d) ₹1880

51. By selling 66 metres of cloth a man loses the selling price of 22 metres. Find the loss per cent.
(a) 20% (b) 25% (c) 30% (d) 35%

52. The profit obtained by selling a book for ₹ 56 is the same as the loss obtained by selling this book for ₹ 42. What is the cost price of the book?
(a) ₹40 (b) ₹50
(c) ₹49 (d) None of these

53. By selling a CD for ₹ 150, a shop owner lost 1/16 of what it costs What is the cost price of CD?
(a) ₹120 (b) ₹140 (c) ₹150 (d) ₹160

54. A shopkeeper professes to sell his articles at CP but gives only 800 gm in the place of 1000 gm. What is his profit percentage?
(a) 20% (b) 25% (c) 16.66% (d) 33.33%

55. In winters, a metre scale shrinks and starts reading 90 cm for every 100 cm. If the shopkeeper is selling the cloth at his CP, what is his loss percentage?
(a) 11.11% (b) 10%
(c) 12.5% (d) None of these

56. A car is sold for ₹ 2,400 at a profit of 20% over SP. What is the CP?
(a) ₹ 2,000 (b) ₹ 1,920 (c) ₹ 1,980 (d) ₹ 1,800

57. A car is sold for ₹ 2, 400 at a profit of 20% over SP. What is the actual profit percentage?
(a) 16.66% (b) 25%
(c) 21.21% (d) 14.28%

58. Which of the following discount options is better for a customer?
i. Successive discounts of 20%, 30% and then pay a service tax of 10%
ii. Successive discounts of 30%, 20% and then pay a service tax of 10%.
iii. Pay service tax of 10% first, then successive discounts of 20% and 30%.
(a) (i) or (ii)
(b) (iii)
(c) Nither of these three
(d) (i) and (iii) or (ii) and (iii)

59. Petrol is purchased @ ₹ 5/litre and sold @ 5 litre/ ₹ What is the profit/loss%?
(a) Loss of 96% (b) No profit, no loss
(c) Profit of 2400% (d) None of these

60. There are two shopkeepers selling the same article at the same price for same quantity. One day, first shopkeeper offers a price discount of 25% for the same quantity whereas second shopkeeper offers 25% more quantity for the same price. From a customer's point of view, which deal is better?
(a) 1st shopkeeper's deal
(b) 2nd shopkeeper's deal
(c) both are equal
(d) cannot be determined

61. A dealer gives as much discount (in per cent) as the markup (in per cent) above the cost price. What is the profit or loss per cent?
(a) 10% (b) 1%
(c) 4% (d) can't be determined

62. Mr. Mittal purchased a car for ₹ 3,00,000 and a bike for his son for ₹ 1,00, 000. He sold the car at a profit of 10% and bike at a loss of 20%. What is the net gain or loss?
(a) 2% gain (b) 1.5% loss
(c) 2.5% loss (d) 2.5% gain

63. The cost price of 19 articles is same as the selling price of 29 article. What is the loss%?
(a) 35% (b) 34.48%
(c) 52.63% (d) None of these

64. The difference between CP and SP of a table fan is ₹ 175 when it gives the profit of 14%. What is the selling price of that fan?
(a) 1225 (b) 1450 (c) 1425 (d) 1275

65. ITC sells one product at a profit of 20% another at a loss of 20% at the same selling price. What is the loss incurred by ITC?
(a) 1% (b) 2% (c) 4% (d) 0%

66. Arun bought toffees at 6 for a rupee. How many for a rupee he should sell to gain 20%?
(a) 3 (b) 4
(c) 5 (d) can't be determined

67. 6% more is gained by selling a coat for ₹ 1425 than by selling it for ₹ 1353. The cost price of the coat is :
(a) ₹ 1000 (b) ₹ 1250 (c) ₹ 1500 (d) ₹ 1200

LEVEL - 2

1. A manufacturer sells a car to a dealer at a profit of 50%, the dealer sells it to a customer at a profit of 20% and the customer sells it to a friend for ₹ 288000 at a loss of 20%. Find the cost of manufacturer.
(a) 200000 (b) 300000 (c) 400000 (d) 50000

2. The difference between a discount of 35% and two successive discounts of 20% and 20% on a certain bill was ₹ 22. Find the amount of the bill.
(a) ₹ 1,100 (b) ₹ 200
(c) ₹ 2,200 (d) None of these

3. Prabhu purchased 30 kg of rice at the rate of ₹ 17.50 per kg and another 30 kg rice at a certain rate. He mixed the two and sold the entire quantity at the rate of ₹ 18.60 per kg and made 20 per cent overall profit. At what price per kg did he purchase the lot of another 30 kg rice?
(a) ₹ 14.50 (b) ₹ 12.50
(c) ₹ 15.50 (d) ₹ 13.50

4. A sells a tube to B at a profit of 20% and B sells it to C at profit of 25 %. If C pays ₹ 225 for it, what did A pay for it?
(a) ₹ 100 (b) ₹ 125 (c) ₹ 150 (d) ₹ 175

5. A trader wants 10% profit on the selling price of a product whereas his expenses amount to 15% on sales. What should be his rate of mark up on an article costing ₹ 9?
(a) 20% (b) $66\frac{2}{3}\%$
(c) 30% (d) $\frac{100}{3}\%$

6. A reduction of 20% in the price of sugar enables a purchaser to obtain $2\frac{1}{2}$ kg more for ₹ 160. Find the original price per kg of sugar.
(a) ₹ 12 (b) ₹ 20 (c) ₹ 16 (d) ₹ 18

GP_3386

7. A man sells an article at 5% profit. If he had bought it at 5% less and sold if for ₹ 1 less, he would have gained 10%. Find the cost price.
(a) 100 (b) 150 (c) 200 (d) 250

8. By selling 9 articles for a rupee, a man incurred a loss of 4%. To make a gain of 44%, the number of articles to be sold for a rupee is **[SSC CGL Tier-II 2012]**
(a) 5 (b) 3 (c) 4 (d) 6

9. By selling a watch at a profit of 10 per cent, a man got ₹15 more than half of its price. What is the price of the watch?
(a) 10 (b) 15 (c) 25 (d) 5

10. If 5% more is gained by selling an article for ₹ 350 than by selling it for ₹ 340 the cost of the article is:
(a) ₹ 50 (b) ₹ 160 (c) ₹ 200 (d) ₹ 225

11. A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 grams *i.e.*, he takes 1100 gram from his supplier and sells only 900 grams to his customer. What is his net profit percentage?
(a) 24.5% (b) 17.33% (c) 25% (d) 32.5%

12. Sweta bought an article with 20% discount on the labelled price. She sold the article with 8% profit on the labelled price. What was her percent profit of the price she bought?
(a) 35% (b) 25% (c) 20% (d) 15%

13. The price of a jewel, passing through three hands, rises on the whole by 65%. If the first and the second sellers earned 20% and 25% profit respectively, find the percentage profit earned by the third seller.
(a) 20% (b) 10%
(c) 25% (d) No gain or loss

14. A firm of readymade garments makes both men's and women's shirts. Its average profit is 6% of the sales. Its profit in men's shirts average 8% of the sales and women's shirts comprise 60% of the output. The average profit per sale rupee in women shirts is
(a) 0.0466 (b) 0.0666
(c) 0.0166 (d) None of these

15. If the manufacturer gains 10%, the wholesale dealer 15% and the retailer 25%, then find the cost of production of a table, the retail price of which is ₹ 1265?
(a) ₹ 800 (b) ₹ 1000 (c) ₹ 900 (d) ₹ 600

16. A sells an article which costs him ₹ 400 to B at a profit of 20%. B then sells it to C, making a profit of 10% on the price he paid to A. How much does C pay to B?
(a) ₹ 472 (b) ₹ 476 (c) ₹ 528 (d) ₹ 532

17. By what % must the cost of goods be marked up so that even after a discount of 20% the same amount is realised as before the discount?
(a) 20 (b) 25 (c) 40 (d) 10

18. Raman expects 20% profit by selling a fan. If the allows 40% discount on marked price then his cost price is what percent of marked price?
(a) 40% (b) 60% (c) 50% (d) 100%

19. A person sells a T.V at 9600 after giving a discount of 20%. If he sells the T.V at marked price then his profit is 50%. What is the cost price of T.V?
(a) ₹10000 (b) ₹12000 (c) ₹8000 (d) ₹9000

20. A person allows 30% discount on his goods and still gains 40%. If he sells a fan on marked price then what is percentage of profit?
(a) 100% (b) 50% (c) 60% (d) 70%

21. For a certain article, if discount is 25%, the profit is 25%. If the discount is 10%, then the profit is
(a) 10% (b) 20% (c) 35% (d) 50%

22. A chair originally costs ₹ 50. It was offered for sale at 108% of its cost. After a week, the price was 10% discounted and was sold. Find the sale price.
(a) ₹ 46.80 (b) ₹ 48.60 (c) ₹ 50 (d) ₹ 52.40

23. Surdeep bought a machinery for ₹800 after getting a discount of 20% from the company. He fixed up the selling price of garments in such a way that he earned a profit of 10% on the original company price. Find the selling price of machinery for Surdeep?
(a) ₹900 (b) ₹1000 (c) ₹1100 (d) ₹1200

24. By selling 66 metres of cloth a person gains the cost price of 22 metres. Find the gain per cent.
(a) 22% (b) $22\frac{1}{2}\%$
(c) 33% (d) $33\frac{1}{3}\%$

25. A shopkeeper fixes the marked price of an item 20% above the cost price. He allows his customers a discount and makes a profit of 8%. Find the rate of discount.
(a) 8% (b) 9% (c) 10% (d) 11%

26. What is the percentage discount (approximately) that a merchant can offer on his marked price so that he ends up selling at no profit or loss, if he initially marked his goods up by 40%? **[SSC CGL Tier II 2014]**
(a) 33.5 % (b) 28.5%
(c) 60% (d) No discount

27. Nitika buys a kinetic for ₹ 16,000 and sells it for ₹ 18,500. If she wants to gain 40%, how much more should she charge for the kinetic?
(a) ₹ 3, 800 (b) ₹ 4,000
(c) ₹ 4, 200 (d) None of these

28. The owner of Book Point does not get either profit or loss by selling 15 books for ₹ 225. How many books should he sell for the same amount to gain 25% profit?
(a) 10 (b) 11 (c) 12 (d) 13

29. A reduction 20% in the price of the rice enables to get 6 kg more of rice for ₹120. What is the initial price of rice (₹/kg)?
(a) 4 (b) 5
(c) 6 (d) None of these

30. CP of 12 apples is equal to the SP of 9 apples and the discount on 10 apples is equal to the profit on 5 apples. What is the percentage difference between the CP and SP of apples?
(a) 20% (b) 25%
(c) 16.66% (d) None of these

31. There is some profit when an article is sold for ₹ 720. However when the same article is sold for ₹ 420, there is some loss. If the quantum of profit is two times the quantum of loss, find the cost price of the article.
(a) ₹ 620 (b) ₹ 700
(c) ₹ 520 (d) None of these

32. When an article is sold for ₹180, a profit of 20% is made. At what price should the article be sold so that profit percentage is double?
(a) ₹ 200 (b) ₹ 210
(c) ₹ 192 (d) ₹ 240

33. A fruit seller declares that he sells fruits at the cost price. However, he uses a weight of 450 g instead of 500 g. His percentage profit is :
(a) 10% (b) $11\frac{1}{9}\%$
(c) 12% (d) $12\frac{2}{9}\%$

GP_3386

Hints and Solutions

Level 1

1. **(b)** Use Shortcut Approach-1

2. **(b)** Let C.P. of 1 article = ₹ 1
then C.P. of 25 articles = ₹ 25
and S.P. of 25 articles = ₹ 20

$$\therefore \text{ loss } \% = \frac{25-20}{25} \times 100 = 20\%$$

3. **(d)** $\text{C.P.} = ₹\left(\frac{100}{105} \times 630000\right) = ₹ 600000.$

$$\therefore \text{ Required loss } \% = \left(\frac{100000}{600000} \times 100\right)\%$$

$$= 16\frac{2}{3}\%.$$

4. **(c)** Use Shortcut Approach 1

5. **(d)** Use Shortcut Approach-2

6. **(a)** Let the CP of the article be ₹ x

Then, SP = $x \times 1.12 \times 1.1$

Now, $x \times 1.12 \times 1.1 = 616$

$$\Rightarrow x = \frac{616}{1.232} = ₹ 500$$

7. **(c)** Use Shortcut Approach-5

8. **(b)** Price of the article after first discount,

$$65 - \left(65 \times \frac{10}{100}\right) = ₹ 58.5$$

Therefore, the second discount

$$= \frac{58.5 - 56.16}{58.5} \times 100 = 4\%$$

9. **(d)** If any two transactions of SP is the same and also gain % and loss % are same then there is always a loss

$$\therefore \text{ loss } \% = \left(\frac{\text{Common gain or loss}\%}{10}\right)^2$$

$$= \left(\frac{10}{10}\right)^2 = 1\%$$

10. **(b)** Use Shortcut Approach-12

11. **(d)** Applying successive discounts of 10%, 12% and 15% on 100, we get

$100 \times 0.9 \times 0.88 \times 0.85 = 67.32$

$\Rightarrow$ Single discount = $100 - 67.32 = 32.68\%$

12. **(a)** Let the C.P. of 150 kg of rice be ₹150.

$\therefore$ S.P. of 50 kg of rice at 10% loss

$$= \frac{90}{100} \times 50 = ₹ 45$$

For 10% of gain on the whole.

$$\text{S.P.} = 150 \times \frac{110}{100} = ₹ 165$$

$\therefore$ 100 kg rice should be sold for ₹ 120.

$\therefore$ Per cent gain = 20%

13. **(c)** $\because$ CP = 3000

Profit = 20%

$$\therefore \text{SP} = 3000 \times \frac{120}{100} = 3600$$

As 5 dozens cup were broken out of 50 dozens.

So, S.P. of remaining 45 dozens cup

$$= \frac{3600}{45} = ₹ 80$$

14. **(b)** Value of consignment $= \dfrac{540 \times 100}{\frac{2}{3} \times 6 + \frac{1}{3}(-3)}$

$$= \frac{540 \times 100}{4-1} = ₹ 18{,}000$$

15. **(c)** Suppose he bought 2 kg, 4 kg and 3 kg of the three varieties. C.P. of 9 kg
= ₹ (2 × 50 + 4 × 20 + 3 × 30) = ₹ 270.
S.P. of 9 kg = ₹ (9 × 33) = ₹ 297.

$$\therefore \quad \text{Profit } \% = \left(\frac{27}{270} \times 100\right)\% = 10\%.$$

16. (b) Let S.P. = ₹100. Then, C.P. = ₹96; Profit = ₹4.

$$\therefore \quad \text{Profit \%} = \left(\frac{4}{96}\times 100\right)\%$$

$$= \frac{25}{6}\% = 4.17\%. \approx 4.2\%$$

17. (b) $(100 - \text{loss}) : S_1 :: (100 + \text{gain}) : S_2$

$\therefore (100 - 12.5) : 420 :: (100 + 12.5) : S_2$

$87.5 : 420 :: 112.5 : S_2$

$\Rightarrow 87.5 \times S_2 = 420 \times 112.5$

$$\Rightarrow S_2 = \frac{420\times 1125}{875} = 540$$

18. (b) S.P. for 1 egg = ₹ $\frac{5}{10}$ = ₹$\frac{1}{2}$

$$\therefore \text{C. P. for 1 egg} = \frac{100}{(100+20)}\times\frac{1}{2} = ₹\ \frac{5}{12}$$

$\Rightarrow$ He bought 12 eggs for 5 rupees.

19. (a) C.P. for one orange = ₹ $\frac{1}{15}$

Then S.P.

$$= \frac{(100+25)}{100}\times\frac{1}{15} = \frac{125}{100\times 15} = \frac{1}{12}$$

Hence S.P. for one orange = ₹ $\frac{1}{12}$

$\therefore$ 12 oranges must be sold for a rupee to gain 25%.

20. (b) $$\frac{\text{True weight}}{\text{False weight}} = \frac{100+\text{gain\%}}{100+x}$$

Here S.P. = C. P. $\therefore x = 0$

$$\Rightarrow \text{False weight} = \frac{1000\times 100}{125} = 800 \text{ gm}$$

21. (d) Use Shortcut Approach-9

22. (b) Use Shortcut Approach-11

23. (c) Let cost price = ₹ 100

$\therefore$ Marked price = ₹ 135

After discount, selling price

$= 135 - 13.5 = 121.5$

$\therefore$ Profit% $= 121.5 - 100 = 21.5\%$

24. (a) Use Shortcut Approach-10

25. (c) Use shortcut Approach-13

26. (a) Gain % or Loss%

$$= \frac{100(20-12)+2\times 20\times(-12)}{200+20-12}$$

$$= \frac{800-480}{208} = \frac{20}{13} = 1.5\% \text{ gain}$$

27. (c) Here, in whole transaction, there is neither gains nor loss, therefore,

Amount of gain in one watch

= Amount of loss in other watch

$\Rightarrow 0.15\times CP_1 = 0.10\times CP_2$

$$\Rightarrow \frac{CP_1}{CP_2} = \frac{0.10}{0.15} = \frac{2}{3}$$

Also $CP_1 + CP_2 = 560$

$$\therefore CP_1 = \frac{2}{(2+3)}\times 560 = ₹\ 224$$

and $CP_2 = 560 - 224 =$ ₹ 336

28. (a) Use Shortcut Approach-16

29. (b) Let the C.P. of horse = ₹ x

Then the C.P. of carriage = ₹ $(3000 - x)$

20% of x – 10% of $(3000 - x)$ = 2% of 3000

$$\Rightarrow \frac{x}{5} - \frac{(3000-x)}{10} = 60$$

$\Rightarrow 2x - 3000 + x = 600$

$\Rightarrow 3x = 3600 \Rightarrow x = 1200$

30. (c) Use Shortcut Approach-14

31. (d) Here, $SP_1 = SP_2$

$$\Rightarrow 140\ CP_1 = 60 CP_2 \Rightarrow \frac{CP_1}{CP_2} = \frac{6}{14} = \frac{3}{7}$$

$$\therefore CP_1 = \frac{3}{(3+7)}\times 8000 = ₹\ 2400$$

and $CP_2 = 8000 - 2400 =$ ₹ 5600

32. (b) Use shortcut Approach-19

33. (b) Let the C.P. be ₹ 100

First S.P. = ₹ 115

Second C.P. = ₹ 90

Second S.P = 125% of ₹ 90 = ₹ 112.50

Difference of two selling prices is

₹ 115 – ₹ 112.50

= ₹ 2.50 and C.P. of the article is ₹ 100

But actual difference is ₹ 4.

$\therefore$ C.P. $= \frac{100}{2.50} \times ₹\, 4 = ₹\, 160.$

34. (d) Use Shortcut Approach-18

35. (b) Equivalent discount $= 10 + 20 - \frac{10 \times 20}{100}$

$= 30 - 2 = 28\%$

36. (b) Let C. P. = ₹ x then profit = S.P. – C. P.

$\Rightarrow \frac{1}{10} \times x = 891 - x \Rightarrow \frac{11x}{10} = 891$

$\Rightarrow x = \frac{891 \times 10}{11} = ₹\, 810$

37. (c) Let the value of radio be ₹ x.

Then C. P. = ₹ $\frac{3}{4}$x,

S. P. $= x\left(1 + \frac{20}{100}\right) = \frac{6}{5}x$

$\therefore$ % profit $= \frac{\frac{6}{5}x - \frac{3}{4}x}{\frac{3x}{4}} \times 100 = 60\%$

38. (c) Let he sells 1 orange at *x* rupees.

$\frac{1}{36} : (100 - 4) :: x : (100 + 8)$

$\Rightarrow x = \frac{108}{96 \times 36} = \frac{1}{32}$

He sells 32 oranges per rupee.

39. (b) C.P. of one litre = ₹ 6

After adding water to it

One has to pay ₹ 7.20 for $\frac{2}{3}$ litre of milk.

So, S.P. of $\frac{2}{3}$ litre of milk = ₹ 7.20

$\Rightarrow$ S.P. of 1 litre of milk

$= ₹ \frac{7.20 \times 3}{2} = ₹\, 10.80$

$\because$ S.P. > C.P.

Hence gain

$= \frac{10.80 - 6}{6} \times 100 = \frac{4.80}{6} \times 100$

$= 0.80 \times 100 = 80\%$

40. (a) Let the CP of the article be ₹ x.

Then, SP = ₹ $\frac{105x}{100}$

Now, new CP = ₹ $\frac{95x}{100}$

and new SP = $\frac{105x}{100} - 1$

According to the question

$\frac{105x}{100} - 1 - \frac{95x}{100} = \frac{10 \times 95x}{100 \times 100}$

$\therefore$ x = ₹ 200

41. (c) Retailer price = list price

$\left(1 - \frac{d_1}{100}\right)\left(1 - \frac{d_2}{100}\right)$

$\Rightarrow 122.40 = 160\left(1 - \frac{10}{100}\right)\left(1 - \frac{d_2}{100}\right)$

$\Rightarrow 1 - \frac{d_2}{100} = \frac{122.40 \times 100}{160 \times 90} = 0.85$

$\Rightarrow d_2 = (1 - 0.85) \times 100 = 15\%$

42. (d) Let C. P. = ₹ x. Then

$S.P_1 - S.P_2$ = ₹ 6

$\frac{(100 + 10)x}{100} - \frac{(100 + 8)x}{100} = 6$

$\Rightarrow \quad 110x - 108x = 600$

$\Rightarrow \quad 2x = 600 \Rightarrow x = ₹\ 300$

43. (c) C.P. $= 50 \times 4 = ₹\ 200$

Remaining eggs $= 600 - 40 = 560$

Let S.P. of eggs $= ₹\ x$ per dozen

$\therefore$ Total S.P. $= ₹\ \frac{560}{12}x$

$\therefore \ \frac{560}{12}x = \frac{(100+5)\%}{100} \times 200$

$\Rightarrow x = \frac{105}{100} \times \frac{2400}{560} = ₹\ 4.5$ per dozen

44. (d) Let C.P. $= ₹\ x$. Then,

$\text{S.P.} = \frac{(100-7)}{100} \times x = \frac{93}{100}x$

Also, $\left(\frac{93}{100}x + 48\right)\frac{100}{(100+5)} = x$

$\Rightarrow \quad 93x + 4800 = 105x$

$\Rightarrow \quad 12x = 4800 \Rightarrow x = ₹\ 400$

45. (c) Let the C.P. be ₹ 100

S.P. $= ₹\ 120$

Discount being 4%, S.P. is 96% of sale price.

$\therefore$ 96% of sale price $= ₹\ 120$

$\Rightarrow$ sale price $= ₹\ 120 \times \frac{100}{96} = ₹\ 125$

$\therefore$ Sale price is 25% higher than the C.P.

46. (c) Real profit % is the profit % on C.P.

Real profit %

$$= \frac{\text{\% profit on S.P.}}{100 - \text{\% profit on S.P.}} \times 100$$

$$= \frac{20}{100-20} \times 100 = 25\%$$

47. (c) Retailer's S.P. = M.P.

Retailer's C.P. for 30 Pens = M.P. of 27 pens

$\therefore$ Retailer's S.P. for 30 pens = M.P of 30 pens

$\therefore$ % gain $= \frac{30-27}{27} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%$

48. (c) SP $= 90 \times 1.2 = ₹\ 108$

Marked price $= \frac{108}{0.85} = ₹\ 127.05$

49. (b) Let the C.P. of the goods be ₹ 100

$\Rightarrow$ Marked price of the goods $= ₹\ 120$

Discount = 10%

$\Rightarrow$ S.P. is 90% of ₹ 120 $= ₹\ 108$

$\therefore$ Gain% $= (108 - 100) = 8\%$.

50. (c) Let C.P. $= ₹\ x$

$$120\% \text{ of} \left(\frac{225}{2}\% \text{ of } x\right) - x = 700$$

$$\Rightarrow \left(\frac{120}{100} \times \frac{225}{2 \times 100} \times x\right) - x = 700 \Rightarrow x = 2000$$

51. (b) Loss = C.P. of 66 metres – S.P. of 66 metres

= S.P. of 22 metres

$\Rightarrow$ C.P. of 66 metres = S.P. of 88 metres

$$\% \text{ loss} = \frac{\text{loss}}{\text{C.P. of 66 metres}} \times 100$$

$$= \frac{\text{S.P of 22 metres}}{\text{C.P of 66 metres}} \times 100$$

$$= \frac{\text{S.P. of 22 metres}}{\text{S.P.of 88 metres}} \times 100$$

$$= \frac{22}{88} \times 100 = 25\%$$

52. (c) Let us assume that the cost price $= ₹\ x$

$56 - x = x - 42$

$2x = 98$

Hence, $x = 49$

53. (d) Assume cost price $= ₹\ x$

$x - 150 = \frac{x}{16} \quad \Rightarrow x = 160$

54. (b) Let CP of 1000 gm $= ₹\ 1000$

CP of 800 gm $= ₹\ 800$

SP of 800 gm $= ₹\ 1000$

Profit percentage $= \frac{200}{800} \times 100 = 25\%$

55. (b) CP of 100 cm $= ₹\ 100$

CP of 90 cm $= ₹\ 90$

Loss $= ₹\ 10$

Loss percentage $= \frac{10}{100} \times 100 = 10\%$

56. **(b)** $\text{Profit} = \frac{2400 \times 20}{100} = 480$

$\text{CP} = ₹(2400 - 480) = ₹ 1920$

57. **(b)** Actual profit percentage

$= \frac{480}{1920} \times 100 = 25\%$

58. **(c)** Let MP = ₹ 100

(i) Customer has to pay = ₹ 61.6

(ii) Customer has to pay = ₹ 61.6

(iii) Customer has to pay = ₹ 61.6

59. **(a)** CP of 1 litre petrol = ₹ 5

SP of 1 litre petrol = $₹ \frac{1}{5}$

$\text{Loss \%} = \frac{5 - \frac{1}{5}}{5} \times 100 = \frac{24}{25} \times 100 = 96\%$

60. **(a)** Let SP of 1 kg article = ₹ 100

For 1st shopkeeper

SP of 1 kg article = ₹ 75

For 2nd shopkeeper

SP of 1.25 kg article = ₹ 100

SP of 1 kg article = ₹ 80

61. **(d)** It is dependent upon the markup (%) or discount (%)

Since, $100 \xrightarrow{+20\%} 120$

$96 \xleftarrow{-20\%}$

Loss = 4%

Again $100 \xrightarrow{50\%} 150$

$75 \xleftarrow{50\%}$

loss = 25%

Hence we can't determine.

62. **(d)** Gain = 30000

Loss = 20000

net gain = 10000 over ₹ 4 lakh

Hence, $\text{profit} = \frac{10000}{400000} \times 100 = 2.5\%$

63. **(b)** $\frac{\text{CP}}{\text{SP}} = \frac{29x}{19x}$

$\text{Loss\%} = \frac{29x - 19x}{29x} \times 100 = 34.48\%$

(To understand the concept assume CP of each article ₹ 29 and SP of each article = ₹ 19)

64. **(c)** SP – CP = 175 = 14% of CP

$\Rightarrow$ CP = 1250

$\therefore$ SP = 1250 + 175 = 1425

65. **(c)** $\text{Loss\%} = \left(\frac{\text{common gain or loss}}{10}\right)^2 \%$

$= \left(\frac{20}{10}\right)^2 \% = 4\%$

66. **(c)** $\text{CP} = \frac{100}{6} = 16.66 \text{ paise}$

$\text{SP} = \frac{100}{5} = 20 \text{ paise}$

$\text{Profit } (\%) = \frac{20 - 16.66}{16.66} \times 100$

$= \frac{3.33}{16.66} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

Hence, option (c) is correct.

Alternatively :

$\text{CP} = \frac{100}{6}$

$\text{SP} = \frac{100}{6} \times 1.2 = \frac{100}{5}$

Hence, he should sell 5 toffees for ₹ 1 (= 100 paise)

67. **(d)** 1425 – 1353 = 72 = 6% of CP $\Rightarrow$ CP = 1200

Level 1

I. **(a)** Let ₹ X be the C.P. of the manufacturer of the car

$\Rightarrow \quad X \times \frac{150}{100} \times \frac{120}{100} \times \frac{80}{100} = 288000$

$\Rightarrow \quad X = 200000$

2. **(c)** Successive discount $= 20\% + \frac{20\times 80}{100}\%$

$= (20+16)\% = 36\%$

Difference in discount $= (36-35)\% = 1\%$

$\therefore$ Bill amount $= 22 \times 100 = ₹2200$

3. **(d)** Let he purchase of ₹ x/kg.

$$\therefore (525+30x)\times\frac{120}{100} = 60\times 18.60$$

$\Rightarrow x = ₹13.5/\text{kg}.$

4. **(c)** Let A paid $= ₹x$

125 % of 120% of $x = 225$

$$\Rightarrow \frac{125}{100}\times\frac{120}{100}\times x = 225$$

$$\Rightarrow x = \frac{225\times 100\times 100}{125\times 120} = ₹150$$

5. **(d)** Let the SP of the article be ₹ x

Expenses $= 15\%$ of $x = ₹0.15x$

Profit $= 10\%$ of $x = ₹0.10x$

CP $= ₹9$ (given)

Therefore, $9 + 0.15x + 0.1x = x \Rightarrow x = 12$

$\therefore$ % increase for marked price

$$= \frac{12-9}{9}\times 100 = \frac{100}{3}\%$$

6. **(c)** Total amount used for purchasing $= ₹160$.

A reduction of 20% in the price means, now a person gets 5/2 kg for ₹ 32 and this is the present price of the sugar.

$$\therefore \text{Present price per kg} = \frac{32}{5}\times 2 = ₹12.8$$

Let the original price be ₹ x. Then new price is arrived after reduction of 20% on it.

$\Rightarrow x \times 0.8 = 12.8$ or $x = ₹16$.

7. **(c)** Let cost price $= x$

Then we have,

$$x\left(\frac{95}{100}\right)\left(\frac{110}{100}\right) = x\left(\frac{105}{100}\right) - 1$$

$$\text{or, } x = \frac{100\times 100}{105\times 100 - 95\times 100}\times 20$$

$\therefore$ Cost price $= ₹ 200$

8. **(d)** Use Shortcut Approach-17

9. **(c)** Price $= ₹x$

SP $= ₹1.1x = 0.5x + 15$, So, $0.6x = 15$

So, $x = 25$

10. **(c)** Let C.P. be ₹ x. Then,

5% of x = (350 – 340) = 10

$$\Rightarrow \frac{x}{20} = 10 \Rightarrow x = 200.$$

11. **(b)** Assume his CP $= ₹1000/1100$ gm

MP $= ₹1200$ and SP $= ₹960/900$ gm

So, SP/1100 gm $= ₹1173.33$

So, profit $= ₹173.33$

Profit percentage $= 17.33\%$

12. **(a)** Use Shortcut Approach-20

13. **(b)** Let the original price of the jewel be ₹ P and let the profit earned by the third seller be x%.

Then, (100 + x)% of 125% of 120% of P = 165% of P

$$\Rightarrow \left[\frac{(100+x)}{100}\times\frac{125}{100}\times\frac{120}{100}\times P\right] = \left(\frac{165}{100}\times P\right)$$

$$\Rightarrow (100+x) = \left(\frac{165\times 100\times 100}{125\times 120}\right) = 110$$

$\Rightarrow x = 10\%$.

14. **(a)** Women's shirt comprise 60% of the output.

$\therefore$ Men's shirts comprise $(100-60) = 40\%$ of the out put.

$\therefore$ Average profit from men's shirt

$= 8\%$ of 40 $= 3.2$ out of 40

Overall average profit $= 6$ out of 100

$\therefore$ Average profit from women's shirts $= 2.8$ out of 60 i.e. 0.0466 out of each shirt.

15. **(a)** Let the cost of production of the table be ₹ x.

Then, 125% of 115% of 110% of x = 1265

$$\Rightarrow \frac{125}{100}\times\frac{115}{100}\times\frac{110}{100}\times x = 1265$$

$$\Rightarrow \frac{253}{160}x = 1265 \Rightarrow x = \left(\frac{1265\times 160}{253}\right)$$

$= ₹800$

16. **(c)** C.P. for B $= 120\%$ of ₹ 400

$$= ₹\left(\frac{120}{100}\times 400\right) = ₹\ 480$$

GP_3386

C.P. for C = 110% of ₹ 480 = ₹ $\left(\frac{110}{100}\times 480\right)$

= ₹ 528.

17. (b) Let C.P. = ₹ 100, Also, let M.P. = ₹ x

Given, C.P. after 20% discount on M.P. = C.P.

$\Rightarrow$ 80 % of x = 100

$\Rightarrow x = \frac{100\times 100}{80} = ₹ 125$

$\therefore$ M.P. = 25 % above C.P.

18. (c) Use Shortcut Approach-22

19. (c) Use Shortcut Approach-23

20. (a) Use Shortcut Approach-24

21. (d) For same article, $\frac{100-d_1}{100-d_2} = \frac{100+g_1}{100+g_2}$

$\Rightarrow \frac{100-25}{100-10} = \frac{100+25}{100+g_2} \Rightarrow \frac{75}{90} = \frac{125}{100+g_2}$

$\Rightarrow 100+g_2 = \frac{90\times 125}{75} = 150 \Rightarrow g_2 = 50\%$

22. (b) Offering price $= \frac{50\times 108}{100} = ₹ 54$

After 10% discount, S.P. = 90% of 54

$= \frac{90\times 54}{100} = ₹ 48.60$

23. (c) Use Shortcut Approach-21

24. (d) Let C.P. of one metre of cloth = ₹ 1

then C.P. of 66 metres of cloth = ₹ 66

Gain = C.P. of 22 metres = ₹ 22

$\%\ \text{gain} = \frac{22}{66}\times 100 = 33\frac{1}{3}\%$

Shortcut method :

If on selling 'x' articles, a man gains equal to the C.P. of 'y' articles, then % gain $= \frac{y}{x}\times 100$ $\therefore$ % gain $= \frac{22}{66}\times 100 = 33\frac{1}{3}\%$

25. (c) Let C.P. = ₹ 100. Then M.P. = ₹ 120 and S.P. = ₹ 108

% discount $= \left(\frac{12}{120}\times 100\right)\% = 10\%$

26. (b) Use Shortcut Approach-15

27. (d) Profit in order to gain 40% = ₹ 6400

More money charged = (₹ 6400 – 2500)

= ₹ 3900

28. (c) CP of 1 book = ₹ 15

SP of 1 book in order to gain 25% profit = ₹18.75

Required no. of books $= \frac{225}{18.75} = 12$

29. (b) Suppose initially x kg of rice was bought.

Now, $\frac{1}{4}x = 6 \Rightarrow x = 24$

Initial price = ₹ $\frac{120}{24}$ = ₹ 5.

30. (d) CP of 1 apple = ₹ 1

CP of 12 apples = ₹ 12,

SP of 9 apples = ₹ 12

Required difference $= \frac{3}{9}\times 100 = 33\frac{1}{3}\%$

31. (c) Let us assume that CP = ₹ x

$720 - x = 2(x - 420) \Rightarrow 720 - x = 2x - 840$

$3x = 1560 \Rightarrow x = 520$

32. (b) CP = ₹ $\left(180 - 180.\frac{1}{6}\right) = ₹ 150$

New profit = 2 ×20 = 40%

New S.P $= 150\times\frac{140}{100} = 210$

33. (b) $\text{Profit}(\%) = \frac{\text{goods left}}{\text{goods sold}}\times 100 = \frac{50}{450}\times 100$

$= 11\frac{1}{9}\%$

Alternatively : Suppose CP of 1 g is ₹ 1 then he sells goods worth ₹ 450 only and charges ₹ 500. So the profit is ₹ 50 over the charge of ₹ 500.

CHAPTER

12

Simple Interest & Compound Interest and Instalments

INTEREST

Interest is the fixed amount paid on borrowed money. The sum lent is called the **Principal**.
The sum of the principal and interest is called the **Amount**.
Interest is of two kinds :

- Simple interest
- Compound interest

Simple Interest

When interest is calculated on the original principal for any length of time, it is called **simple interest**.

★ Simple interest $= \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

i.e. $\text{S.I.} = \frac{P \times R \times T}{100}$

★ Principal (P) $= \frac{100 \times \text{S.I.}}{R \times T}$

★ Rate (R) $= \frac{100 \times \text{S.I.}}{T \times P}$

★ Time (T) $= \frac{100 \times \text{S.I.}}{P \times R}$

★ If rate of simple interest differs from year to year, then

$$\text{S.I.} = P \times \frac{(R_1 + R_2 + R_3 + ...)}{100}$$

★ Amount = Principal + Interest

i.e. $A = P + I = P + \frac{PRT}{100} = P\left(1 + \frac{RT}{100}\right)$

Illustration – 1

Find the interest to be paid on a loan of ₹ 6000 at 5% per year for 5 years

Solution : P = ₹ 6000, R = 5% and T = 5 years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{6000 \times 5 \times 5}{100} = ₹ 1500$$

Illustration – 2

Find the amount to be paid back on a loan of ₹ 18,000 at 5.5% per annum for 3 years

Solution : P = ₹ 18000, R = 5.5%, T = 3 years

$$\text{S.I.} = \frac{P \times R \times T}{100} = \frac{18000 \times 5.5 \times 3}{100} = ₹ 2970$$

Amount = P + I = 18000 + 2970 = ₹ 20970

Illustration – 3

In how many years will a sum of money triple itself, at 25% per annum simple interest?

Solution : Let the sum of money be ₹ P.
So, A = 3P
and S.I. = A – P = 3P – P = 2P R = 25%

$$\therefore T = \frac{100 \times \text{S.I.}}{P \times R} = \frac{100 \times 2P}{P \times 25} = 8 \text{ years}$$

Illustration – 4

What rate per cent per annum will produce ₹ 250 as simple interest on ₹ 6000 in 2.5 years

Solution : P=₹6000; Time(T)=2.5 years; S.I.=₹250

$$\therefore \text{Rate} = \frac{\text{S.I.} \times 100}{P \times T} = \frac{250 \times 100}{6000 \times 2.5} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}\%$$

Illustration – 5

To buy furniture for a new apartment, Sylvia Chang borrowed ₹5000 at 11 % simple interest for 11 months. How much interest will she pay?

Solution : From the formula, I = Prt, with P = 5000, r = 0.11, and t = 11/12 (in years). The total interest she will pay is I = 5000 (0.11) (11/12)
= 504.17 or ₹ 504.17

Compound Interest (C.I.)

If the borrower and the lender agree to fix up a certain interval of time (a year, a half year or a quarter of a year etc.) called **conversion period** or time unit, so that the amount (= principal + interest) at the end of an conversion period becomes the principal for the next conversion period, then the total interest over all the loan period calculated in this way is called the **compound interest**.

NOTE: The main difference between the simple interest and the compound interest is that the principal in the case of simple interest remains constant throughout the loan period where as in the case of compound interest, the principal changes periodically (i.e. after each conversion period) throughout the loan period.

(I) Computation of compound interest when interest is compounded annually :

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$\therefore \text{C.I.} = \text{A} - \text{P} = \text{P}\left[\left(1+\frac{r}{100}\right)^n - 1\right],$$ where A is the amount, P is the principal, r is the rate of interest in percent per conversion period and n is the number of conversion periods in the whole loan period.

(i) When interest is compounded half-yearly,

$$A = P\left[1+\frac{\frac{r}{2}}{100}\right]^{2t} = P\left[1+\frac{r}{200}\right]^{2t}$$

(ii) When interest is compounded quaterly,

$$A = P\left[1+\frac{\frac{r}{4}}{100}\right]^{4t} = P\left[1+\frac{r}{400}\right]^{4t}$$

(iii) When interest is compounded monthly,

$$A = P\left[1+\frac{\frac{r}{12}}{100}\right]^{12t} = P\left[1+\frac{r}{1200}\right]^{12t}$$

Note : Case (i), (ii) and (iii) can be obtained from case (I) by dividing the rate of interest in percent per annum by the no. of time units (at which the interest is compounded) contained in an year and converting the time period in years into number of equivalent new time units.

Illustration – 6

Roohi deposited ₹ 7,000 with a finance company for 3 years at an interest of 15% per annum compounded annually. What is the compound interest and the amount that Roohi gets after 3 years ?

Solution : Principal, P = ₹ 7,000, n = 3 years, r = 15% per annum

$$\text{A} = P\left(1+\frac{r}{100}\right)^n = 7{,}000\left(1+\frac{15}{100}\right)^3 = 7{,}000\left(\frac{115}{100}\right)^3$$

$$= 7{,}000 \times \frac{115}{100}\times\frac{115}{100}\times\frac{115}{100} = 10{,}646.125$$

= ₹ 10,646.13 = ₹ 10,646 (approx.)

Compound interest = $A - P$ = 10,646 – 7,000 = ₹ 3,646

Illustration – 7

Find the compound interest on ₹ 1000 at the rate of 10% per annum for 18 months when interest is compounded half-yearly.

Solution : Here, P = ₹ 1000, R = 10% per annum and,

$$n = \frac{18}{12}\text{ years} = \frac{3}{2}\text{ years}$$

$$\therefore \quad \text{Amount after 18 months} = P\left(1+\frac{R}{200}\right)^{2n}$$

$$= ₹\,1000 \times \left(1+\frac{10}{200}\right)^{2\times\frac{3}{2}} = ₹\,1000 \times \left(1+\frac{1}{20}\right)^3$$

$$= ₹\,1000 \times \left(\frac{21}{20}\right)^3 = ₹\,1000 \times \frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

= ₹ 1157.63

Hence, Compound interest = Amount – Principal

= ₹ 1157.63 – ₹ 1000 = ₹ 157.63

(II) Computation of compound interest when interest is compounded annually but rate of interest in percent being different for different years :

$$A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\ldots\left(1+\frac{R_n}{100}\right),$$

where $R_1, R_2, \ldots, R_n$ are rate of interest in percent per year for different years.

Illustration – 8

Ram Singh bought a refrigerator for ₹ 4000 on credit. The rate of interest for the first year is 5% and of the second year is 15%. How much will it cost him if he pays the amount after two years?

Solution : Here, $P = ₹\ 4000$, $R_1 = 5\%$ per annum and $R_2 = 15\%$ per annum.

$\therefore$ Amount after 2 years $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)$

$= ₹\ 4000 \times \left(1+\frac{5}{100}\right)\left(1+\frac{15}{100}\right)$

$= ₹\ 4000 \times \left(1+\frac{1}{20}\right)\left(1+\frac{3}{20}\right)$

$= ₹\ 4000 \times \frac{21}{20} \times \frac{23}{20} = ₹\ 4830$

Thus, the refrigerator will cost ₹ 4830 to Ram Singh.

(III) If P be the value of an article (or population of a town or country etc.) at a certain time and R % per annum is the rate of depreciation, then the value A at the end of n years is given by,

$$A = P\left(1-\frac{R}{100}\right)^n$$

Illustration – 9

The population of a town 2 years ago was 62500. Due to migration to cities, it decreases every year at the rate of 4% per annum. Find its present population.

Solution : We have,
Population two years ago = 62500
Rate of decrease of population = 4% per annum.

$\therefore$ Present population $= 62500 \times \left(1-\frac{4}{100}\right)^2$

$= 62500 \times \left(1-\frac{1}{25}\right)^2 = 62500 \times \left(\frac{24}{25}\right)^2$

$= 62500 \times \frac{24}{25} \times \frac{24}{25} = 57600.$

Hence, present population = 57600.

(IV) If P be the population of a country (or value of an article etc.) at a certain time, which increases at the rate R_1% per year for first n_1 years and decreases at the rate of R_2% per year for next n_2 years, then the population at the end of $(n_1 + n_2)$ years is given by

$$A = P\left(1+\frac{R_1}{100}\right)^{n_1} \cdot \left(1-\frac{R_2}{100}\right)^{n_2}$$

This formula can be extended for more than 2 different periods and rates.

Illustration – 10

10000 workers were employed to construct a river bridge in four years. At the end of first year, 10% workers were retrenched. At the end of the second year, 5% of the workers at that time were retrenched. However to complete the project in time, the number of workers was increased by 10% at the end of the third year. How many workers were working during the fourth year?

Solution : We have,
Initial number of workers = 10000
Reduction of workers at the end of first year = 10%
Reduction of workers at the end of second year = 5%
Increase of workers at the end of third year = 10%
$\therefore$ Number of workers working during the fourth year

$= 10000\left(1-\frac{10}{100}\right)\left(1-\frac{5}{100}\right)\left(1+\frac{10}{100}\right)$

$= 10000 \times \frac{9}{10} \times \frac{19}{20} \times \frac{11}{10} = 9405.$

Hence, the number of workers working during the fourth year was 9405.

SHORTCUTS

Shortcut Approach - 1

If a sum is put at SI at a certain rate r_1 % for t_1 year, again at rate r_2% at t_2 years and then so on, then

$$\text{Principal} = \frac{\text{Interest} \times 100}{t_1r_1 + t_2r_2 + t_3r_3 + \ldots}$$

Illustration – 11

The rate of interest for the first 2 years is 3% per annum, for the next 3 years is 8% per annum and for the period beyond 5 years 10% per annum. If a man gets ₹ 1520 as a simple interest for 6 years, how much money did he deposit?
(a) 4000 (b) 3800 (c) 3900 (d) 4100

Shortcut Solution : (b)
Detail Method: Let his deposit = ₹ 100
Interest for first 2 years = ₹ 6

GP_3386

Interest for next 3 years = ₹ 24
Interest for the last year = ₹ 10
Total interest = ₹ 40
When interest is ₹ 40, deposited amount is ₹ 100
∴ when interest is ₹ 1520, deposited amount

$= \frac{100}{40} \times 1520 = ₹\ 3800$

Direct formula : Principal $= \frac{\text{Interest} \times 100}{t_1r_1 + t_2r_2 + t_3r_3 + \ldots}$

$= \frac{1520 \times 100}{2 \times 3 + 3 \times 8 + 1 \times 10} = \frac{1520 \times 100}{40} = ₹\ 3800$

Note: Here $t_3 = 6 - 5 = 1$ year

Illustration – 12

Manish borrowed some money at the rate of 7 per cent per annum for the first three years, 9 per cent per annum for the next six years and 10 per cent per annum for the period beyond nine years. If the total interest paid by him at the end of fifteen years is ₹ 4050, how much money did he borrow?
(a) ₹ 2800 (b) ₹ 3600 (c) ₹ 3000 (d) ₹ 3500

Shortcut Solution : (c)

$\text{Principal} = \frac{\text{Interest} \times 100}{t_1r_1 + t_2r_2 + t_3r_3 + \ldots}$

$= \frac{4050 \times 100}{7 \times 3 + 9 \times 6 + 6 \times 10} = \frac{4050 \times 100}{135} = ₹\ 3000.$

Shortcut Approach - 1

A sum was put at SI at a certain rate for t years. Had it been put at x% higher rate, it would have fetched ₹ 'A' more, then the sum is

$₹ \left[\frac{A \times 100}{t \times x}\right]$ **or** $\frac{\text{More interest} \times 100}{\text{Time} \times \text{More rate}}$.

Illustration – 13

A sum was put at SI at a certain rate for 5 years. Had it been put at 5% higher rate, it would have fetched ₹ 500 more. Find the sum.
(a) ₹ 2500 (b) ₹ 2000 (c) ₹ 1500 (d) ₹ 1800

Shortcut Solution : (b)

$\text{Sum} = \left[\frac{A \times 100}{t \times x}\right] = \left[\frac{500 \times 100}{5 \times 5}\right] = ₹\ 2000.$

Shortcut Approach - 1

A sum was put at SI at a certain rate for t years. Had it been put at x% lower rate, it would have fetched ₹ A less, then the sum is

$₹ \left[\frac{A \times 100}{t \times x}\right]$ **or** $\frac{\text{Less interest} \times 100}{\text{Time} \times \text{Less rate}}$.

Illustration – 14

A sum was put at SI at a certain rate for 5 years. Had it been put at 2% lower rate, it would have fetched ₹ 150 less. Find the sum.
(a) ₹ 1000 (b) ₹ 1500 (c) ₹ 1800 (d) ₹ 2000

Shortcut Solution : (b)

$\text{Sum} = \left[\frac{A \times 100}{t \times x}\right] = \left[\frac{150 \times 100}{5 \times 2}\right] = ₹\ 1500$

Shortcut Approach - 1

If a person deposits ₹ x_1 in a bank at r_1% per annum and ₹ x_2 in another bank at r_2% per annum, then the rate of interest for the whole sum is $\left[\frac{x_1r_1 + x_2r_2}{x_1 + x_2}\right]$.

Illustration – 15

A man deposits ₹ 1350 in a bank at 5% per annum and ₹ 1150 in another bank at 6% per annum. Find the rate of interest for the whole sum.
(a) 5.40% (b) 6.40% (c) 5.46% (d) 11%

Shortcut Solution : (c)

$\text{Rate of Interest} = \left[\frac{x_1r_1 + x_2r_2}{x_1 + x_2}\right]$

$= \left[\frac{1350 \times 5 + 1150 \times 6}{1350 + 1150}\right] = \left[\frac{6750 + 6900}{2500}\right] = 5.46\%$

Shortcut Approach - 1

A certain sum is invested for certain time. It amounts to ₹ A_1 at r_1% per annum. But when invested at r_2% per annum, it amounts to ₹ A_2, then the sum is given by $₹ \left[\frac{A_2r_1 - A_1r_2}{r_1 - r_2}\right]$ **and the time is given by** $\left[\frac{A_1 - A_2}{A_2r_1 - A_1r_2}\right] \times 100$ **years.**

Illustration – 16

A certain sum is invested for certain time. It amounts to ₹ 500 at 8% per annum. But when invested at 3 % per annum, it amounts to ₹ 200. Find the sum.
(a) ₹ 20 (b) ₹ 50 (c) ₹ 25 (d) ₹ 35

Shortcut Solution : (a)

$$\text{Sum} = \left[\frac{A_2r_1 - A_1r_2}{r_1 - r_2}\right] = \left[\frac{200\times 8 - 500\times 3}{8-3}\right]$$

$$= \frac{100}{5} = ₹\,20$$

Illustration – 17

A certain sum is invested for certain time. It amounts to ₹ 450 at 7% per annum. But when invested at 5% per annum, it amounts to ₹ 350. Find the time.

(a) 50 years (b) 60 years (c) 45 years (d) 40 years

Shortcut Solution : (a)

$$\text{Time} = \left[\frac{A_1 - A_2}{A_2r_1 - A_1r_2}\right]\times 100$$

$$= \left[\frac{450-350}{350\times 7 - 450\times 5}\right]\times 100 = \left[\frac{100\times 100}{200}\right] = 50 \text{ years.}$$

Shortcut Approach - 1

If a certain sum of money at simple interest amounts to ₹ A_1 in t_1 years and to ₹ A_2 in t_2 years, then the sum is given by $\left[\frac{A_2t_1 - A_1t_2}{t_1 - t_2}\right]$.

Illustration – 18

A certain sum of money amounts to ₹ 625 in 4 years and to ₹ 680 in 5 years. Find the sum.

(a) ₹ 505 (b) ₹ 305 (c) ₹ 405 (d) ₹ 504

Shortcut Solution : (c)

$$\text{Sum} = \left[\frac{A_2t_1 - A_1t_2}{t_1 - t_2}\right] = \left[\frac{680\times 4 - 625\times 5}{4-5}\right]$$

$$= \left[\frac{2720-3125}{-1}\right] = ₹\,405$$

Shortcut Approach - 1

If a sum of money becomes 'n' times in 't' years at a simple interest, then the time in which it will amount to 'm' times itself is given by $\left[\frac{m-1}{n-1}\right]t$ **years.**

Illustration – 19

A sum of money becomes 5 times itself in 5 years at a simple interest. In how many years will it amount to 7 times itself?

(a) 7 years (b) 6 years 6 months
(c) 6 years (d) 7 years 6 months

Shortcut Solution : (d)

$$\text{Time} = \left[\frac{m-1}{n-1}\right]t \text{ years} = \left[\frac{7-1}{5-1}\right]\times 5 \text{ years}$$

$$= \frac{6}{4}\times 5 \text{ years} = 7.5 \text{ years} = 7 \text{ years, 6 months.}$$

Shortcut Approach - 8

If the difference between the simple interest received from two different banks on Rs X for t years is ₹ I_d, then the difference between their rates is given by $\left[\frac{I_d \times 100}{X \times t}\right]$ **per cent.**

Illustration – 20

The difference between the interest received from two different banks on ₹ 600 for 6 years is ₹ 72. Find the difference between their rates.

(a) 3% (b) 3.5% (c) 4% (d) 2%

Shortcut Solution : (d)

$$\text{Difference between their rates} = \left[\frac{I_d\times 100}{X\times t}\right]$$

$$= \left[\frac{72\times 100}{600\times 6}\right] = 2\%$$

Shortcut Approach - 8

A certain sum of money at SI amounts to ₹ A_1 in t years at r% per annum, then the time in which it will amount to ₹ A_2 at the same rate of interest is given by $\left[\frac{A_2}{A_1}\left(t + \frac{100}{r}\right) - \frac{100}{r}\right]$ **years.**

Illustration – 21

A certain sum of money amounts to ₹ 5000 in 5 years at 10% per annum. In how many years will it amount to ₹ 6000 at the same rate?

(a) 8 years (b) 6 years (c) 10 years (d) 9 years

Shortcut Solution : (a)

$$\text{Time} = \left[\frac{A_2}{A_1}\left(t + \frac{100}{r}\right) - \frac{100}{r}\right]$$

GP_3386

$$= \left[\frac{6000}{5000}\left(5+\frac{100}{10}\right)-\frac{100}{10}\right]$$

$$= \left[\frac{6}{5}\times 15-10\right] = 8 \text{ years.}$$

Shortcut Approach - 10

If a sum of money becomes 'n' times at the simple interest rate of r% per annum, then it will become 'm' times at the simple interest rate of $\left[\left(\frac{m-1}{n-1}\right)\times r\right]$ per cent

Illustration – 22

A sum of money becomes two times at the simple interest rate of 2% per annum. At what rate per cent will it become five fold?

(a) 10% (b) 8% (c) 6% (d) 9%

Shortcut Solution : (b)

$$\text{Rate of Interest} = \left[\left(\frac{m-1}{n-1}\right)\times r\right]$$

$$= \left[\left(\frac{5-1}{2-1}\right)\times 2\right] = 8\%$$

Shortcut Approach - 8

₹ X is divided into two parts such that if one part be invested at r_1% and the other at r_2%, the annual interest from both the investments is ₹ A. Then the first part is given by $\left[\frac{100A - r_2X}{r_1 - r_2}\right]$.

Illustration – 23

Aniket deposited two parts of a sum of ₹ 25000 in different banks at the rates of 15% per annum and 18% per annum respectively. In one year he got ₹4050 as the total interest. What was the amount deposited at the rate of 18% per annum?

(a) ₹ 9000 (b) ₹ 18000
(c) ₹ 15000 (d) None of these

Shortcut Solution : (d)

Applying the formula, we have the amount deposited at the rate of 15% per annum

$$= \frac{100\times 4050 - 18\times 25000}{15-18} = ₹15000$$

$\therefore$ Amount deposited at the rate of 18% per annum

$= ₹25000 - ₹15000 = ₹10000.$

Shortcut Approach - 8

Some amount out of ₹ P was lent at r_1% per annum and the remaining at r_2% per annum. If the total simple interest from both the fractions in t years was ₹ 'A' then the sum lent at r_1% per annum was given by ₹ $\left[\frac{100A - Pr_2t}{(r_1 - r_2)t}\right]$

Illustration – 24

Some amount out of ₹ 950 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 years was ₹ 200, find the sum lent at 6% per annum.

(a) ₹ 700 (b) ₹ 100 (c) ₹ 250 (d) ₹ 450

Shortcut Solution : (b)

$$\text{Sum} = \left[\frac{100A - Pr_2t}{(r_1 - r_2)t}\right] = \left[\frac{100\times 200 - 950\times 4\times 5}{(6-4)\times 5}\right]$$

$$= ₹100$$

Shortcut Approach - 8

If a person borrows ₹ X from a bank at simple interest and after t_1 years he paid ₹ x_1 to the bank and at the end of t_2 years from the date of borrowing he paid ₹ x_2 to the bank to settle the account, then the rate of interest is given by $\left[\frac{x_1 + x_2 - X}{x_1t_1 + t_2(X - x_1)}\right]\times 100\%$

Illustration – 25

Uday borrows ₹ 5000 from a bank at SI. After 5 years he paid ₹ 1700 to the bank and at the end of 5 years from the date borrowing he paid ₹ 3550 to the bank to settle the account. Find the rate of interest.

(a) 1% (b) 1.5% (c) 2% (d)10%

Shortcut Solution : (a)

$$\text{Rate of Interest} = \left[\frac{x_1 + x_2 - X}{x_1t_1 + t_2(X - x_1)}\right]\times 100\%$$

$$= \left[\frac{1700 + 3550 - 5000}{1700\times 5 + 5(5000 - 1700)}\right]\times 100\%$$

$$= \frac{250}{25000}\times 100\% = 1\%$$

Shortcut Approach - 10

A person invested $\frac{1}{n_1}$ of his capital at $x_1\%$, $\frac{1}{n_2}$ at $x_2\%$ and the remainder $\frac{1}{n_3}$ at $x_3\%$. If his annual income is ₹A, the capital is given by ₹ $\left[\frac{A\times 100}{\frac{x_1}{n_1}+\frac{x_2}{n_2}+\frac{x_3}{n_3}}\right]$.

Illustration – 26

Out of a certain sum, $\frac{1}{3}$rd is invested at 3%, $\frac{1}{6}$th at 6% and the rest at 8%. If the simple interest for 2 years from all these investments amounts to ₹ 600, find the original sum.
(a) ₹ 5000 (b) ₹ 6000 (c) ₹ 5200 (d) ₹ 5500

Shortcut Solution : (a)

Here A = Annual income = ₹ $\frac{600}{2}$ = ₹ 300

$$\text{Original sum} = \left[\frac{A\times 100}{\frac{x_1}{n_1}+\frac{x_2}{n_2}+\frac{x_3}{n_3}}\right] = \left[\frac{300\times 100}{\frac{3}{3}+\frac{6}{6}+\frac{8}{2}}\right]$$

$$= \left[\frac{30000}{6}\right] = ₹5000$$

Shortcut Approach - 10

Let, Amount = ₹ A
Principal = ₹ P
Time = t years

Rate of interest $(r) = n\times 100\left[\left(\frac{A}{P}\right)^{\frac{1}{t}\times n}-1\right]\%$ per annum.

Where, n = 1, when the interest is compounded yearly
n = 2, when the interest is compounded half-yearly
n = 4, when the interest is compounded quaterly
n = 12, when the interest is compounded monthly
If the interest is compounded yearly, then n = 1

$$\therefore\ r = 100\left[\left(\frac{A}{P}\right)^{1/t}-1\right]\% \text{ per annum}$$

Illustration – 27

At what rate per cent compound interest will ₹ 625 amount to ₹ 676 in 2 years?
(a) 3% (b) 2% (c) 4% (d) 5%

Shortcut Solution : (c)

$$\text{Rate of Interest} = 100\left[\left(\frac{A}{P}\right)^{1/t}-1\right]\%$$

$$= 100\left[\left(\frac{676}{625}\right)^{\frac{1}{2}}-1\right] = 100\left[\frac{26}{25}-1\right] = 4\%$$

Shortcut Approach - 10

A sum of money, placed at compound interest, becomes n times in t years and m times in x years. We calculate the value of x from the equation given below $\left(n^{1/t} = m^{1/x}\right)$.

Illustration – 28

A sum of money placed at compound interest doubles itself in 6 years. In how many years will it amount to 16 times itself?
(a) 24 years (b) 26 years (c) 22 years (d) 20 years

Shortcut Solution : (a)

We can calculate the value of x by the given equation

$$(n)^{1/t} = (m)^{1/x}$$

From question, putting the values of n, m and t

$$\Rightarrow (2)^{1/6} = (16)^{1/x} \Rightarrow (2)^{1/6} = (2)^{4/x}$$

By comparing both side

$$\Rightarrow \left[\frac{1}{6}=\frac{4}{x}\right] \Rightarrow x = 24$$

Shortcut Approach - 8

If a certain sum becomes 'm' times in 't' years, the rate of compound interest r is equal to $\left[(m)^{1/t}-1\right]\times 100$.

Illustration – 29

At what rate per cent will the compound interest, does a sum of money become 27 times in 3 years?
(a) 100% (b) 150% (c) 75% (d) 200%

Shortcut Solution : (d)

$$\text{Rate of compound interest} = \left[(m)^{1/t}-1\right]\times 100\%$$

$$= \left[(27)^{1/3}-1\right]\times 100\% = \left[(3)-1\right]\times 100\%$$

$$= 2\times 100 = 200\%$$

GP_3386

Shortcut Approach - 18

Let Principal = P
Rate = r% per annum
Amount = A and

Note: Time must be given in the form of fraction like $2\frac{1}{2}$ years.

$$\text{Amount (A)} = P\left[1+\frac{r}{100}\right]^2 \times \left[1+\frac{\frac{r}{2}}{100}\right]$$

Illustration – 30

On what sum will the compound interest for $2\frac{1}{2}$ years at 10% amount to ₹ 6352.50 ?
(a) ₹ 4900 (b) ₹ 5500 (c) ₹ 5000 (d) ₹ 5800

Shortcut Solution : (c)

$$A = P\left[1+\frac{r}{100}\right]^2 \times \left[1+\frac{\frac{r}{2}}{100}\right]$$

$$\Rightarrow 6352.50 = P\left[1+\frac{10}{100}\right]^2 \times \left[1+\frac{10}{2\times 100}\right]$$

$$\Rightarrow 6352.50 \times \left(\frac{10}{11}\right)^2 \times \frac{20}{21} = P$$

$$\Rightarrow P = ₹\ 5000$$

Shortcut Approach - 19

On a certain sum of money, the difference between compound interest and simple interest for 3 years at r% per annum is given by $P\left(\frac{R}{100}\right)^2\left(\frac{R}{100}+3\right)$ and the difference between CI and SI for 2 years = $P\left(\frac{R}{100}\right)^2 = \frac{SI \times R}{200}$

Note : SI and CI are given on the same sum, at the same rate and for the same time duration.

Illustration – 31

Find the difference between the simple and compound interest on ₹ 10000 for 3 years at 3 per cent.
(a) ₹ 27.8 (b) ₹ 27.27 (c) ₹ 37.27 (d) ₹ 37.8

Shortcut Solution : (b)

$$\text{Difference} = \frac{\text{Sum} \times r^2(300+r)}{(100)^3}$$

$$= \frac{10000 \times (3)^2(300+3)}{(100)^3} = 27.27$$

Illustration – 32

On a certain sum of money, the simple interest for 2 years is ₹ 160 at the rate of 5 % per annum. Find the difference in CI and SI.
(a) ₹ 4 (b) ₹ 5 (c) ₹ 6 (d) ₹ 8

Shortcut Solution : (a)

$$\text{Difference in C.I and S.I} = \left(\frac{xr}{200}\right) = \left(\frac{160\times 5}{200}\right) = ₹\ 4.$$

Shortcut Approach - 20

To find the ratio of Compound Interest (CI) to Simple Interest (SI) on a certain sum at r% for 2 years, we use the following formula

$$\frac{CI}{SI} = \frac{r}{200} + 1.$$

Note : This formula is applicable only for 2 years.

Illustration – 33

Find the ratio of CI to SI on a certain sum at 10% per annum for 2 years.
(a) 7 : 5 (b) 21 : 20 (c) 8 : 5 (d) 20 : 19

Shortcut Solution : (b)

$$\text{Ratio of CI to SI} = \frac{r}{200} + 1 = \left[\frac{10}{200} + 1\right] = \frac{210}{200}$$

$$= 21 : 20$$

Shortcut Approach - 21

If the compound interest on a certain sum for 2 years is ₹ 'C' and simple interest is ₹ 'S', then the rate of interest per annum is $\left[\frac{2\times(C-S)}{S}\times 100\right]\%$ per or $\frac{2\times \text{Diff.} \times 100}{SI}$ and sum = $\frac{\text{Diff.} \times (100)^2}{r^2}$

Illustration – 34

The compound interest on a certain sum for 2 years is ₹ 105 and simple interest is ₹ 100. Find the rate of interest per annum and the sum.

(a) 10%, ₹ 500 (b) 10%, ₹ 1000
(c) 20%, ₹ 1000 (d) None of these

Shortcut Solution : (a)

$$\text{Rate of Interest} = \left[\frac{2(C-S)}{S}\times 100\right]\%$$

$$= \left[\frac{2(105-100)}{100}\times 100\right] = \left[\frac{2\times 5}{100}\times 100\right] = 10\% \text{ p.a.}$$

$$\text{and Sum} = \frac{\text{Diff}\times 10000}{r^2} = \frac{5\times 10000}{(10)^2} = ₹\,500$$

Shortcut Approach - 10

If an amount of money grows upto ₹ A_1 in n years and upto ₹ A_2 in $(n+1)$ years on compound interest, then the sum is given by ₹ $\left[A_1\left(\frac{A_1}{A_2}\right)^n\right]$

and the rate $= \left(\frac{A_2 - A_1}{A_1}\times 100\right)\%$

Illustration – 35

An amount of money grows upto ₹ 2750 in 2 years and upto ₹ 3125 in 3 years on compound interest. Find the sum

(a) ₹ 2129.6 (b) ₹ 1229.6
(c) ₹ 2219.6 (d) Data inadequate

Shortcut Solution : (a)

$$\text{Sum} = \left[A_1\left(\frac{A_1}{A_2}\right)^n\right] = \left[2750\left(\frac{2750}{3125}\right)^2\right]$$

$$= 2750\times\left[\frac{7562500}{9765625}\right] = ₹\,2129.60.$$

Shortcut Approach - 21

If the compound rate of interest for the first t_1 years is r_1%, for the next t_2 year is r_2%, for the next t_3 years is r_3%... and the last t_n years is r_n%, then compound interest on ₹ x for $(t_1 + t_2 + t_3 + ...t_n)$ years is

$$\left[x\left(1+\frac{r_1}{100}\right)^{t_1}\left(1+\frac{r_2}{100}\right)^{t_2}...\left(1+\frac{r_n}{100}\right)^{t_n}\right] - x.$$

Illustration – 36

Find the compound interest on ₹ 8000 in 2 years, the rate of interest being 5% for the first year and 10% for the second year.

(a) ₹ 1340 (b) ₹ 1420 (c) ₹ 1240 (d) None of these

Shortcut Solution : (c)

Compound Interest

$$= \left[x\left(1+\frac{r_1}{100}\right)^{t_1}\cdot\left(1+\frac{r_2}{100}\right)^{t_2}.....\left(1+\frac{r_n}{100}\right)^{t_n}\right] - x$$

$$= \left[8000\left(1+\frac{5}{100}\right)^1\cdot\left(1+\frac{10}{100}\right)^1\right] - 8000$$

$$= 9240 - 8000 = ₹\,1240$$

INSTALMENTS

To increase their sales, different business organisations offer different schemes to the buyers to enable them to buy costly things even if their income is very low. One such scheme is the **instalment scheme**.

While purchasing any thing under instalment scheme, some payment has to be made initially. This payment is called **Cash Down Payment**. The remaining amount is paid in equal monthly, quaterly or annual instalments as decided at the time of purchase. Sometime there is no cash down payment, then whole amount is paid under instalment scheme.

In instalment scheme, the buyer has to pay more amount because in addition to the selling price/ cash price, the buyer also pay the interest on it. Here selling price/ cash price means the amount for which the article can be purchased on full payment at the time of purchase.

In general, the value of each instalment is kept constant even when the interest charged on each instalment vary. When the period over which the instalment scheme is operative is less than a year. The payment is made in a specified number of equal monthly instalments. In such cases normally simple interest is charged. In all other cases compound interest is charged.

Shortcut Approach - 10

Let an article of ₹ P is purchased on cash down payment of ₹ X and n equal montly instalments of ₹ x. If r% simple interest is charged in this instalment scheme, then the value of each instalment is given by

$$(P-X) + \frac{(P-X)\times r\times n}{100\times 12} = x + \left(x + \frac{x\times r\times 1}{100\times 12}\right)$$

GP_3386

$$+\left(x+\frac{x\times r\times 2}{100\times 12}\right)+.....+\left(x+\frac{x\times r\times(n-1)}{100\times 12}\right)$$

or

Remaining amount $\left[1+\frac{r\times \text{no. of instalments}}{12\times 100}\right]$

= (Value of each instalments × Number of instalments) + $\frac{\text{Value of each instalments}}{12\times 100}(r+2r+3r+...+(n-1)r)$

or

$(P-X)\left[1+\frac{r\times n}{1200}\right]=(x\times n)+\frac{x}{1200}[r+2r+3r+.....+(n-1)r]$

Illustration – 1

A scooty is sold by an automobile agency for ₹19200 cash or for ₹ 4800 cash down payment together with five equal monthly instalments. If the rate of interest charged by the company is 12% per annum (simple interest), find the value of an instalment.

(a) ₹ 2548.50 **(b) ₹ 2640.60**
(c) ₹ 2899.75 **(d) ₹ 2964.70**

Shortcut Solution : (d)

$$(P-X)+\frac{(P-X)\times r\times n}{100\times 12}=x+\left(x+\frac{x\times r\times 1}{100\times 12}\right)+\left(x+\frac{x\times r\times 2}{100\times 12}\right)+.....+\left(x+\frac{x\times r\times(n-1)}{100\times 12}\right)$$

Here, P = ₹ 19200, X = ₹ 4800, n = 5, r = 12, x = ?

$$\therefore \quad (19200-4800)+\frac{(19200-4800)\times 12\times 5}{100\times 12}$$

$$=x+\left(x+\frac{x\times 12\times 1}{100\times 12}\right)+\left(x+\frac{x\times 12\times 2}{100\times 12}\right)+\left(x+\frac{x\times 12\times 3}{100\times 12}\right)+\left(x+\frac{x\times 12\times 4}{100\times 12}\right)$$

$$\Rightarrow \quad 14400+144\times 5=5x+\frac{x}{100}(1+2+3+4)$$

$$\Rightarrow \quad 14400+720=5x+\frac{x}{10}$$

$$\Rightarrow \quad 15120=\frac{51x}{10}\Rightarrow x=2964.70$$

Hence, value of an instalment = ₹ 2964.70

Illustration – 2

A television is marked at ₹10000 cash or ₹ 4000 as cash down payment and three monthly instalments of ₹ 2100. Find the simple rate of interest for this instalment plan.

(a) 27.9% per annum **(b) 28.5% per annum**
(c) 30.8% per annum **(d) 31% per annum**

Shortcut Solution : (c)

$$(P-X)+\frac{(P-X)\times r\times n}{100\times 12}=x+\left(x+\frac{x\times r\times 1}{100\times 12}\right)+\left(x+\frac{x\times r\times 2}{100\times 12}\right)+.....+\left(x+\frac{x\times r\times(n-1)}{100\times 12}\right)$$

Here, P = ₹ 10000, X = ₹ 4000, x = ₹ 2100, n = 3, r = ?

$$\therefore \quad (10000-4000)+\frac{6000\times r\times 3}{100\times 12}$$

$$=2100+\left(2100+\frac{2100\times r\times 1}{100\times 12}\right)+\left(2100+\frac{2100\times r\times 2}{100\times 12}\right)$$

$$\Rightarrow 6000+\frac{180r}{12}=6300+\frac{21r}{12}+\frac{42r}{12}$$

$$\Rightarrow r\left(\frac{60}{4}-\frac{7}{4}-\frac{14}{4}\right)=300 \Rightarrow r\times\frac{39}{4}=300$$

$$\therefore \quad r=\frac{300\times 4}{39}=30.8\%\text{ per annum}$$

Shortcut Approach - 10

The annual payment that will discharge a debt of ₹ P due in t years at the rate of simple interest r% per annum $=\frac{100P}{100t+\frac{rt(t-1)}{2}}$

Illustration – 3

Find the annual payment that will discharge a debt of ₹ 5400 due in 5 years at 4% per annum simple interest.

(a) ₹ 950 **(b) ₹ 960** **(c) ₹ 990** **(d) ₹ 1000**

Shortcut Solution : (d)

$$\text{Annual payment}=\frac{100P}{100t+\frac{rt(t-1)}{2}}$$

$$=\frac{100\times 5400}{100\times 5+\frac{4\times 5(5-1)}{2}}=\frac{100\times 5400}{540}=₹\,1000$$

Shortcut Approach - 10

A sum of ₹ P is to be paid back in n equal annual instalments. If the interest is compounded annually at r% per annum, then the value of each instalment

$$= \frac{P}{\left(\frac{100}{100+r}\right)+\left(\frac{100}{100+r}\right)^2+....+\left(\frac{100}{100+r}\right)^n}$$

Illustration – 4

A sum of ₹ 2600 is to paid back in 2 equal annual instalments. What is the annual instalment, if the rate of interest is 8% per annum compounded annually?

(a) ₹ 1448 (b) ₹ 1728 (c) ₹ 1458 (d) ₹ 1878

Shortcut Solution : (c)

$$\text{Annual instalment} = \frac{P}{\left(\frac{100}{100+r}\right)+\left(\frac{100}{100+r}\right)^2}$$

$$= \frac{2600}{\left(\frac{100}{100+8}\right)+\left(\frac{100}{100+8}\right)^2} = \frac{2600}{\frac{25}{27}\left(1+\frac{25}{27}\right)}$$

$$= \frac{2600\times 27\times 27}{25\times 52} = 2\times 27\times 27 = ₹\,1458$$

Shortcut Approach - 21

If a man borrows some money on compound interest at r% per annum compounded annually and returns it in n yearly equal instalments of ₹ x, then

(i) Amount borrowed

$$= x\left[\left(\frac{100}{100+r}\right)+\left(\frac{100}{100+r}\right)^2+...+\left(\frac{100}{100+r}\right)^n\right]$$

(ii) Total interest charged

$$= x\left[n-\left\{\left(\frac{100}{100+r}\right)+\left(\frac{100}{100+r}\right)^2+...+\left(\frac{100}{100+r}\right)^n\right\}\right]$$

Illustration – 5

A man borrows some money on compound interest and returns it in two equal yearly instalments. If the rate of interest is 5% and yearly instalment is ₹ 441, find the amount borrowed and total interest paid.

(a) ₹ 820, ₹ 62 (b) ₹ 800, ₹ 70

(c) ₹ 800, ₹ 62 (d) ₹ 924, ₹ 62

Shortcut Solution : (a)

(i) Amount borrowed

$$= x\left[\left\{\left(\frac{100}{100+r}\right)+\left(\frac{100}{100+r}\right)^2+...+\left(\frac{100}{100+r}\right)^n\right\}\right]$$

$$= 441\left[\left\{\left(\frac{100}{100+5}\right)+\left(\frac{100}{100+5}\right)^2\right\}\right] = 441\left[\frac{20}{21}+\frac{400}{441}\right]$$

= 420 + 400 = ₹ 820

(ii) Total interest paid

= Total amount paid – Amount borrowed

= 2 × 441 – 820 = 882 – 820 = ₹ 62

Shortcut Approach - 18

A sum of ₹ P is to be paid back in n annual instalments. If the interest is compounded annually on the balance at r% per annum and is to be included in each instalment, then

Value of m^{th} instalment $= \frac{P}{n}\left[1+\frac{r(n-m+1)}{100}\right]$

i.e. Value of 1^{st} instalment $= \frac{P}{n}\left[1+\frac{rn}{100}\right]$

Value of 2^{nd} instalment $= \frac{P}{n}\left[1+\frac{r(n-1)}{100}\right]$ **and so on.**

Illustration – 6

A sum of ₹ 5600 is paid back in four yearly instalments. How much is each instalment, if the interest is compounded annually on the balance at 8% per annum and is to be included in each instalment?

Shortcut Solution :

$$\text{Value of first instalment} = \frac{P}{n}\left[1+\frac{rn}{100}\right]$$

$$= \frac{5600}{4}\left[1+\frac{8\times 4}{100}\right] = ₹\,1848$$

$$\text{Value of second instalment} = \frac{P}{n}\left[1+\frac{r(n-1)}{100}\right]$$

$$= \frac{5600}{4}\left[1+\frac{8\times 3}{100}\right] = ₹\,1736$$

$$\text{Value of third instalment} = \frac{P}{n}\left[1+\frac{r(n-2)}{100}\right]$$

$$= \frac{5600}{4}\left[1+\frac{8\times 2}{100}\right] = ₹1624$$

$$\text{Value of fourth instalment} = \frac{P}{n}\left[1+\frac{r(n-3)}{100}\right]$$

$$= \frac{5600}{4}\left[1+\frac{8\times 1}{100}\right] = ₹1512$$

Shortcut Approach - 10

A person buys an item for ₹ P cash or on the terms that he is required to pay ₹ X cash down payment followed by ₹ x_1 at the end of first year, ₹ x_2 at the end of second year, ..., ₹ x_n at the end of the nth year. Interest is charged at the rate of r% per annum, then

(i) Cash price of the item (P)

$$= X + \frac{100}{100+r}\left[x_1 + x_2\left(\frac{100}{100+r}\right) + x_3\left(\frac{100}{100+r}\right)^2 + ... + x_n\left(\frac{100}{100+r}\right)^n\right]$$

(ii) Total interest charged

= (Cash down payment) + (Sum of all instalments) − (Cash Price)

$$= X + (x_1 + x_2 + ... + x_n) - P$$

Illustration – 7

A sewing machine is available at ₹ 240 cash down payment followed by 3 annual instalments of ₹ 380, ₹ 240 and ₹ 200 payable at the end of first year, second year and third year respectively. If the rate of interest is 25% per annum compound interest, find the cash price and total interest being charged under the instalment plan.

(a) ₹ 800, ₹ 260 **(b) ₹ 900, ₹ 240**

(c) ₹ 800, ₹ 460 **(d) ₹ 900, ₹ 260**

Shortcut Solution : (a)

Cash price

$$= X + \frac{100}{100+r}\left[x_1 + x_2\left(\frac{100}{100+r}\right) + x_3\left(\frac{100}{100+r}\right)^2 + ... + x_n\left(\frac{100}{100+r}\right)^n\right]$$

$$= 240 + \frac{100}{100+25}\left[380 + 240\left(\frac{100}{100+25}\right) + 200\left(\frac{100}{100+25}\right)^2\right]$$

$$= 240 + \frac{4}{5}\left[380 + 240\times\frac{4}{5} + 200\times\frac{16}{25}\right]$$

$$= 240 + \frac{4}{5}[380 + 192 + 128]$$

$$= 240 + \frac{4}{5}\times 700 = ₹800$$

$$\text{Total interest charged} = X + (x_1 + x_2 + ... + x_n) - P$$

$$= 240 + (380 + 240 + 200) - 800 = ₹260$$

LEVEL - 1

1. A certain sum of money triple itself in 8 years. In how many years it will be five times?
(a) 22 years (b) 16 years
(c) 20 years (d) 24 years

2. In how many minimum number of complete years, the interest on ₹ 212.50 *P* at 3% per annum will be in exact number of rupees?
(a) 6 (b) 8 (c) 9 (d) 7

3. At a simple interest ₹ 800 becomes ₹ 956 in three years. If the interest rate, is increased by 3%, how much would ₹ 800 become in three years?
(a) ₹ 1020.80 (b) ₹ 1004
(c) ₹ 1028 (d) Data inadequate

4. An amount of ₹ 12820 due 3 years hence, is fully repaid in three annual instalments starting after 1 year The first instalment is 1/2 the second instalment and the second instalment is 2/3 of the third instalment. If the rate of interest is 10% per annum, find the first instalment.
(a) ₹ 2400 (b) ₹ 1800 (c) ₹ 2000 (d) ₹ 2500

5. What will be the ratio of simple interest earned by certain amount at the same rate of interest for 6 years and that for 9 years?
(a) 1 : 3 (b) 1 : 4
(c) 2 : 3 (d) None of these

6. Two equal sums were lent, one at the rate of 11% p.a. for five years and the other at the rate of 8% p.a. for six years, both under simple interest. If the difference in interest accrued in the two cases is ₹ 1008. find the sum.
(a) ₹ 11,200 (b) ₹ 5,600
(c) ₹ 12,600 (d) ₹ 14,400

7. A sum is invested at compound interest payable annually. The interest in two successive years was ₹ 225 and ₹ 236.25. Find the rate of interest
(a) 2% (b) 3% (c) 5% (d) 9%

8. A sum of money invested at simple interest triples itself in 8 years. How many times will it become in 20 years time?
(a) 8 times (b) 7 times (c) 6 times (d) 9 times

9. What will be the difference in simple and compound interest on ₹ 2000 after three years at the rate of 10 percent per annum?
(a) ₹ 160 (b) ₹ 42 (c) ₹ 62 (d) ₹ 20

10. Mr. *X* invested an amount for 2 years at 15 percent per annum at simple interest. Had the interest been compounded, he would have earned ₹ 450/- more as interest. What was the amount invested?
(a) ₹ 22000 (b) ₹ 24000
(c) ₹ 25000 (d) None of these

11. Two friends *A* and *B* jointly lent out ₹ 81,600 at 4% per annum compound interest. After 2 years *A* gets the same amount as *B* gets after 3 years. The investment made by *B* was
(a) ₹ 40,000 (b) ₹ 30,000
(c) ₹ 45,000 (d) ₹ 38,000

12. Two equal sums of money were invested, one at 4% and the other at 4.5%. At the end of 7 years, the simple interest received from the latter exceeded to that received from the former by ₹ 31.50. Each sum was :
(a) ₹ 1,200 (b) ₹ 600 (c) ₹ 750 (d) ₹ 900

13. Nitin borrowed some money at the rate of 6% p.a. for the first three years, 9% p.a. for the next five years and 13% p.a. for the period beyond eight years If the total interest paid by him at the end of eleven years is ₹ 8160, how much money did he borrow?
(a) ₹ 8000 (b) ₹ 10,000
(c) ₹ 12,000 (d) Data inadequate

14. A lent ₹ 5000 to B for 2 years and ₹ 3000 to C for 4 years on simple interest at the same rate of interest and received ₹ 2200 in all from both of them as interest. The rate of interest per annum is:
(a) 5% (b) 7% (c) $7\frac{1}{8}\%$ (d) 10%

15. A sum of money lent out at simple interest amounts to ₹ 720 after 2 years and to ₹ 1020 after a further period of 5 years. The sum is
[SSC CGL Tier I 2014]
(a) ₹ 500 (b) ₹ 600 (c) ₹ 700 (d) ₹ 710

16. The difference between the simple interest received from two different sources on ₹ 1500 for 3 years is ₹ 13.50. The difference between their rates of interest is:
(a) 0.1% (b) 0.2% (c) 0.3% (d) 0.4%

17. An amount of ₹ 1,00,000 is invested in two types of shares. The first yields an interest of 9% p.a. and the second, 11% p.a. If the total interest at

GP_3386

the end of one year is $9\frac{3}{4}\%$, then the amount invested in each share was:
(a) ₹ 52,500; ₹ 47,500 (b) ₹ 62, 500; ₹ 37,500
(c) ₹ 72,500: ₹ 27,500 (d) ₹ 82, 500; ₹ 17,500

18. A man invests ₹ 3000 at a rate of 5% per annum. How much more should he invest at a rate of 8%, so that he can earn a total of 6% per annum?
(a) ₹ 1200 (b) ₹ 1300 (c) ₹ 1500 (d) ₹ 2000

19. If the difference between S.I and C.I for 2 years on a sum of money lent at 5% is ₹ 6, then the sum be.
(a) ₹ 2200 (b) ₹ 2400 (c) ₹ 2600 (d) ₹ 2000

20. What will ₹ 1000 be worth after three years if it earns interest at the rate of 5% compounded annually?
(a) ₹ 1075 (b) ₹ 1257 (c) ₹ 1157 (d) ₹ 1300

21. If ₹ 1 produces ₹ 9 in 60 years at simple interest, the rate % p.a. is
(a) $13\frac{1}{3}\%$ (b) 14% (c) 15% (d) $12\frac{1}{2}\%$

22. Mr. Thomas invested an amount of ₹ 13,900 divided in two different schemes A and B at the simple interest rate of 14% p.a. and 11% p.a. respectively. If the total amount of simple interest earned in 2 years be ₹ 3508, what was the amount invested in Scheme B?
(a) ₹ 6400 (b) ₹ 6500 (c) ₹ 7200 (d) ₹ 7500

23. A milk man borrowed ₹ 2,500 from two money lender For one loan, he paid 5% p.a. and for the other, he paid 7% p.a. The total interest paid for two years was ₹ 275. How much did he borrow at 7% rate?
(a) ₹ 600 (b) ₹ 625 (c) ₹ 650 (d) ₹ 675

24. In what time will ₹ 72 become ₹ 81 at $6\frac{1}{4}\%$ p.a. simple interest?
(a) 1 year 6 months (b) 2 years
(c) 1 years (d) $2\frac{1}{2}$ years

25. A man borrows ₹ 4000 and pays back after 5 years at 15% simple interest. The amount paid by the man is :
(a) ₹ 1800 (b) ₹ 4800 (c) ₹ 7500 (d) ₹ 7000

26. A man loses ₹ 55.50 yearly when the annual rate of interest falls from 11.5 % to 10%. His capital (in rupees) is: **[Bank PO 2014]**
(a) ₹3700 (b) ₹7400 (c) ₹8325 (d) ₹11100

27. A man borrowed ₹ 2500 at 18% p.a. and ₹ 1500 at 10% simple interest for the same period. The rate of interest for the whole is **[Bank Clerk 2015]**
(a) 10% (b) 12% (c) 15% (d) 20%

28. What is the rate of simple interest at which ₹ 14000 gives interest of ₹ 1960 in two years?
(a) 4 % (b) 5 % (c) 7 % (d) 10 %

29. If the rate of simple interest is 12% per annum, the amount that would fetch interest of ₹ 6000 per annum is :
(a) ₹ 7200 (b) ₹ 4800
(c) ₹ 50000 (d) ₹ 72000

30. In what time will a sum of money double itself @ 20% per annum (p.a.) simple interest?
(a) 10 years (b) 5 years
(c) 2 years (d) 4 years

31. Out of a sum of ₹ 625, a part was lent at 5% SI and the other at 10% SI. If the interest on the first part after 2 years is equal to the interest on the second part after 4 years, then the second sum (in ₹) is :
(a) 250 (b) 300 (c) 125 (d) 275

32. A sum was put at a SI at certain rate for 2 years. Had it been put at 3% higher rate, it would fetched ₹450 more as interest. The sum is. **[SSC CGL Tier I 2015]**
(a) ₹ 7500 (b) ₹ 6000 (c) ₹ 5000 (d) ₹ 4500

33. A sum was put at SI at a certain rate for 2 years. Had it been put at 4% lower rate, it would have fetched ₹100 less. Find the sum. **[Bank LDC 2015]**
(a) ₹1000 (b) ₹1250 (c) ₹1500 (d) ₹1750

34. A certain sum in certain time becomes ₹500 at the rate of 8% per annum SI and the same sum amounts to ₹200 at the rate of 2% p.a. SI in the same duration. Find the sum and time. **[CDS 2010]**
(a) ₹400; 10 yrs (b) ₹300; 20 yrs
(c) ₹200; 30 yrs (d) ₹100; 50 yrs

35. A sum of money trebles (i.e., 3 times) in 15 years at r% of simple interest per annum. What is the value of r?
(a) 12% (b) $\frac{40}{3}\%$
(c) $\frac{50}{3}\%$ (d) can't be determined

36. A sum of money doubles itself in 12 years. In how many years would it treble itself?
(a) 36 years (b) 18 years
(c) 24 years (d) 15 years

37. What annual payment will discharge a debt of ₹ 580 in 5 years, the rate being 8% p.a.?
(a) ₹ 120 (b) ₹ 100 (c) ₹ 80 (d) ₹ 78

38. The compound interest on ₹ 10000 at 20% p.a. in 4 years:
(a) ₹ 10736 (b) ₹ 736
(c) ₹ 20736 (d) ₹ 7280

39. Find the amount of ₹ 1700 invested at 16% half yearly at simple interest for one year :
(a) ₹ 2100 (b) ₹ 2244 (c) ₹ 2200 (d) ₹ 2500

40. The compound interest on ₹ 1000 at 10% p.a. in 3 years is :
(a) ₹ 331 (b) ₹ 1331 (c) ₹ 133 (d) ₹ 300

41. A sum of ₹ 400 would become ₹ 441 after 2 years at r% compound interest, find the value of 'r' :
(a) 10% (b) 5% (c) 15% (d) 20%

42. Find the compound interest on ₹ 2000 at 15% p.a. for 2 yr 4 months, compounded annually. **[SSC CHSL 2014]**
(a) ₹ 776 .05 (b) ₹ 777. 25
(c) ₹ 707 .25 (d) ₹ 717. 65

43. A sum, at the compound rate of interest, becomes $2\frac{1}{2}$ times in 6 yrs. The same sum becomes what times in 18 yrs.? **[SSC CGL Tier I 2016]**
(a) $\frac{5}{2}$ (b) $\frac{25}{4}$ (c) $\frac{125}{8}$ (d) $\frac{625}{16}$

44. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is ₹ 631. Find the sum. **[Bank PO 2012]**
(a) ₹ 63,100 (b) ₹ 73,100
(c) ₹ 53,100 (d) ₹ 68,100

45. A sum of money becomes eight times of itself in 3 years at compound interest. The rate of interest per annum is : **[SSC CGL 2011]**
(a) 100% (b) 80% (c) 20% (d) 10%

46. What is the difference between the compound interest and the simple interest calculated on an amount of ₹16200 at the end of 3 yrs at 25% p.a.? **[IBPS Clerk 2011]**
(a) ₹3213.44 (b) ₹3302.42
(c) ₹3495.28 (d) ₹3290.63

47. On what sum does the difference between the compound interest and the simple interest for 3 years at 10% is ₹31? **[SSC CHSL 2011]**
(a) ₹1500 (b) ₹1200 (c) ₹1100 (d) ₹1000

48. Compound interest on certain sum for 2 years at 3% p.a. compounded annually, is ₹101.50. The simple interest on the same sum at the same rate and for the same time duration will be: **[SSC CGL 2010]**
(a) ₹96 (b) ₹80 (c) ₹90 (d) ₹100

49. The compound interest on a sum for 2 years is ₹832 and the simple interest on the same sum at the same rate for the same period is ₹800. What is the rate of interest? **[CDS 2009]**
(a) 6% (b) 8% (c) 10% (d) 12%

50. On a certain sum, compound interest and simple interest for two years are ₹520 and ₹500 respectively. Find the principal. **[SBI PO 2013]**
(a) ₹ 2000 (b) ₹ 3000 (c) ₹ 3125 (d) ₹ 3500

51. A sum of money invested at compound interest amount in 3 years to ₹2400 and in 4 years to ₹2520. The interest rate per annum is : **[SSC CGL 2002]**
(a) 5% (b) 6% (c) 10% (d) 12%

52. A sum of money invested amounts to ₹800 in 2 years and ₹ 840 in 3 years. Find the rate of interest per annum and the sum. **[SSC CGL Tier I 2014]**
(a) 10%, ₹ 825.62 (b) 10%, ₹ 800
(c) 5%, ₹720 (d) 5%, ₹ 725-62

53. Find the compound interest on ₹10000 for 3 years if the rate of interest is 4% for the first year, 5% for the second year and 6% for the third year. **[Bank PO 2016]**
(a) ₹ 1000.50 (b) ₹ 1620.25
(c) ₹ 1700 (d) ₹ 1575.20

54. The difference between simple and compound interest on ₹ 6000 for 1 year at 20% per annum reckoned half yearly is :
(a) ₹ 120 (b) ₹ 60 (c) ₹ 180 (d) ₹ 72

55. Amit borrowed ₹ 800 at 10% rate of interest. He repaid ₹ 400 at the end of first year. What is the amount required to pay at the end of second year to discharge his loan which was calculated at compound interest?
(a) ₹ 420 (b) ₹ 440 (c) ₹ 450 (d) ₹ 528

56. Ravi borrowed some money at the rate of 4% p.a. for the first three years, at the rate of 8% p.a. for next two years and at the rate of 9% p.a for the period 5 years. If the pays a total simple interest of ₹19550 at the end of 7 years. How much money did he borrow? **[Bank PO 2005]**
(a) ₹39500 (b) ₹42500 (c) ₹41900 (d) ₹43000

57. Indicom cell – phone is available for ₹ 2500 cash or ₹ 520 cash down payments followed by 4 equal monthly instalments. If the rate of interest charged is 25% per annum, calculate the monthly instalment :
(a) ₹ 520 (b) ₹480
(c) ₹ 550 (d) None of these

58. The difference between interest received by A and B is ₹ 18 on ₹ 1500 for 3 year. What is the difference in rate of interest?
(a) 1% (b) 2.5 % (c) 0.5% (d) 0.4%

GP_3386

59. What annual payment will discharge a debt of ₹1092 due in 2 years at 12% simple interest?
(a) ₹725 (b) ₹325 (c) ₹515 (d) ₹900

60. The annual payment of ₹160 in 5 years at 5% per annum simple interest will discharge a debt of
(a) ₹980 (b) ₹880 (c) ₹440 (d) ₹220

61. Kamal took ₹ 6800 as a loan which along with interest is to be repaid in two equal annual instalments. If the rate of interest is $12\frac{1}{2}\%$, compounded annually, then the value of each instalment is : **[SSC CGL Tier I 2016]**
(a) ₹8100 (b) ₹4150 (c) ₹4050 (d) ₹4000

62. A man borrows ₹ 1820 and undertakes to pay back with compound interest @ 20% pa in 3 equal yearly instal ments at the end of first, second and third years. What is the amount of each instalment?
(a) ₹500 (b) ₹560 (c) ₹750 (d) ₹864

LEVEL - 2

1. A person invested some amount at the rate of 12% simple interest and a certain amount at the rate of 10% simple interest. He received yearly interest of ₹ 130. But if he had interchanged the amounts invested, he would have received ₹ 4 more as interest. How much did he invest at 12% simple interest ?
(a) ₹ 700 (b) ₹ 500 (c) ₹ 800 (d) ₹ 400

2. A man invested $\frac{1}{5}$ of his money at 5%, $\frac{1}{6}$ of the money at 8% and remaining money at 10% p.a. If his annual income from interest is ₹260, what is total money he had? **[SSC CGL 2009]**
(a) ₹ 1000 (b) ₹ 2000 (c) ₹ 3000 (d) ₹ 4000

3. Akram Ali left an amount of ₹ 340000 to be divided between his two sons aged 10 years and 12 years such that both of them would get an equal amount when each attain 18 years age. What is the share of elder brother if the whole amount was invested at 10% simple interest ?
(a) 120000 (b) 140000
(c) 160000 (d) 180000

4. The ratio of the amount for two years under C.I. annually and for one year under S.I. is 6 : 5. When the rate of interest is same, then the value of rate of interest is
(a) 12.5% (b) 18% (c) 20% (d) 16.66%

5. The comound interest on a certain sum for 2 years at 10% per annum is ₹ 1260. The simple interest on the same sum for double the time at half the rate per cent per annum is
(a) ₹ 1200 (b) ₹ 1160 (c) ₹ 1208 (d) ₹ 1175

6. A sum was put at simple interest at a certain rate for 2 years. Had it been put at 1% higher rate, it would have fetched ₹ 24 more? The sum is
(a) 1200 (b) 1500 (c) 1800 (d) 2000

7. A man borrows ₹ 12,500 at 20% compound interest. At the end of every year he pays ₹ 2000 as part repayment. How much does he still owe after three such instalments?
(a) ₹ 12,000 (b) ₹ 12,864
(c) ₹ 15,600 (d) None of these

8. A man lends ₹ 10,000 in four parts. If he gets 8% on ₹ 2000; $7\frac{1}{2}\%$ on ₹ 4000 and $8\frac{1}{2}\%$ on ₹ 1400; what percent must he get for the remainder, if his average annual interest is 8.13% ?
(a) 7% (b) 9% (c) $9\frac{1}{4}\%$ (d) $10\frac{1}{2}\%$

9. If the rate increases by 2%, the simple interest received on a sum of money increases by ₹ 108. If the time period is increased by 2 years, the simple interest on the same sum increases by ₹ 180. The sum is :
(a) ₹ 1800 (b) ₹ 3600
(c) ₹ 5400 (d) Data inadequate

10. A person lent a certain sum of money at 4% simple interest; and in 8 years the interest amounted to ₹ 340 less than the sum lent. Find the sum lent.
(a) 500 (b) 600 (c) 1000 (d) 1500

11. An automobile financier claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest of 10%, the effective rate of interest becomes :

(a) 10% (b) 10.25%
(c) 10.5% (d) None of these

12. A sum of ₹ 725 is lent in the beginning of a year at a certain rate of interest. After 8 months, a sum of ₹ 362.50 more is lent but at the rate twice the former. At the end of the year, ₹ 33.50 is earned as interest from both the loans. What was the original rate of interest?
(a) 3.6% (b) 4.5%
(c) 5% (d) None of these

13. A father left a will of ₹ 68,000 to be divided between his two sons aged 10 years and 12 years such that they may get equal amount when each attains the age of 18 years If the money is reckoned at 10% p.a., find how much each gets at the time of the will.
(a) ₹ 30,000, ₹ 38,000 (b) ₹ 28,000, ₹ 40,000
(c) ₹ 32,000, ₹ 36,000 (d) cannot be determined.

14. If there are three sum of money P,Q and R so that P is the simple interest of Q and Q is the simple interest of R, rate % and time are same in each case, then the relation of P, Q and R is given by
(a) $P^2 = QR$ (b) $Q^2 = PR$
(c) $R^2 = PQ$ (d) $PQR = 100$

15. What annual instalment will discharge a debt of ₹ 4,200 due in 5 years at 10% simple interest?
(a) ₹ 500 per year (b) ₹ 600 per year
(c) ₹ 700 per year (d) ₹ 800 per year

16. A certain amount earns simple interest of ₹ 1750 after 7 years Had the interest been 2% more, how much more interest would it have earned?
(a) ₹ 35 (b) ₹ 245
(c) ₹ 350 (d) Cannot be determined

17. Peter invested an amount of ₹ 12,000 at the rate of 10 p.c.p.a simple interest and another amount at the rate of 20 p.c.p.a. simple interest. The total interest earned at the end of one year on the total amount invested became 14 p.c.p.a. Find the total amount invested?
(a) ₹ 20,000 (b) ₹ 22,000
(c) ₹ 24,000 (d) ₹ 25,000

18. A sum of money doubles itself in 6 years at a certain rate of compound interest. In how many years will be 16 times at the same rate of interest?
(a) 18 years (b) 24 years
(c) 30 years (d) 12 years

19. A sum of money placed at compound interest doubles itself in 5 years Find in how many years it will become 8 times.
(a) 15 years (b) 20 years
(c) 25 years (d) 30 years

20. At simple interest, a sum becomes 3 times in 20 years. Find the time, in which the sum will be double at the same rate of interest.
[RRB 2007]
(a) 8 yrs (b) 10 yrs (c) 12 yrs (d) 14 yrs

21. A sum amounts to ₹5750 in 3 years at 5% p.a. simple interest. In how many years will the same sum amount to ₹ 7000 at the same rate of interest?
[RRB 2009]
(a) 5 years (b) 7 years (c) 8 years (d) 10 years

22. A certain sum becomes 4 times at the rate of 5% p.a. At what rate of simple interest the same sum becomes 8 times in the same duration?
[SBI PO 2011]
(a) $11\frac{2}{3}\%$ (b) $10\frac{3}{4}\%$ (c) $8\frac{4}{5}\%$ (d) $9\frac{4}{7}\%$

23. A certain sum becomes 3 fold at 4% annual rate of interest. At what rate, it will become of 6 fold?
[CBI 2008]
(a) 10% (b) 12% (c) 8% (d) 9%

24. The rates of simple interest in two banks A and B are in the ratio 5 : 4. A person wants to deposit his total savings in two banks in such a way that he received equal half-yearly interest from both. He should deposit the savings in banks A and B in the ratio.
(a) 2 : 5 (b) 4 : 5 (c) 5 : 2 (d) 5 : 4

25. The price of a T.V. set worth ₹ 20,000 is to paid in 20 instalments of ₹ 1000 each. If the rate of interest be 6% per annum, and the first instalment be paid at the time of purchase, then the value of the last instalment covering the interest as well will be :
(a) ₹ 1050 (b) ₹ 2050
(c) ₹ 3000 (d) None of these

26. Two equal sums of money are lent at the same time at 8% and 7% per annum simple interest. The former is recovered 6 months earlier than the later and the amount in each case is ₹ 2560. The sum and the time for which the sum of money were lent out, are:
(a) ₹ 2500, 3.5 years and 4 years
(b) ₹ 2000, 3.5 years and 4 years
(c) ₹ 2000, 4 years and 5.5 years
(d) ₹ 3000, 4 years and 4.5 years

27. Divide ₹ 2379 into 3 parts so that their amounts after 2, 3 and 4 years respectively may be equal, the rate of interest being 5% per annum at simple interest. The first part is:
(a) 759 (b) 792 (c) 818 (d) 828

GP_3386

28. Simple interest on a certain amount is $\frac{9}{16}$ of the principal. If the numbers respresenting the rate of interest in percent and time in years be equal, then time, for which the principal is lent out, is:

(a) $5\frac{1}{2}$ years (b) $6\frac{1}{2}$ years

(c) 7 years (d) $7\frac{1}{2}$ years

29. David invested certain amount in three different Schemes A, B and C with the rate of interest 10% p.a., 12% p.a. and 15% p.a. respectively. If the total interest accrued in one year was ₹ 3200 and the amount invested in Scheme C was 150 % of the amount invested in Scheme A and 240% of the amount invested in Scheme B, what was the amount invested in Scheme B?

(a) ₹ 5000 (b) ₹ 6500

(c) ₹ 8000 (d) cannot be determined

30. The simple interest on a sum of money is $\frac{1}{9}$ of the principal and the number of years is equal to the rate % p.a. The rate % p.a. is

(a) 3 % (b) $3\frac{1}{3}$% (c) 10% (d) $\frac{1}{3}$%

31. A person takes a loan of ₹10000 partly from a bank at 8% p.a. and remaining from another bank at 10% p.a. He pays a total interest of ₹950 per annum. Amount of loan taken from the first bank (in ₹) is **[SSC CGL Tier I 2012]**

(a) 2500 (b) 5200 (c) 2050 (d) 5020

32. A person lent ₹5000 partly at the rate of 4 percent and partly at the rate of 5 per cent per annum simple interest. The total interest after 2 years is ₹440. To find the sum of money lent at each of the above rates, ₹5000 is to be divided in the ratio. **[SSC CGL 2002]**

(a) 4 : 5 (b) 3 : 2 (c) 5 : 4 (d) 2 : 3

33. Ramesh borrows ₹ 7000 from a bank at SI. After 3 yrs, he paid ₹ 3000 to the bank and at the end of 5 yrs from the date of borrowing he paid ₹ 5450 to the bank to settle the account. Find the rate of interest. **[CDS 2008]**

(a) 5% (b) 15% (c) 0.05% (d) 10%

34. A person takes a loan of ₹ 200 at 5% simple interest. He returns ₹100 at the end of one year. In order to clear his dues at the end of 2 years, he would pay :

(a) ₹ 125.50 (b) ₹ 110

(c) ₹ 115.50 (d) ₹ none of these

35. Consider the following statements.
If a money is loaned at simple interest then the:

(i) money gets doubled in 5 years if the rate of interest is $16\frac{2}{3}$%

(ii) money gets doubled in 5 years if the rate of interest is 20%

(iii) money becomes four times in 10 years if the gets doubled in 5 years.

Of these statements :

(a) (i) and (iii) are correct

(b) (iii) alone is correct

(c) (ii) alone is correct

(d) (ii) and (iii) are correct

36. The CI on ₹ 5000 for 3 years at 8% for first year, 10% for second year and 12% for third year will be:

(a) ₹ 1750 (b) ₹ 1652.80

(c) ₹ 1575 (d) ₹ 1685.20

37. The difference between simple and compound interest for the fourth years is ₹ 7280 at 20% p.a. What is the principal sum?

(a) ₹ 10000 (b) ₹ 50000

(c) ₹ 1 lakh (d) ₹ 40000

38. The ratio of CI for 3 years and SI for 1 year for a fixed amount at a rate of r% is 3.64. What is the value of r?

(a) 10% (b) 15%

(c) 20% (d) None of these

39. The rate of interest in two banks DNB and HBI are in the ratio of 7 : 8. If a person invested some amount in both the banks and received equal interest from both the banks in two year. The ratio of interest invested in DNB and HBI respectively is :

(a) 15 : 1 (b) 8 : 7

(c) 7 : 8 (d) 108 : 107

40. Mr. Bajaj invested $\frac{1}{7}$ of his total investment at 4% and $\frac{1}{2}$ at 5% and rest at 6% for the one year and received total interest of ₹ 730. What is the total sum invested?

(a) ₹ 70000 (b) ₹ 14000

(c) ₹ 24000 (d) ₹ 38000

41. The difference between the simple interest received from two different banks on ₹ 500 for 2 years is ₹ 2.50. The difference between their rates is _____. **[SSC CHSL 2010]**

(a) 1% (b) 2% (c) 0.25% (d) 1.5%

42. A man ₹16000 part of which he lent at 4% and the rest at 5% p.a. simple interest. If the total interest received was ₹700 in one year, the money lent at 4% p.a. was.

[SSC CGL Tier -I 2015]

(a) ₹12000 (b) ₹8000
(c) ₹10000 (d) ₹6000

43. A man borrows a certain sum and pays it back in two equal annual instalments. If he pays back ₹ 676 annually and rate of interest is 4%, what is the sum borrowed?

(a) ₹1352 (b) ₹1300
(c) ₹1275 (d) ₹1250

44. The price of a television is ₹ 10,000. It can be purchased on a down payment of ₹ 3000 and 6 equal monthly instalment of ₹ 1300. Find the rate of interest.

(a) 42% (b) $42\frac{2}{3}\%$ (c) $42\frac{1}{3}\%$ (d) $42\frac{1}{2}\%$

45. A sum of ₹ 7200 has to be paid in three annual instalments of ₹ 2400 with interest. If the rate of interest is 8% per annum, then what amount will be paid at the end of every year?

(a) ₹2976, ₹2784, ₹2592
(b) ₹2782, ₹2529, ₹2425
(c) ₹2594, ₹2463, ₹2218
(d) ₹1800, ₹1712, ₹1690

GP_3386

Hints and Solutions

Level 1

1. (b) It triple itself in 8 years, which makes interest equal to 200% of principal.
So, 200% is added in 8 years
Hence, 400% which makes the whole amount equal to five times of the principal, which will be added in 16 years

2. (b) Interest for one year

$$= ₹\,212.50 \times \frac{3}{100} \times 1 = ₹\,\frac{51}{8}$$

Thus in 8 years, the interest is ₹ 51.

3. (c) Rate of interest

$$= \frac{956-800}{3\times 800} \times 100 = 6.50\%$$

$$\therefore \text{ Amount} = 800 + \frac{800 \times 9.5 \times 3}{100}$$

$$= 800 + 228 = ₹1028$$

4. (c) Solve using options. Option (c) fits the situation as:
12820 = 2000 + 2 years interest on 2000 + 4000 + 1 years interest on 4000 + 6000 (use 10% compound interest for calculation of interest) →
12820 = 2000 + 420 + 4000 + 400 + 6000.
Thus, option (c) fits the situation perfectly.

5. (c) Let the principal be P and rate of interest be R%.

$$\therefore \text{ Required ratio} = \left[\frac{\left(\frac{P\times R\times 6}{100}\right)}{\left(\frac{P\times R\times 9}{100}\right)}\right] = \frac{6PR}{9PR} = \frac{6}{9} = 2:3.$$

6. (d) Let the sum be ₹ x.

$$\therefore \frac{x\times 11\times 5}{100} - \frac{x\times 8\times 6}{100} = 1008$$

$$\Rightarrow \frac{7x}{100} = 1008 \Rightarrow x = 14400$$

7. (c) Difference in interest = 236.25 − 225 = ₹ 11.25
This difference is the simple interest over ₹ 225 for one year. Hence, rate of interest

$$= \frac{11.25\times 100}{225\times 1} = 5\%$$

8. (c) Tripling in 8 years means that the interest earned in 8 years is equal to 200% of the capital value. Thus, interest per year (simple interest) is 25% of the capital. In 20 years, total interest earned = 500% of the capital and hence the capital would become 6 times it's original value.

9. (c) **For 3 years:**

$$\text{Diff.} = \frac{\text{Sum}\times(\text{rate})^2(300+\text{rate})}{(100)^3}$$

$$= \frac{2000\times 10\times 10\times 310}{100\times 100\times 100} = ₹\,62$$

10. (d) $\frac{30p}{100} + 450 = \left[p\left(1+\frac{15}{100}\right) - p\right]$

$\Rightarrow p = ₹\,20{,}000.$

11. (a) Let A lent ₹ x and B lent ₹ y
Since, A and B together lent out ₹ 81600
$\therefore x + y = 81{,}600$
Now, given (r) Rate = 4%

$$\therefore 1 + r = 1 + \frac{4}{100} = \frac{26}{25}$$

According to the question, we have

$$\frac{x}{y} = \left(\frac{26}{25}\right)^{3-2} = \frac{26}{25}$$

$\therefore$ Investment made by B

$$= 81600 \times \frac{25}{51} = 40{,}000$$

12. (d) Difference of S.I. = ₹ 31.50
Let each sum be ₹ x. Then

$$\frac{x\times 4\frac{1}{2}\times 7}{100} - \frac{x\times 4\times 7}{100} = 31.50$$

$$\text{or} \quad \frac{7x}{100}\times\frac{1}{2} = \frac{63}{2}$$

$$\text{or} \quad x = ₹\,900$$

13. (a) Let the sum be ₹ x. Then,

$$\left(\frac{x\times 6\times 3}{100}\right) + \left(\frac{x\times 9\times 5}{100}\right) + \left(\frac{x\times 13\times 3}{100}\right) = 8160$$

$$\Rightarrow 18x + 45x + 39x = (8160\times 100)$$

$$\Rightarrow 102x = 816000 \Rightarrow x = 8000.$$

14. (d) Let the rate be R% p.a. Then,

$$\left(\frac{5000\times R\times 2}{100}\right)+\left(\frac{3000\times R\times 4}{100}\right)=2200$$

$$\Rightarrow 100R+120R=2200 \Rightarrow R=\left(\frac{2200}{220}\right)=10.$$

15. (b) Use Shortcut Approach - 6

16. (c) $\left(\frac{1500\times R_1\times 3}{100}\right)-\left(\frac{1500\times R_2\times 3}{100}\right)=13.50$

$$\Rightarrow 4500(R_1-R_2)=1350$$

$$\Rightarrow R_1-R_2=\frac{1350}{4500}=0.3\%$$

17. (b) Let the sum invested at 9% be ₹ x and that invested at 11% be ₹ (100000 – x).
Then,

$$\left(\frac{x\times 9\times 1}{100}\right)+\left[\frac{(100000-x)\times 11\times 1}{100}\right]$$

$$=\left(100000\times\frac{39}{4}\times\frac{1}{100}\right)$$

$$\Rightarrow \frac{9x+1100000-11x}{100}=\frac{39000}{4}=9750$$

$\Rightarrow 2x=(1100000-975000)=125000$
$\Rightarrow x=62500.$
$\therefore$ Sum invested at 9% = ₹ 62500.
Sum invested at 11% = ₹ (100000 – 62500)
= ₹ 37500.

18. (c) $3000(1+0.05)+x(1+0.08)=(3000+x)(1+0.06)$

or $3150+1.08x=3180+1.06x$

or $0.02x=30$ or $x=\frac{30}{0.02}=1500$

19. (b) Difference $=\frac{PR^2}{10000}$

$$\Rightarrow \quad 6=\frac{P\times 5\times 5}{10000}$$

$$\Rightarrow \quad 6\times 400 = ₹2400.$$

20. (c) $A=P\left(1+\frac{r}{100}\right)^n$; A = Amount, P = Principal
r = rate of interest
n = time

Required amount $=1000\left(\frac{105}{100}\right)^3$

$$=1000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

= ₹ 1000 × 1.157 = ₹ 1157

21. (c) Here S.I. = ₹ 9, P = ₹ 1

$$\therefore R=\frac{9\times 100}{1\times 60}=15\%$$

22. (a) Let the sum invested in Scheme A be ₹ x and that in Scheme B be ₹ (13900 – x).
Then,

$$\left(\frac{x\times 14\times 2}{100}\right)+\left[\frac{(13900-x)\times 11\times 2}{100}\right]=3508$$

$\Rightarrow 28x-22x=350800-(13900\times 22)$
$\Rightarrow 6x=45000 \Rightarrow x=7500.$
So, sum invested in Scheme B = ₹ (13900 – 7500) = ₹ 6400.

23. (b) Let he borrowed at 5% = ₹ x
$\therefore$ He borrowed at 7% = ₹ (2500 – x)
Now $I_1+I_2=275$

$$\frac{x\times 5\times 2}{100}+\frac{(2500-x)\times 7\times 2}{100}=275$$

$\Rightarrow 10x+14(2500-x)=27500$
$\Rightarrow 4x=35000-27500=7500 \Rightarrow x=$ ₹ 1875
$\therefore$ Sum borrowed at 7% rate = 2500 – 1875
= ₹ 625

24. (b) S. I. = 81 – 72 = ₹ 9

$$\therefore T=\frac{9\times 100\times 4}{72\times 25}=2 \text{ years}$$

25. (d) 4000 + (4000 × 5 × 0.15) = Rs. 7000

26. (a) Use Shortcut Approach-3

27. (c) Use Shortcut Approach-4

28. (c) $1960=\frac{14000\times r\times 2}{100} \Rightarrow r=7\%$

29. (c) $6000=\frac{P\times 12\times 1}{100}$

$\Rightarrow P =$ ₹ 50000

30. (b) SI = 2P – P = P (interest = Amount – Principal)

$$P=\frac{P\times 20\times t}{100} \Rightarrow t=5 \text{ years}$$

31. (c) $\frac{P_1\times 5\times 2}{100}=\frac{P_2\times 10\times 4}{100}$

$\Rightarrow P_1:P_2=4:1$
Therefore second principal is

$$\Rightarrow\left(=625\times\frac{1}{5}\right)=₹125$$

32. **(a)** Use Shortcut Approach- 2

33. **(b)** Use Shortcut Approach - 3

34. **(d)** Use Shortcut Approach - 5

35. **(b)** $2P = \frac{P \times 15 \times r}{100}$ $(\because 2P = 3P - P)$

$r = \frac{40}{3}\%$ p.a.

36. **(c)** $P = \frac{P \times r \times 12}{100} \Rightarrow r = \frac{100}{12}\%$ p.a.

Now, $2p = \frac{P \times 100 \times t}{12 \times 100} = 24$ years

37. **(b)** $580 = \left[(x) + \left(x + \frac{x \times 1 \times 8}{100}\right) + \left(x + \frac{x \times 2 \times 8}{100}\right) + \left(x + \frac{x \times 3 \times 8}{100}\right) + \left(x + \frac{x \times 4 \times 8}{100}\right)\right]$

$580 = 5x + \frac{8x}{10} = \frac{58x}{10} \Rightarrow x = ₹100$

38. **(a)** $A = 10000 \times (1.2)^4$

$A = 10000 \times 2.0736$

$A = 20736$

$CI = A - P$

$CI = 20736 - 10000 = ₹\,10736$

39. **(b)** $1700 + \frac{1700 \times 16 \times 2}{100} = ₹\ 2244$

40. **(a)** $A = P\left(1 + \frac{r}{100}\right)^n$

$A = 1000\left(1 + \frac{10}{100}\right)^3$

$A = 1000 \times (1.1)^3$

$A = 1331$

$\therefore$ $CI = A - P = 1331 - 1000 = ₹\,331$

41. **(b)** $441 = 400\left(1 + \frac{r}{100}\right)^2$

$\Rightarrow \frac{21}{20} = 1 + \frac{r}{100}$

$\Rightarrow r = 5\%$

42. **(b)** Use Shortcut Approach-18

43. **(c)** Use Shortcut Approach-16

44. **(a)** Use Shortcut Approach-19

45. **(a)** Use Shortcut Approach -17

46. **(d)** Use Shortcut Approach -19

47. **(d)** Use Shortcut Approach-19

48. **(d)** Use Shortcut Approach -20

49. **(b)** Use Shortcut Approach -21

50. **(c)** Use Shortcut Approach -21

51. **(a)** Use Shortcut Approach -22

52. **(d)** Use Shortcut Approach -22

53. **(d)** Use Shortcut Approach -23

54. **(b)** $6000\left(\frac{10}{100}\right)^2 = ₹60$

55. **(d)** Amount to be paid at the end of 1 year

$= \frac{800 \times 10 \times 1}{100} + 800 = 880$

Amount left as principal for the second year $= 480 = (880 - 400)$

Amount to be paid after 2nd year

$= 480 + \frac{480 \times 10}{100} = ₹\,528$

56. **(b)** Use Shortcut Approach-1

57. **(a)** Balance price to be paid in instalments = 1980

At the rate of r% per annum after 4 months, ₹ 1980 will amount to

$₹\left(1980 + \frac{1980 \times 4 \times 25}{12 \times 100}\right) = ₹\ 2145$ ...(i)

Now, the total amount for the 4 instalments at the end of fourth month will be

$\left[x + \left(x + \frac{25 \times x}{12 \times 100}\right) + \left(x + \frac{25 \times 2x}{12 \times 100}\right) + \left(x + \frac{25 \times 3x}{12 \times 100}\right)\right]$

$= 4x + \frac{25x}{1200}(1 + 2 + 3)$

$= \frac{33x}{8}$ ...(ii)

From (i) and (ii) $\frac{33x}{8} = 2145$

$x = ₹\,520$

58. **(d)** $\frac{1500 \times 3}{100}(r_1 - r_2) = 18$

$\Rightarrow r_1 - r_2 = 0.4$

59. **(c)** Use Shortcut Approach-25

60. **(b)** Use Shortcut Approach-25

61. **(c)** Use Shortcut Approach-26

62. **(d)** Use Shortcut Approach-26

Level 1

1. (b) Let amount invested at 12% be x and amount invested at 10% be y.
According to question

$$130 = \frac{x \times 12 \times 1}{100} + \frac{y \times 10 \times 1}{100}$$

$$\Rightarrow 13000 = 12x + 10y \quad \ldots(1)$$

$$\text{And } 134 = \frac{x \times 10 \times 1}{100} + \frac{y \times 12 \times 1}{100}$$

$$\Rightarrow 13400 = 10x + 12y \quad \ldots(2)$$

From equations (1) and (2)
$x = 500$

2. (c) Use Shortcut Approach -14

3. (d) Go through options

$$1.8 + \frac{1.8 \times 6 \times 10}{100} = 1.6 + \frac{1.6 \times 8 \times 10}{100}$$

Hence (d) is correct.
Alternatively :

$$P_1 + \frac{P_1 \times 6 \times 10}{100} = P_2 + \frac{P_2 \times 8 \times 10}{100}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{9}{8}$$

Share of elder brother

$$= \frac{340000 \times 9}{17} = ₹180000$$

4. (c) On the second year (in terms of C.I.) is

$$\frac{P\left(1+\frac{r}{100}\right)^2}{\left(P+\frac{Pr}{100}\right)} = \frac{6}{5} \quad \Rightarrow \left(1+\frac{r}{100}\right) = \frac{6}{5}$$

$$\Rightarrow r = 20\%$$

5. (a) $\text{Sum} = \dfrac{\text{Difference in interest} \times 100}{\text{Time} \times \text{Difference in rate}}$

$$= \frac{24 \times 100}{2 \times 1} = ₹1200$$

6. (a) Let the sum be ₹ P. Then,

$$\Rightarrow \left[P\left(1+\frac{10}{100}\right)^2 - P\right] = 1260$$

$$\Rightarrow P\left[\left(\frac{11}{10}\right)^2 - 1\right] = 1260$$

$\therefore$ Sum = ₹ 6000

So, $\text{S.I.} = ₹\left(\dfrac{6000 \times 4 \times 5}{100}\right) = ₹\ 1200$

7. (d) $\text{Balance} = ₹\left[\left\{12500 \times \left(1+\frac{20}{100}\right)^3\right\} - \left\{2000 \times \left(1+\frac{20}{100}\right)^2 + 2000 \times \left(1+\frac{20}{100}\right) + 2000\right\}\right]$

$$= ₹\left[\left(12500 \times \frac{5}{6} \times \frac{6}{5} \times \frac{6}{5}\right) - \left(2000 \times \frac{6}{5} \times \frac{6}{5} + 2000 \times \frac{6}{5} + 2000\right)\right]$$

= ₹ [21600 – (2880 + 2400 + 2000)]
= ₹14320.

8. (b) Let the required rate be R. Then,

$$\left(\frac{20000 \times 8 \times 1}{100}\right) + \left(4000 \times \frac{15}{2} \times \frac{1}{100}\right) + \left(1400 \times \frac{17}{2} \times \frac{1}{100}\right) + \left(2600 \times R \times \frac{1}{100}\right) = \left(\frac{813}{10000} \times 10000\right)$$

$$\Leftrightarrow 160 + 300 + 119 + 26R = 813 \Leftrightarrow R = 9\%$$

9. (d) Let the sum be ₹ x, rate be R% p.a. and time be T years.

Then, $\left[\dfrac{x \times (R+2) \times T}{100}\right] - \left(\dfrac{x \times R \times T}{100}\right) = 108$

$$\Leftrightarrow 2xT = 10800 \quad \ldots(i)$$

$$\left[\frac{x \times R \times (T+2)}{100}\right] - \left(\frac{x \times R \times T}{100}\right) = 180$$

$$\Leftrightarrow 2xR = 18000 \quad \ldots(ii)$$

Clearly, from (i) and (ii), we cannot find the value of x.
So, the data is inadequate.

10. (a) Let the sum be ₹ x.

$$\therefore \text{Interest} = \frac{x \times 8 \times 4}{100} = \frac{32x}{100}$$

$$x - \frac{32x}{100} = \frac{68x}{100}$$

When interest is $\frac{68x}{100}$ less, the sum is ₹ x.

∴ When interest is ₹ 340 less, the sum is $\frac{x}{68x}\times 100\times 340$ = ₹ 500

Direct Formula:

$$\text{Sum} = \frac{100}{100-8\times 4}\times 340 = \frac{100\times 340}{68} = ₹\,500$$

11. (b) Let the sum be ₹ 100. Then,
S.I. for first 6 months

$$= ₹\left(\frac{100\times 10\times 1}{100\times 2}\right) = ₹\,5$$

S.I. for last 6 months

$$= ₹\left(\frac{105\times 10\times 1}{100\times 2}\right) = ₹\,5.25.$$

So, amount at the end of 1 year = ₹ (100 + 5 + 5.25) = ₹ 110.25.
∴ Effective rate = ₹ (110.25 – 100) = 10.25%.

12. (d) Let the original rate be R%. Then, new rate = (2R)%.

$$\therefore \left(\frac{725\times R\times 1}{100}\right)+\left(\frac{362.50\times 2R\times 1}{100\times 3}\right) = 33.50$$

$$\Rightarrow (2175+725)R = 33.50\times 100\times 3 = 10050$$

$$\Rightarrow R = \frac{10050}{2900} = 3.46\%$$

13. (c) Let one gets = ₹ x
then, second gets = ₹ (68,000 – x)
Given : $A_1 = A_2$

$$x+\frac{x\times 10\times 8}{100} = (68,000-x)+\frac{(68000-x)\times 10\times 6}{100}$$

$$\Rightarrow x[100+80] = (68,000-x)[100+60]$$

$$\Rightarrow \frac{180x}{160} = 68,000-x$$

$$\Rightarrow 34x = 68000\times 16 \Rightarrow x = ₹\,32,000$$

∴ second gets = ₹ 36,000

14. (b) $P = \frac{Q\times r\times t}{100}$ and $Q = \frac{R\times r\times t}{100}$

$$\Rightarrow \frac{P}{Q} = \frac{Q}{R} = \frac{r\times t}{100}$$

∴ $Q^2 = PR$.

15. (c) Shortcut method :
If borrowed amount be ₹ M and it is to be paid in equal instalments, then

$$M = na + \frac{ra}{100\times Y}\times\frac{n(n-1)}{2}$$

where Y = no. of instalments per annum
a = annual instalment
Here, M = 4200, Y = 1, r = 10, n = 5, a = ?

$$4200 = 5a + \frac{10a}{100}\times\frac{5(5-1)}{2}$$

$$\Rightarrow 4200 = a[5+1] \Rightarrow 6a = 4200$$

$$\Rightarrow a = ₹\,700$$

16. (d) We need to know the S.I., principal and time to find the rate. Since the principal is not given, so data is inadequate.

17. (a) Let the second amount be ₹ x. Then,

$$\frac{12,000\times 10\times 1}{100}+\frac{x\times 20\times 1}{100} = \frac{(12000+x)\times 14\times 1}{100}$$

$$\Rightarrow 120000+20x = (12000+x)\,14$$

$$\Rightarrow 6x = 168000-120000$$

$$\Rightarrow x = ₹\,8000$$

∴ Total investment = 12,000 + 8000 = ₹ 20,000

18. (b) P becomes 2P in 6 years at r% p.a.

$$\Rightarrow 2P = P\left(1+\frac{r}{100}\right)^6$$

$$\therefore \left(1+\frac{r}{100}\right)^6 = 2$$

$$\Rightarrow 2^4 = \left(1+\frac{r}{100}\right)^{6\times 4} = \left(1+\frac{r}{100}\right)^{24}$$

∴ P becomes 16 P in 24 years

19. (a) $P\left(1+\frac{R}{100}\right)^5 = 2P \Rightarrow \left(1+\frac{R}{100}\right)^5 = 2$ … (i)

Let $P\left(1+\frac{R}{100}\right)^n = 8P$

$$\Rightarrow \left(1+\frac{R}{100}\right)^n = 2^3 = \left\{\left(1+\frac{R}{100}\right)^5\right\}^3$$

[By (i)]

$$\Rightarrow \left(1+\frac{R}{100}\right)^n = \left(1+\frac{R}{100}\right)^{15}$$

$$\Rightarrow n = 15 \text{ years}$$

20. (b) Use Shortcut Approach -7

21. (c) Use Shortcut Approach-9

22. **(a)** Use Shortcut Approach - 10

23. **(a)** Use Shortcut Approach - 10

24. **(b)** Let the savings be X and Y and the rates of simple interest be 5x and 4x respectively. Then,

$$X \times 5x \times \frac{1}{2} \times \frac{1}{100} = Y \times 4x \times \frac{1}{2} \times \frac{1}{100},$$

$$\text{or } \frac{X}{Y} = \frac{4}{5}, \text{ i.e., } X : Y = 4 : 5.$$

25. **(d)** Money paid in cash = ₹ 1000.
Balance payment = ₹ (20000 – 1000)
= ₹ 19000.

26. **(b)** Let the sum be ₹ x.
∴ ₹ x is lent at 8% for t years and ₹ x is also lent at 7% for $\left(t + \frac{1}{2}\right)$ years

$$\Rightarrow \frac{x \times t \times 8}{100} + x = 2560 \ldots(1)$$

$$\text{and } \frac{x \times (2t+1) \times 7}{2 \times 100} + x = 2560 \ldots(2)$$

Solving (1) and (2),
⇒ x = ₹ 2,000, t = 3.5 years and 4 years

27. **(d)** Let the parts be x, y and z

$$x + \left(x \times 2 \times \frac{5}{100}\right) = y + \left(y \times 3 \times \frac{5}{100}\right)$$

$$= z + \left(z \times 4 \times \frac{5}{100}\right)$$

$$\Rightarrow \frac{11x}{10} = \frac{23y}{20} = \frac{6z}{5} = k$$

$$\Rightarrow x = \frac{10k}{11}, y = \frac{20k}{23}, z = \frac{5k}{6}$$

But x + y + z = 2379.

$$\Rightarrow \frac{10k}{11} + \frac{20k}{23} + \frac{5k}{6} = 2379$$

$$\Rightarrow 1380k + 1320k + 1256k = 2379 \times 11 \times 23 \times 6$$

$$\Rightarrow k = \frac{2379 \times 11 \times 23 \times 6}{3965} = \frac{3 \times 11 \times 23 \times 6}{5}$$

$$\therefore x = \left(\frac{10}{11} \times \frac{3 \times 11 \times 23 \times 6}{5}\right) = 828.$$

Hence, the first part is ₹ 828.

28. **(d)** $\frac{9}{16}P = \frac{P \times T \times T}{100}$

$$\Rightarrow T^2 = \frac{900}{16} \Rightarrow T = \frac{30}{4} = 7\frac{1}{2} \text{ years}$$

29. **(a)** Let x, y and z be the amounts invested in schemes A, B and C respectively. Then,

$$\left(\frac{x \times 10 \times 1}{100}\right) + \left(\frac{y \times 12 \times 1}{100}\right) + \left(\frac{z \times 15 \times 1}{100}\right) = 3200$$

$$\Rightarrow 10x + 12y + 15z = 320000 \quad \ldots\ldots (i)$$

$$\text{Now, } z = 240\% \text{ of } y = \frac{12}{5}y \quad \ldots\ldots (ii)$$

$$\text{And, } z = 150\% \text{ of } x = \frac{3}{2}x$$

$$\Rightarrow x = \frac{2}{3}z = \left(\frac{2}{3} \times \frac{12}{5}\right)y = \frac{8}{5}y \quad \ldots\ldots (iii)$$

From (i), (ii) and (iii), we have :
16y + 12y + 36y = 320000 ⇒ 64y = 320000
⇒ y = 5000.
∴ Sum invested in Scheme B = ₹ 5000.

30. **(b)** Here S.I. $= \frac{1}{9}P$ and R = T

$$R = \frac{P}{9} \times \frac{100}{P \times R}$$

$$\Rightarrow R^2 = \frac{100}{9} \Rightarrow R = \frac{10}{3} = 3\frac{1}{3}\%$$

31. **(a)** Use Shortcut Approach -12

32. **(b)** Use Shortcut Approach -12

33. (a) Use shortcut Approach -13

34. **(c)** Amount to be paid in first year

$$= \frac{200 \times 5 \times 1}{100} + 200 = 210$$

Amount left as a principal for the second year
= 210 – 100 = 110
∴ Amount paid in second year

$$= 110 + \frac{110 \times 5 \times 1}{100} = ₹ 115.50$$

35. **(c)** **Statement (i)**

$$\frac{P \times 5 \times 50}{100 \times 3} \neq P, \text{ hence wrong}$$

Statement (ii)

$$SI = \frac{P \times 5 \times 20}{100} = P, \text{ hence correct}$$

Statement (iii) $P = \frac{P \times 5 \times r}{100} \Rightarrow r = 20\%$

Again SI $= \frac{P \times 10 \times 20}{100} = 2P$ hence wrong

36. (b) $A = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)\ldots$

$A = 5000\,(1.08)\,(1.1)\,(1.12)$

$A = 6652.8$

$CI = (6652.8 - 5000) = 1652.8$

37. (b) Difference between CI and SI for n^{th} year

$$= \frac{Pr}{100}\left[\left(1+\frac{r}{100}\right)^{n-1} - 1\right]$$

$$7280 = \frac{P \times 20}{100}\left[(1.2)^3 - 1\right]$$

$\Rightarrow P =$ ₹ 50000

Alternatively :

	Initially	Ist year	IInd year	IIIrd year	IVth year
SI	10000	12000	14000	16000	18000
CI	10000	12000	14400	17280	20736

CI for 4th year = 20736 – 17280 = 3456

SI for 4th year = 2000

Difference between CI and SI = 1456 for ₹ 10000

So, the difference of ₹ 7280 is for ₹ 50000

38. (c)

$$\frac{P\left[\left(1+\frac{r}{100}\right)^3 - 1\right]}{\frac{pr}{100}} = \frac{\left(1+\frac{r}{100}\right)^3 - 1}{\frac{r}{100}} = 3.64$$

By taking the option (c), r = 20%

Now, $\frac{\left(1+\frac{20}{100}\right)^3 - 1}{\frac{20}{100}} = \frac{(1.2)^3 - 1}{0.2}$

$$= \frac{0.728}{0.2} = 3.64$$

Hence, r = 20%

39. (b) $\frac{P_1 \times 2 \times 7x}{100} = \frac{P_2 \times 2 \times 8x}{100}$

$\Rightarrow \frac{P_1}{P_2} = \frac{8}{7}$

40. (b) Let the principal be x, then

$$\frac{\left(\frac{x}{7} \times 4 + \frac{x}{2} \times 5 + \frac{5x}{14} \times 6\right)}{100} = 730$$

$\Rightarrow x =$ ₹ 14000

41. (c) Use Shortcut Approach -8

42. (c) Use Shortcut Approach -11

43. (c) Use Shortcut Approach-27

44. (b) Use Shortcut Approach -24

45. (a) Use Shortcut Approach -28

CHAPTER

13

Sequence and Series

(AP, GP, Sum of Special Sequences and Number Series)

SEQUENCE

An arrangement of numbers $x_1, x_2, x_3, ..., x_n$ according to definite rule or a set of rules is called a **sequence**. The various numbers $x_1, x_2, x_3, ..., x_n, ...$ occuring in a sequence are called its **terms**.

For example :

(i) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...$ **(ii)** $1, 1, 2, 3, 5, 8, ...$

(iii) $3, 5, 7, 9, ...$

SERIES

By adding or subtracting the terms of a sequence, we get an expression, which is called a **series**. If $a_1, a_2, a_3, ..., a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + ... + a_n$ is a series.

ARITHMETIC PROGRESSIONS (A.P.)

A sequence of numbers which are either continuously increased or continuously decreased by a common difference found by subtracting any term of the sequence from the next term.

The following sequences of numbers are arithmetic progressions:

(i) 5, 8, 11, 14, ...

(ii) – 6, – 1, 4, 9, 14, ...

(iii) 10, 7, 4, 1, – 2, – 5, ...

(iv) $p, p + q, p + 2q, p + 3q, ...$

In the arithmetic progression (i); 5, 8, 11 and 14 are first term, second term, third term and fourth term respectively. Common difference of this A.P. is found out either by subtracting 5 from 8, 8 from 11 or 11 from 14. Thus common difference = 3. Similarly, common difference of arithmetic progression (ii), (iii) and (iv) are 5, – 3 and q respectively. First term and common difference of an A.P. are denoted by a and d respectively. Hence, d of (i) A.P.= 3, d of (ii) A.P.= 5, d of (iii) A.P. = – 3 and d of (iv) A.P. = q

n^{th} Term of an A.P.

To find an A.P. if first term and common difference are given, we add the common difference to first term to get the second term and add the common difference to second term to get the third term and so on.

The standard form of an A.P. is

$$a, a + d, a + 2d, a + 3d, ...$$

Here 'a' is the first term and 'd' is the common difference. Also we see that coefficient of d is always less by one than the position of that term in the A.P. Thus n^{th} term of the A.P. is given by

$$T_n = a + (n - 1)\, d \qquad ...(1)$$

This equation (1) is used as a formula to find any term of the A.P. If l be the last term of a sequence containing n terms, then

$$l = T_n = a + (n - 1)\, d$$

To find any particular term of any A.P., generally we put the value of a, n and d in the formula (1) and then calculate the required term.

For example: To find the 25^{th} term of the A.P. 6, 10, 14, 18, ... ; using the formula (1), we put the value of $a = 6$, $n = 25$ and $d = 4$ in formula and calculate as

$$T_{25} = 6 + (25 - 1)\, 4 = 102$$

Illustration – 1

In an A.P. if $a = -7.2$, $d = 3.6$, $a_n = 7.2$, then find the value of n.

Solution : $a_n = a + (n - 1)\, d$

$\Rightarrow \quad 7.2 = -7.2 + (n - 1)\,(3.6)$

$\Rightarrow \quad 14.4 = (n - 1)\,(3.6)$

$\Rightarrow \quad n - 1 = 4 \Rightarrow n = 5.$

GP_3386

Illustration – 2

Which term of the A.P. 21, 42, 63, ... is 420 ?

Solution : $420 = a_n = a + (n-1)\,d$

[Here $a = 21, d = 42 - 21 = 21$]

$= 21 + (n-1)\,21 = 21n$

$\Rightarrow \quad n = \dfrac{420}{21} = 20$

$\therefore$ required term is 20^{th} term.

Illustration – 3

Is – 150 a term of the A.P. 11, 8, 5, 2, ... ?

Solution : Here $a = 11,\ d = -3$

$-150 = a_n = a + (n-1)\,d$

$= 11 + (n-1)(-3)$

$= 11 - 3n + 3 = 14 - 3n$

$3n = 14 + 150 \Rightarrow n = \dfrac{164}{3} = 54\dfrac{2}{3}$,

which is not possible.

$\therefore$ – 150 is not a term of the given A.P.

Sum of First *n* Terms of an A.P.

Sum of first n terms means sum of from first term to n^{th} term.

Consider an A.P. whose first term and common difference are 'a' and 'd' respectively.

Sum of first n terms S_n of this A.P. is given by

$$S_n = \frac{n}{2}\,[2a + (n-1)\,d]$$

If last term of an A.P. containing n terms be l, then n^{th} term $l = a + (n-1)\,d$.

$\therefore \quad S_n = \dfrac{n}{2}\,[2a + (n-1)\,d]$

$= \dfrac{n}{2}\,[a + \{a + (n-1)\,d\}]$

$\Rightarrow \quad S_n = (a + l)$

Considering the Terms in an A.P.

If sum of three consecutive terms of an A.P. is given, then if required consider the three consecutive terms as $(a-d)$, a and $(a+d)$. This reduces one unknown d thereby making the solution easier.

Similarly, we consider the four consecutive terms as $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$ and five consecutive terms as $(a-2d)$, $(a-d)$, a, $(a+d)$ and $(a+2d)$; if their sums are given otherwise consider three terms as a, $a+d, a+2d$; four terms as a, $a+d$, $a+2d$, $a+3d$ and five terms as a, $a+d$, $a+2d$, $a+3d$, $a+4d$.

Arithmetic Mean of n Numbers

Arithmetic mean of n numbers $a_1, a_2, a_3, a_4, ..., a_n$

$$= \frac{a_1 + a_2 + a_3 + a_4 + ... + a_n}{n}$$

To Find a given Number of Arithmetic Mean(s) between Two given Numbers

Between any two given numbers, it is always possible to insert any number of terms such that whole series thus formed shall be an A.P. The terms thus inserted are called **Arithmetic Means**.

(a) A Single Arithmetic Mean between any Two given Numbers

If A be the arithmetic mean between any two given numbers a and b; then a, A, b will be in A.P.

$\therefore \quad b - A = A - a \Rightarrow A = \dfrac{a+b}{2}$

(b) More than One Arithmetic Mean Between any Two given Numbers

If a and b are two given numbers and n be the number of arithmetic means between them, then a will be the first term and b will be the $(n+2)^{th}$ term of the A.P. formed when n arithmetic means inserted between a and b.

$\therefore b = a + (n+1)\,d$, where d is the common difference

$\Rightarrow \quad d = \dfrac{b-a}{n+1}$

Since required arithmetic means are the second, third, fourth, ..., $(n+1)^{th}$ terms of the A.P. whose first term is a and common difference is $\dfrac{b-a}{n+1}$,

Hence the required arithmetic means are

$$a + \frac{b-a}{n+1},\ a + \frac{2(b-a)}{n+1},\ a + \frac{3(b-a)}{n+1},\ ...,\ a + \frac{n(b-a)}{n+1}.$$

Illustration – 4

Find three arithmetic means between 3 and 19.

Solution : $d = \dfrac{b-a}{n+1} = \dfrac{19-3}{3+1} = 4$

$\therefore$ First arithmetic mean $= 3 + 4 = 7$

Second arithmetic mean $= 7 + 4 = 11$

and Third arithmetic mean $= 11 + 4 = 15$.

Illustration – 5

Find the A.M. between $(x - y)$ and $(x + y)$.

Solution : Let A be A.M. between $(x - y)$ and $(x + y)$

$$\therefore A = \frac{x - y + x + y}{2} = x$$

Illustration – 6

For what value of n, A.M. between a and b

$$= \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

Solution : A.M. between a and b is $\frac{a + b}{2}$

$$\therefore \quad \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a + b}{2}$$

$$\Rightarrow \quad 2\left(a^{n+1} + b^{n+1}\right) = \left(a^n + b^n\right)(a + b)$$

$$\Rightarrow \quad a^n = b^n$$

$$\therefore \quad n = 0$$

GEOMETRIC PROGRESSION (G.P.)

A sequence of numbers whose each term (except first term) is found out by multiplying the just previous term by the same number. The number by which we multiply to any term to get its next term is called **common ratio** of the G.P.

For example:

5, 10, 20, 40, ... is a G.P. whose first term is 5, second term is 10, third term is 20 and so on. Its common ratio is 2, because to get any term (except first term) we multiply its just previous term by 2.

Common ratio is also found out by dividing any term (except first term) by its just previous term, thus

$$\text{common ratio} = \frac{10}{5} = \frac{20}{10} = \frac{40}{20} = \ldots = 2$$

First term of a G.P. is denoted by 'a' and its common ratio is denoted by 'r'.

$\therefore \quad a = 5, r = 2$

Standard form of a G.P. is

$a, ar, ar^2, ar^3, ar^4, \ldots$

(i) n^{th} term of a G.P., $a_n = ar^{n-1}$

(ii) Sum of first n terms of a G.P.,

$$S_n = \frac{a(r^n - 1)}{r - 1}, \text{ if } |r| > 1$$

$$\text{and } S_n = \frac{a(1 - r^n)}{1 - r}, \text{ if } |r| < 1$$

(iii) If $|r| < 1$, then sum of infinite terms of the G.P.,

$$S_\infty = \frac{a}{1 - r}$$

If $|r| > 1$, then sum of infinite terms cannot exist.

Illustration – 7

Which term of the G.P. 2, 1, $\frac{1}{2}, \frac{1}{4}$, ... is $\frac{1}{128}$?

Solution :

Let the nth term be $\frac{1}{128}$.

Then, $a_n = \frac{1}{128}$

$$\Rightarrow \quad ar^{n-1} = \frac{1}{128}$$

$$\Rightarrow \quad 2\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^7 \Rightarrow \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^8$$

$$\therefore \quad n - 1 = 8$$

$$\Rightarrow \quad n = 9.$$

Illustration – 8

The third term of a G.P. is 4. Find the product of its first five terms.

Solution : Let a be the first term and r the common ratio. Then,

$a_3 = 4 \quad \Rightarrow ar^2 = 4$

Product of first five terms $= a_1 a_2 a_3 a_4 a_5$

$= a(ar)(ar^2)(ar^3)(ar^4)$

$= a^5 r^{10} = (ar^2)^5 = (4)^5 = 1024.$

Illustration – 9

Find the sum of the series

$2 + 6 + 18 + \ldots + 4374$.

GP_3386

Solution : Required sum

$$= \frac{a(r^n-1)}{r-1} = \frac{(ar^{n-1})r-a}{r-1} = \frac{4374\times 3-2}{3-1} = 6560.$$

[Here $a=2, r=3, ar^{n-1}=4374$]

Illustration – 10

Find the sum of the G.P. 5 + 55 + 555 + ... + to *n* terms.

Solution : 5 + 55 + 555 + ... to *n* terms

$$= \frac{5}{9}[9+99+999+\ldots \text{ to } n \text{ terms}]$$

$$= \frac{5}{9}[(10-1)+(10^2-1)+(10^3-1)+\ldots+(10^n-1)]$$

$$= \frac{5}{9}[(10+10^2+10^3+\ldots+10^n)-(1+1+1\ldots \text{to } n \text{ terms})]$$

$$= \frac{5}{9}\left[10\times\frac{(10^n-1)}{10-1}-n\right] = \frac{5}{81}[10^{n+1}-10-9n].$$

Illustration – 11

Find the sum of all terms of the G.P.

4, 2, 1, $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

Solution : $a=4, |r| = \frac{1}{2} < 1$

Sum of all terms, $S_\infty = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8$

Illustration – 12

Find the sum of infinite terms of the G.P.

$\frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \frac{-32}{5}, \ldots$

Solution : $a = \frac{4}{5}, r = \frac{\frac{-8}{5}}{\frac{4}{5}} = -2$

$\therefore |r| = |-2| = 2 \not< 1$

Hence sum of infinite terms of the given G.P. will not exist.

Considering the Terms In a G.P.

If product of three consecutive terms of a G.P. is given, then if required consider the three consecutive term of the G.P. as, $\frac{a}{r}$, a and ar. This reduces one unknown, r, which makes the solution easier.

Similarly, we consider the four consecutive terms as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ and five consecutive terms as $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$; if their products are given. Otherwise we consider three terms as a, ar, ar^2; four terms as a, ar, ar^2, ar^3 and five terms as a, ar, ar^2, ar^3, ar^4.

Illustration – 13

If the sum of three consecutive terms of a G.P. is 38 and their product is 1728, then find these three consecutive terms.

Solution : Let the three consecutive terms be $\frac{a}{r}, a, ar$.

Then, $\frac{a}{r}\cdot a\cdot ar = 1728 \Rightarrow a^3 = 1728$

$\Rightarrow \quad a = 12$

And $\frac{a}{r} + a + ar = 38$

$$a\left(\frac{1}{r}+1+r\right) = 38$$

$6r^2 - 13r + 6 = 0$

$r = 3/2$ or $r = 2/3$

$\therefore$ Numbers are 8, 12, 18 or 18, 12, 8.

Geometric Mean of n Numbers

Geometric mean of n positive numbers $a_1, a_2, a_3, a_4, \ldots, a_n = (a_1 . a_2 . a_3 . a_4 \ldots a_n)^{1/n}$.

To Find a given Number of Geometric Mean(s) between Two given Numbers

Between any two given numbers, it is always possible to insert any number of terms such that whole series thus formed will be a G.P. The terms thus inserted are called **Geometric Means**.

(a) A Single Geometric Mean between any Two given Numbers

Let G be the geometric mean (G.M.) between any two given numbers a and b; then a, G, b are in G.P.

$$\therefore \quad \frac{b}{G} = \frac{G}{a} \Rightarrow G = \sqrt{ab}$$

(b) More than One Geometric Means between any Two given Numbers

If a and b are two given numbers and n be the number of geometric means between them, then a will be the first term and b will be the $(n+2)^{th}$ term of the G.P. formed when n geometric means inserted between a and b.

$$\therefore\ b = a\,.\,r^{n+1} \Rightarrow r^{n+1} = \frac{b}{a} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}};$$

where r is the common ratio.

Since required geometric means are the second, third, fourth, ..., $(n+1)^{th}$ term of the G.P. whose first term is a and common ratio is $\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$.

Hence required geometric means are

$$a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}, a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}, ..., a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

Illustration – 14

Find 5 G.M. between 576 and 9.

Solution : Let G_1, G_2, G_3, G_4, G_5 be 5 G.M. between 576 and 9.

$$\therefore\quad r = \left(\frac{9}{576}\right)^{\frac{1}{5+1}} = \left(\frac{1}{64}\right)^{\frac{1}{6}} = \frac{1}{2}$$

$$G_1 = ar = 576 \times \frac{1}{2} = 288\,;\ G_2 = ar^2 = 576 \times \frac{1}{4} = 144$$

$$G_3 = ar^3 = 576 \times \frac{1}{8} = 72,\ G_4 = ar^4 = 576 \times \frac{1}{16} = 36;$$

$$G_5 = ar^5 = 576 \times \frac{1}{32} = 18$$

RELATION BETWEEN ARITHMETIC MEAN AND GEOMETRIC MEAN

Arithmetic mean is always greater than the Geometric mean.

SHORTCUTS

Shortcut Approach - 1

a, b, c, d, ... are in A.P., if a – b = b – c = c – d = ...

Illustration – 15

Check whether the series $2, \frac{5}{2}, 3, \frac{7}{2}$, ... is an AP or not.

Shortcut Solution :

Here $2 - \frac{5}{2} = \frac{4}{2} - \frac{5}{2} = \frac{-1}{2}$; $\frac{5}{2} - 3 = \frac{5}{2} - \frac{6}{2} = \frac{-1}{2}$

& $3 - \frac{7}{2} = \frac{6}{2} - \frac{7}{2} = \frac{-1}{2}$

$\therefore$ Given series is an A.P.

Shortcut Approach - 2

If the same number be added or subtracted from each term of an A.P., the resulting terms are also in A.P. with the same common difference as earlier.

Illustration – 16

If the series : 5, 8, 11, 14, ... is an A.P., then 11, 14, 17, 20, ... is

(a) an A.P. with common difference 6
(b) G.P.
(c) an A.P. with common difference 3
(d) None of these

Shortcut Solution : (c)

Here, it is clear that if we add 6 to every term of the first series we get second series.

$\therefore$ 11, 14, 17, 20, ... is also an AP with common difference as 3.

Shortcut Approach - 2

In an A.P., the sum of terms equidistant from the beginning and end is constant and equal to the sum of first term and last term.

Illustration – 17

Consider an A.P. : 2, 6, ..., 46, with 12 terms. The sum of 4^{th} & 9^{th} term is

(a) 46 **(b) 48**
(c) 50 **(d) None of these**

Shortcut Solution : (b)

Since the give A.P. has 12 terms

$\therefore$ 9^{th} term from beginning is the 4^{th} term from end.

$\therefore$ Sum of 4^{th} and 9^{th} terms = 4^{th} term from beginning + 4^{th} term from end

= 1st term + last term = 2 + 46 = 48

Shortcut Approach - 2

If in an A.P., sum of first p terms is equal to sum of first q terms, then sum of (p + q) terms is zero.

Illustration – 18

Consider an A.P. : 8, 6, 4, ... If $S_2 = 14$, $S_7 = 14$, then S_9 is equal to

(a) 28 (b) 14 (c) 7 (d) 0

Shortcut Solution : (d)

Here, $S_2 = S_7 = 14$

$\therefore$ $S_9 = S_{(2+7)} = 0$

Shortcut Approach - 5

If in an A.P., p^{th} term is q and q^{th} term is p, then n^{th} term is $(p + q - n)$.

Illustration – 19

If the 6^{th} term of an AP : 15, 14, 13, ... is 10 & 10^{th} term is 6, then 20^{th} term of this AP is

(a) 0 (b) 2 (c) – 2 (d) – 4

Shortcut Solution : (d)

Here $T_6 = 10$

& $T_{10} = 6$

$\therefore$ $T_{20} = 10 + 6 - 20 = -4$

Shortcut Approach - 1

If in an A.P., sum of p terms is q and sum of q terms is p, then sum of $(p + q)$ terms is $-(p + q)$.

Illustration – 20

If sum of 10 terms of an AP is 15 & sum of 15 terms of the same AP is 10. Then sum of its 25 terms is

(a) – 10 (b) – 25 (c) 25 (d) – 15

Shortcut Solution : (b)

Here $S_{10} = 15$

and $S_{15} = 10$

$\therefore$ $S_{25} = S_{(10+15)} = -(10 + 15) = -25$

Shortcut Approach - 7

If $m_1, m_2, m_3, ..., m_k$ and $n_1, n_2, n_3, ..., n_k$ are two A.P.'s, then $(m_1 + n_1)$, $(m_2 + n_2)$, $(m_3 + n_3)$, ... $(m_k + n_k)$ are also in A.P.

Illustration – 21

Consider two AP's

3, 5, 7, 9, 11, ...

& 7, 11, 15, 19, 23, ...

then 10, 16, 22, 28, 34 ..... is

(a) an A.P. (b) G.P.

(c) both A.P. & G.P. (d) None of these

Shortcut Solution : (a)

Here, 3rd given series is obtained by adding the corresponding terms of 1st & 2nd series.

$\therefore$ Third series is an A.P.

Shortcut Approach - 2

$a, b, c, d,$ ..... are in G.P., if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = ...$

Illustration – 22

Check whether the given series in G.P. or not.

2, 6, 18, 54, ...

Shortcut Solution :

Here $\frac{2}{6} = \frac{1}{3}$; $\frac{6}{18} = \frac{1}{3}$ & $\frac{18}{54} = \frac{1}{3}$

$\therefore$ Given series is a G.P.

Shortcut Approach - 1

If each term of a G.P. be multiplied or divided by the same quantity, the resulting sequence will be also in G.P. with the same common ratio as before.

Illustration – 23

If 5, 15, 45, 135, ... is a G.P. with common ratio 3. Then 1, 3, 9, 27, ... is

(a) GP with common ratio 3

(b) GP with common ratio 5

(c) GP with common ratio –3

(d) None of these

Shortcut Solution : (a)

It is clear that every element of 1st series is divided by 5, then the 2nd series is obtained.

$\therefore$ 2nd series is also a G.P. with common ratio 3.

Shortcut Approach 10

If $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ are two geometric progressions, then the sequence $a_1b_1, a_2b_2, a_3b_3, ...$ is also G.P.

Illustration – 24

Consider two GP's 2, 6, 18, 54, ... &4, 8, 16, 32, ..., then 8, 48, 288, 1728, ... is

(a) a G.P. (b) an A.P.

(c) both A.P. & G.P. (d) None of these

Shortcut Solution : (a)

Here 3rd series is obtained by multiplying corresponding terms of 1st & 2nd series

$\therefore$ Third series is a G.P.

Shortcut Approach - 2

Sum of first n natural numbers $= \frac{n(n+1)}{2}$

i.e. $1+2+3+4+5+.....+n = \frac{n(n+1)}{2}$

Illustration – 25

$1+2+3+4+5+6+.....+15=$

(a) 220 **(b)** 120 **(c)** 200 **(d)** 180

Shortcut Solution : (b)

Since the given series is a set of 1st 15 natural numbers.

$\therefore \quad \text{Sum} = \frac{15(15+1)}{2} = 120$

Shortcut Approach - 12

Sum of first n odd natural numbers $= n^2$

i.e. $1+3+5+7+.....+(2n-1)=n^2$

Illustration – 26

$1+3+5+7+.....$ upto 20 terms =

(a) 200 **(b)** 360

(c) 400 **(d)** None of these

Shortcut Solution : (c)

Since the given series is a set of first 20 odd natural numbers.

$\therefore \text{Sum} = 20^2 = 400$

Shortcut Approach 10

Sum of first n even natural numbers $= n(n+1)$

i.e. $2+4+6+8+.....+2n = n(n+1)$

Illustration – 27

$2+4+6+8+.....$ upto 15 terms =

(a) 225 **(b)** 240 **(c)** 300 **(d)** 280

Shortcut Solution : (b)

Since the given series is a set of 1st 15 even natural numbers.

$\therefore \quad \text{Sum} = 15(15+1) = 240$

Shortcut Approach 10

Sum of squares of first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

i.e. $1^2+2^2+3^2+.....+n^2 = \frac{n(n+1)(2n+1)}{6}$

Illustration – 28

$1^2+2^2+3^2+4^2+5^2=$

(a) 50 **(b)** 60 **(c)** 55 **(d)** 65

Shortcut Solution : (c)

Here n = 5

$\therefore \quad \text{Sum} = \frac{5(5+1)(10+1)}{6} = \frac{5\times6\times11}{6} = 55$

Shortcut Approach 10

Sum of cubes of first n natural numbers

$$= \left[\frac{n(n+1)}{2}\right]^2$$

i.e. $1^3+2^3+3^3+4^3+.....+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

Illustration – 29

$1^3+2^3+3^3+4^3+5^3+6^3=$

(a) 325 **(b)** 365 **(c)** 425 **(d)** 441

Shortcut Solution : (d)

Here n = 6

$\therefore \text{Sum} = \left[\frac{6(6+1)}{2}\right]^2 = \left(\frac{6\times7}{2}\right)^2 = 441$

Shortcut Approach - 16

If $m_1, m_2, m_3, ..., m_k$ and $n_1, n_2, n_3, ..., n_k$ are in A.P., then $(m_1-n_1), (m_2-n_2), (m_3-n_3), ... (m_k-n_k)$ are also an A.P.

Illustration – 30

Consider two AP's 8, 19, 30, 41, 52, ... &3, 5, 7, 9, 11, ..., then 5, 14, 23, 32, 41 ..... is

(a) a G.P. **(b)** an A.P.

(c) both A.P. & G.P. **(d)** None of these

Shortcut Solution : (b)

Here, 3rd given series is obtained by subtracting terms of the 2nd series from the corresponding elements of 1st series.

$\therefore$ Third series is an A.P.

GP_3386

LEVEL - 1

1. How many terms of the series 1 + 3 + 5 + 7 + ... amount to 123454321?
(a) 11101 (b) 11011 (c) 10111 (d) 11111

2. Find the value of $1-2-3+2-3-4+...$ + upto 100 terms.
(a) –694 (b) –626 (c) –624 (d) –549

3. What is the sum of all the two-digit numbers which when divided by 7 gives a remainder of 3?
(a) 94 (b) 676
(c) 696 (d) None of these

4. Find the sum of all numbers in between 10–50 excluding all those numbers which are divisible by 8. (include 10 and 50 for counting.)
(a) 1070 (b) 1220 (c) 1320 (d) 1160

5. A number 15 is divided into three parts which are in AP and the sum of their squares is 83. Find the smallest number.
(a) 5 (b) 3 (c) 6 (d) 8

6. If sixth term of a H. P. is $\frac{1}{61}$ and its tenth term is $\frac{1}{105}$, then the first term of that H.P. is
(a) $\frac{1}{28}$ (b) $\frac{1}{39}$ (c) $\frac{1}{6}$ (d) $\frac{1}{17}$

7. If $1^3 + 2^3 + ... + 9^3 = 2025$, then the value of $(0.11)^3 + (0.22)^3 + ... + (0.99)^3$ is close to:
(a) 0.2695 (b) 0.3695 (c) 2.695 (d) 3.695

8. Find the sum of the given series:
$1^3+2^3+3^3+4^3+..... (30)^3 =$
(Bank Clerk 2013)
(a) 206225 (b) 226225 (c) 236225 (d) 216225

9. Find the value of $5^3+6^3+..... +10^3$.
(SBI Clerk 2015)
(a) 2295 (b) 2425 (c) 2495 (d) 2925

10. If $1^3+2^3+3^3+..... +10^3=3025$, then $4+32+108+..... +400$ is equal to:
[SSC CGL Tier-I 2012]
(a) 12000 (b) 12100 (c) 12200 (d) 12400

11. If $1^2+2^2+3^2+..... +x^2=\frac{x(x+1)(2x+1)}{6}$ then $1^2+3^2+5^2+..... +19^2$ is equal to:
[SSC CGL Tier-I 2013]
(a) 1330 (b) 2100 (c) 2485 (d) 2500

12. Find the correct option for the given statement consider two AP's **(Bank PO 2014)**
15 19 23 27 31 .....
2 5 8 11 14 .....
then 13, 14, 15, 16, 17 .....
(a) an A.P (b) a G.P
(c) both A.P and G.P (d) None of these

13. The sum to infinity of the progression
$9-3+1-\frac{1}{3}+\cdots$ is
(a) 9 (b) 9/2 (c) 27/4 (d) 15/2

14. The sum of an infinite GP is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, then how many values of common ratio is/are possible, common ratio is greater than 0?
(a) 0 (b) 1 (c) 2 (d) 3

15. Find the value of $25^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.....\infty\right)}$
(a) 16 (b) 9 (c) 25 (d) 5

16. The 7th and 21th term of an AP are 6 and – 22 respectively. Find the 26th term.
(a) –34 (b) –32 (c) – 12 (d) – 10

17. The 4th and 10th term of a GP are 1/3 and 243 respectively. Find the 2nd term.
(a) 3 (b) 1 (c) 1/27 (d) 1/9

18. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?
(a) 6 (b) 9 (c) 7 (d) 8

19. Find the sum of the series $2\sqrt{3}, 2\sqrt{2}, \frac{4}{\sqrt{3}}, ...$
(a) $6\left(\sqrt{3}+\sqrt{2}\right)$ (b) $6\left(\sqrt{3}-\sqrt{2}\right)$
(c) $\frac{6}{\left(\sqrt{3}+\sqrt{2}\right)}$ (d) None of these

20. Find the number of terms in the A.P. 22, 28, 34, ...616:
(a) 80 (b) 78 (c) 99 (d) 100

21. How many terms of the series 20 + 16 + 12 + ... amounts to 48?
(a) 3 (b) 5
(c) 8 (d) both (a) and (c)

22. How many terms are there in A.P. whose first and fifth terms are –14 and 2 respectively and the sum is 40 :
(a) 12 (b) 10 (c) 16 (d) 8

23. Find the number of terms of the A.P. 98, 91, 84, ... must be taken to give a sum of zero :
(a) 14 (b) 15 (c) 31 (d) 29

24. Check whether the series 0.6, 1.7, 2.8, 3.9, ... is **(CDS 2012)**
(a) an A.P (b) a G.P
(c) an H.P (d) None of these

25. Determine the sum of 6th and 19th term of A.P. 4, 9, 14, ..., 119 with 24 terms. **(SSC CGL Tier II 2016)**
(a) 124 (b) 123 (c) 120 (d) 122

26. Find the sum of middle term (s) of the A.P. 7, 13, 19, ..., 241. **(Bank Clerk 2010)**
(a) 248 (b) 208 (c) 320 (d) 410

27. Determine the sum of the middle terms for A.P. 1, 7, 13, 19, ... 415. **(SSC CGL 2011)**
(a) 316 (b) 320 (c) 416 (d) 420

28. If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then find the sum of (m + n) terms, **(SSC CGL Tier I 2015)**
(a) – mn (b) – (m + n)
(c) mn (d) O

29. In a certain A.P, it the sum of m terms is the same as the sum of its n terms, then find the sum of its (m + n) terms. **(SSC CGL Tier I 2017)**
(a) – (m + n) (b) mn
(c) $\frac{1}{mn}$ (d) 0

30. In an A.P, it the sum of first 23 terms is 40 and sum of 40 terms is 23, then find the sum of its 63 terms. **(SSC CHSL 2013)**
(a) – 163 (b) – 263
(c) – 363 (d) – 63

31. What is the sum of 100 terms of the series
1 – 2 + 3 – 4 + 5 – 6 + ... ?
(a) 100 (b) 50
(c) 550 (d) None of these

32. Find the sum of the given series
1 + 2 + 3 + 4 + ... 120. **(SSC CGL 2010)**
(a) 5260 (b) 6260 (c) 7260 (d) 8260

33. Find the sum of the gvien series.
1 + 2 + 3 + 4 + 5 + ... 5600 **(Bank Clerk 2012)**
(a) 15682600 (b) 15682800
(c) 15682700 (d) 15682900

34. Find the sum of the given series.
1 + 3 + 5 + 7 + ... upto 100 terms. **(SSC CGL Tier I 2013)**
(a) 10000 (b) 9000 (c) 8000 (d) 11000

35. Find the sum of the given series:
2 + 4 + 6 + 8 + ... upto 50 terms. **(SSC CHSL 2014)**
(a) 2450 (b) 2560 (c) 2460 (d) 2550

Directions (36-40) : What should come in place of the question mark (?) in the following number series ?

36. 12, 14, 31, ?, 393, 1971
(a) 96 (b) 80 (c) 97 (d) 82

37. 96, 144, 216, ?, 486, 729
(a) 316 (b) 402
(c) 344 (d) None of these

38. 12, 48, ?, 89, 98, 102
(a) 73 (b) 79 (c) 82 (d) 94

39. 1212, 1176, 1151, 1135, ?, 1122
(a) 1155 (b) 1126 (c) 1118 (d) 1148

40. 5, 19, 41, 71, ?, 155
(a) 106 (b) 125 (c) 115 (d) 109

Directions (41-45): What will come in place of question mark (?) in the following number series?

41. 9, 19, 40, 83, 170, 345, ?
(a) 687 (b) 696 (c) 706 (d) 745

42. 1980, 1484, 1236, 1112, 1050, ?
(a) 1025 (b) 917
(c) 1029 (d) None of these

43. 18, 19, 40, 123, 496, 2485, ?
(a) 16430 (b) 17450
(c) 14916 (d) 14746

44. 1115, 608, 355, 229, 166.5, ?
(a) 134.50 (b) 135
(c) 135.30 (d) 135.75

45. 4, 10, 33, 136, 685, ?
(a) 3430 (b) 4110 (c) 4116 (d) 3435

GP_3386

LEVEL - 2

1. If $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in an A.P., where a, b and c are in a GP, then a, b and c, are the lengths of sides of
(a) An isosceles triangle
(b) An equilateral triangle
(c) A scalene triangle
(d) None of these

2. In a geometric progression the sum of the first and the last term is 66 and the product of the second and the last but one term is 128. Determine the first term of the series.
(a) 64 (b) 64 or 2 (c) 2 or 32 (d) 32

3. The middle term of arithmetic series 3, 7, 11...147, is
(a) 71 (b) 75 (c) 79 (d) 83

4. If $x + y + z = 1$ and x, y, z are positive numbers such that $(1-x)(1-y)(1-z) \geq kxyz$, then $k =$
(a) 2 (b) 4 (c) 8 (d) 16

5. How many terms are identical in the two APs 1,3, 5,... up to 120 terms and 3, 6, 9, ... up to 80 terms ?
(a) 38 (b) 39 (c) 40 (d) 41

6. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ is
(a) less than 1 (b) equal to 1
(c) greater than 1 (d) any real number

7. The sum of the series

$1/\left(\sqrt{2}+\sqrt{1}\right)+1/\left(\sqrt{2}+\sqrt{3}\right)+.....$

$+1/\left(\sqrt{120}+\sqrt{121}\right)$ is:

(a) 10 (b) 11
(c) 12 (d) None of these

8. If $\log_x a$, $a^{x/2}$, and $\log_b x$ are in GP, then x is
(a) $\log_a(\log_b a)$
(b) $\log_a(\log_e a) + \log_a(\log_e b)$
(c) $-\log_a(\log_a b)$
(d) $\log_a(\log_e b) - \log_a(\log_e a)$

9. If a, b and c are in HP, then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in.
(a) AP (b) GP
(c) HP
(d) Cannot be determined uniquely

10. If a man saves ₹ 4 more each year than he did the year before and if he saves ₹ 20 in the first year, after how many years will his savings be more than ₹ 1000 altogether?
(a) 19 years (b) 20 years
(c) 21 years (d) 18 years

11. After striking a floor a rubber ball rebounds $(7/8)^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of 420 meters?
(a) 2940 (b) 6300 (c) 1080 (d) 3360

12. If the sum of first 11 terms of A.P is equal to sum of first 19 terms of that A.P, find the sum of first 30 terms of that A.P.
(a) 0 (b) –2 (c) 2 (d) 5

13. Find the sum of the integers between 1 and 200 that are multiples of 7?
(a) 2742 (b) 2842 (c) 2642 (d) 2546

14. How many terms are there in the G.P 5, 20, 80, 320, ..., 20480?
(a) 5 (b) 6 (c) 7 (d) 8

15. The sides of a right angled Δ are 6, 8 and 10 cm respectively. A new right angled Δ is made by joining the mid-points of all the sides. This process continues for infinite then calculate the area of all the Δ so made.
(a) 16 cm^2 (b) 32 cm^2
(c) 24 cm^2 (d) 48 cm^2

16. The side of a square is 16 cm. Infinite squares are made by joining the mid points of the square. Calculate the area of all the square made.
(a) 512 cm^2 (b) 256 cm^2
(c) 128 cm^2 (d) 1024 cm^2

17. Find the sum to n terms of the series $11 + 103 + 1005 + ...$
(a) $\frac{10}{9}(10^n + 1) + n^2$ (b) $\frac{10}{9}(10^n - 1) + n^2$
(c) $\frac{10}{9}(10^n + 1) + n$ (d) $\frac{10}{9}(10^n - 1) + n$

18. Two numbers A and B are such that their GM is 20% lower than their AM. Find the ratio between the numbers.
(a) 3 : 2 (b) 3 : 1 (c) 4 : 1 (d) 2 : 1

19. A ball is thrown from a height of 600 m on the ground. The ball bounce 2/3 times of its every last bounce then calculate the total distance travelled by the ball till it stop.
(a) 1000 m (b) 3000m
(c) 2000m (d) 4000m

20. If the p^{th} term of an A.P. is q and its q^{th} term is p then its m^{th} term is :
(a) $\frac{pq}{m}$ (b) $pq - m$
(c) $p + q - m$ (d) pqm

21. Find the sum of all the integers between 55 and 5555 which are divisible by 7 :
(a) 678 (b) 786
(c) 876 (d) None of these

22. What is the least possible sum of the A.P. $-23, -19, -15...$:
(a) -3 (b) -78
(c) -87 (d) None of these

23. Find the sum of the first hundred even natural numbers divisible by 5 :
(a) 50500 (b) 55000
(c) 50050 (d) 50005

24. Find the sum of all odd numbers of four digits which are divisible by 9 :
(a) 2784491 (b) 2478429
(c) 2754000 (d) 2448729

25. The third term of a G.P. is 4. The product of first five terms is :
(a) 4^3 (b) 4^4
(c) 4^5 (d) None of these

26. Sum of three consecutive terms in a G.P. is 42 and their product is 512. Find the largest of these numbers :
(a) 28 (b) 16
(c) 32 (d) None of these

27. The sum of first three terms of a G.P. is to the sum of the first six terms is 125 : 152. Find the common ratio of the G.P. :
(a) 3/5 (b) 3 (c) 2/5 (d) 5/8

28. If a, b, c are respectively the x^{th}, y^{th} and z^{th} terms of a G. P. then the value of
$(y-z)\log a + (z-x)\log b + (x-y)\log c$:
(a) 0 (b) 1
(c) 3 (d) None of these

29. The sum of first two terms of a G. P. is $\frac{5}{3}$ and the sum to infinity of the series is 3. Find the first term :
(a) 1 (b) $\frac{2}{3}$
(c) 5 (d) both (a) and (c)

30. Find the correct option for the given series. 5, 10, 20, 40, ... **(CDS 2013)**
(a) A. P with difference 5
(b) G.P with ratio 2
(c) G.P with ratio $\left(\frac{1}{2}\right)$
(d) A.P. with difference 2

31. Find the correct option for the given series. 3, 9, 27, 81, ... **(SSC CGL Tier I 2012)**
(a) A. P with difference (3)
(b) A.P with difference (6)
(c) G.P with ratio (3)
(d) G.P with difference $\left(\frac{1}{3}\right)$

32. Find the correct option for the given statement. If 4, 12, 36, 108, 324 ..... is a G.P, with common ratio 3. Then, 1, 3, 9, 27, 81 .... is **(Bank Clerk 2011)**
(a) G. P with common ratio 4
(b) G.P with common ratio $\left(\frac{1}{3}\right)$
(c) G.P with common ratio 6
(d) None of these

33. Find the correct option for the given statement consider two GP's 6, 12, 24, 48 ..... and 1, 2, 4, 8 ..... then. 6, 24, 96, 384, ... is **(CDS 2014)**
(a) an A.P
(b) both A.p and G.P
(c) a G.P
(d) neither an AP nor a G.P

34. If A.M. and G.M. of two numbers is 10 and 8 respectively, then find their H.M. :
(a) 9 (b) 12 (c) 6 (d) 6.4

35. The value of $3^{1/3}.\ 9^{1/18}.\ 27^{1/81.....}$:
(a) 3 (b) 9
(c) 27 (d) None of these

36. Three non - zero real numbers form an A.P. and the squares of these numbers taken in the same order form a G.P. Then the number of all possible common ratio of the G.P. is :
(a) 1 (b) 2
(c) 3 (d) None of these

37. The value of $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+....\right)}$:
(a) 2 (b) $\frac{1}{2}$
(c) 4 (d) None of these

38. The sum of the series
$\frac{5}{13}+\frac{55}{(13)^2}+\frac{555}{(13)^3}+\frac{5555}{(13)^4}+.....\infty$ is :

GP_3386

(a) $\frac{55}{39}$ (b) $\frac{65}{39}$ (c) $\frac{110}{169}$ (d) $\frac{65}{36}$

39. What will be the 33rd term of the sequence 1, 7, 25, 79, ... ?
(a) 33^3-1 (b) $3^{33}-2$
(c) $3(33^3-2)$ (d) None of these

40. Find the sum of
$1+\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\frac{10}{243}+....\infty$.
(a) $\frac{2}{3}$ (b) $\frac{5}{2}$ (c) $\frac{19}{45}$ (d) $\frac{81}{17}$

41. The sum of the infinite series
$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+.....\infty$ is equal to :
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{38}{27}$ (d) None of these

Directions (42 and 43) : In the zoolozical park Lucknow there are four kinds of animals viz. Elephant, monkey, lion and tiger which are in increasing G.P and in the local zoo in Kanpur there are the same kinds of animals but in A.P. The number of monkeys in each of the places is same but just greater than that of elephants. Total number of animals in zoolozical park is 50% more than that of local zoo. Also the common ratio of the G.P is same as the common difference of the A.P Number of tigers in both the places is maximum.

42. What is the number of elephants in each of the places?
(a) 1 (b) 2 (c) 3 (d) 4

43. What is the number of lions in zoological park Lucknow ?
(a) 5 (b) 6
(c) 8 (d) can not be determine

44. Find the sum of the series
$1.3^2+2.5^2+3.7^2+.....$ to 20 terms :
(a) 12896 (b) 187898
(c) 98970 (d) 188090

45. A bouncing tennis ball rebounds each time to a height equal to one half the height of the previous bounce. If it is dropped from a height of 16 m, find the total distance it has travelled when it hits the ground for the 10 th time :
(a) $47\frac{15}{16}$ (b) $37\frac{5}{16}$
(c) $67\frac{11}{16}$ (d) None of these

Directions (46–50) : Find out the wrong number in the following given series.

46. 9 5 8 10.5 23 60 183
(a) 8 (b) 5 (c) 23 (d) 10.5

47. 14 21 48 105 315 1102.5 4410
(a) 315 (b) 48 (c) 21 (d) 4410

48. 23 24 33 58 107 186 309
(a) 58 (b) 33 (c) 107 (d) 186

49. 24 25 49 148 492 2956 17735
(a) 25 (b) 49 (c) 492 (d) 148

50. 12 26 56 118 276 498 1008
(a) 26 (b) 56 (c) 498 (d) 276

Directions (51–55): What will come in the place of the question mark (?) in the following number series?

51. 15, 19, 28, 44, 69, 105, (?)
(a) 172 (b) 154 (c) 140 (d) 174

52. 2400, 960, 384, 153.6, 61.44, (?)
(a) 25.91 (b) 28.321 (c) 24.576 (d) 22.234

53. 33, 34, 70, 213, 856, (?)
(a) 3848 (b) 4225 (c) 4150 (d) 4285

54. 305, 338, 404, 503, 635, (?)
(a) 820 (b) 880 (c) 800 (d) 890

55. 16, 34, 19, 40, 22, (?)
(a) 46 (b) 30 (c) 52 (d) 48

Directions (56-60): What should come in place of question mark (?) in the following number series?

56. 300 331 364 399 436 ?
(a) 455 (b) 475 (c) 465 (d) 595

57. 10 22 8 24 6 ?
(a) 20 (b) 18 (c) 26 (d) 156

58. 262144 32768 4096 512 64 ?
(a) 7 (b) 8 (c) 12 (d) 18

59. 2197 2744 3375 4096 4913 ?
(a) 5832 (b) 5459 (c) 5842 (d) 5852

60. 196 207 229 262 306 ?
(a) 351 (b) 355 (c) 361 (d) 364

Directions (61-65): In each of the following number series, a wrong number is given. Find out the wrong number

61. –16 –15 –7 20 84 219 425
(a) –15 (b) 219 (c) 84 (d) 425

62. –11 –10 –38 –349 –5420 –135495
(a) –10 (b) –38 (c) –349 (d) –5420

63. 48 24 28 36 72 180 540
(a) 24 (b) 28 (c) 36 (d) 72

64. 16 17 33 53 83 133 216 349
(a) 349 (b) 33 (c) 53 (d) 216

65. 288 144 96 72 58 48
(a) 144 (b) 96 (c) 72 (d) 58

Hints and Solutions

Level 1

1. (d) It can be seen that for the series the average of two terms is 2, for 3 terms the average is 3 and so on. Thus, the sum for 2 terms is 2^2, for 3 terms it is 3^2 and so on. For 11111 terms it would be 11111^2 = 123454321.

2. (b) The first 100 terms of this series can be viewed as:
$(1-2-3)+(2-3-4)+....+(33-34-35)+34$
The first 33 terms of the above series (indicated inside the brackets) will give an A.P: $-4, -5, -6 ... -36$
Sum of this A.P. = $33 \times (-20) = -660$
Answer = $-660 + 34 = -626$

3. (b) This series is like → 10, 17, 24, ...94.
Here $n = 13$, $d = 7$ and $a = 10$
Using the formula for the sum

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right], \text{sum} = 676$$

Alternatively, Using the average method,
Average = (1st number + last number)/2

$$= \frac{10+94}{2} = 52$$

So, the sum = average × number of numbers
$= 52 \times 13 = 676$

4. (a) The answer will be given by:
$[10 + 11 + 12 + ... + 50] - [16 + 24 + ... + 48]$
$= 41 \times 30 - 32 \times 5$
$= 1230 - 160 = 1070.$

5. (b) The three parts are 3, 5 and 7 since $3^2 + 5^2 + 7^2 = 83$. Since, we want the smallest number, the answer would be 3.

6. (c) Let six term of H.P. = $\frac{1}{61}$
$\Rightarrow$ Six term of A.P. = 61
Similarly, tenth term of A.P. = 105
Let first term of AP is a and common diff. = d
$\therefore \quad a + 5d = 61$
and $\quad a + 9d = 105$
Solving these equation, we get
$a = 6, d = 11$
Hence, first term of H.P. = $\frac{1}{6}$

7. (c) $(0.11)^3 + (0.22)^3 + ...+ (0.99)^3$
$= (0.11)^3 (1^3 + 2^3 + ... + 9^3)$
$= 0.001331 \times 2025 = 2.695275 = 2.695.$

8. (d) Use Shortcut Approach-15

9. (d) Use Shortcut Approach-15

10. (b) Use Shortcut Approach-15

11. (a) Use Shortcut Approach-14

12. (a) Use Shortcut Approach-16

13. (c) Given series is,

$9 - 3 + 1 - \frac{1}{3} + \ldots$ an infinite G.P.

Here $a = 9, r = -\frac{1}{3}$

where a = first term and r = common ratio

We know, $S_\infty = \frac{a}{1-r} = \frac{9}{1-\left(-\frac{1}{3}\right)} = \frac{9}{\frac{4}{3}} = \frac{27}{4}$

14. (c) $a/(1-r) = 162$ and $a(1-r^n)/(1-r) = 160$
$\Rightarrow \quad 1 - r^n = 160/162 \Rightarrow r^n = 1/81$
Hence, there will be only two values of n, *i.e.*, 2 and 4.

15. (d) $25^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.....\infty\right)}$

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...\infty$. It is an infinite GP with comman ratio $\frac{1/9}{1/3} = \frac{1}{3}$ and first term $\frac{1}{3}$.

Sum of infinite terms of this GP = $\frac{a}{1-r}$

$$= \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{2/3} = \frac{1}{2}$$

So, $\quad (25)^{1/2} = \sqrt{25} = 5.$

GP_3386

16. (b) $T_7 = 7\text{th term} = a + 6d = 6$ ...(i)

$T_{21} = 21\text{st term} = a + 20d = -22$ ...(ii)

From (i) and (ii)

$a + 20d - a - 6d = -22 - 6$

$\Rightarrow 14d = -28$

$d = \frac{-28}{14} = -2$

$T_{21} = a + 20(-2) = -22$

$\Rightarrow a = -22 + 40$

$a = 18$

$26\text{th term} = T_{26} = a + 25d$

$= 18 + 25(-2) = 18 - 50 = -32.$

17. (c) $T_4 = ar^3 = \frac{1}{3}$ ...(i)

$T_{10} = ar^9 = 243$ ...(ii)

From (i) and (ii)

$$\frac{T_{10}}{T_4} = \frac{ar^9}{ar^3} = \frac{243}{\frac{1}{3}}$$

$\Rightarrow r^6 = 243 \times 3 \Rightarrow r^6 = 3^6$

$r = 3$

$\Rightarrow a(3)^3 = \frac{1}{3}, \quad a = \frac{1}{81} \Rightarrow T_2 = ar = \frac{1}{81}.3 = \frac{1}{27}$

18. (d) $T_3 = a + 2d = 9$

$T_7 = a + 6d$

$T_2 = a + d$

According to question,

$a + 6d - (a + d) = 20 \Rightarrow 5d = 20$

$d = 4$

$T_3 = a + 2 \times 4 = 9$

$a = 1$

$\Rightarrow 120 = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 240 = n[2a + (n-1)d]$

$\Rightarrow 240 = n[2 + (n-1)4]$

$\Rightarrow 120 = n[1 + (n-1)2]$

$\Rightarrow 120 = n + 2n^2 - 2n$

$\Rightarrow n = 8$

Hence, 8 terms.

19. (a) $r = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\frac{4}{\sqrt{3}}}{2\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

$\therefore \quad S_\infty = \frac{a}{1-r}; \ |r| < 1$

$$= \frac{2\sqrt{3}}{1 - \sqrt{\frac{2}{3}}} = \frac{2\sqrt{3} \times \sqrt{3}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{6}{\sqrt{3} - \sqrt{2}} = 6(\sqrt{3} + \sqrt{2})$$

20. (d) Number of terms $= \left(\frac{616 - 22}{6}\right) + 1$

$= \frac{594}{6} + 1 = 100$

Alternatively :

22, 28, 34, ... 616 = (4, 10, 16, 22, 28 ,....., 616) – (4, 10, 16)

$\Rightarrow$ (6, 12, 18, 24, 30, ..., 618) – (6, 12, 18)

$\Rightarrow$ (1, 2, 3, 4, 5, ..., 103) – (1, 2, 3)

$\Rightarrow 103 - 3 = 100$

Alternatively :

22, 28, 34, ..., 616

$\Rightarrow$ 24, 30, 36, ..., 618

$\Rightarrow$ 4, 5, 6, ..., 103

$\Rightarrow 103 - 3 = 100$

21. (d) $20 + 16 + 12 = 48$

Hence number of terms = 3

Again $20 + 16 + 12 + 8 + 4 + 0 + (-4) + (-8) = 48$

Hence number of terms = 8

Alternatively :

$S_n = 48 = \frac{n}{2}[2 \times 20 + (n-1)(-4)]$

$\Rightarrow 96 = 40n - 4n^2 + 4n$

$\Rightarrow n^2 - 11n + 24 = 0$

$\Rightarrow (n-8)(n-3) = 0$

$\Rightarrow n = 8$ or $n = 3$

22. (b) $T_1 = a = -14$

$T_5 = a + 4d = 2$

$\Rightarrow d = 4$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$

$40 = \frac{n}{2}[-28 + (n-1)4]$

$\Rightarrow n^2 - 8n - 20 = 0$

$\Rightarrow n = 10$ or $n = -2$

Since number of terms can not be negative, hence $n = 10$

Alternatively : Since S_n is very small, therefore we can solve it physically within seconds.

$T_1 = -14, T_5 = 2$

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
– 14	– 10	– 6	– 2	2	6	10	14	18	22
								40	

$\Rightarrow T_1 + T_2 + ... + T_{10} = 0 + 40 = 40$

23. (d) $S_n = \frac{n}{2}[2a + (n-1)d]$

$0 = \frac{n}{2}[196 + (n-1)\times(-7)]$

$\Rightarrow 0 = n(203 - 7n)$

$\Rightarrow 7n = 203$ or $n = 0$

$\Rightarrow n = 29$

Since n = 0 is not acceptable.

The A.P. is 98, 91, 84, ... 0, ..., –91, –98

24. (a) Use Shortcut Approach-1

25. (b) Use Shortcut Approach-3

26. (a) Use Shortcut Approach-3

27. (c) Use Shortcut Approach-3

28. (b) Use Shortcut Approach-6

29. (d) Use Shortcut Approach-4

30. (d) Use Shortcut Approach-6

31. (d) $(1-2)+(3-4)+(5-6)+(7-8)+.....(99-100)$
$=(-1)+(-1)+(-1)+(-1)+.....$ 50 times
$= -50$

32. (c) Use Shortcut Approach-11

33. (b) Use Shortcut Approach-11

34. (a) Use Shortcut Approach-12

35. (d) Use Shortcut Approach-13

36. (c) The series is as follows :
$12 \times 1 + 2 = 12 + 2 = 14$
$14 \times 2 + 3 = 28 + 3 = 31$
$31 \times 3 + 4 = 93 + 4 = 97$
$97 \times 4 + 5 = 388 + 5 = 393$
$393 \times 5 + 6 = 1965 + 6 = 1971$

37. (d) The series is as follows :
$96 \times 3/2 = 144, 144 \times 3/2 = 216, 216 \times 3/2 = 324,$
$324 \times 3/2 = 486, 486 \times 3/2 = 729$

38. (a) The series is as follows :
$12 + 6^2 = 12 + 36 = 48$
$48 + 5^2 = 48 + 25 = 73$
$73 + 4^2 = 73 + 16 = 89$
$89 + 3^2 = 89 + 9 = 98$
$98 + 2^2 = 98 + 4 = 102$

39. (b) The series is as follows :
$1212 - 6^2 = 1212 - 36 = 1176$
$1176 - 5^2 = 1176 - 25 = 1151$
$1151 - 4^2 = 1151 - 16 = 1135$
$1135 - 3^2 = 1135 - 9 = 1126$
$1126 - 2^2 = 1126 - 4 = 1122$

40. (d) The series is as follows :
$1 + 2^2 = 5$
$3 + 4^2 = 19$
$5 + 6^2 = 41$
$7 + 8^2 = 71$
$9 + 10^2 = 109$
$11 + 12^2 = 155$

41. (c) 19 29 50 93 180 355
10 21 43 87 175 351
Here, $21 = 10+10+1$, $43 = 21+21+1$,
$87 = 43+43+1$, $351 = 175+175+1$
So next number = 355 + 351 = 706

42. (d) 1980 1484 1236 1112 1050 1019
496 248 124 62 31
Divide difference from 2 then the result is 31
$= 1050 - 31 = 1019$

43. (c) 18
$18 \times 1 + 1 = 19$
$19 \times 2 + 2 = 40$
$40 \times 3 + 3 = 123$
$123 \times 4 + 4 = 496$
$496 \times 5 + 5 = 2485$
$2485 \times 6 + 6 = 14916$

44. (d) 1115 608 355 229 166.5 135.75
507 253 126 62.5 30.75
254 127 63.5 31.75

45. (c) $(13 \times 1) + 1 = 13 + 1 = 14$
$(14 \times 2) + 2 = 28 + 2 = 30$
$(30 \times 3) + 3 = 90 + 3 = 93$
$(93 \times 4) + 4 = 372 + 4 = 376$
$(376 \times 5) + 5 = 1880 + 5 = 1885$

Level 1

1. (d) $\because \log\frac{5c}{a}, \log\frac{3b}{5c}$ and $\log\frac{a}{3b}$ are in AP

So, $\frac{5c}{a}, \frac{3b}{5c}$ and $\frac{a}{3b}$ are in G.P

$\left(\frac{3b}{5c}\right)^2 = \left(\frac{5c}{a}\right)\left(\frac{a}{3b}\right) \Rightarrow 3b = 5c$ ...(1)

$\because a, b, c$ are in G.P

Therefore, $b^2 = ac$ ...(2)

From (1) and (2)

$25c = 9a$

GP_3386

Hence, $9a = 15b = 25c$

or $\frac{a}{25} = \frac{b}{15} = \frac{c}{9}$

$\because$ $25 > 15 + 9$, so triangle will not be possible

2. (b) Let a be the first term and r be the common ratio of the G.P.
Also assume that n^{th} term is the last term of the GP.

Then, $a + ar^{n-1} = 66$ .....(1)

and $ar.ar^{n-2} = 128$

or $a^2 r^{n-1} = 128$ .....(2)

From (1) and (2), $a + \frac{128}{a} = 66$

or $a^2 - 66a + 128 = 0$ $\therefore$ $a = 64, 2$.

3. (b) 3, 7, 11,....., 147
It is an arithmetic series whose
first term, $a = 3$
last term, $x_n = 147$
common difference, $d = 4$
$x_n = a + (n-1)\,d$
$147 = 3 + (n-1) \times 4$

$n - 1 = \frac{147 - 3}{4}$

$n - 1 = 36 \Rightarrow n = 37$
The given series consists of 37 terms.

Therefore, its middle term will be $\frac{37+1}{2}$

= 19th term
$x_{19} = 3 + (19-1)4 = 3 + 18 \times 4 = 75$
$\therefore$ The middle term of the given arithmetic series is 75.

4. (c) Since A.M. $\geq$ G.M.

$\therefore$ $\frac{y+z}{2} \geq \sqrt{yz}$ ...(1)

$\frac{z+x}{2} \geq \sqrt{zx}$ ...(2)

$\frac{y+x}{2} \geq \sqrt{yx}$ ...(3)

Multiplying (1), (2) and (3), we get

$\frac{(y+z)(z+x)(x+y)}{8} \geq xyz$

or $(1-x)(1-y)(1-z) \geq 8xyz$

$(\because x + y + z = 1)$

5. (c) The first series is 1, 3, 5, 7,....,239
While the second series is 3, 6, 9, ..., 240
Hence, the last common term is 237.

Thus our answer becomes $\frac{237-3}{6} + 1 = 40$

6. (a) Since a and b are unequal,

$\frac{a^2 + b^2}{2} > \sqrt{a^2 b^2}$

(A.M. > G.M. for unequal numbers)
$\Rightarrow a^2 + b^2 > 2ab$
Similarly $b^2 + c^2 > 2bc$ and $c^2 + a^2 > 2ca$
Hence $2(a^2 + b^2 + c^2) > 2(ab + bc + ca)$
$\Rightarrow ab + bc + ca < 1$

7. (a) The first term sum of the series is $\sqrt{2} - 1$, for 2 terms we have the sum as $\sqrt{3} - 1$ and so on. For the given series of 120 terms the sum would be $\sqrt{121} - 1 = 10$.

8. (a) Since, $\log_x a, a^{x/2}$ and $\log_b x$ are in GP,
$\therefore$ $(a^{x/2})^2 = (\log_x a) \times (\log_b x)$
or $a^x = \log_b a$
Taking log of both sides with base a, we get, $x \log_a a = \log_a (\log_b a)$
or $x = \log_a (\log_b a)$

9. (c) a, b, c are in H.P, so, $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ will be in A.P.

or, $\frac{a+b+c}{a}, \frac{a+b+c}{b}$ and $\frac{a+b+c}{c}$ will be in A.P

or, $1 + \frac{b+c}{a}, 1 + \frac{a+c}{b}$ and $1 + \frac{a+b}{c}$

will be in A.P.

Hence $\frac{b+c}{a}, \frac{a+c}{b}$ and $\frac{a+b}{c}$ are in A.P.

So, $\frac{a}{b+c}, \frac{b}{c+a}$ and $\frac{c}{a+b}$ will be in H.P.

Let us take 1, 1/2, 1/3 (which are in H.P)

$\frac{a}{b+c} = \frac{6}{5}, \frac{b}{a+c} = \frac{3}{8}, \frac{c}{a+b} = \frac{2}{9}$

Now, when we check these values of A.P, G.P and H.P, we find that $\frac{8}{3}$ is the AM of $\frac{5}{6}$ and $\frac{9}{2}$.

10. (a) We need the sum of the series 20 + 24 + 28 to cross 1000. Trying out the options, we can see that in 20 years the sum of his savings would be: 20 + 24 + 28 + ... + 96. The sum of this series would be 20 × 58 = 1160. If we remove the 20th year we will get the series for savings for 19 years. The series would be 20 + 24 + 28 + ... 92. Sum of the series would be 1160 – 96 = 1064. If we remove the 19th years's savings the savings would be 1064 – 92 which would go below 1000. Thus, after 19 years his savings would cross 1000.

11. (b) The sum of the total distance it travels would be given by the infinite sum of the series:
$420 \times 8/1 + 367.5 \times 8/1$
$= 3360 + 2940 = 6300.$

12. (a) Sum of first 11 terms = Sum of first 19 terms.

$$S_{11} = \frac{11}{2}[2a+10d] \quad S_{19} = \frac{19}{2}[2a+18d]$$

$$= \frac{22a+110\,d}{2} = \frac{38a+342\,d}{2}$$

According to question $S_{11} = S_{19}$

$\Rightarrow 22a + 110d = 38a + 342d$

$\Rightarrow -232\,d = 16a \Rightarrow -29d = 2a$

$\Rightarrow 2a + 29d = 0$

$$S_{30} = \frac{30}{2}[2a+29d]$$

$$= \frac{30}{2} \times 0 = 0$$

$\therefore$ Sum of 30 terms of the AP = 0.

13. (b) Multiple of 7 between 1 and 200 are

7, 14, 21, 28, ..., 196.

$$\text{Number of terms} = \left[\frac{196-7}{7}\right] + 1 = 28$$

$$\text{Sum} = \frac{28}{2}[7+196] = 2842.$$

14. (c) 5, 20, 80, 320, ..., 20480

$a = 5$

$$r = \frac{20}{5} = 4$$

$T_n = 20480$

$T_n = a.r^{(n-1)}$

$\Rightarrow 20480 = 5 \times 4^{(n-1)}$

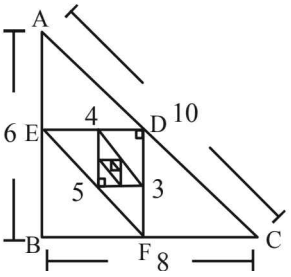
$$\Rightarrow \frac{20480}{5} = 4^{(n-1)} \Rightarrow 4096 = 2^{2(n-1)}$$

$\Rightarrow 2^{12} = 2^{2n-2} \Rightarrow 2n - 2 = 12$

$$\Rightarrow n = \frac{14}{2} = 7$$

Hence, there are 7 terms.

15. (b)



Area of the largest right angle triangle

$$= \frac{1}{2} \times 6 \times 8 = 24.$$

Area of the second largest

$$\Delta = \frac{1}{2} \times 4 \times 3 = 6$$

Area of the third largest $\Delta = \frac{1}{2} \times 2 \times \frac{3}{2} = \frac{3}{2}$

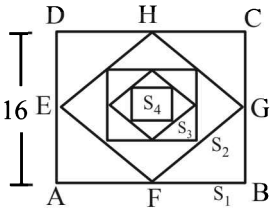
$(A_\Delta)_1$	$(A_\Delta)_2$	$(A_\Delta)_3$	$(A_\Delta)_4$ .....
↓	↓	↓	↓
24	6	$\frac{3}{2}$	$\frac{3}{8}$

$$r = \frac{(A_\Delta)_2}{(A_\Delta)_1} = \frac{(A_\Delta)_4}{(A_\Delta)_3} \Rightarrow r = \frac{6}{24} = \frac{1}{4}$$

$$S_\infty = \frac{a}{1-r} = \frac{24}{1-\frac{1}{4}} = 24 \times \frac{4}{3} = 32\ \text{cm}^2$$

GP_3386

16. (a)



Square	Side	Area
S_1	16	256
S_2	$\frac{\text{Diagonal}(S_1)}{2} = 8\sqrt{2}$	128
S_3	$\frac{\text{Diagonal}(S_2)}{2} = 8$	64
S_4	$\frac{\text{Diagonal}(S_3)}{2} = 4\sqrt{2}$	32
⋮	⋮	⋮

Sum of area of these squares
$= 256 + 128 + 64 + 32 + ...$
First term $\rightarrow a = 256$

Common ratio $\rightarrow \frac{128}{256} = \frac{1}{2}$

$$\text{Sum} = \frac{a}{1-r} = \frac{256}{1-1/2} = 2 \times 256 = 512 \text{ cm}^2$$

17. (b) $11 + 103 + 1005 + ...$

$10 + 1 \quad 100 + 3 \quad 1000 \quad 5$

$= (10 + 100 + 1000 + 10000 + ...) + (1 + 3 + 5 + 7 + ...)$

$= (10 + 10^2 + 10^3 + 10^4 + ... + 10^n) + (1 + 3 + 5 + 7 + ... + 2n-1)$

GP; AP or sum of first n odd numbers

$$S_n = a.\frac{(r^n - 1)}{r - 1} + n^2$$

$$= 10.\frac{(10^n - 1)}{(10 - 1)} + n^2 = \frac{10}{9}(10^n - 1) + n^2$$

18. (c) GM of A and B $= \sqrt{AB}$

AM of A and B $= \frac{A + B}{2}$

Let the AM of A and B be 100.
then GM = 80
AM : GM = 100 : 80

$$\frac{AM}{GM} = \frac{5}{4} \Rightarrow \frac{\frac{A+B}{2}}{\sqrt{AB}} = \frac{5}{4} \Rightarrow \frac{A+B}{2\sqrt{AB}} = \frac{5}{4}$$

$$\Rightarrow \frac{A + B + 2\sqrt{AB}}{A + B - 2\sqrt{AB}} = \frac{5+4}{5-4}$$

[Componendo & Dividendo Theorem]

$$\Rightarrow \frac{(\sqrt{A} + \sqrt{B})^2}{(\sqrt{A} - \sqrt{B})^2} = \frac{9}{1} \Rightarrow \frac{\sqrt{A} + \sqrt{B}}{\sqrt{A} - \sqrt{B}} = \frac{3}{1}$$

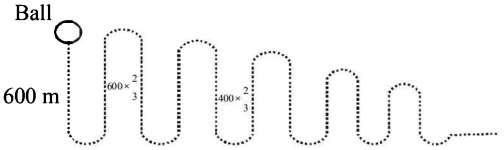
Again using componendo and dividendo, theorem

$$\frac{\sqrt{A} + \sqrt{B} + \sqrt{A} - \sqrt{B}}{\sqrt{A} + \sqrt{B} - \sqrt{A} + \sqrt{B}} = \frac{3+1}{3-1} = \frac{\sqrt{A}}{\sqrt{B}} = \frac{4}{2} = \frac{2}{1}$$

Squaring both sides $\frac{A}{B} = \frac{4}{1}$

Hence, A : B = 4 : 1.

19. (b)



Distance travelled :-

$$(600 + 400) + \left(400 + \frac{800}{3}\right) + ...$$

$$= 1000 + \frac{2000}{3} + \frac{4000}{9} + ...$$

$a = 1000, \; r = \frac{2}{3}$

$$\text{Sum of infinite terms} = \frac{a}{1-r} = \frac{1000}{\left(1 - \frac{2}{3}\right)}$$

$$= \frac{1000}{1/3} = 3000 \text{ m.}$$

20. (c) $T_p = a + (p-1)d = q$ ....(i)
and $T_q = a + (q-1)d = p$ ....(ii)
and $T_m = a + (m-1)d$ .... (iii)
From (i) and (ii)
$a = q - (p-1)d$ and $a = p - (q-1)d$
$\Rightarrow q - (p-1)d = p - (q-1)d$
$\Rightarrow (q-p) = -d[(q-1) - (p-1)]$
$\Rightarrow d = -1$

Again $a+(p-1)d=q$

$\Rightarrow a+(p-1)\times(-1)=q$

$\Rightarrow a=p+q-1$

$\therefore T_m=(p+q-1)+(m-1)\times(-1)$

$\Rightarrow T_m=p+q-m$

21. (d) The required numbers are 56, 63, 70, ..., 5551

Number of terms = 786

$\therefore S_{786}=56+63+.....+5551$

$$=\left(\frac{56+5551}{2}\right)\times 786=2203551$$

22. (b) The least possible sum can be obtained only when all the terms of A. P. be non – positive.

Let the last term be T_n which must be non positive.

$\therefore T_n\le 0$

$a+(n-1)d\le 0$

$-23+(n-1)4\le 0$

$4n\le 27$

$n\le 6$

$\therefore n=6$

$\therefore T_n=T_6=-23+5\times 4=-3$

$$\therefore S_6=\left[\frac{(-23)+(-3)}{2}\right]\times 6=-78$$

(The required A.P. is –23, –19, –15, –11, –7, –3).

23. (a) The numbers which are even and divisible by 5 also are 10, 20, 30 ..... 1000.

$\therefore S_{100}=10+20+30+.....+1000$

$=10(1+2+3+.....+100)$

$$=10\times\frac{100\times 101}{2}=50500$$

24. (c) The required numbers are 1017, 1035, 1053, ... 9999

Total number of terms = 500

$$\left[\because n=\left(\frac{9999-1017}{18}\right)+1\right]$$

$\therefore$ Sum of all the numbers

$$=\left(\frac{1017+9999}{2}\right)\times 500=2754000$$

25. (c) Let a, b, c, d, e be five consecutive terms is G. P. then

$$a=\frac{c}{r^2},\ b=\frac{c}{r},\ d=cr,\ e=cr^2$$

$$\therefore a.b.c.d.e=\frac{c}{r^2}.\frac{c}{r}.c.cr.cr^2=c^5$$

$\therefore a.b.c.d.e=4^5$ $(\because c=4)$

26. (c) $a+b+c=42$

$abc=512$

$\Rightarrow b=8(\because (abc)^{1/3}=b=8)$

$\therefore ac=64$

and $a+c=34$

Now, either a = 2, then c = 32

or a = 32 then c = 2

27. (a)
$$\frac{S_3}{S_6}=\frac{125}{152}=\frac{\frac{a\left(r^3-1\right)}{(r-1)}}{\frac{a\left(r^6-1\right)}{(r-1)}}=\frac{r^3-1}{r^6-1}$$

$$\Rightarrow \frac{r^3-1}{\left(r^3\right)^2-1}=\frac{125}{152}$$

$$\Rightarrow \frac{\left(r^3-1\right)}{\left(r^3+1\right)\left(r^3-1\right)}=\frac{125}{152}$$

$$\Rightarrow \frac{1}{r^3+1}=\frac{125}{152}$$

$$\Rightarrow r^3+1=\frac{152}{125}$$

$$\Rightarrow r^3=\frac{27}{125}$$

$$\Rightarrow r=\frac{3}{5}$$

28. (a) Let the G. P. be 1, 2, 4, 8, 16, 32, 64, ...

Let x = 3, y = 5 and z = 6

$\therefore T_x=4=a$

$T_y=16=b$

$T_z=32=c$

$\therefore (y-z)\log a+(z-x)\log b+(x-y)\log c$

$=-1\log 4+3\log 16+(-2)\log 32$

$=\log 2^{-2}+\log 2^{12}+\log 2^{-10}$

$=\log 2^0=\log 1=0$

29. (d) $a+ar=\frac{5}{3}\Rightarrow a(1+r)=\frac{5}{3}$ ....(i)

and $\frac{a}{1-r}=3$ ....(ii)

GP_3386

Dividing equation (i) by equation (ii), we get

$$(1+r)(1-r)=\frac{5}{9}\Rightarrow 1-r^2=\frac{5}{9}$$

$$\Rightarrow \quad r^2=\frac{4}{9}\Rightarrow r=\pm\frac{2}{3}$$

If $r=\frac{2}{3}$, then $a=1$ and if $r=-\frac{2}{3}$, then $a=5$

30. **(c)** Use Shortcut Approach-8

31. **(d)** Use Shortcut Approach-8

32. **(b)** Use Shortcut Approach-9

33. **(c)** Use Shortcut Approach-10

34. **(d)** $\frac{a+b}{2}=10\Rightarrow a+b=20$

$\sqrt{a.b}=8\Rightarrow ab=64$

$\therefore$ $a, b=16, 4$ or $4, 16$

$\therefore$ $\text{H.M.}=\frac{2\times4\times16}{(4+16)}=6.4$

35. **(d)** $3^{1/3}.9^{1/18}.27^{1/81}\ldots\ldots=3^{1/3}.3^{1/9}.3^{1/27}\ldots..$

$=3^{1/3+1/9+1/27+..}$

$=3^{\left(\frac{1/3}{1-1/3}\right)}=3^{1/2}=\sqrt{3}$

36. **(c)** Let the numbers be $a-d$, a, $a+d$

then $(a-d)^2$, a^2, $(a+d)^2$ are in G. P.

$\therefore$ $a^4=(a-d)^2(a+d)^2$

$\Rightarrow$ $d^4-2a^2d^2=0$

$\Rightarrow$ $d=0,\pm\sqrt{2}a$

Hence d has three values.

37. **(c)** $(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)}=(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)}$

$=\left(5^{-1}\right)^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)}$

$=(5)^{\log_{\sqrt{5}}2}\qquad\left(\because m\log n=\log(n)^m\right)$

$=\left[\left(\sqrt{5}\right)^2\right]^{\log_{\sqrt{5}}(2)}\qquad\left(\because {}_a\log_a b=b\right)$

$={}_{\sqrt{5}}\log_{\sqrt{5}}4=4$

38. **(d)** Let $S=\frac{5}{13}+\frac{55}{(13)^2}+\frac{555}{(13)^3}+\frac{5555}{(13)^4}+\ldots..\infty$

$\therefore$ $\frac{S}{13}=\frac{5}{(13)^2}+\frac{55}{(13)^3}+\frac{555}{(13)^4}+\frac{5555}{(13)^5}+\ldots..\infty$

$\therefore$ $\left(S-\frac{S}{13}\right)=\frac{5}{13}+\frac{50}{(13)^2}+\frac{500}{(13)^3}+\frac{5000}{(13)^4}+\ldots..\infty$

$\therefore$ $S\left(1-\frac{1}{13}\right)=\frac{5}{13}+\frac{5}{13}\left(\frac{10}{13}\right)+\frac{5}{13}\left(\frac{10}{13}\right)^3+\frac{5}{13}\left(\frac{10}{13}\right)^3+\ldots.\infty$

$\frac{12}{13}S=\frac{5/13}{3/13}\qquad\left[\because S_\infty=\frac{a}{1-r}\right]$

$\frac{12}{13}S=\frac{5}{3}\Rightarrow S=\frac{65}{36}$

39. **(b)** $T_1=3^1-2$

$T_2=3^2-2$

$T_3=3^3-2$

$T_4=3^4-2$

40. **(b)** Let $S=1+\frac{2}{3}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\frac{10}{243}+\ldots\ldots..\infty$

$\therefore$ $3(S)=3+2+\frac{4}{3}+\frac{6}{9}+\frac{8}{27}+\frac{10}{81}+\ldots\ldots..\infty$

$\therefore$ $2(S)=4+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\frac{2}{81}+\ldots\ldots..\infty$

$\therefore$ $2(S)=4+2\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots.\infty\right)$

$\Rightarrow$ $2(S)=4+2\left[\frac{\frac{1}{3}}{1-\left(\frac{1}{3}\right)}\right]$

$\Rightarrow$ $2(S)=4+1\Rightarrow 2(S)=5$

$\therefore$ $S=\frac{5}{2}$

41. **(a)** Let $S=\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots.\infty$

$\Rightarrow$ $S=\frac{1}{3}\left[\frac{3}{1.4}+\frac{3}{4.7}+\frac{3}{7.10}+\ldots.\infty\right]$

$\Rightarrow$ $S=\frac{1}{3}\left[\left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\ldots..\infty\right]$

$\Rightarrow$ $S=\frac{1}{3}[1]\quad\therefore S=\frac{1}{3}$

Solution (42 and 43)

Let us consider choice (a)

	E	M	L	T		
Lucknow (G.P.)	1	1	1	1	}	4
Kanpur (A.P.)	1	2	3	4	}	10

It is wrong since M = E

Again	E	M	L	T		
Lucknow (G.P.)	1	2	4	8	}	15
Kanpur (A.P.)	1	3	5	7	}	16

It is wrong since $15 \neq \frac{3}{2} \times 16$, also $2 \neq 3$

Again	E	M	L	T		
Lucknow (G.P.)	1	3	9	27	}	40
Kanpur (A.P.)	1	4	7	10	}	22

It is also wrong since $40 \neq \frac{3}{2} \times 22$, also $3 \neq 4$

Again	E	M	L	T		
Lucknow (G.P.)	2	4	8	16	}	30
Kanpur (A.P.)	2	4	6	8	}	20

It is correct since number of monkeys and number of elephants in both the places are equal respectively. Also total number of animals in zoological park is 50% more than that of local zoo in Kanpur.

42. (b) **43. (d)**

44. (d) $t_n = n(2n+1)^2$

$= 4n^3 + 4n^2 + n$

$\therefore S_n = \Sigma t_n = 4\Sigma n^3 + 4\Sigma n^2 + \Sigma_n$

then put n = 20

45. (a) Distance travelled before the first hit = 16 m

Distance travelled before the second hit

$= 16 + \frac{1}{2} \times 16 \times 2$

$= (16 + 16)$ m

Distance travelled before the third hit

$= 16 + 16 + \frac{1}{2} \times 8 \times 2$

$= (16 + 16 + 8)$ m

Distance travelled before the fourth hit

$= 16 + 16 + 8 + \frac{1}{2} \times 4 \times 2 = (16 + 16 + 8 + 4)$ m

and so on.

Hence the required distance

$= 16 + 16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

$= 16 + \left(16 + 8 + 4 + ... + \frac{1}{16}\right)$

$= 16 + \frac{16\left(1 - \left(\frac{1}{2}\right)^9\right)}{1 - \frac{1}{2}}$

$= 16 + 16 \times \left(\frac{511}{512}\right) \times \frac{2}{1}$

$= 16 + \frac{511}{16} = 47\frac{15}{16}$

46. (a) The given number series is based on the following pattern

$9 \times 0.5 + 0.5 = 5$

$5 \times 1 + 1 = 6$

$6 \times 1.5 + 1.5 = 10.5$

$10.5 \times 2 + 2 = 23$

$23 \times 2.5 + 2.5 = 60$

$60 \times 3 + 3 = 183$

Hence the wrong number is 8.

47. (b) The given number series is based on the following pattern :

$14 \times 1.5 = 21$

$21 \times 2 = 42$

$42 \times 2.5 = 105$

$105 \times 3 = 315$

$315 \times 3.5 = 1102.5$

$1102.5 \times 4 = 4410$

Hence, the wrong number is 48.

48. (d) The given number series is based on the following pattern:

$24 - 23 = 1^2$

$33 - 24 = 9 = 3^2$

$58 - 33 = 25 = 5^2$

$107 - 58 = 49 = 7^2$

$188 - 107 = 81 = 9^2$

$309 - 188 = 121 = 11^2$

Obviously, 186 is the wrong number.

49. (c) The number series follows the rule as mentioned below:

$24 \times 1 + 1 = 25$

$25 \times 2 - 1 = 49$

$49 \times 3 + 1 = 148$

$148 \times 4 - 1 = 591$

$591 \times 5 + 1 = 2956$

$29566 \times 6 - 1 = 17735$

Hence 492 is the wrong number.

50. **(d)** The followed pattern is:
$12 \times 2 + 2 = 26$
$26 \times 2 + 4 = 56$
$56 \times 2 + 6 = 118$
$118 \times 2 + 8 = 244$
$244 \times 2 + 10 = 498$
$498 \times 2 + 12 = 1008$
Hence, the wrong number is 276.

Solution (51-55):

51. **(b)** 15, 19, 28, 44, 69, 105, (154)
+4 +9 +16 +25 +36 +49

52. **(c)** 2400, 960, 384, 153.6, 61.44, (24.576)
×2/5 ×2/5 ×2/5 ×2/5 ×2/5

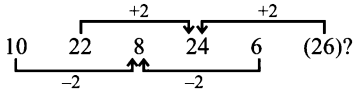
53. **(d)** 33, 34, 70, 213, 856, (4285)
×1+1 ×2+2 ×3+3 ×4+4 ×5+5

54. **(c)** 305, 338, 404, 503, 635, (800)
+33 +66 +99 +132 +165

55. **(a)** 16, 34, 19, 40, 22, (46)
×2+2 ÷2+2 ×2+2 ÷2+2 ×2+2

56. **(b)** The series is:
$300 + 31 = 331$
$331 + 33 = 364$
$364 + 35 = 399$
$399 + 37 = 436$
$436 + 39 = 475$

57. **(c)**



58. **(b)** The series is:
$262144 \div 8 = 32768$
$32768 \div 8 = 4096$
$4096 \div 8 = 512$
$512 \div 8 = 64$
$64 \div 8 = 8$

59. **(a)** $13^3 = 2197$
$14^3 = 2744$
$15^3 = 3375$
$16^3 = 4096$
$17^3 = 4913$
$18^3 = 5832$

60. **(c)** The series is:
$196 + 11 = 207$
$207 + 22 = 229$
$229 + 33 = 262$
$262 + 44 = 306$
$306 + 55 = 361$

61. **(b)** –16 –15 –7 20 84 219 425
$-16 + 1^3 = -15$
$-15 + 2^3 = -7$
$-7 + 3^3 = 20$
$20 + 4^3 = 84$
$84 + 5^3 = 209$
$209 + 6^3 = 425$
So 219 is the wrong number.

62. **(c)** –11 –10 –38 –349 –5420 –135495
$-11 \times 1^2 + 1 = -10$
$-10 \times 2^2 + 2 = -38$
$-38 \times 3^2 + 3 = -339$
$-339 \times 4^2 + 4 = -5420$
$-5420 \times 5^2 + 5 = -135495$
So –349 is the wrong number.

63. **(b)** 48 24 28 36 72 180 540
$48 \times 0.5 = 24$
$24 \times 1 = 24$
$24 \times 1.5 = 36$
$36 \times 2 = 72$
$72 \times 2.5 = 180$
$180 \times 3 = 540$
So 28 is wrong number.

64. **(c)** 16 17 33 53 83 133 216 349
$16 + 17 = 33$
$17 + 33 = 50$
$33 + 50 = 83$
$50 + 83 = 133$
$83 + 133 = 216$
$133 + 216 = 349$
So 53 is wrong number

65. **(d)** 288 144 96 72 58 48
$288 \times 1/2 = 144$
$144 \times 2/3 = 96$
$96 \times 3/4 = 72$
$72 \times 4/5 = 57.6$
$57.6 \times 5/6 = 48$
So 58 is wrong number.

CHAPTER

14

Allegation & Mixture

More At @Aj_ebooks

ALLIGATION

Alligation is the simplified, faster technique to solve the problems based on weighted average. This method plays a vital role insaving the time in solving the problems related to weighted average situation.

We know that

$$\text{Weighted Average} = \frac{\text{Sum total of all numbers of all groups}}{\text{Total number of numbers in all groups together}}$$

Therefore weighted average A_w of two groups having n_1 and n_2 numbers with averages A_1 and A_2 respectively is $A_w = \dfrac{n_1 A_1 + n_2 A_2}{n_1 + n_2}$

$$\Rightarrow \quad (n_1 + n_2) A_w = n_1 A_1 + n_2 A_2$$
$$\Rightarrow \quad n_1 (A_w - A_1) = n_2 (A_2 - A_w)$$
$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$$

Equation $\dfrac{n_1}{n_2} = \dfrac{A_2 - A_w}{A_w - A_1}$ is called Alligation Formula.

For convenient, we take $A_1 < A_2$. Hence $A_1 < A_w < A_2$.

Solving the Problems of Alligations Using Alligation Formula

Illustration – 1

10 kg of wheat costing ₹ 12 per kg and 15 kg of wheat costing ₹ 20 per kg are mixed. Find the average cost of the mixture per kg.

Solution : $\dfrac{n_1}{n_2} = \dfrac{A_2 - A_w}{A_w - A_1} \Rightarrow \dfrac{10}{15} = \dfrac{20 - A_w}{A_w - 12}$

$$\Rightarrow \quad \frac{2}{3} = \frac{20 - A_w}{A_w - 12} \Rightarrow 5A_w = 84$$
$$\Rightarrow \quad A_w = \frac{84}{5} = 16.8$$

Hence average cost of the mixture = ₹ 16.8 per kg.

Illustration – 2

A mixture worth ₹ 3.25 per kg is formed by mixing two types of salts, one costing ₹ 3.10 per kg while the other ₹ 3.60 per kg. 1n what ratio must they have been mixed?

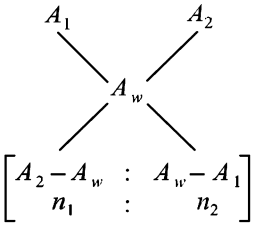
Solution : $\dfrac{n_1}{n_2} = \dfrac{A_2 - A_w}{A_w - A_1}$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{3.60 - 3.25}{3.25 - 3.10} = \frac{35}{15}$$
$$\Rightarrow \quad n_1 : n_2 = 7 : 3$$

Hence required ratio = 7 : 3.

Graphical Representation of Alligation– Cross Method

The alligation formula $\dfrac{n_1}{n_2} = \dfrac{A_2 - A_w}{A_w - A_1}$ is graphically represented by the following cross-diagram:



The ratios in the bracket [ ] are equal i.e.

$n_1 : n_2 = A_2 - A_w : A_w - A_1$.

In the above graphical representation five variables A_1, A_2, A_w, n_1 and n_2 are involved.

GP_3386

Based on the problem situation, one of the following three cases may occur with respect to the known and the unknown out of the five variables A_1, A_2, A_w, n_1 and n_2 involved in the problem.

Case	Known	Unknown
I	(a) A_1, A_2, A_w	(a) $n_1 : n_2$
	(b) A_1, A_2, A_w, n_1	(b) n_2 and $n_1 : n_2$
II	A_1, A_2, n_1, n_2	A_w
III	A_1, A_w, n_1, n_2	A_2

Solving the problem using graphical representation of alligation is called cross method.

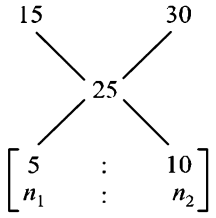
Let us solve some problems in each of the three cases using cross method.

Case I: When A_1, A_2, A_w are known and one of n_1 and n_2 may be also known then to find $n_1 : n_2$ and n_2 if n_1 is known OR n_1 if n_2 is known.

Illustration – 3

If the average weight of the students of a class is 15 kg, the average weight of the students of another class is 30 kg and average weight of the students of both the classes is 25 kg, then find the ratio of the number of students of the first class to the another class.

Solution :



$\therefore n_1 : n_2 = 5 : 10 = 1 : 2$

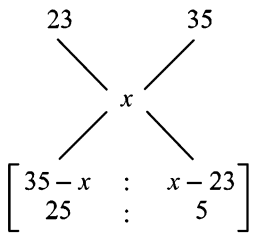
Hence required ratio = 1 : 2

Case II: When A_1, A_2, n_1, n_2 are known and A_w is unknown then to find A_w.

Illustration – 4

5 kg of superior quality of sugar is mixed with 25 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is ₹ 35 and ₹ 23 respectively. Find the average price per kg of the mixture.

Solution :



$$\therefore \frac{35-x}{x-23} = \frac{25}{5}$$

$\Rightarrow 30x = 175 + 575 \Rightarrow x = 25$

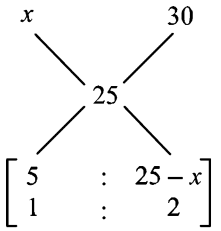
Hence average price per kg of the mixture = ₹ 25

Case-III: When A_1, A_w, n_1, n_2 are known and A_2 is unknown, then to find the value of A_2.

Illustration – 5

The ratio of number of girls to number of boys is 1 : 2. If the average weight of the boys is 30 kg and the average weight of both the boys and girls is 25 kg, then find the average weight of the girls.

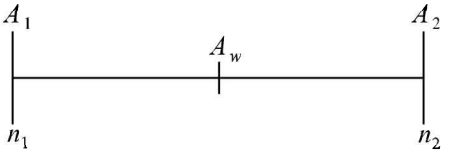
Solution :



Hence average weight of the girls = 15 kg.

The Straight Line Approach to Solve the Problems Related to Alligations

The straight line approach is actually the cross method.



The above diagram is the straight line diagram in which the symbols A_1, A_2, A_w, n_1 and n_2 denote the same quantity as shown in cross method. Here

$A_1 < A_w < A_2$.

In the above diagram,

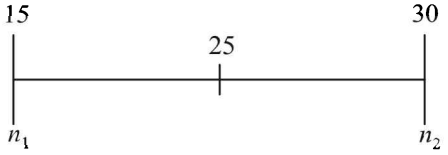
(a) n_1 corresponds to $(A_2 - A_w)$

(b) n_2 corresponds to $(A_w - A_1)$

(c) $(n_1 + n_2)$ corresponds to $(A_2 - A_1)$

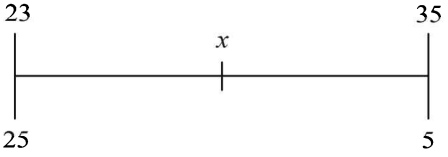
Now, we again solve the examples 3, 4 and 5 given in case-I, II and III respectively of cross-method using straight line approach.

Sol. of illustration 3 by straight line approach.



n_1 corresponds to $5 = (30 - 25)$
and n_2 corresponds to $10 = (25 - 15)$
$\therefore \quad n_1 : n_2 = 5 : 10 = 1 : 2$
Hence required ratio = 1 : 2

Sol. of illustration 4 by straight line approach.

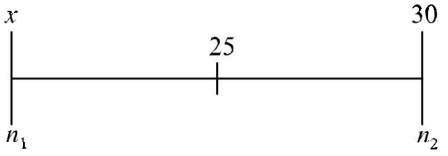


25 corresponds to $(35 - x)$
Also 5 corresponds to $(x - 23)$

$$\frac{25}{5} = \frac{35 - x}{x - 23} \Rightarrow x = 25$$

Hence average price per kg of mixture = ₹ 25

Sol. of illustration 5 by straight line approach.



Here $n_1 : n_2 = 1 : 2$
Now, n_1 corresponds to $5 = (30 - 25)$
and n_2 corresponds to $(25 - x)$

$$\therefore \quad \frac{n_1}{n_2} = \frac{5}{25 - x} \Rightarrow \frac{1}{2} = \frac{5}{25 - x} \Rightarrow x = 15$$

Hence average weight of the girls = 15 kg

Recognition of Different Situations Where Alligation can be Used

There are many types of situations where alligation can be used, which must be recognised by the students. Here you are given some situations (or problems) which help you to recognise different alligation situations and identify A_1, A_2, n_1, n_2 and A_w in each alligation situation.
In each of the following problems
$A_1 = 20, A_2 = 35, n_1 = 20, n_2 = 40$ and answer as $A_w = 30$.

Weighted Mean

An average weight of students of a class of 40 students is 35 kg and an average weight of students of a class of 20 students is 20 kg. Find the average weight of the students of both the combined classes. **(30 kg)**

Mixture

(i) 20 litres of one variety of soda water is mixed with 40 litres of other variety of soda water. The price of first variety of soda water is ₹ 20 per litre and price of other variety of soda water is ₹ 35 per litre. Find the cost of the mixture per litre. **(₹ 30)**

(ii) A 40 litres mixture of water and milk contains 35% of milk and in another 20 litres of mixture of water and milk contains 20% of milk. If a new mixture is formed by mixing the both mixtures, then find the percentage of milk in new mixture. **(30%)**

Time and Speed

A car travels at 20 km/h for 20 minutes and at 35 km/h for 40 minutes. Find the average speed of the car for the whole journey. **(30 km/hr)**

Profit and Loss

(i) A car agency sold 20 cars at 20% profit and 40 cars at 35% profit. Find the gain percent on the sale of all these cars. **(30%)**

(ii) A trader earns a profit of 20% on 20% of his goods sold while he earns a profit of 35% on 40% of his goods sold. Find the percentage profit on the total goods sold. **(30%)**

(iii) A shopkeeper sold the 40% hardware at the profit of 35% and 20% software at a profit of 20%. Find the average profit% on the whole goods sold, if he sells only these two kind of things. **(30%)**

Some Keys to Identify A_1, A_2 & A_w and Differentiate these from n_1 and n_2

1. Normally, there are 3 averages mentioned in the problem, while there are only 2 quantities. This is not foolproof. Sometimes the question might confuse the students by giving 3 values for quantities representing n_1, n_2 and $n_1 + n_2$ respectively.
2. A_1, A_2 and A_w are always rate units, while n_1 and n_2 are quantity units.

Rate units are like ₹ x/kg, y km/hour, etc. and corresponding quantity units are kg, hour etc.

3. The denominator of the average unit corresponds to the quantity unit (i.e., unit for n_1 and n_2).
For example, denominator kg and hour of rate units ₹ x/kg and y km/hour are the units of quantity corresponding to rates.

A Typical Problem

Let's discuss the solution of a typical problem given below:

Illustration – 6

A person used to draw out 20% of the honey from a jar containing 10 kg honey and replaced it with sugar solution. He has repeated the same process three times. Find the final amount of honey left in the jar and the final ratio of honey to sugar solution finally left in the jar.

Solution : In each step 20% of the remaining honey drawn out. So, after each step 80% of the honey remains is left.

Hence, final amount of honey left in the jar

$$= 10 \times \frac{80}{100} \times \frac{80}{100} \times \frac{80}{100} \text{ or } 10 \times \left(\frac{80}{100}\right)^3$$

$= 5.12$ kg

Now the final amount of sugar solution left in the jar $= 10 - 5.12\text{ kg} = 4.88$ kg

Hence final ratio of honey to the sugar solution left in the jar

$$= \frac{5.12}{4.88} = 64 : 61.$$

MIXTURE OF MORE THAN TWO INGREDIENTS

Let understand this through the following Illustrations.

Illustration – 7

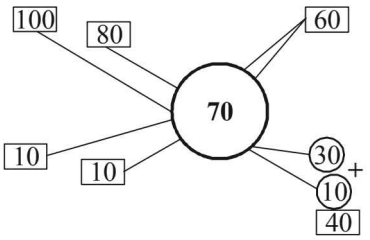
In what ratio may three kinds of tea prices @ ₹ 100, ₹ 80 and ₹ 60 per kg be mixed to produce a mixture worth ₹ 70 per kg.

Solution : Write the prices in ascending or descending order like

100 80 70 60

(Mean Price)

Now make pairs by choosing one from each side of the mean price and apply alligation rule each time. Then add the quantity obtained under ecah price. Thus will give the ratio in which the ingredients should be mixed as shown below.



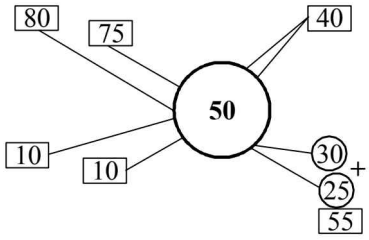
Required ratio = 10 : 10 : 40 = 1 : 1: 4

Illustration – 8

In what ratio three kinds of rice @ ₹ 80, ₹ 75 and ₹ 40 be mixed to produce a mixture which would earn 20% on selling it at ₹ 60 per kg ?

Solution : S.P. = ₹ 70 , profit = 20%

C.P. of mixture $= \frac{100}{120} \times 60 =$ ₹ 50 per kg.



Required ratio = 10 : 10 : 55 = 2 : 2 : 11

SHORTCUTS

Shortcut Approach - 1

The proportion in which rice at ₹ x per kg must be mixed with rice at ₹ y per kg, so that the mixture be worth ₹ z a kg, is given by $\left(\frac{y-z}{z-x}\right)$

Illustration – 9

How must a grocer mix wheat at ₹ 6 a kg and ₹ 6.50 a kg, so that the mixture may be worth ₹ 6.20 a kg.
(a) 2 : 3 (b) 3 : 2 (c) 3 :1 (d) 1 : 3

Shortcut Solution : (b)

Mixture ratio $= \left[\frac{y-z}{z-x}\right] = \left[\frac{6 \cdot 50 - 6.20}{6 \cdot 20 - 6 \cdot 0}\right]$

$= \left[\frac{0 \cdot 30}{0 \cdot 20}\right] = 3 : 2$

Shortcut Approach - 1

The quantity of salt m kg at ₹ x per kg that a man must mix with n kg of salt at ₹ y per kg, so that he may, on selling the mixture at ₹ z per kg, gain p% on the outlay is given by $n \times \left[\frac{100z - y(100+p)}{x(100+p) - 100z}\right] kg.$

Note: If we suppose that the quantity of salt at ₹x be m, then we have

$$\frac{m}{n} = \frac{100z - y(100+p)}{x(100+p) - 100z}.$$

Illustration – 10

A grocer purchased 20 kg of rice at the rate of ₹ 15 per kg and 30 kg of rice at the rate of ₹ 13 per kg. At what price per kg should he sell the mixture to earn $33\frac{1}{3}$% profit on the cost price ?

(a) ₹ 28. 00 (b) ₹ 20. 00 (c) ₹ 18.40 (d) ₹ 17.40

Shortcut Solution : (c)

$$\text{Profit} = \left[\frac{100z - y(100+P)}{x(100+P) - 100z}\right] n$$

$$20 = \left[\frac{100z - 13(100 + \frac{100}{3})}{15(100 + \frac{100}{3}) - 100z}\right] 30$$

$$\therefore z = ₹18 \cdot 40$$

Shortcut Approach - 1

n gm of sugar solution has x% sugar in it. The quantity of sugar should be added to make it y% in the solution is given by $n\left(\frac{y-x}{100-y}\right)$ gm or quantity of sugar

$$\text{added} = \frac{\text{solution (required \% value – present \% value)}}{\text{(100 – required \% value)}}$$

Illustration – 11

A petrol pump owner mixed leaded and unleaded petrol in such a way that the mixture contains 10% unleaded petrol. What quantity of unleaded petrol should be added to 1 litre mixture so that the percentage of unleaded petrol becomes 20%?

(a) 125 ml (b) 400 ml (c) 250 ml (d) 100 ml

Shortcut Solution : (a)

Quantity of unleaded petrol

$$\text{added} = \frac{1000(20-10)}{(100-20)} \text{ml} = \left(1000 \times \frac{10}{80}\right) \text{ml} = 125\text{ml}$$

Shortcut Approach - 1

A trader has N kg of certain item, part of which he sells at x% profit and the rest at y% profit. He gains P% on the whole. The quantity of item sold at x% profit is $\left[\left(\frac{y-P}{y-x}\right)N\right]$ kg and the quantity of item sold at y% profit is given by $\left[\left(\frac{P-x}{y-x}\right)N\right] kg.$.

Illustration – 12

A trader has 100 kg of wheat, part of which he sells at 16% profit and the rest at 36% profit. He gains 28% on the whole. What is the quantity sold at 36% profit ?

(a) 50 kg (b) 60 kg
(c) 45 kg (d) 65 kg

Shortcut Solution : (b)

$$\text{Quantity of item sold at y \% profit} = \left[\left(\frac{P-x}{y-x}\right)N\right] \text{kg}$$

$$= \left[\frac{28-16}{36-16}\right] \times 100 = 60\text{kg}$$

Shortcut Approach - 1

A trader has N kg of a certain item, a part of which he sells at x% profit and the rest of y% loss. He gains P % on the whole. Then the quantity sold at x% profit is given by $\left[\left(\frac{P+y}{x+y}\right)N\right]$ kg and the quantity sold at y% loss is given by $\left[\left(\frac{x-P}{x+y}\right)N\right]$ kg

Illustration – 13

A trader has 40 kg of tea, a part of which he sells at 12% profit and the rest at 8% loss. He gains 9% on the whole. What is the quantity sold at 12% gain and 8% loss?

(a) 30 kg, 10 kg (b) 32 kg, 8 kg
(c) 33 kg, 7 kg (d) 34 kg, 6 kg

Shortcut Solution : (d)

Quantity sold at x% profit $= \left[\left(\frac{P+y}{x+y}\right)N\right]$

$= \left[\frac{9+8}{12+8}\right]\times 40 = \frac{17}{20}\times 40 = 34\text{ kg}$

Quantity sold at y % profit $= \left[\left(\frac{x-p}{x+y}\right)N\right]$

$= \left[\frac{12-9}{12+8}\right]40 = \left[\frac{3}{20}\times 40\right] = 6\text{ kg}$

Shortcut Approach - 6

A trader has N kg of a certain item, a part of which he sells at x% profit and the rest at y% loss. On the whole his loss is P%. Then the quantity sold at x% profit is $\left(\frac{y-P}{x+y}\right)$N kg and the quantity sold at y % loss is given by $\left(\frac{x+P}{x+y}\right)$N kg.

Illustration – 14

A trader has 48 kg of rice, a part of which he sells at 16% profit and the rest at 8% loss. On the whole his loss is 6% What is the quantity sold at 16% profit and that at 8% loss ?

(a) 42 kg, 6 kg (b) 44 kg, 4 kg
(c) 4 kg, 44 kg (d) 6 kg, 42 kg

Shortcut Solution : (c)

Profit $= \left(\frac{y-P}{x+y}\right)N$

$= \left(\frac{8-6}{16+8}\right)48 = \frac{2}{24}\times 48 = 4\text{kg}$

Loss $= \left(\frac{x+P}{x+y}\right)N = \left[\frac{16+6}{16+8}\right]\times 48 = \frac{22}{24}\times 48 = 44\text{kg}$

Shortcut Approach - 6

If a person buys n kg of an item at the rate of ₹ P per kg. If he sells m kg at a profit of x%, then the rate per kg, at which he should sell the remaining to get a profit of y% on the total deal, is given by ₹ $P\left[1+\frac{ny-mx}{(n-m)100}\right]$

Illustration – 15

Mala purchased 75 kg of salt at the rate of ₹ 8 per kg. She sold 25 kg at a profit of 5%. At what rate per kg should she sell the remaining to get a profit of 10% on the total deal?

(a) ₹ 8.25 (b) ₹ 9. 50 (c) ₹ 9 (d) ₹ 9. 75

Shortcut Solution : (c)

Profit $= P\left[1+\frac{ny-mx}{(n-m)100}\right]$

$= 8\left[1+\frac{75\times 10-25\times 5}{(75-25)\times 100}\right] = 8\left[1+\left[\frac{750-125}{50\times 100}\right]\right]$

$= 8\left[1+\frac{625}{5000}\right] = 8\left[\frac{5625}{5000}\right] = ₹ 9$

Shortcut Approach - 6

Amount of liquid A left after n operations, when the container originally contains x units of liquid A from which y units is taken out and replaced by liquid B each time is $x\left(\frac{x-y}{x}\right)^n$ units.

Illustration – 16

From a cask full of spirits one-hundredth part is drawn and the cask filled with water. From the mixture one hundredth part is drawn and the cask again filled with water, and a similar operation is again performed. Find the ratio of the quantity of wine left in the cask to the original quantity after the third operation.

(a) 970299 : 1000000 (b) 29701 : 1000000
(c) 970399 : 1000000 (d) 971099 :1000000

Shortcut Solution : (a)

$\therefore$ Required ratio $\left(\frac{y}{x}\right) = \left(\frac{x-y}{x}\right)^n$

$\left(\frac{y}{x}\right) = \left(\frac{100-1}{100}\right)^3 = \left(\frac{99}{100}\right)^3 = 970299 : 1000000$

Shortcut Approach - 1

Consider a container containing only ingredient 'A' of x_0 unit. From this, x_r unit is taken out and replaced by an equal amount of ingredient B. This process is repeated n times, then after n operations.

$$\frac{\text{Amount of A left}}{\text{Amount of B left}} = \frac{\left(1-\frac{x_r}{x_0}\right)^n}{1-\left(1-\frac{x_r}{x_0}\right)^n}$$

Illustration – 17

From a cask of wine, containing 64 litres, 8 litres are drawn out and the cask is filled up with water. If the same process is repeated a second, then a third time, what will be the proportion of wine to water in the resulting mixture?

(a) 343 : 169 (b) 343 :512
(c) 169 : 343 (d) 512 : 343

Shortcut Solution : (a)

Required proportion

$$= \frac{\left(1-\frac{1}{8}\right)^3}{1-\left(1-\frac{1}{8}\right)^3} = \frac{\frac{343}{512}}{1-\frac{343}{512}} = 343:169$$

Shortcut Approach - 12

Suppose a container contians M units of mixture of A and B. From this, R unit of mixture is taken out and replaced by an equal amount of ingredient B only. This process (of taking out and replacing it) is repeated *n* times, then after *n* operations,

$$\frac{\text{Amount of A left}}{\text{Amount of A originally present}} = \left(1-\frac{R}{M}\right)^n$$

and the amount of B left = M – amount of A left.

Illustration – 18

A 12-litres cylinder contains a mixture of oxygen and nitrogen, the volume of oxygen being 40% of total volume. A few litres of the mixture is released and an equal amount of nitrogen is added. Then the same amount of the mixture as before is released and replaced by nitrogen for the second time. As a result, the oxygen content becomes 10% of the total volume. How many litres of mixture is released each time ?

(a) 3 litres (b) 9 litres
(c) 6 litres (d) None of these

Shortcut Solution : (c)

$$\frac{\text{Amount of A left}}{\text{Amount of originally present}} = \left(1-\frac{R}{M}\right)^n$$

$$\frac{10}{40} = \left[1-\frac{R}{M}\right]^2 = \frac{1}{2} = 1-\frac{R}{M} \Rightarrow \frac{R}{M} = \frac{1}{2}$$

$$\text{Required value} = \frac{1}{2}\times 12 = 6 \text{ litres}$$

Shortcut Approach - 11

A person's expenditure and savings are in the ratio *a* : *b*. His income increases by *x*%. His expenditure also increases by *y*%. His percentage increase in saving is given by $\left[\left(\frac{a}{b}+1\right)x-\frac{a}{b}y\right]\%$

Illustration – 19

Ritu's expennditure and saving are in the ratio 5 : 2. Her income increases by 12%. Her expenditure also increases by 14%. By how many % does her saving increase?

(a) 14% (b) 7% (c) 8% (d) 9%

Shortcut Solution : (b)

$$\text{Percentage} = \left[\left(\frac{a}{b}+1\right)x-\frac{a}{b}y\right]\%$$

$$= \left[\left(\frac{5}{2}+1\right)12-\frac{5}{2}\times 14\right] = \left[\frac{7}{2}\times 12-\frac{5}{2}\times 14\right] = 42-35 = 7\%$$

Shortcut Approach - 11

In a group, there are some 4-legged creatures and some 2-legged creatures. If heads are counted, there are x and if leggs are counted there are y, then the no. of 4 – legged creatures are given by $\left(\frac{y-2x}{2}\right)$ **or** $\left(\frac{\text{Total legs} - 2\times \text{Total heads}}{2}\right)$ **and the no. of 2– legged creatures are given by** $\left(\frac{4x-y}{2}\right)$ **or** $\left(\frac{4\times \text{Total heads} - \text{Total legs}}{2}\right)$

Illustration – 20

In a zoo, there are rabbits and pigeons. If heads are counted, there are 100 and if legs are counted, there are 290. How many rabbits are there ?

(a) 55 (b) 45 (c) 40 (d) 50

Shortcut Solution : (b)

$$\text{No. of rabbits} = \left(\frac{y-2x}{2}\right) = \left(\frac{290-2\times 100}{2}\right) = 45.$$

GP_3386

LEVEL - 1

1. The average weight of girls is 15 and the average weight of boys is 30 and the average weight of boys and girls both is 25. If the number of boys are 12, then the number of girls is :
(a) 4 (b) 6 (c) 10 (d) 18

2. 5 kg of superior quality of sugar is mixed with 25 kg of inferior quality sugar. The price of superior quality and inferior quality sugar is ₹ 18 and ₹ 12 respectively. The average price per kg of the mixture is :
(a) ₹ 13 (b) ₹ 15 (c) ₹ 18 (d) ₹ 21

3. In what proportion, must wheat at ₹ 6.20 per kg be mixed with wheat at ₹ 7.20 per kg, so that the mixture be worth ₹ 6.50 per kg? **(SBI PO 2011)**
(a) 4 : 7 (b) 2 : 5 (c) 3 : 4 (d) 7 : 3

4. Amit travels 30 minutes at the speed of 25 km/hr. Further he travels 20 minutes at the speed of 40 km/hr. Find his average speed.
(a) 25 km/hr (b) 30 km/hr
(c) 31 km/hr (d) None of these

5. In what ratio should freely available water be mixed with the wine worth ₹ 60 per litre so that after selling the mixture at ₹ 50 per litre, the profit will be 25%?
(a) 1 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 5

6. A shopkeeper has 180 kg of sugar. Some sugar he sells at 10% profit and rest of sugar at 15% profit. If his overall profit is 12%, then what is quantity of sugar sold at 15% discount? **(SSC-CHSL 2012)**
(a) 72 kg (b) 75 kg (c) 90 kg (d) 108 kg

7. In a mixture of milk and water, there is only 26% water. After replacing the mixture with 7 litres of pure milk, the percentage of milk in the mixture become 76%. The quantity of mixture is :
(a) 65 litres (b) 91 litres
(c) 38 litres (d) None of these

8. The ratio of petrol and kerosene in the container is 3 : 2 when 10 litres of the mixture is taken out and is replaced by the kerosene, the ratio becomes 2 : 3. The total quantity of the mixture in the container is :
(a) 25 (b) 30
(c) 45 (d) cannot be determined

9. A shopkeeper buys 50 kg sugar at the rate of 40/kg and 100kg sugar at the rate 70/kg. At what price per kg should he sell the mixture to earn 20% profit on cost price? **(CDS 2013)**
(a) 72/kg (b) 75/kg (c) 84/kg (d) 90/kg

10. The shopkeeper mixed 40 kg refined oil with vegetable oil worth ₹ 60 per kg. Thus he gains ₹ 10 after selling the mixture of the two oils. The price of the first oil is :
(a) 20 (b) 25
(c) 45 (d) can't be determined

11. In my office the average age of all the female employees is 21 years and that of male employees is 32 years, where the average of all the (male and female) employees is 28 years. The total no. of employees in my office could be:
(a) 35 (b) 78 (c) 231 (d) 90

12. A milkman sells the milk at the cost price but he mixes the water (freely available) in it and thus he gains 9.09%. The quantity of water in the mixture of 1 litres is :
(a) 83.33 mL (b) 90.90 mL
(c) 99.09 mL (d) Can't be determined

13. 140 litres of a mixture of milk and water contains 15% of water. How much water must be added to make the water 25% in the mixture?
(a) 18 lit (b) 25 lit (c) 15 lit (d) 12.5 lit

14. How much water must be added to a flask which contains 25 litres of milk at a cost price of ₹ 18 per litre so that the cost of milk reduces to ₹ 15 per litre?
(a) 2 litres (b) 5 litres (c) 6 litres (d) 8 litres

15. The cost of 26 kg of sugar is ₹ 390. The cost of 17kg of rice is ₹ 544 and the cost of 42 kg of wheat is ₹672. What is the total cost of 24 kg of sugar, 36 kg of rice and 29 kg of wheat?
(a) ₹ 1850 (b) ₹ 1976 (c) ₹ 1880 (d) ₹ 1936

16. The wheat sold by a grocer contained 10% low quality wheat. What quantity of good quantity wheat should be added to 250kg of wheat so that the percentage of low quality wheat becomes 5%?
(a) 285 kg (b) 250 kg (c) 135 kg (d) 150 kg

17. 120 litres of a mixture contains milk and water in the ratio 2 : 3. If 40 liters of water is mixed in it, the percentage of milk in the new mixture will be

(a) 40% (b) 36% (c) 24% (d) 30%

18. If 50 litres of milk solution has 40% milk in it, then how much milk should be added to make it 60% in the solution?

(SSC-CGL Tier-I 2014)

(a) 25 L (b) 30 L (c) 35 L (d) 40 L

19. A mixture of 120 litre of milk and water contains 20% of water. How much water must be added to make the water 40% in the new mixture?

(a) 10 litres (b) 45 litres (c) 40 litres (d) 30 litres

20. 60 litres of a mixture contains milk and water in the ratio 2 : 3. If 20 litres of water is mixed in it, the percentage of milk in the new mixture will be

(a) 40% (b) 36% (c) 24% (d) 30%

21. A mixture contains 80% acid and rest water. Part of the mixture that should be removed and replaced by same amount of water to make the ratio of acid and water 4 : 3 is-

(a) $1/3^{rd}$ (b) $3/7^{th}$ (c) $2/3^{rd}$ (d) $2/7^{th}$

22. A jar contains 10 red marbles and 30 green ones. How many red marbles must be added to the jar so that 60% of the marbles will be red?

(a) 25 (b) 30 (c) 35 (d) 40

23. In a drum there is 100 litres of milk. From this 10 litres of milk taken out and replaced with water this process is repeated one more time then what is respective ratio of milk and water in new mixture?

(a) 19 : 81 (b) 81 : 19 (c) 81 : 100 (d) 100 : 81

24. A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

(a) 1/3 (b) 1/4 (c) 1/5 (d) 1/7

25. Find the ratio in which rice at ₹ 7.20 a kg be mixed with rice at ₹ 5.70 a kg to produce a mixture worth ₹ 6.30 a kg.

(a) 4 : 3 (b) 3 : 4 (c) 2 : 3 (d) 3 : 2

26. A person buys 100 kg of sugar at the rate of ₹ 40/kg. He sells 30 kg sugar at 8% profit. Then what is rate per kg of sugar is he need 10% overall profit?

(a) 34.44 (b) 37.84 (c) 40.84 (d) 44.34

27. How many kilograms of rice costing ₹ 9 per kg must be mixed with 27 kg of rice costing ₹ 7 per kg so that there may be a gain of 10 % by selling the mixture at ₹ 9.24 per kg?

(a) 60 kg (b) 63 kg (c) 58 kg (d) 56 kg

28. In what ratio should rice at ₹ 9.30 per kg be mixed with rice at ₹ 10.80 per kg so that the mixture be worth ₹10 per kg ?

(a) 7 : 8 (b) 8 : 7 (c) 6 : 7 (d) 7 : 6

29. In what ratio must tea worth ₹ 60 per kg be mixed with tea worth ₹ 65 a kg such that by selling the mixture at ₹ 68.20 a kg , there can be a gain 10%?

(a) 3 : 2 (b) 2 : 3 (c) 4 : 3 (d) 3 : 4

30. In what ratio must water be mixed with milk costing ₹12 per litre in order to get a mixture worth of ₹ 8 per litre?

(a) 1 : 3 (b) 2 : 2 (c) 1 : 2 (d) 3 : 1

31. From a container full of milk 20 % is drawn and the container filled with water and this process is performed two times more. What would be the ratio of milk in the mixture in present and original?

(a) 64 : 125 (b) 216 : 125
(c) 125 : 64 (d) 64 : 81

32. In what ratio must tea at ₹62 per kg be mixed with tea at ₹ 72 per kg so that the mixture must be worth ₹ 64.50 per kg?

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

33. In what ratio must a grocer mix two varieties of pulses costing ₹15 and ₹ 20 per kg respectively to obtain a mixture worth ₹16.50 per kg?

(a) 1 : 2 (b) 2 : 1 (c) 3 : 7 (d) 7 : 3

34. A merchant has 1000 kg of sugar part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is

(a) 300 (b) 400 (c) 600 (d) 500

35. In what ratio must rice at ₹ 7.10 be mixed with rice at ₹ 9.20 so that the mixture may be worth ₹ 8 per kg?

(a) 5 : 4 (b) 2 : 1 (c) 3 : 2 (d) 4 : 3

36. How many kg of rice at ₹ 6.60 per kg be mixed with 56 kg of rice at ₹ 9.60 per kg to get a mixture worth ₹ 8.20 per kg?

(a) 56 kg (b) 52 kg (c) 44 kg (d) 49 kg

37. We have a 630 ml mixture of milk and water in the ratio 7:2. How much water must be added to make the ratio 7:3?

(a) 70 ml (b) 60 ml (c) 80 ml (d) 50 ml

38. 3 litre of water is added to 11 litre of a solution containing 42% of alcohol in the water. The percentage of alcohol in the new mixture is

(a) 25% (b) 20% (c) 30% (d) 33%

39. Respective ratio of expenditure and saving of Ravi is 7 : 3. His income is increases by 12% and his expenditure is increases by 15% then what is increament in his saving?

(a) 15% (b) 8% (c) 5% (d) 7%

40. On a place some persons and some horses are moving. If heads are counted, there are 60 and if legs are counted there are 190. What is number of persons?

(a) 40 (b) 20 (c) 35 (d) 25

GP_3386

LEVEL - 2

1. If the average weight of a class of students is 15 and the average weight of another class of students is 30, then find the ratio of the students of the first class to the another class of 30 students when the average weight of both the classes is 25 :
(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 4

2. A milkman has two types of milk. In the first container the percentage of milk is 80% and in the second container the percentage of milk is 60%. If he mixes 28 litres of milk of the first container to the 32 litres of milk of the second container, then the percentage of milk in the mixture is :
(a) 63.99 (b) 69.33 (c) 72.5 (d) 75.2

3. Avinash covered 150 km distance in 10 hourse. The first part of his journey he covered by car, then he hired a rickshaw. The speed of car and rickshaw is 20 km/hr and 12 km/hr respectively. The ratio of distances covered by car and the rickshaw respectively are :
(a) 2 : 3 (b) 4 : 5
(c) 1 : 1 (d) None of these

4. A mixture of rice is sold at ₹ 3.00 per kg. This mixture is formed by mixing the rice of ₹ 2.10 and ₹ 2.52 per kg. What is the ratio of cheaper to the costlier quality in the mixture if the profit of 25% is being earned.
(a) 5 : 2 (b) 2 : 7 (c) 2 : 5 (d) 15 : 8

5. In a 50 litres mixture of milk and water volume of water is 60% total mixture. A few litres of mixture is released and an equal amount of water is added and this process is reapeted once again. If volume of water was 15% in final mixture then how many litres of minture released each time?
(SSC-CHSL 2009)
(a) 15 (b) 20 (c) 25 (d) 30

6. There are three types of milk, Parag. Amul and Nestle. The ratio of fat to the non–fat contents in milk is 4 : 5, 5 : 6, 6 : 7 respectively. If all these three types of milk is mixed in equal quantity, the ratio of fat to the non-fat contents in the mixture will be :
(a) 1751 : 2110 (b) 175 : 543
(c) 3 : 5 (d) 10 : 18

7. Some amount out of ₹ 6000 was lent out at 10% per annum and the rest amount at 20% per annum and thus in 4 years the total interest from both the amounts collected was ₹ 3400. What is the amount which was lent out @ 10% per annum?
(a) ₹ 2500 (b) ₹ 2800 (c) ₹ 3200 (d) ₹ 3500

8. From a container of wine, a theif has stolen 15 litres of wine and replaced it with same quantity of water. He again repeated the same process. Thus in three attempts the ratio of wine and water became 343 : 169. The initial amount of wine in the container was :
(a) 75 litres (b) 100 litres
(c) 150 litres (d) 120 litres

9. The ratio of water and alcohol in two different containers is 2 : 3 and 4 : 5. In what ratio we are required to mix the mixtures of two containers in order to get the new mixture in which the ratio of alcohol and water be 7 : 5?
(a) 7 : 3 (b) 5 : 3 (c) 8 : 5 (d) 2 : 7

10. The diluted wine contains only 8 litres of wine and the rest is water. A new mixture whose concentration is 30% is to be formed by replacing wine. How many litres of mixture shall be replaced with pure wine if there was initially 32 litres of water in the mixture?
(a) 4 (b) 5
(c) 8 (d) None of these

11. Mr. Mittal purchased two steel factories, one in India and other one in Malaysia for total ₹ 72 crores. Laters on he sold the Indian factory at 16 % profit and Malasian factory at 24% profit. Thus he gained a total profit of 19%. The selling price of India factory is:
(a) 45 crore (b) 52.2 crore
(c) 8.55 crore (d) Can not be determinded

12. A person has 216 books in his stock. He sells some books at 24% profit and remaining books at 8% loss. If he got a profit of 12% on whole stock then what is number of books sold at 24% profit and 8% loss? **(SSC CGL 2010)**
(a) 116, 100 (b) 120, 96
(c) 135, 81 (d) 132, 84

13. A person has 200 litres of milk. He sold some quantity of milk at 20% profit and some quantity at 30% loss. If his loss was 6% on whole stock then what quantity of milk is sold at 20% profit and 30% loss? **(SSC CGL Tier I 2014)**
(a) 100 litres, 100 litres (b) 96 litres, 104 litres
(c) 80 litres, 120 litres (d) 125 litres, 75 litres

14. A butler stole wine from a butt of sherry containing 50% of spirit, then he replenished it by different wine containing 20% spirit. Thus there was only 30% strength (spirit) in the new mixture. How much of the original wine did he steal?
(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/4

15. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?
(a) 26 litres (b) 29.16 litres
(c) 28 litres (d) 28.2 litres

16. Tea worth ₹ 126 per kg and ₹ 135 per kg are mixed with a third variety of tea in the ratio 1 : 1 : 2. If the mixture is worth ₹ 153 per kg, what is the price of the third variety per kg ?
(a) ₹ 182.50 (b) ₹ 170.5 (c) ₹ 175.50 (d) ₹ 180

17. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?
(a) 5litres, 7 litres (b) 7litres, 4 litres
(c) 6litres, 6 litres (d) 4litres, 8 litres

18. Two vessels A and B contain spirit and water in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixtures be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5 ?
(a) 3:4 (b) 4 : 3 (c) 9 : 7 (d) 7 : 9

19. The cost of type 1 material is ₹ 15 per kg and type 2 material is ₹ 20 per kg. If both type 1 and type 2 are mixed in the ratio of 2 : 3, then what is the price per kg of the mixed variety of material?
(a) ₹ 19 (b) ₹ 16 (c) ₹ 18 (d) ₹ 17

20. Tea worth ₹ 252 per kg and ₹ 270 per kg are mixed with a third variety in the ratio 1 : 1 : 2. If the mixture is worth ₹ 306 per kg, the price of the third variety per kg will be:
(a) ₹ 359.5 (b) ₹ 340 (c) ₹ 351 (d) ₹ 360

21. A can contains a mixture of two liquids A and B is the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A were contained by the can initially?
(a) 10 (b) 20 (c) 21 (d) 25

22. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask originally hold?
(a) 30 litres (b) 26 litres
(c) 24 litres (d) 32 litres

23. A jar full of whiskey contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is
(a) 4/3 (b) 3/4 (c) 3/2 (d) 2/3

24. A container contains a mixture of two liquids P and Q in the ratio 7 : 5. When 9 litres of mixture are drawn off and the container is filled with Q, the ratio of P and Q becomes 7 : 9. How many litres of liquid P was contained in the container initially?
(a) 23 (b) 21 (c) 19 (d) 17

25. A vessel is filled with liquid, 3 parts of which are water and 5 parts are syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?
(a) 1/3 (b) 1/4 (c) 1/5 (d) 1/6

26. A dishonest milkman sells his milk at cost price but he mixes it with water and thereby gains 25%. What is the percentage of water in the mixture?
(a) 25% (b) 20% (c) 22% (d) 24%

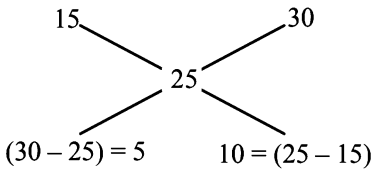
27. ₹ 460 was divided among 41 boys and girls such that each boy got ₹12 and each girl got ₹8. What is the number of boys?
(a) 33 (b) 30 (c) 36 (d) 28

GP_3386

Hints and Solutions

Level 1

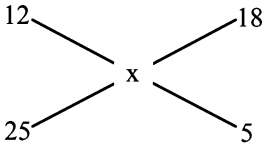
1. (b)



Since the ratio of girls to boys is $\frac{5}{10}=\frac{1}{2}$

Hence if there are 12 boys, so there will be 6 girls.

2. (a)



$\frac{(x-12)}{(18-x)}=\frac{5}{25}\Rightarrow x=13$

Shortcut :

The corresponding ratio is 25 : 5 = 5 : 1

Now reverse the ratio, which is 1 : 5

Now divide the differences of 12 and 18 in the ratio of 1 : 5

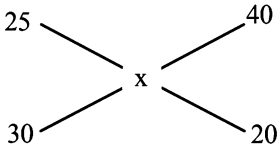
i.e., divide 6 into two parts in the ratio of 1 : 5.

then the average price of mixture is 12 + 1 = 13

or 18 – 5 = 13

3. (d) Use Shortcut Approach-1.

4. (c)



Since the actual ratio is 3 : 2, so reverse it, i.e., the required ratio is 2 : 3.

Now divided the difference of both the speed i.e., 25 and 40 in the ratio of 2 : 3.

i.e., $(40-25)\times\frac{2}{5}=6$ and $(40-25)\times\frac{3}{5}=9$

So the average speed is 25 + 6 = 31

or 40 – 9 = 31.

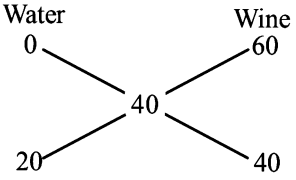
5. (a) Selling price = ₹ 50

Therefore, the cost price

= ₹ 40, which is the average price.

It means the wine, worth ₹ 60 becomes worth ₹ 40 when the water was mixed in it. So we can conclude that in the mixture of ₹ 60, there is wine worth ₹ 40 and the rest is water. Therefore, the ratio of water and wine is 20 : 40 i.e., 1 : 2 :

Alternatively :



$\Rightarrow 1:2$

6. (a) Use Shortcut Approach-4.

7. (b)

Milk	Water	
74%	26%	(initially)
76%	24%	(after replacement)

Left amount = Initial amount

$\left(1-\frac{\text{replaced amount}}{\text{total amount}}\right)\Rightarrow 24=26\left(1-\frac{7}{K}\right)$

$\Rightarrow \frac{12}{13}=\left(1-\frac{7}{K}\right)\Rightarrow\frac{1}{13}=\frac{7}{K}\Rightarrow K=91$ litres

8. (b) Petrol : Kerosene

3 : 2 (initially)

2 : 3 (after replacement)

$\frac{\text{Remaining (or left) quantity}}{\text{Initial quantity}}$

$=\left(1-\frac{\text{replaced quantity}}{\text{total quantity}}\right)$

$(\text{for petrol})\frac{2}{3}=\left(1-\frac{10}{K}\right)$

$\Rightarrow\frac{1}{3}=\frac{10}{K}\Rightarrow K=30$ litres

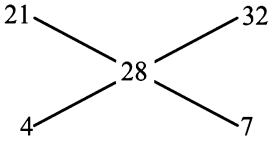
Therefore, the total quantity of the mixture in the container is 30 litres.

9. (a) Use Shortcut Approach-2.

10. (d) Since there is insufficient data.

11. (c) Since the ratio of no. of female and male employees is 4 : 7, so the total no, of

employees must be the multiples of 11. Hence the possible answer is 231.



12. (a) Profit (%) $= 9.09\% = \frac{1}{11}$

Since the ratio of water and milk is 1 : 11, Therefore the ratio of water is to mixture = 1 : 12

Thus the quantity of water in mixture of 1 litre

$= 1000 \times \frac{1}{12} = 83.33$ ml

13. (a) Milk = 119, Water = 21

75% = 119; 100% = 158 (approx.)

158 – 140 = 18 litres

14. (b) Let x litres is the water added

$15(25 + x) = 25 \times 18 \Rightarrow 15x = 75 \Rightarrow x = 5$

15. (b) Cost of 24 kg of sugar = 360

Cost of 36 kg of rice = 1152

Cost of 29 kg of wheat = 464

So total cost = (360 + 1152 + 464) = ₹1976

16. (b) Here, 10% of 250 kg = 25 kg

So good quality of wheat is 225.

To become the 5% low quality of wheat, we add 250 kg of more wheat.

17. (d) Quantity of milk in original solution

$= \frac{2}{5} \times 120 = 48$ liters and

Quantity of water = 120 – 48 = 72 liters

Percentage of milk in new solution

$= \frac{48}{120 + 40} \times 100 = 30\%$

18. (a) Use Shortcut Approach-3.

19. (c) Total water in mixture $= 120 \times \frac{20}{100} = 24$ litres

Total milk in mixture = 120 – 24 = 96 litres

Let x liters water must be added

$\therefore \quad \frac{24 + x}{96} = \frac{40}{60} = \frac{2}{3} \Rightarrow 72 + 3x = 192$

$\Rightarrow x = 40$ litres

20. (d) Quantity of milk in original solution

$= \frac{2}{5}$ of 60 = 24 litres and

Quantity of water = 60 – 24 = 36 litres

Percentage of milk in new solution

$= \frac{24}{60 + 20} \times 100 = 30\%$

21. (d) Let the initial wt. of mixture be 1 kg and x kg of mixture is taken out and replaced by same amount of water.

$\because$ Amt. of acid/Amt. of water

$= (0.8 - 0.8x)/(0.2 - 0.2x + x) = 4/3$

$\Rightarrow \quad 2.4 - 2.4x = 0.8 + 3.2x \Rightarrow 5.6x = 1.6$

$\therefore \quad x = 1.6/5.6 = 2/7$th part

22. (c) Let x red marbles be added so that they will be 60% of the total.

$\because \quad (10+x)/(10 + x + 30) = 60/100$

$\Rightarrow \quad (10 + x)/(40 + x) = 3/5$

$\Rightarrow \quad 50 + 5x = 120 + 3x \Rightarrow x = 70/2 = 35$

23. (b) Use Shortcut Approach-9.

24. (c) Suppose the vessel initially contains 8 litres of liquid.

Let x litres of this liquid be replaced with water.

Quantity of water in new mixture

$= \left(3 - \frac{3x}{8} + x\right)$ litres

Quantity of syrup in new mixture

$= \left(5 - \frac{5x}{8}\right)$ litres

$\therefore \quad \left(3 - \frac{3x}{8} + x\right) = \left(5 - \frac{5x}{8}\right)$

$\Rightarrow \quad 5x + 24 = 40 - 5x \Rightarrow 10x = 16 \Rightarrow x = \frac{8}{5}$

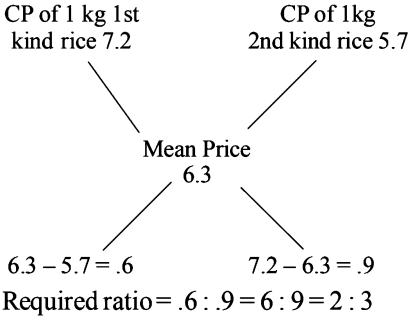
So, part of the mixture replace $= \left(\frac{8}{5} \times \frac{1}{8}\right) = \frac{1}{5}$

25. (c) CP of 1kg 1st kind rice = ₹7.20

CP of 1kg 2nd kind rice = ₹5.70

CP of 1kg mixed rice = ₹6.30

By rule of alligation,



Required ratio = .6 : .9 = 6 : 9 = 2 : 3

26. (d) Use Shortcut Approach-7.

27. (b) Selling Price (SP) of 1 kg mixture = ₹9.24

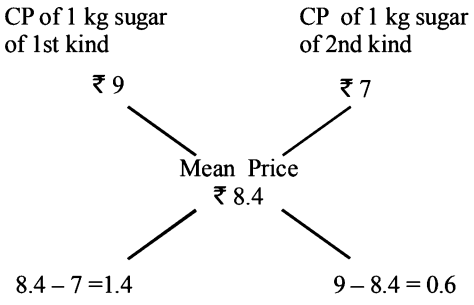
Profit = 10%

Cost Price (CP) of 1 kg mixture

$$= \frac{100}{(100+\text{Profit}\%)} \times SP = \frac{100}{(100+10)} \times 9.24$$

$$= \frac{100}{110} \times 9.24 = \frac{92.4}{11} = ₹\ 8.4$$

By rule of alligation,



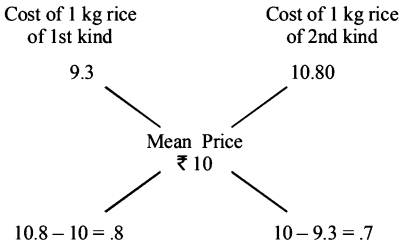
i.e., to get a cost price of 8.4, the sugars of kind1 and kind 2 should be mixed in the ratio 1.4 : 0.6 = 14 : 6 = 7 : 3

Suppose x kg of kind1 sugar is mixed with 27 kg of kind 2 sugar.

then, x : 27 = 7 : 3

$\Rightarrow$ 3x = 27 × 7 $\Rightarrow$ x = 9 × 7 = 63

28. (b) By rule of alligation,



Required ratio = .8 : .7 = 8 : 7.

29. (a) Selling Price (SP) of 1 kg mixture = ₹ 68.20

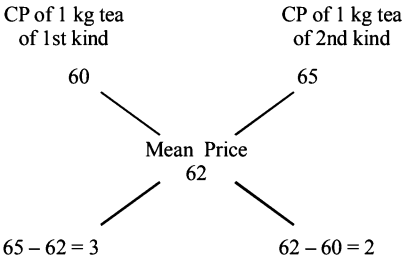
Profit = 10%

Cost Price (CP) of 1 kg mixture

$$= \frac{100}{(100+\text{Profit}\%)} \times SP = \frac{100}{(100+10)} \times 68.20$$

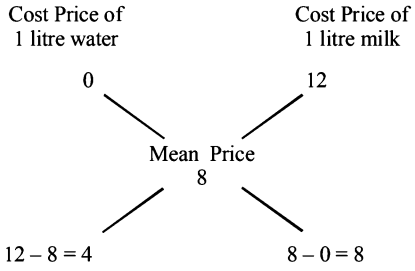
$$= \frac{100}{110} \times 68.20 = \frac{682}{11} = ₹ 62$$

By rule of alligation



Hence required ratio = 3 : 2

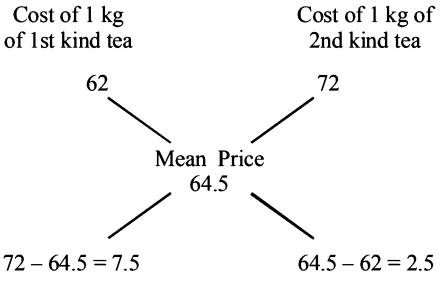
30. (c) By rule of alligation,



Required ratio = 4 : 8 = 1 : 2

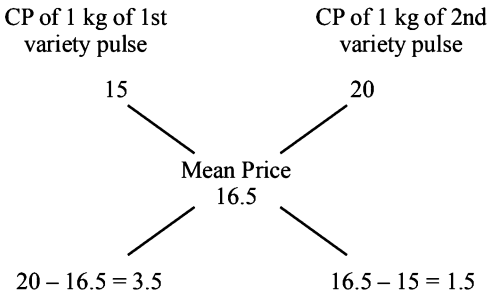
31. (a) Use Shortcut Approach-8.

32. (c) By rule of alligation,



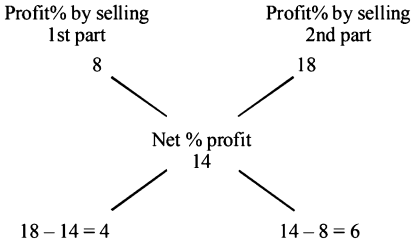
Required ratio = 7.5 : 2.5 = 3 : 1

33. (d) By rule of alligation,



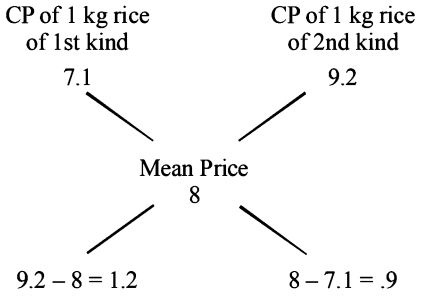
Required ratio = 3.5 : 1.5 = 35 : 15 = 7 : 3

34. **(c)** By rule of alligation,



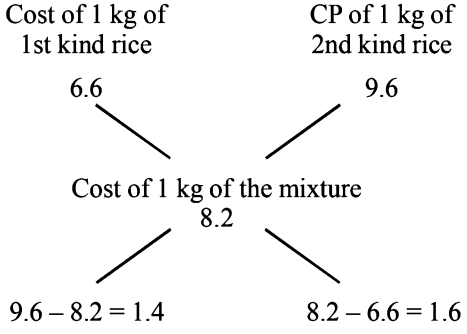
$\Rightarrow$ Quantity of part 1 : Quantity of part 2 = 4 : 6 = 2 : 3
Total quantity is given as 1000 kg. So quantity of part 2 (quantity sold at 18% profit) = $1000 \times 3/5 = 600$ kg

35. **(d)** By rule of alligation,



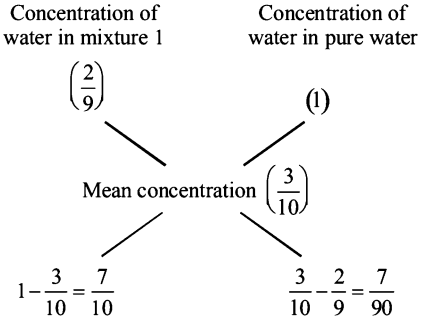
Required ratio = 1.2 : .9 = 12 : 9 = 4 : 3

36. **(d)** By rule of alligation,



Quantity of 1st kind rice : Quantity of 2nd kind rice = 1.4 : 1.6 = 7 : 8
$\Rightarrow$ Quantity of 1st kind rice = $56 \times 7/8 = 49$

37. **(a)** Concentration of water in mixture 1 = 2/9 (since the ratio of milk and water = 7:2)
Concentration of water in pure water = 1
Now the above mentioned items are mixed to form mixture 2 where milk and water ratio = 7 : 3
$\Rightarrow$ Concentration of water in mixture 2 = 3/10
By rule of alligation,



$\Rightarrow$ Quantity of mixture 1 : Quantity of water = 7/10 : 7/90 = 1/10 : 1/90 = 1 : 1/9
Given that quantity of mixture 1 = 630 ml
$\Rightarrow$ 630 : Quantity of water = 1 : 1/9
$\Rightarrow$ Quantity of water = $630 \times 1/9 = 70$ ml

38. **(d)** We have a 11 litre solution containing 42% of alcohol in the water.
Quantity of alcohol in the solution

$$= \frac{11 \times 42}{100}$$

Now 3 litres of water is added to the solution.
$\Rightarrow$ Total quantity of the new solution = 11 + 3 = 14
Percentage of alcohol in the new solution

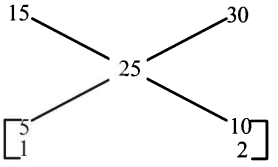
$$= \frac{\frac{11 \times 42}{100}}{14} \times 100 = \frac{11 \times 3}{100} = 33\%$$

39. **(c)** Use Shortcut Approach-11

40. **(d)** Use Shortcut Approach-12

Level 1

1. **(a)** Let the ratio of the students with 15 students to 30 students be x : y, then

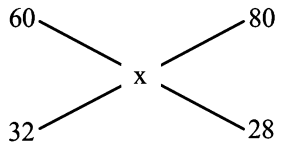


Here we know that the averages of individual classes is 15 and 30. Again we know that the averages of both the

combined classes is 25. So the difference between 25 and 15 i.e., 10 and 30 & 25, i.e., 5 is written diagonally opposite.

Thus the required ratio $= \frac{5}{10} = \frac{1}{2}$

2. (b)



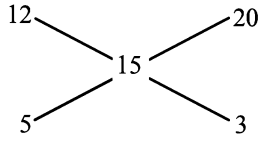
8 : 7
7 : 8 (reversed ratio)

So $(80-60)\times\frac{7}{7+8} = 20\times\frac{7}{15} = 9.33$

$\therefore$ Required percentage $= 60 + 9.33 = 69.33$

3. (c) The average speed of Avinash

$= \frac{150}{10} = 15$ km/hr



It means the rickshaw took 5/8 and car took 3/8 of the total time i.e., the ratio of time taken by rickshaw to car is 5 : 3

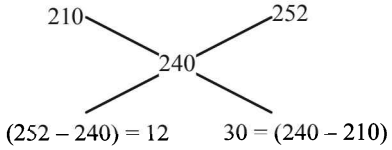
So the ratio of distances covered by rickshaw to car is $5 \times 12 : 3 \times 20 \Rightarrow 1 : 1$

4. (c) Let the cost price of the mixture be ₹x per kg,

then selling price $= x + \frac{25 \times x}{100} = 3$

$\Rightarrow 1.25x = 3 \quad \Rightarrow x = 2.4$

Now the average cost price of mixture = ₹2.4



Therefore, the ratio of cheaper to costiler rice is 12 : 30, i.e., 2 : 5.

5. (c) Use Shortcut Approach-10.

6. (a) In this type of questions, we consider only one item (viz, either fat or non-fat) as the fraction of the total quantity. Again we equate the denominators.

Let us consider fraction of fat

	Parag	Amul	Nestle
Fraction of fat →	$\frac{4}{9}$	$\frac{5}{11}$	$\frac{6}{13}$
	$\frac{4\times11\times13}{9\times11\times13}$	$\frac{5\times9\times13}{11\times9\times13}$	$\frac{6\times9\times11}{13\times9\times11}$
	$\frac{572}{1287}$	$\frac{585}{1287}$	$\frac{594}{1287}$

So the total fat in the total mixture

$= \frac{572+585+594}{1287+1287+1287} = \frac{1751}{3861}$

Hence, the ratio of fat to the non-fat contents in the mixture

$= 1751 : 2110 \quad \because (3861 - 1751 = 2110)$

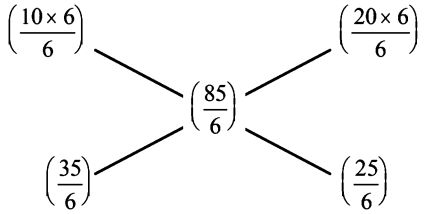
7. (d) The total interest of one year

$= ₹850 = \left(\frac{3400}{4}\right)$

Therefore, the average rate of interest

$= 14\frac{1}{6}\% = \frac{85}{6}\%$

Thus,



Hence the ratio of amount which is lent @ 10% per annum to the amount lent @ 20% per annum is 35 : 25, i.e., 7 : 5.

Therefore, the amount which is lent out @ 10% per annum is ₹3500.

8. (d) $\frac{\text{Wine (left)}}{\text{Water (added)}} = \frac{343}{169}$

It means $\frac{\text{wine (left)}}{\text{wine (initial amount)}} = \frac{343}{512}$

$(\because 343 + 169 = 512)$

Thus, $343x = 512x\left(1-\frac{15}{K}\right)^3$

$\Rightarrow \quad \frac{343}{512}=\left(\frac{7}{8}\right)^3=\left(1-\frac{15}{K}\right)^3$

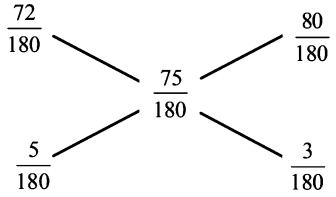
$\Rightarrow \quad \left(1-\frac{15}{K}\right)=\frac{7}{8}=\left(1-\frac{1}{8}\right)$

$\Rightarrow \quad K=120$

Thus the initial amount of wine was 120 litres.

9. (b)

$W_1 : A_1$	$W_2 : A_2$	$W_N : A_N$
2 : 3	4 : 5	5 : 7
$\frac{W_1}{W_1+A_1}=\frac{2}{5}$	$\frac{W_2}{W_2+A_2}=\frac{4}{9}$	$\frac{W_N}{W_N+A_N}=\frac{5}{12}$
$=\frac{72}{180}$	$=\frac{80}{180}$	$=\frac{75}{180}$



$\Rightarrow 5 : 3$

Therefore, the ratio is 5 : 3

10. (b)

Wine		Water
8L		32L
1	:	4
20%		80% (original ratio)
30%		70% (required ratio)

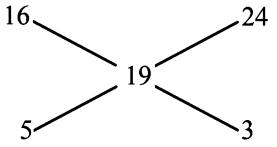
In this case, the percentage of water being reduced when the mixture is being replaced with wine.

So the ratio of left quantity to the initial quantity is 7 : 8.

Therefore $\frac{7}{8}=\left[1-\frac{K}{40}\right] \Rightarrow \frac{7}{8}=\left[\frac{40-K}{40}\right]$

$\Rightarrow \quad K = 5$ litres

11. (b)



Thus the cost price of Indian factory is ₹45, crore.

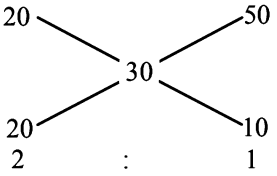
Therefore, the selling price of Indian factory is

$= 45+\frac{45\times 16}{100} = 52.2$ crore

12. (c) Use Shortcut Approach-5.

13. (b) Use Shortcut Approach-6.

14. (b)



Since the ratio of 20% wine to 50% wine is 2 : 1, it means there is $\frac{2}{3}$ wine is replaced with wine in which the concentration of spirit is 20%.

15. (b) Suppose a container contains x units of a liquid from which y units are taken out and replaced by water. After n operations,

quantity of pure liquid $= x\left(1-\frac{y}{x}\right)^n$ units.

milk contained by the container now

$=40\left(1-\frac{4}{40}\right)^3 \quad =40\left(1-\frac{1}{10}\right)^3$

$=40\times\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}=\frac{4\times9\times9\times9}{100}=29.16$

16. (c) Tea worth ₹ 126 per kg and ₹ 135 per kg are mixed in the ratio 1 : 1

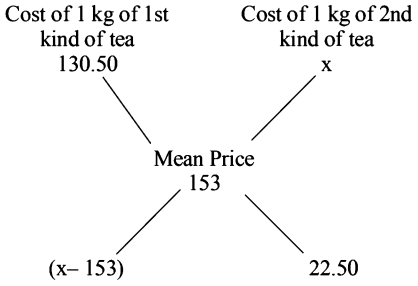
So their average price

$=\frac{(126+135)}{2}=130.5$

Hence let's consider that the mixture is formed by mixing two varieties of tea, one at ₹ 130.50 per kg and the other at

₹ x per kg in the ratio 2 : 2, i.e., 1 : 1. Now let's find out x.

By rule of allegation,



$(x-153):22.5=1:1$

$\Rightarrow \quad x-153=22.50 \Rightarrow x=153+22.5=175.5$

17. (c) Let x and (12 – x) litres of milk be mixed from the first and second container respectively.

Amount of milk in x litres of the the first container = .75x

Amount of water in x litres of the the first container = .25x

Amount of milk in (12 – x) litres of the the second container = .5(12 – x)

Amount of water in (12 – x) litres of the the second container = .5(12 – x)

Ratio of water to milk

$=[.25x+.5(12-x)]:[.75x+.5(12-x)]$

$=3:5$

$$\Rightarrow \quad \frac{(0.25x+6-0.5x)}{(0.75x+6-0.5x)}=\frac{3}{5}$$

$$\Rightarrow \quad \frac{(6-.25x)}{(.25x+6)}=\frac{3}{5} \Rightarrow 30-1.25x=.75x+18$$

$\Rightarrow \quad 2x=12 \Rightarrow x=6$

Since $x=6, 12-x=12-6=6$

Hence 6 and 6 litres of milk should mixed from the first and second container respectively.

18. (d) Let Cost Price (CP) of 1 litre spirit be ₹1

Quantity of spirit in 1 litre mixture from vessel A = 5/7

Cost Price(CP) of 1 litre mixture from vessel A = ₹ 5/7

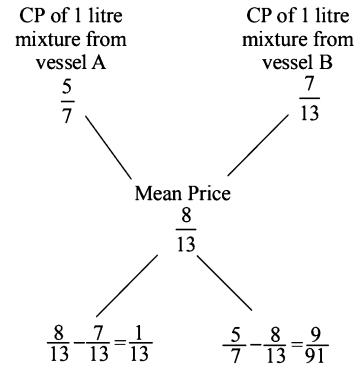
Quantity of spirit in 1 litre mixture from vessel B = 7/13

Cost Price(CP) of 1 litre mixture from vessel B = ₹ 7/13

Quantity of spirit to be obtained in 1 litre mixture from vessel C = 8/13

Cost Price(CP) of 1 litre mixture from vessel C(Mean Price) = ₹ 8/13

By rule of allegation,



$\Rightarrow$ Mixture from vessel A : Mixture from vessel B

$$=\frac{1}{13}:\frac{9}{91}=7:9$$

19. (c) Cost Price (CP) of type 1 material is ₹ 15 per kg

Cost Price (CP) of type 2 material is ₹ 20 per kg

Type 1 and type 2 are mixed in the ratio of 2 : 3.

Hence Cost Price (CP) of the resultant mixture

$$=\frac{(15\times2)+(20\times3)}{(2+3)}=\frac{(30+60)}{5}=\frac{90}{5}=18$$

$\Rightarrow$ Price per kg of the mixed variety of material = ₹18

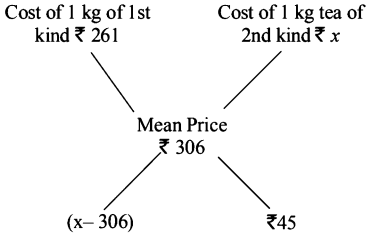
20. (c) Since first and second varieties are mixed in equal proportions.

So, their average price = ₹ (252 + 270)/2 = ₹ 261

So, the mixture is formed by mixing two varieties, one at ₹ 261 per kg and the other at say, ₹ x per kg in the ratio 2 : 2, i.e., 1 : 1.

We have to find x.

By the rule of alligation, we have:



$$\therefore \quad \frac{x-306}{45}=1 \Rightarrow x-306=45 \Rightarrow x=351$$

21. (c) Suppose the can initially contains $7x$ and $5x$ of mixtures A and B respectively.

Quantity of A in mixture left

$$= \left(7x - \frac{7}{12} \times 9\right) \text{litres} = \left(7x - \frac{21}{4}\right) \text{litres.}$$

Quantity of B in mixture left

$$= \left(5x - \frac{5}{12} \times 9\right) \text{litres} = \left(5x - \frac{15}{4}\right) \text{litres.}$$

$$\therefore \quad \frac{\left(7x - \frac{21}{4}\right)}{\left(5x - \frac{15}{4}\right) + 9} = \frac{7}{9} \Rightarrow \frac{28x - 21}{20x + 21} = \frac{7}{9}$$

$\Rightarrow 252x - 189 = 140x + 147 \Rightarrow x = 3.$

So, the can contained 21 litres of A

22. (c) Let the quantity of the wine in the cask originally be x litres then, quantity of wine left in cask after 4 operations

$$= \left[x\left(1 - \frac{8}{x}\right)^4\right] \text{ litres}$$

$$\therefore \quad \left[\frac{x\left(1 - \frac{8}{x}\right)^4}{x}\right] = \frac{16}{81} \Rightarrow \left[1 - \frac{8}{x}\right]^4 = \left(\frac{2}{3}\right)^4$$

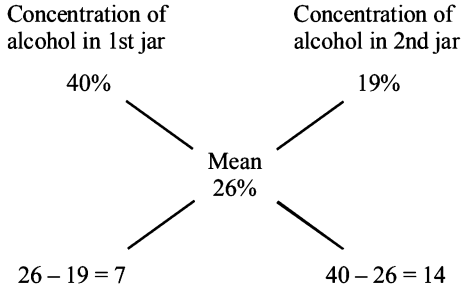
$$\Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3} \Rightarrow 3x - 24 = 2x \Rightarrow x = 24.$$

23. (d) Concentration of alcohol in 1st jar = 40%

Concentration of alcohol in 2nd jar = 19%

After the mixing, concentration of alcohol in the mixture = 26%

By rule of alligation,



Hence ratio of 1st and 2nd quantities = 7 : 14 = 1 : 2

i.e., 2/(1+ 2) = 2/3 part of the whisky is replaced.

24. (b) Let initial quantity of P in the container be 7x and initial quantity of Q in the container be 5x

Now 9 litres of mixture is drawn off from the container.

Quantity of P in 9 litres of the mixture drawn off = 9 × 7/12 = 63/12 = 21/4

Quantity of Q in 9 litres of the mixture drawn off = 9 × 5/12 = 45/12 = 15/4

Hence,

Quantity of P remaining in the mixture after 9 litres is drawn off = 7x – 21/4

Quantity of Q remaining in the mixture after 9 litres is drawn off = 5x – 15/4

Since the container is filled with Q after 9 litres of mixture is drawn off, quantity of Q in the mixture

$$= 5x - \frac{15}{4} + 9 = 5x + \frac{21}{4}$$

Given that the ratio of P and Q becomes 7 : 9

$$\Rightarrow \left(7x - \frac{21}{4}\right) : \left(5x + \frac{21}{4}\right) = 7 : 9$$

$$\Rightarrow 9\left(7x - \frac{21}{4}\right) = 7\left(5x + \frac{21}{4}\right)$$

$$\Rightarrow 63x - \left(\frac{9 \times 21}{4}\right) = 35x + \left(\frac{7 \times 21}{4}\right)$$

$$\Rightarrow 28x = \left(\frac{16 \times 21}{4}\right) \Rightarrow x = \left(\frac{16 \times 21}{4 \times 28}\right)$$

Litres of P contained in the container initially

$$= 7x = \left(\frac{7 \times 16 \times 21}{4 \times 28}\right) = \frac{16 \times 21}{4 \times 4} = 21$$

25. (c) Let the quantity of the liquid in the vessel = 8 litre. Then, quantity of water in the liquid = 3 litre, and quantity of syrup in the liquid = 5 litre.

Suppose x litre of the mixture is drawn off and replaced with water. Then, quantity of

water in the new mixture $= 3 - \frac{3x}{8} + x$

Quantity of syrup in the new mixture

$= 5 - \frac{5x}{8}$

Given that in the new mixture, quantity of water = quantity of syrup

$\Rightarrow 3 - \frac{3x}{8} + x = 5 - \frac{5x}{8}$

$\Rightarrow \frac{10x}{8} = 2 \Rightarrow \frac{5x}{4} = 2 \Rightarrow x = \frac{8}{5}$

i.e., if the quantity of the liquid is 8 litre, 8/5 litre of the mixture needs to be drawn off and replaced with water so that the mixture may be half water and half syrup.

It means 1/5 of the mixture needs to be drawn off and replaced with water so that the mixture may be half water and half syrup.

26. (b) If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True Value} - \text{Error})} \times 100\right]\%$$

Here Gain = 25%

error = quantity of water he mixes in the milk = x

true value = true quantity of milk = T

So the formula becomes, $25 = \frac{x}{(T-x)} \times 100$

$\Rightarrow 1 = \frac{x}{(T-x)} \times 4 \Rightarrow T - x = 4x$

$\Rightarrow T = 5x$

Percentage of water in the mixture

$= \frac{x}{T} \times 100 = \frac{x}{5x} \times 100 = \frac{1}{5} \times 100 = 20\%$

27. (a) Assume that the number of boys = b and number of girls = g

According to question,

Number of boys + Number of girls = 41

$\Rightarrow$ b + g = 41 ...(1)

Given that each boy got ₹12 and each girl got ₹8. Then the total amount is ₹460

$\Rightarrow$ 12b + 8g = 460 ...(2)

Now we need to solve these equations to get b and g.

(1) × 8 $\Rightarrow$ 8b + 8g = 8 × 41 = 328 ...(3)

From equation (2) – (3)

$\Rightarrow$ 4b = 460 – 328 = 132

$\Rightarrow$ b = 132/4 = 33

CHAPTER

15

Time, Work and Wages
(Including Pipes & Cistern)

CONCEPT OF EFFICIENCY

Efficiency means rate of doing work. This means that more the efficiency, less will be the number of days required to complete a certain work and less the efficiency, more will be the number of days required to complete a certain work.

Aliza is twice as efficient as Binny.

⇒ Aliza does twice as much work as Binny in the same time interval

⇒ Aliza will require half the time as required by Binny to do the same work.

CONCEPT OF NEGATIVE WORK

Suppose two persons A and B are working to build a wall while C is working to demolish the wall. If we consider the work as the building of the wall, then breaking the wall (by C) is negative work.

The concept of negative work generally appears in the problems based on pipes and cisterns, where there are inlet pipes and outlet pipes/leaks, which are working against each other.

If we consider the work of filling a tank, the inlet pipe does positive work while the outlet pipes/leaks does negative work.

Illustration – 1

A can build a wall in 15 days and B can build it in 10 days, while C can completely demolish the wall in 12 days. If they start working at the same time, in how many days will the work be completed.

Solution : Work per day by $A = \frac{1}{15}$

Work per day by $B = \frac{1}{10}$

Work per day by $C = -\frac{1}{12}$

(negative sign is taken for negative work)

The net combined work per day by A, B and C

$$= \frac{1}{15} + \frac{1}{10} - \frac{1}{12} = \frac{1}{12}$$

Since, total work done = (Work done per day) × (No. of days required to complete the work)

$\therefore$ No. of days required to complete the work

$$= \frac{\text{Total work done}}{\text{Work done per day}} = \frac{1}{\frac{1}{12}} = 12$$

CONCEPT OF MAN-DAYS

If 'M' men working together can complete a work in 'D' days, then the product of number of men (M) and number of days (D) i.e. $M \times D$ is known as the number of MAN-DAYS.

Number of man days to complete a specific task always remains constant.

Suppose 30 persons working together for 20 days to complete a job, then the total work done is equal to ($30 \times 20 = 600$) man-days. If we change the number of days in which the work is to be completed, then the other factor i.e. the number of persons will change accordingly, so that the product of the factors becomes equal to 600 man-days.

WORK DONE

Consider a whole work as the unit work.

1. Work Done by Two Persons

Let A can do a whole work in x days and B can do the same one unit work in y days.

Hence work done by A in one day $= \frac{1}{x}$

GP_3386

and work done by B in one day $= \frac{1}{y}$

Then work done in one day when A and B work together

$= \frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}$ or $\frac{x+y}{xy}$

Whole work = (Work done in one day) × (Number of days required to complete the whole work)

Hence, number of days required to complete the whole work $= \frac{\text{Whole work}}{\text{Work done in one day}}$

$\Rightarrow$ Number of days required to complete the whole work when A and B are working together

$= \frac{1}{\frac{x+y}{xy}} = \frac{xy}{x+y}$, because a whole work is considered as one unit of work.

Illustration – 2

If A can do a work in 10 days and B can do the same work in 15 days, then how many days will they take to complete the work both while working together?

Solution : Work done by A in one day $= \frac{1}{10}$

Work done by B in one day $= \frac{1}{15}$

Work done in one day when A and B work together

$= \frac{1}{10}+\frac{1}{15}=\frac{3+2}{30}=\frac{5}{30}=\frac{1}{6}$

Hence required number of days $= \frac{1}{\frac{1}{6}} = 6$

We can find the required number of days directly by using the formula,

Number of days $= \frac{xy}{x+y}$, derived above

$= \frac{10\times 15}{10+15}=\frac{150}{25}=6.$

Illustration – 3

'A' completes a work in 12 days. 'B' completes the same work in 15 days. 'A' started working alone and after 3 days B joined him. How many days will they now take together to complete the remaining work?

(a) 5 (b) 8 (c) 6 (d) 4

Solution : (a) Work done by 'A' in 3 days

$= \frac{1}{12}\times 3 = \frac{1}{4}$

$\therefore$ Remaining work $= 1-\frac{1}{4}=\frac{3}{4}$

Work done by A and B together $= \frac{12\times 15}{27}=\frac{20}{3}$

$\therefore$ Remaining work done by A and B together in

$= \frac{3}{4}\times\frac{20}{3} = 5$ days

---

2. Work Done by Three Persons

As we derived the formula for two persons, you can also derived the formula for three persons in the same way.

If A, B, C can do a work in x, y and z days respectively, then all of them working together can finish the work in $\frac{xyz}{xy+yz+zx}$ days.

Illustration – 4

If A, B, C can do a work in 12, 15 and 20 days respectively, then how many days will they take to complete the work when all the three work together.

Solution : (a) Required number of days

$= \frac{xyz}{xy+yz+zx}=\frac{12\times 15\times 20}{12\times 15+15\times 20+20\times 12}$

$= \frac{3600}{180+300+240}=\frac{3600}{720}=5$

---

3. If A and B Together Can do a Work in x Days and A Alone can do it in y Days, then B alone can do the Work in $\frac{xy}{y-x}$ Days

Illustration – 5

A and B can do a work in 8 days and A alone can do it in 12 days. In how many days can B alone do it?

Solution : Work done by A and B working together in one day $= \frac{1}{8}$

Work done by A in one day $= \frac{1}{12}$

$\therefore$ Work done by B in one day

$= \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24}$

Hence number of days in which B alone can do the whole work

$= \frac{1}{\frac{1}{24}} = 1 \times \frac{24}{1} = 24$

You can find required number of days directly by using the above formula as

Required number of days

$= \frac{xy}{y-x} = \frac{8 \times 12}{12-8} = \frac{8 \times 12}{12-8} = \frac{8 \times 12}{4} = 24$

Work Done Equation

If M_1 men each of efficiency E_1 can do W_1 works in D_1 days working T_1 hours per day and M_2 men each of efficiency E_2 can do W_2 works in D_2 days working T_2 hours per day, then

$M_1D_1T_1E_1W_2 = M_2D_2T_2E_2W_1$

This is the general equation in two work situations. Suffix 1 indicate first work situation while the suffix 2 indicate the second work situation.

If one or more items in both work situations are same, then no need to write them in the general equation in two work situations. **For examples**

- If E_1 and E_2 are same, then $M_1D_1T_1W_2 = M_2D_2T_2W_1$
- If E_1 & E_2 and T_1 & T_2 are same, then $M_1D_1W_2 = M_2D_2W_1$
- If E_1 & E_2, T_1 & T_2 and W_1 & W_2 are same, then $M_1D_1 = M_2D_2$

Illustration – 6

25 men can make 120 chairs in 10 days. How many chairs can be made by 35 men in 15 days.

Solution : $M_1D_1W_2 = M_2D_2W_1$

Here $M_1 = 25$, $D_1 = 10$, $W_1 = 120$

$M_2 = 35$, $D_2 = 15$, $W_2 = ?$

$\therefore 25 \times 10 \times W_2 = 35 \times 15 \times 120$

$\Rightarrow W_2 = \frac{35 \times 15 \times 120}{25 \times 10} = 252$

Hence required number of chairs = 252.

Illustration – 7

12 men can make 80 tables in 20 days working 8 hours a day. In how many days 30 men can make 120 tables working 6 hours a day?

Solution : $M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$

Here $M_1 = 12$, $D_1 = 20$, $T_1 = 8$, $W_1 = 80$

$M_2 = 30$, $D_2 = ?$ $T_2 = 6$, $W_2 = 120$

$\therefore \quad 12 \times 20 \times 8 \times 120 = 30 \times D_2 \times 6 \times 80$

$\Rightarrow \quad D_2 = \frac{12 \times 20 \times 8 \times 120}{30 \times 6 \times 80} = 16$

Hence required number of days = 16 days.

Illustration – 8

8 men can do a work in 12 days while 20 women can do it in 10 days. In how many days can 12 men and 15 women complete the same work?

Solution : 8 men can do a work in 12 days while 20 women can do it in 10 days.

$M_1D_1E_1 = M_2D_2E_2$, where E_1 and E_2 are efficiency of a man and a woman respectively

Here $M_1 = 8$, $D_1 = 12$, $E_1 = ?$,

$M_2 = 20$, $D_2 = 10$, $E_2 = ?$

$\therefore \quad 8 \times 12 \times E_1 = 20 \times 10 \times E_2$

$\Rightarrow \quad \frac{E_1}{E_2} = \frac{25}{12}$ or $\frac{E_2}{E_1} = \frac{12}{25}$

Since 12 men and 15 women have to complete the same work as can be done by 8 men in 12 days (or 20 women in 10 days)

$\therefore (12 \times E_1 + 15 \times E_2) \times D_3 = 8 \times 12 \times E_1$

$\Rightarrow \frac{12 \times E_1 + 15 \times E_1}{E_1} \times D_3 = 8 \times 12$

$\Rightarrow \left(12 + 15 \times \frac{E_2}{E_1}\right) \times D_3 = 8 \times 12$

$\Rightarrow \left(12 + 15 \times \frac{12}{25}\right) \times D_3 = 8 \times 12 \Rightarrow D_3 = 5$

Hence 12 men and 15 women can complete the work in 5 days.

GP_3386

EXTENSION OF THE CONCEPT OF TIME AND WORK

1. Pipes and Cisterns

Problems related to Pipes and Cisterns are almost the same as those of Time and Work. Statement 'pipes A and B can fill a tank in 2 hours and 3 hours working individually' is similar to the statement 'A and B can do a work in 2 hours and 3 hours respectively working individually'.

If a pipe fills a tank in 3 hours, then the pipe fills $\frac{1}{3}rd$ of the same tank in 1 hour.

The only difference with the pipes and cisterns problems is that there are inlets as well as outlets. Inlet is a pipe connected with a tank (or a cistern or a reservoir) that fills it. Outlet is a pipe connected with a tank (or a cistern or a reservoir) that empties it.

Hence, if we consider filling the tank by inlet as positive work, then empting the tank by outlet will be considered as negative work.

(a) Let a pipe fill a tank in x hours and another pipe can empty the full tank in y hours. Then the net part of the tank filled in 1 hour, when both the pipes are opened, if x is less than y.

$$= \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$$

$\therefore$ time taken to fill the tank, when both the pipes are opened

$$= \frac{1}{\frac{y-x}{xy}} = \frac{xy}{y-x}.$$

(b) Let a pipe fill a tank in x hours while another fills the same tank in y hours but a third one empties the full tank in z hours. If all the three pipes are opened together, then the net part of the tank filled in 1 hour $= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$

$$= \frac{yz + zx - xy}{xyz}$$

$\therefore$ time taken to fill the tank $= \frac{xyz}{yz + zx - xy}$

(c) Let a pipe fill a tank in x hours but due to the leak in the bottom, the tank is filled in y hours and when the tank is filled, the time taken by the leak to empty the tank is z hours.

Net part of the tank filled in 1 hour by the pipe when there is the leak in the bottom

$$= \frac{1}{x} - \frac{1}{z} = \frac{z-x}{xz}$$

Since the tank will be filled completely in y hours by the pipe when there is the leak in the bottom, therefore

$$\left(\frac{z-x}{xz}\right) \times y = 1 \Rightarrow y = \frac{xz}{z-x} \Rightarrow yz - xy = xz$$

$$\Rightarrow z(y-x) = xy \Rightarrow z = \frac{xy}{y-x}$$

Hence, if a pipe can fill a tank in x hours but due to the leak in the bottom, the tank is filled in y hours, then the fully filled tank will be emptied in $\frac{xy}{y-x}$ hours.

(d) Let a pipe A fill a tank in x hrs while pipe B can fill the tank in y hrs alone. When both the pipes are opened together, then time required to fill the tank $= \frac{xy}{y+x}$ hrs.

(e) Let pipes A, B and C fill a tank alone in x, y and z hrs respectively. When all the three pipes open together, then time required to fill the tank $= \frac{xyz}{xy + yz + zx}$ hrs.

Illustration – 9

If a pipe fills a tank in 4 hrs and another pipe can empty the full tank in 6 hrs. When both the pipes are opened together, then find the time required to completely fill the tank.

Solution : Required time $= \frac{xy}{y-x}$

Here $x = 4$, $y = 6$

$\therefore$ Required time $= \frac{4 \times 6}{6-4} = 12$ hrs.

Illustration – 10

Pipe A can fill a tank in 6 hrs while pipe B alone can fill it in 5 hrs and pipe C can empty the full tank in 8 hrs. If all the pipes are opened together, how much time will be needed to completely fill the tank?

Solution : Required time

$$= \frac{xyz}{yz - zx - xy} = \frac{6 \times 5 \times 8}{5 \times 8 + 8 \times 6 - 6 \times 5}$$

$$= \frac{6 \times 5 \times 8}{58} = \frac{120}{29} = 4\frac{4}{29} \text{ days.}$$

Illustration – 11

A pipe can fill a tank in 10 hrs. Due to a leak in the bottom, it is filled in 15 hrs. If the tank is full, how much time will the leak take to empty it.

Solution : Required time $= \frac{xy}{y - x} = \frac{10 \times 15}{15 - 10} = 30$ hrs.

Illustration – 12

If three pipes *A*, *B* and *C* can fill the tank alone in 5, 6 and 8 hrs, then when all the three pipes are opened together, find the time to fill the tank completely.

Solution : Required time $= \frac{xyz}{xy + zx + xy}$

$$= \frac{6 \times 5 \times 8}{5 \times 8 + 6 \times 8 + 8 \times 5} = \frac{240}{30 + 48 + 40}$$

$$= \frac{240}{118} = 2\frac{2}{59} \text{hrs}$$

2 Alternate Work

In some problems two or more people of different efficiencies work alternatively or in some particular pattern. You can understand the method to solve these types of problems through the following illustration.

Illustration – 13

Sanjeev can build a wall in 20 days and Parveen can demolish the same wall in 30 days. If they work on alternate days with Sanjeev starting the job on the 1st day, then in how many days will the wall be built for the first time ?

Solution : Let us assume the total units of work = 60 units (i.e. LCM of 20 and 30)

So, the wall built by Sanjeev in one day = 3 units

And wall demolished by Parveen in one day = 2 units

So, effectively in two days, total wall built = 1 unit

Now, they work on alternate days, so days taken to built 57 units = 57 days

On 58th day Sanjeev will add another 3 units and so completing the construction of wall in 58 days.

(This problem can be understood well with another very traditional problem—A frog climbs up a pole 3 inches in 1 minute and slips 2 inches in next minute. If height of the pole is 120 inches, then how much time is taken by the frog to reach the top of the pole ?)

SHORTCUTS

Shortcut Approach - 1

If A and B can do a piece of work in *x* days, B and C in y days, C and A in z days, then (*A* + *B* + *C*) working together will do the same work in $\left[\frac{2xyz}{xy + yz + xz}\right]$ days.

Illustration – 14

A and B can finish a piece of work in 30 days, B and C in 40 days while C and A in 60 days. How long will they take to finish it together?

(a) $26\frac{2}{3}$ **days** **(b)** $16\frac{2}{3}$ **days**

(c) 25 days **(d) 24 days**

Shortcut Solution : (a)

$$\text{Required time} = \frac{2 \times 30 \times 40 \times 60}{30 \times 40 + 40 \times 60 + 60 \times 30}$$

$$= \frac{2 \times 30 \times 40 \times 60}{5400} = \frac{80}{3} = 26\frac{2}{3} \text{days}$$

Shortcut Approach - 1

A certain number of men can do a work in 'D' days. If there were '*x*' men less it could be finished in 'd' days more, then the number of men originally are $\left[\frac{x(D + d)}{d}\right]$.

Illustration – 15

A certain number of men can do a work in 30 days. if there were 6 men less it could be finished in 20 days more. How many men are there?

(a) 15 men (b) 12 men (c) 18 men (d) 20 men

GP_3386

Shortcut Solution : (a)

$$\text{Number of men} = \left[\frac{x(D+d)}{d}\right] = \left[\frac{6(30+20)}{20}\right] = \frac{300}{20} = 15$$

Shortcut Approach - 3

If A is 'n' times as fast (or slow) as B, and is therefore able to finish a work in 'D' days less (or more) than B, then the time in which they can do it working together is given by $\left(\frac{Dn}{n^2-1}\right)$ days.

Illustration – 16

A is thrice as fast as B, and is therefore able to finish a work in 40 dyas less than B. Find the time in which they can do it working together.

(a) 16 days (b) 10 days
(c) 15 days (d) None of these

Shortcut Solution : (c)

$$\text{Required time} = \left(\frac{Dn}{n^2-1}\right) = \left(\frac{40\times3}{3^2-1}\right) = \frac{40\times3}{9-1} = \frac{120}{8}$$

$= 15$ days

Shortcut Approach - 4

If x_1 men or y_1 women can reap a field in 'D' days, then x_2 men and y_2 women take to reap it $\left[\frac{D(x_1y_1)}{x_2y_1 + x_1y_2}\right]$ days.

Illustration – 17

If 6 men or 10 women can reap a field in 86 days, how long will 10 men and 12 women take to reap it?

(a) 30 days (b) 35 days (c) 25 days (d) 40 days

Shortcut Solution : (a)

Required number of days

$$= \left[\frac{D(x_1y_1)}{x_2y_1 + x_1y_2}\right] = \left[\frac{86(6\times10)}{10\times10 + 6\times12}\right] = \frac{86\times60}{100+72}$$

$$= \frac{86\times60}{172} = \frac{60}{2} = 30 \text{ days}$$

Shortcut Approach - 3

If a_1 men and b_1 boys can do a piece of work in x days and a_2 men and b_2 boys can do it in y days, then the following relationship is obtained.

$$1\ man = \left[\frac{yb_2 - xb_1}{xa_1 - ya_2}\right] \text{boys}$$

Illustration – 18

If 12 men and 16 boys can do a piece of work in 5 days and 13 men and 24 boys can do it in 4 days, compare the daily work done by a man with that done by a boy.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

Shortcut Solution : (b)

Applying the above theorem,

$$\frac{\text{a man's work}}{\text{a boy's work}} = \frac{24\times4 - 16\times5}{12\times5 - 4\times13} = \frac{16}{8} = 2:1$$

Shortcut Approach - 3

If A and B can do a work in x and y days respectively, they began the work together, but A left after some time and B finished the remaining work in z days; then the no. of days after which A left is given by $\left(\frac{xy}{x+y}\right)\left(\frac{y-z}{y}\right)$ days.

Illustration – 19

A and B can do a work in 20 and 15 days respectively. They began the work together, but A left after sometime and B finished the remaining work in 8 days. After how many days did A leave?

(a) 4 days (b) 5 days (c) 3 days (d) 6 days

Shortcut Solution : (a)

Required number of days

$$= \left(\frac{xy}{x+y}\right)\left(\frac{y-z}{y}\right) = \left(\frac{15\times20}{15+20}\right)\left(\frac{15-8}{15}\right)$$

$$= \frac{300}{35}\times\frac{7}{15} = \frac{140}{35} = 4 \text{ days}$$

Shortcut Approach - 4

A certain number of men can do a work in 'D' days. If there were 'x' men more it could be finished in 'd' days less, then the number of men originally are $\left[\frac{x(D-d)}{d}\right]$

Illustration – 20

A certain number of men can do a work in 75 days. If

there were 6 men more it could be finished in 15 days less. How many men are there?

(a) 20 men (b) 24 men (c) 28 men (d) 32 men

Shortcut Solution : (b)

Required number of men

$$=\left[\frac{x(D-d)}{x}\right]=\left[\frac{6(75-15)}{15}\right]=\frac{6\times 60}{15}=24 \text{ men}$$

Shortcut Approach - 3

A builder decided to build a farmhouse in 'D' days. He employed 'x' men in the beginning and 'y' more men after 'd' days and completed the construction in stipulated time. If he had not employed the additional men, then the men in the beginning would have finished it in $\left[\frac{D(x+y)-yd}{x}\right]$ days and it would have been $\left[\frac{y(D-d)}{x}\right]$ days behind the schedule.

Illustration – 21

A builder decided to build a farmhouse in 60 days. He employed 150 men in the begining and 130 more after 45 days and completed the construction in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

(a) 10 days (b) 23 days (c) 13 days (d) 15 days

Shortcut Solution : (c)

Required number of days

$$=\left[\frac{y(D-d)}{n}\right]=\left[\frac{130(60-45)}{150}\right]=\frac{130\times 15}{150}=13 \text{ days}$$

Shortcut Approach - 3

A team of x persons is supposed to do a work in 'D' days. After 'd_1' days, 'y' more persons were employed and the work was finished 'd_2' days earlier, then the number of days it would have been delayed if 'x' more persons were not employed is given by $\left[\frac{y\{D-(d_1+d_2)\}-d_2x}{x}\right]$ days and the number of days in which the work would have been finished is given by $\left[\frac{(x+y)(D-d_2)-d_1y}{x}\right]$ days

Illustration – 22

A team of 20 men is supposed to do a work in 30 days. Afte 12 days, 5 more men were employed and the work finished 2 days earlier. In how many days would it have been finished if 5 more men were not employed?

(a) 30 days (b) 28 days
(c) 32 days (d) 34 days

Shortcut Solution : (c)

Required number of days

$$=\left[\frac{(x+y)(D-d_2)-yd_1}{x}\right]=\left[\frac{(20+5)(30-2)-12\times 5}{20}\right]$$

$$=\frac{25\times 28-60}{20}=\frac{700-60}{20}=\frac{640}{20}=32 \text{ days}$$

Shortcut Approach - 10

A group of men decided to do a work in x days, but 'n' if them became absent. If the rest of the group did the work in 'y' days, then the original number of men is given by $\left(\frac{ny}{y-x}\right)$ men.

Illustration – 23

A group of men decided to do a work in 15 days, but 2 of them became absent. If the rest of the group did the work in 25 days, find the original number of men.

(a) 5 men (b) 4 men (c) 7 men (d) 6 men

Shortcut Solution : (a)

$$\text{Required number of men}=\frac{ny}{y-x}$$

$$=\frac{2\times 25}{25-15}=\frac{50}{10}=5 \text{ men}$$

Shortcut Approach - 10

A can do a work in x days and B can do the same work in y days. If they work together for 'd' days and A goes away, then the number of days in which B finishes the work is given by $y-\left(1+\frac{y}{x}\right)d$ days.

Illustration – 24

A can do a piece of work in $12\frac{1}{2}$ days and B in 10 days. They work together for $2\frac{1}{2}$ days and then A leaves B to finish the work alone. How long will B take to finish it ?

GP_3386

(a) $\frac{11}{3}$ days (b) $\frac{11}{5}$ days (c) 6 days (d) $\frac{11}{2}$ days

Shortcut Solution : (d)

Required number of days

$$= y - \left(1+\frac{y}{x}\right)d = 10 - \left(1+\frac{10}{\frac{25}{2}}\right)\frac{5}{2}$$

$$= 10 - \left(1+\frac{10\times 2}{25}\right)\frac{5}{2} = 10 - \frac{45}{25}\times\frac{5}{2} = \frac{11}{2} \text{ days}$$

Shortcut Approach - 12

A and B can do a piece of work in x and y days respectively and both of them starts the work together. If A leaves the work 'a' days before the completion of the work, then the total time in which the whole work is completed $= \frac{(x+a)y}{(x+y)}$ days.

A and B can do a piece of work in x and y days respectively and both of them starts the work together. If B leaves the work 'a' days before the completion of work, then the total time, in which the whole work is completed $= \left[\frac{(y+a)x}{x+y}\right]$ days

Illustration – 25

A can do a piece of work in 15 days and B in 25 days. They begin together. But 5 days before the completion of the work, A leaves off. In how many days is the work completed?

(a) $12\frac{1}{2}$ days (b) $13\frac{1}{2}$ days

(c) $11\frac{3}{4}$ days (d) 25 days

Shortcut Solution : (a)

Required number of days $= \left[\frac{(x+a)y}{(x+y)}\right]$

$$\left[\frac{(15+5)25}{15+25}\right] = \frac{20\times 25}{40} = \frac{25}{2} = 12\frac{1}{2} \text{ days}$$

Illustration – 26

A and B can do a piece of work in 16 days and 24 days. Both starts the work together for sometime, but B leaves the job 6 days before the work is completed. Find the time in which work is finished.

(a) 18 days (b) 14 days

(c) 12 days (d) 8 days

Shortcut Solution : (c)

Required number of days

$$= \left[\frac{(y+a)x}{x+y}\right] = \left[\frac{(24+6)16}{16+24}\right] = \frac{30\times 16}{40} = 12 \text{ days}$$

Shortcut Approach - 10

If A working alone takes 'x' days more than A and B, and B working alone takes 'y' days more than A and B together then the number of days taken by A and B working together is given by $\left[\sqrt{xy}\right]$ days.

Illustration – 27

A alone would take 16 hours more to complete the job than if both A and B would together. If B worked alone, he took 4 hours more to complete the job than A and B worked toether. What time, would they take if both A and B worked together?

(a) 5 hours (b) 8 hours

(c) 9 hours (d) None of these

Shortcut Solution : (b)

Required number of hours

$$= \left[\sqrt{xy}\right] = \left[\sqrt{16\times 4}\right] = \sqrt{64} = 8 \text{ hours}$$

Shortcut Approach - 12

There is a sufficient food for 'M' men for 'D' days. If after 'd' days 'm' men leave the place, then the rest of the food will last for the rest of the men for $\left[\frac{D-d}{M-m}\times M\right]$ days.

There is a sufficient food for 'M' persons for 'D' days. If after 'd' days 'm' more men join them the food will last in $\frac{D-d}{M+m}\times M$ days.

Illustration – 28

There is a suffficient food for 300 men for 32 days. After 29 days, 210 men leave the place. For how many days will the rest of the food last for the rest of the men?

(a) 12 days (b) 14 days (c) 15 days (d) 10 days

Shortcut Solution : (d)

Required number of days

$$\left[\frac{D-d}{M-m}\times M\right] = \left[\frac{32-29}{300-210}\times 300\right]$$

$= \frac{3}{90} \times 300 = \frac{900}{90} = 10$ days

Shortcut Approach - 10

A can do a work in x days and B can do the same work in y days. If the contract for the work is ₹ X, and both of them work together, then the share of A and B is given by ₹ $\left(\frac{X}{x+y} \times y\right)$ and $\left(\frac{X}{x+y} \times x\right)$ respectively.

Illustration – 29

A can do a work in 10 days while B can do it in 12 days. Both of them work together to do the work. If the total amount paid for the work is ₹ 440, how much is A's share in it?

(a) ₹ 200 (b) ₹ 250 (c) ₹ 240 (d) ₹ 260

Shortcut Solution : (c)

Share of A

$= \left(\frac{X}{x+y} \times y\right) = \left(\frac{440}{10+12} \times 12\right) = \frac{440}{22} \times 12 = 240$

Shortcut Approach - 10

A person A can do a work in x days. With the help of another person B, he can do the same work in y days. If they get ₹ X for that work, then the share of A and B is given by ₹ $\left[\frac{Xy}{x}\right]$ and ₹ $\left[\frac{X(x-y)}{x}\right]$ repsectively.

Illustration – 30

Sohan can do a work in 25 days. Sohan and Mohan together do the same work in 20 days. If they are paid ₹ 625 for the work, how much money should Mohan get?

(a) ₹ 400 (b) ₹ 200
(c) ₹ 100 (d) ₹ 125

Shortcut Solution : (d)

Share of Mohan

$= \left[\frac{X(x-y)}{x}\right] = \left[\frac{625(25-20)}{25}\right] = \frac{625 \times 5}{25} = 125$

Shortcut Approach - 3

If a pipe can fill a tank in x hours and another can fill the same tank in y hours, then time taken to fill the tank = $\left[\frac{xy}{x+y}\right]$ hours.

Or

A tap 'A' can empty a cistern in x hours and the other tap 'B' can empty is in y hours. If both emptying taps are opened together, then the time taken to empty the full cistern is given by $\left[\frac{xy}{x+y}\right]$ hrs.

Illustration – 31

A pipe can empty a tank in 15 hrs and another pipe can empty it in 10 hours. If both the pipes are opened simultaneously, find the time in which a full tank is emptied.

(a) 8 hrs (b) 6 hrs (c) 4 hrs (d) 5 hrs

Shortcut Solution : (b)

Required time

$= \left[\frac{xy}{x+y}\right] = \left[\frac{15 \times 10}{15+10}\right] = \frac{150}{25} = 6$ hours

Illustration – 32

Two pipes A and B can fill a tank in 30 minutes and 15 minutes respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

(a) 10 minutes (b) 12 minutes
(c) 8 minutes (d) 9 minutes

Shortcut Solution : (c)

Required time

$= \left(\frac{xy}{x+y}\right) = \left(\frac{30 \times 15}{30+15}\right) = \frac{450}{45} = 10$ minutes

Shortcut Approach - 10

A, B and C are three pipes connected to a tank. A and B together fill the tank in x hours. B and C together fill the tank in y hours. A and C together fill the tank in z hours.

(i) Time taken by A to fill the tank = $\left(\frac{2xyz}{xy+yz-xz}\right)$ hrs,

(ii) Time taken by B to fill the tank = $\left(\frac{2xyz}{yz+xz-xy}\right)$ hrs,

(iii) Time taken by C to fill the tank = $\left(\frac{2xyz}{xz+xy-yz}\right)$ hrs

Illustration – 33

Three pipes A, B and C are connected to a tank. A and B together can fill the tank in 12 hrs, B and C together in 20 hrs and C and A together in 15 hrs. In

GP_3386

how much time will each pipe fill the tank separately?
(a) 10 hrs, 15 hrs, 30 hrs
(b) 20 hrs, 15 hrs, 60 hrs
(c) 20 hrs, 30 hrs, 60 hrs
(d) 20 hrs, 30 hrs, 45 hrs

Shortcut Solution : (c)

Time taken by A $= \left(\frac{2xyz}{xy+yz-xz}\right)$

$= \left(\frac{2\times12\times20\times15}{12\times20+20\times15-12\times15}\right) = \frac{7200}{360} = 20$ hrs

Time taken by B

$= \left(\frac{2xyz}{yz+xz-xy}\right) = \left(\frac{2\times12\times20\times15}{20\times15+15\times12-12\times20}\right) = \frac{7200}{240}$

$= 30$ hrs

Time taken by C

$= \left(\frac{2xyz}{xz+xy-yz}\right) = \left(\frac{2\times12\times20\times15}{12\times15+12\times20-15\times20}\right) = \frac{7200}{120} = 60$ hrs

Shortcut Approach - 10

One filling pipe A is *n* times faster than the other filling pipe B. If B can fill a cistern in *x* hours, then the time in which the cistern will be full, if both the filling pipes are opened together, is $\left(\frac{x}{n+1}\right)$ hours.

Note: Value of the slower filling pipe is given.

Illustration – 34

One filling pipe A is 5 times faster than second filling pipe B. If B can fill a cistern in 36 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

(a) 6 minutes (b) 8 minutes
(c) 4 minutes (d) 12 minutes

Shortcut Solution : (a)

Required time $= \frac{x}{n+1} = \frac{36}{5+1} = 6$ minutes

Shortcut Approach - 11

One filling pipe A is *n* times faster than the other filling pipe B. If A can fill a cistern in *x* hours, then the time in which the cistern will be full, if both the filling pipes are opened together, is $\left(\frac{n}{n+1}\right)x$ hours.

Note: Value of the faster filling pipe is given.

Illustration – 35

One filling pipe A is 6 times faster than second filling pipe B. If A can fill a cistern in 28 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

(a) 20 minutes (b) 24 minutes
(c) 18 minutes (d) 12 minutes

Shortcut Solution : (b)

Required time $= \left(\frac{n}{n+1}\right)x$

$= \left(\frac{6}{6+1}\right)28 = \frac{6}{7}\times28 = 24$ minutes

Shortcut Approach - 12

If one filling pipe A is *n* times faster and takes *x* minutes less time than the other *filling* pipe B, then the time, they will take to fill a cistern, if both the pipes are opened together, is $\left[\frac{nx}{(n^2-1)}\right]$ minutes.

Illustration – 36

One fill pipe A is 5 times faster than second fill pipe B and takes 48 minutes less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

(a) 12 min (b) 8 min (c) 10 min (d) 15 min

Shortcut Solution : (c)

Required time $= \left[\frac{nx}{n^2-1}\right] = \left[\frac{5\times48}{5^2-1}\right] = \frac{240}{24} = 10$ minutes

Illustration – 37

One fill pipe A is 4 times slower than the second fill pipe B and takes 30 minutes more time than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

(a) 15 min (b) 5 min (c) 10 min (d) 20 min

Shortcut Solution : (c)

Required time $= \frac{4\times30}{(4^2-1)} = 8$ minutes

A will fill the cistern in $\frac{4\times30}{4-1} = 40$ minutes and B will fill the cistern in $\frac{30}{4-1} = 10$ minutes.

Shortcut Approach - 10

If a pipe A fills a cistern in *x* hours and suddenly a leak develops through which every hour *n* part of the water filled by the pipe A leaks out, then the time in

which tank is full $= \left(\frac{x}{1-n}\right)$ hours.

Illustration – 38

A pipe A can fill a tank in 27 minutes. Due to development of a whole in the bottom of the tank every hour $\frac{1}{10}$ of the water filled by the pipe A leaks out. Find the time when the tank will be full.

(a) 32 min (b) 34 min (c) 36 min (d) 30 min

Shortcut Solution : (d)

Required time

$$= \left(\frac{x}{1-n}\right) = \frac{27}{1-\frac{1}{10}} = \frac{27}{\frac{10-1}{10}} = \frac{27\times 10}{9} = 30 \text{ minutes.}$$

Shortcut Approach - 10

A cistern is normally filled in x hrs but takes t hrs longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in $\left[\frac{x\times(x+t)}{t}\right]$ hrs.

Illustration – 39

A cistern is normally filled in 6 hrs but takes 3 hrs longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in _____ hr.

(a) 24 hrs (b) 18 hrs (c) 30 hrs (d) 21 hrs

Shortcut Solution : (b)

Required time $= \left[\frac{x\times(x+t)}{t}\right]$

$$= \left[\frac{6(6+3)}{3}\right] = \frac{6\times 9}{3} = 18 \text{ hours}$$

Shortcut Approach - 12

If a pipe fills a tank in x hours and another fills the same tank in y hours, but a third one empties the full tank in z hours when all of them are opened together, then the time taken to fill the tank $= \left[\frac{xyz}{yz+xz-xy}\right]$ hours.

Illustration – 40

Tap A can fill a water tank in 25 minutes, tap B can fill the same tank in 40 minutes and tap C can empty the tank in 30 minutes. If all the three taps are opened together, in how many minutes will the tank be completely filled up or emptied?

(a) $3\frac{2}{13}$ (b) $15\frac{5}{13}$ (c) $8\frac{2}{13}$ (d) $31\frac{11}{19}$

Shortcut Solution : (d)

Required time $= \left[\frac{xyz}{yz+xz-xy}\right]$

$$= \left[\frac{25\times 40\times 30}{40\times 30+25\times 30-25\times 40}\right] = \frac{25\times 40\times 30}{950}$$

$$= \frac{600}{19} = 31\frac{11}{19} \text{ minutes}$$

Shortcut Approach - 10

A pipe can fill a tank in x hours. Due to a leak in the bottom it is filled in y hours. If the tank is full, the time taken by leak to empty the tank $= \left(\frac{xy}{y-x}\right)$ hrs.

Illustration – 41

A cistern is normally filled in 8 hrs but takes 2 hrs longer to fill because of a leak in its bottom. If the cistern is full, the leak will empty it in :

(a) 16 hrs (b) 40 hrs (c) 25 hrs (d) 20 hrs

Shortcut Solution : (b)

Required time $= \left(\frac{xy}{y-x}\right) = \frac{8\times 10}{2} = 40 \text{ hrs}$

Shortcut Approach - 12

A cistern has a leak which would empty it in x hours. If a tap is turned on which admits water at the rate of w litres per hour into the cistern, and the cistern is now emptied in y hours, then the capacity of the cistern is $w\times\left(\frac{xy}{y-x}\right)$ litres.

Illustration – 42

A cistern has a leak which would empty it in 10 hours. A tap is turned on which admits 2 litres per hr into the cistern, and it is now emptied in 15 hours. How many litres does the cistern hold?

(a) 50 litres (b) 60 litres
(c) 45 litres (d) 360 litres

Shortcut Solution : (b)

Here w = 2 litres per hour

$\therefore$ required answer $= \frac{15\times 10}{15-10}\times 2 = 60$ litres.

GP_3386

LEVEL - 1

1. A and B together can do a job in 12 days. B alone can finish it in 28 days. In how many days can A alone finish the work?
(a) 21 days (b) 19 days
(c) 20 days (d) None of these

2. A is 30% more efficient than B. How much time will they take working together, to complete a job which A alone could have done in 23 days?
(a) 11 days (b) 13 days
(c) $20\frac{3}{17}$ days (d) None of these

3. A and B can do a piece of work in 12 days, B and C in 8 days, C and A in 6 days. How long would B take to do the same work alone? **[SSC CGL 2005]**
(a) 24 days (b) 32 days
(c) 40 days (d) 48 days

4. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day?
(a) $\frac{1}{6}$ (b) $\frac{1}{9}$
(c) $\frac{2}{5}$ (d) $\frac{2}{7}$

5. If 6 BSF or 10 CRPF companies can demolish a terrorist outfit in Kashmir in 2 days, find how long will 4 BSF and 9 CRPF companies take to do the same ?
(a) 1.27 days (b) 2.27 days
(c) 3.27 days (d) 4.27 days

6. If 18 binders bind 900 books in 10 days, how many binders will be required to bind 660 books in 12 days ?
(a) 14 (b) 13 (c) 22 (d) 11

7. X and Y can do a piece of work in 72 days. Y and Z can do it in 120 days. X and Z can do it in 90 days. In how many days all the three together can do the work ?
(a) 100 days (b) 150 days
(c) 60 days (d) 80 days

8. Two men A and B started a job in which A was thrice as good as B and therefore took 60 days less than B to finish the job. How many days will they take to finish the job if they start working together? **[SSC CGL 1999, 2012]**
(a) 15 days (b) 20 days
(c) $22\frac{1}{2}$ days (d) 25 days

9. A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there ?
(a) 75 men (b) 40 men (c) 48 men (d) 45 men

10. A certain number of men can do a work in 60 days. If there were 8 men more it could be finished in 10 days less. How many men are there? **[CDS 2016]**
(a) 40 (b) 45 (c) 50 (d) 55

11. A can do a piece of work in 25 days and B in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the remaining work ?
(a) 17 days (b) 11 days
(c) 10 days (d) 15 days

12. A certain number of women can do a work in 12 days. If there were 4 women less it could be finished in 16 days more. How many women are there? **[SSC CHSL 2016]**
(a) 5 women (b) 6 women
(c) 7 women (d) None of these

13. 12 men complete a work in 18 days. Six days after they had started working, 4 men joined them. How many days will all of them take to complete the remaining work ?
(a) 10 days (b) 12 days
(c) 15 days (d) 9 days

14. A work could be completed in 100 days by some workers. However, due to the absence of 10 workers, it was completed in 110 days. The original number of worker was: **[SSC CGL 2005]**
(a) 100 (b) 110 (c) 55 (d) 50

15. 12 men and 16 boys can do a piece of work in 5 days, 13 men and 24 boys can do it in 4 days. Then the ratio of daily work done by a man to that of a boy is
(a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 5 : 4

16. If m man can do a work in r days, then the number of days taken by $(m + n)$ men to do it is:
(a) $\frac{m+n}{mn}$ (b) $\frac{m+n}{mr}$
(c) $\frac{mr}{(m+n)}$ (d) $\frac{(m+n)r}{mn}$

17. In a fort there was sufficient food for 200 soldiers for 31 days. After 27 days 120 soldiers left the fort. For how many extra days will the rest of the food last for the remaining soldiers?
(a) 12 days (b) 10 days
(c) 8 days (d) 6 days

18. There is a sufficient rice for 280 men for 20 days. After 18 days 200 men leave the place. For how many days will the rest of the rice last for the rest of the men? **[SSC CGL 2010]**
(a) 6 days (b) 7 days
(c) 9 days (d) 11 days

19. Sambhu can do 1/2 of the work in 8 days while kalu can do 1/3 of the work in 6 days. How long will it take for both of them to finish the work?
(a) $\frac{88}{17}$ days (b) $\frac{144}{17}$ days
(c) $\frac{72}{17}$ days (d) 8 days

20. A can do a piece of work in 20 days and B can do the same work in 30 days. They start work together but A left after 10 days. How many days required to B for remaining work? **[CDS 2013]**
(a) 10 days (b) 2.5 days
(c) 15 days (d) 5 days

21. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work?
(a) 5 days (b) 6 days
(c) 8 days (d) 9 days

22. A man can do a piece of work in 10 days but with the assistance of his son, the work is done in 8 days. In how many days, his son alone can do the same piece of work?
(a) 15 days (b) 22 days
(c) 30 days (d) 40 days

23. X and Y can do job in 25 days and 30 days respectively. They work together for 5 days and then X leaves. Y will finish the rest of the work in how many days?
(a) 18 days (b) 19 days
(c) 20 days (d) 21 days

24. A can do 50% more work as B can do in the same time. B alone can do a piece of work in 20 hours. A, with help of B, can finish the same work in how many hours ?
(a) 12 (b) 8 (c) $13\frac{1}{3}$ (d) $5\frac{1}{2}$

25. If 15 women or 10 men can complete a project in 55 days, in how many days will 5 women and 4 men working together complete the same project ?
(a) 75 (b) 8 (c) 9 (d) 85

26. 24 men working 8 hours a day can finish a work in 10 days. Working at the rate of 10 hours a day, the number of men required to finish the same work in 6 days is :
(a) 30 (b) 32 (c) 34 (d) 36

27. 12 men or 15 women can reap a field in 14 days. Find the number of days that 7 men and 5 women will take to reap it. **[SSC CHSL 2015]**
(a) $12\frac{3}{11}$ days (b) $13\frac{3}{11}$ days
(c) $14\frac{3}{11}$ days (d) $15\frac{3}{11}$ days

28. A sum of ₹ 25 was paid for a work which A can do in 32 days, B in 20 days, B and C in 12 days and D in 24 days. How much did C receive if all the four work together ?
(a) ₹ $\frac{14}{3}$ (b) ₹ $\frac{16}{3}$
(c) ₹ $\frac{15}{3}$ (d) ₹ $\frac{17}{3}$

29. X can do a piece of work in 15 days. If he is joined by Y who is 50% more efficient, in what time will X and Y together finish the work?
(a) 10 days (b) 6 days
(c) 18 days (d) Data insufficient

30. A can do a certain job in 12 days. B is 60% more efficient than A. How many days B alone take to do the same job?
(a) $7\frac{1}{2}$ (b) 11 (c) $8\frac{1}{2}$ (d) 8

31. *x* is 3 times as faster as y and is able to complete the work in 40 days less than *y*. Then the time in which they can complete the work together?
(a) 15 days (b) 10 days
(c) $7\frac{1}{2}$ days (d) 5 days

32. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in :
(a) $1\frac{13}{17}$ hours (b) $2\frac{8}{11}$ hours
(c) $3\frac{9}{17}$ hours (d) $4\frac{1}{2}$ hours

GP_3386

33. Two pipes A and B can fill a tank in 18 h and 12 h, repectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank: **[SSC CGL Tier II 2016]**

(a) $7\frac{1}{5}$ hr (b) $6\frac{1}{5}$ hr (c) $7\frac{2}{5}$ hr (d) $6\frac{2}{5}$ hr

34. One fill pipe A is 3 times faster than second fill pipe B and takes 10 minutes less time to fill a cistern than B takes. Find when the cistern will be full if fill pipe B is only opened.

(a) 20min (b) 18min (c) 15min (d) 10min

35. Two fill pipes A and B can fill a cistern in 12 and 16 minutes respectively. Both fill pipes are opened together, but 4 minutes before the cistern is full, one pipe A is closed. How much time will the cistern take to fill ?

(a) $9\frac{1}{7}$ min. (b) $3\frac{1}{3}$ min.

(c) 5min. (d) 3min.

36. Three fill pipes A, B and C can fill separately a cistern in 3, 4 and 6 minutes respectively. A was opened first. After 1 minute, B was opened and after 2 minutes from the start of A, C was also opened. Find the time when the cistern will be full ?

(a) $2\frac{1}{9}$min (b) $4\frac{1}{2}$ min

(c) $3\frac{3}{4}$ min (d) None of these

37. A hot pipe takes 3 minutes longer to fill a tank than the cold pipe. Together they take 6 minutes 40 seconds. Time taken by the cold pipe alone to fill the tank is :

(a) 6 minutes (b) 18 minutes
(c) 9 minutes (d) 12 minutes

38. Sagar is 20% more efficient than Pranesh. If Pranesh can complete a piece of work in 30 days, then in how many days can both Sagar and Pranesh complete the work?

(a) (11 + 5/11) days (b) (13 + 7/11) days
(c) (15 + 5/11) days (d) (17 + 7/11) days

39. Chandni and Divakar can do a piece of work in 9 days and 12 days respectively. If they work for a day alternatively, Chandni beginning, in how many days, the work will be completed?

(a) $10\frac{1}{4}$ (b) $9\frac{1}{5}$ (c) 11.11 (d) 10

40. Raja can do a piece of work in 14 days, while Rani can do the same work in 21 days. They started the work together but 3 days before the completion of the work, Raja left the work. The total number of days to complete the work is :

(a) 7 (b) 8.5

(c) 5 (d) $10\frac{1}{5}$

41. A and B can separately complete a piece of work in 20 days and 30 days respectively. They worked together for sometime, then B left the work. If A completed the rest of the work in 10 days, then B worked for: **[SSC CGL 2008]**

(a) 6 days (b) 8 days (c) 12 days(d) 16 days

42. A and B can complete a piece of work in 30 days and 40 days respectively. Both of them start the work together but A left the work 10 days before the completion of work then what is total time? **[SSC CGL 2011]**

(a) $21\frac{4}{5}$days (b) $22\frac{1}{7}$days

(c) $22\frac{6}{7}$days (d) 25 days

43. A can finish a piece of work in 12 days while B can do it in 15 days. If both work at it together, what time will they take to do the work?

(a) 6 days (b) 8 days

(c) $6\frac{2}{3}$ days (d) $10\frac{1}{5}$ days

44. Pipe A can fill the tank in 35 minutes. Due to some leakage 40% water is leakes out. What is time required to fill the tank? **[SSC CHSL 2012]**

(a) 40 minutes (b) 30 minutes
(c) 60 minutes (d) 45 minutes

45. A pipe can fill a tank in 15 hrs. Due to a leak it takes 5 hrs more to fill it. If tank is full the leak will empty it in: **[SSC CHSL 2013]**

(a) 20 hrs (b) 40 hrs
(c) 45 hrs (d) 60 hrs

46. 6 children and 2 men complete a certain piece of work in 6 days. Each child takes twice the time taken by a man to finish the work. In how many days will 5 men finish the same work?

(a) 6 (b) 8
(c) 9 (d) 15

47. Mr. Modi can copy 40 pages in 10 minutes, Mr Xerox and Mr. Modi both working together can copy 250 pages in 25 minutes. In how many minutes Mr. Xerox can copy 36 pages?

(a) 5 minutes (b) 6 minutes
(c) 3 minutes (d) 12 minutes

48. There was a leakage in the container of the refined oil. If 11 kg oil is leakage out per day

then it would have lasted for 50 days, if the leakage was 15 kg per day, then it would have lasted for only 45 days. For how many days would the oil have lasted, if there was no leakage and it was completely used for eating purpose?
(a) 80 days (b) 72 days
(c) 100 days (d) 120 days

49. A contractor undertook to complete the work in 40 days and he deployed 20 men for his work. 8 days before the scheduled time he realised that $\frac{1}{3}$rd of the work was still to be done. How many more men were required to complete the work in stipulated time?
(a) 16 b) 15 (c) 20 (d) 25

50. Tap A can fill the empty tank in 12 hours, but due to a leak in the bottom it is filled in 15 hours. If the tank is full and then tap A is closed then in how many hours the leak can empty it?
(a) 45 hours (b) 48 hours
(c) 52 hours (d) 60 hours

51. A pipe of diameter d can empty a tank in 40 min, then how much time a pipe of diameter 2d will take to empty it?
(a) 5 min (b) 10 min
(c) 20 min (d) 80 min

52. 5 men can compelete any work in 6 days while 10 women can complete it in 5 days. In how many days will 5 women and 3 men complete this work?
(a) 4 (b) 5 (c) 6 (d) 8

53. If 20 persons complete one–third of a work in 20 days then how many more persons will be required to complete the rest work in 25 days?
(a) 6 (b) 12 (c) 18 (d) 24

54. A tank has three pipes. The first pipe can fill $\frac{1}{2}$ part of the tank in 1 hour and the second pipe can fill $\frac{1}{3}$ part in 1 hour. The third pipe is for making the tank empty. When all the three pipes are open, $\frac{7}{12}$th part of the tank is filled in 1 hour. How much time will the third pipe take to empty the completely filled tank?
(a) 3 hours (b) 4 hours (c) 5 hours (d) 6 hours

55. A pipe can fill the tank in 20 hrs. Due to a leak it takes 25 hrs to fill the tank then what is time required for leak to empty the full tank? **[RBI 2014]**
(a) 100 hrs (b) 80 hrs (c) 75 hrs (d) 50 hrs

56. A outlet can empty the tank in 20 hrs. A inlet pumps 5 litres water in 1 hr and it is now emptied in 30 hrs then what is capacity of tank? **[CDS 2016]**
(a) 600 litres (b) 200 litres
(c) 300 litres (d) 500 litres

57. A can complete a work in 24 days. B is 60% more efficient than A. How many days will B take to complete that work?
(a) 12 (b) 15 (c) 16 (d) 18

58. George takes 8 hours to copy a 50 page manuscript while Sonia can copy the same manuscript in 6 hours. How many hours would it take them to copy a 100 page manuscript, if they work together?
(a) $6\frac{6}{7}$ (b) 9 (c) $9\frac{5}{7}$ (d) 14

59. A man is twice as fast as a woman. Together the man and the woman do the piece of work in 8 days. In how many days each will do the work if engaged alone?
(a) man-14 days, woman-28 days
(b) man-12 days, woman-24 days
(c) man-10 days, woman-20 days
(d) None of these

60. Two pipes A and B can fill a cistern in 10 and 15 minutes respectively. Both fill pipes are opened together, but at the end of 3 minutes, 'B' is turned off. How much time will the cistern take to fill ?
(a) 6 min (b) 8 min (c) 10 min (d) 12 min

61. Two inlet pipes A and B and outlet pipe C is connected with a tank. A and B can fill the tank in 30 hrs and 40 hrs and C can empty it in 20 hrs. If all pipes are opened together then what is time required to fill the tank? **[CDS 2015]**
(a) 100 hrs (b) 120 hrs (c) 60 hrs (d) 80 hrs

62. Pratibha is thrice as efficient as Sonia and is therefore able to finish a piece of work in 60 days less than Sonia. Pratibha and Sonia can individually complete the work respectively in
(a) 30, 60 days (b) 60, 90 days
(c) 30, 90 days (d) 40, 120 days

63. 40 men can finish a piece of work in 60 days. After some days, 10 men leave the work so that the work is finished in 70 days. The number of days after which 10 men left the work is
(a) 20 days (b) 25 days
(c) 30 days (d) 40 days

GP_3386

LEVEL - 2

1. After working for 8 days, Anil finds that only $\frac{1}{3}$ of the work has been done. He employs Rakesh who is 60% efficient as Anil. How many more days will Anil take to complete the job?
(a) 15 days (b) 12 days
(c) 10 days (d) 8 days

2. A group of some persons decided to complete a piece of work is 20 days, but 4 of them became absent rest of the group takes 25% extra time to complete the work then what is original number of men? **[SSC CHSL 2014]**
(a) 15 men (b) 20 men (c) 25 men (d) 10 men

3. A takes 18 days more than A and B together to complete a piece of work and B takes 50 days more than A and B together. What is time required to complete the work for A and B together? **[SSC CGL Tier II 2014]**
(a) 30 days (b) 45 days
(c) 15 days (d) 50 days

4. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy to the work ?
(a) $12\frac{1}{2}$ days (b) $11\frac{1}{2}$ days
(c) $15\frac{1}{2}$ days (d) $13\frac{1}{2}$ days

5. Two pipes can fill a cistern in 6 minutes and 7 minutes respectively. Both the pipes are opened alternatively for 1 minute each. In what time will they fill the cistern.
(a) 6 minutes (b) $6\frac{2}{3}$ minutes
(c) $6\frac{3}{7}$ minutes (d) $3\frac{1}{2}$ minutes

6. Three pipes are connected with a tank. A and B can fill the tank in 40 hrs and B and C in 50 hrs. If A and C can fill the tank in 30 hrs then find the time required for each pipe? **[CDS 2015]**
(a) $\frac{400}{11}$ hrs, $\frac{1200}{7}$ hrs, $\frac{1200}{17}$ hrs
(b) $\frac{500}{3}$ hrs, $\frac{200}{3}$ hrs, $\frac{700}{17}$ hrs
(c) 60 hrs, 50 hrs, 80 hrs
(d) 90.5 hrs, 83.3 hrs, 62.5 hrs

7. Two pipes *A* and *B* when working alone can fill a tank in 36 min. and 45 min. respectively. *A* waste pipe *C* can empty the tank in 30 min. First *A* and *B* are opened. After 7 min., *C* is also opened. In how much time will the tank be full ?
(a) 46min (b) 60min (c) 39min (d) 43min

8. Two inlet pipes A and B are connected with a tank. A is 4 times faster than B. If B can fill the tank in 20 hrs then find the time when tank will be full if both A and B are opened together?
(a) 4 hrs (b) 5 hrs (c) 16 hrs (d) $6\frac{2}{3}$ hrs

9. Two inlet pipes A and B are connected with a tank. A is 3 times faster than B. If A can fill the tank in 24 minutes, then what is time required for A and B together to fill the tank?
(a) 15 minutes (b) 20 minutes
(c) 18 minutes (d) 12 minutes

10. Two inlets pipes A and B are connected with a tank. A is 6 times faster than B. A takes 35 minutes less than B to fill the tank then what is time required for A and B together to fill the tank?
(a) 35 minutes (b) 6 minutes
(c) 10 minutes (d) 7.5 minutes

11. *A* and *B* can do a job in 16 days and 12 days respectively. *B* has started the work alone 4 days before finishing the job, *A* joins *B*. How many days has B worked alone?
(a) 6 days (b) 4 days
(c) 5 days (d) 7 days

12. Three diggers dug a ditch of 324 m deep in six days working simultaneously. During one shift, the third digger digs as many metres more than the second as the second digs more than the first. The third digger's work in 10 days is equal to the first digger's work in 14 days. How many metres does the first digger dig per shift?
(a) 15m (b) 18m (c) 21m (d) 27m

13. A tyre has two punctures. The first puncture along would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how

long does it take both the punctures together to make it flat?

(a) $1\frac{1}{2}$ minutes (b) $3\frac{1}{2}$ minutes

(c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes

14. A builder decided to build a farmhouse in 40 days. He employed 100 men in the beginning and 100 more after 35 days and completed the construction in stipulated time. If he had not employed the additional men, how many days behind schedule would it have been finished?

[SSC CGL Tier I 2017]

(a) 2 days (b) 4 days (c) 5 days (d) 6 days

15. A contractor employed 30 men to do a work in 38 days. After 25 days, he employed 5 men more and the work was finished 1 day earlier. How many days he would have behind, if he had not employed additional men?

[SSC CGL Tier II 2016]

(a) 1 (b) 1.5 (c) 2 (d) 2.5

16. A contractor undertakes to built a walls in 50 days. He employs 50 peoples for the same. However after 25 days he finds that only 40% of the work is complete. How many more man need to be employed to complete the work in time?

(a) 25 (b) 30 (c) 35 (d) 20

17. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in 12 days. If all the 10 men and 15 women work together, in how many days will the work get completed?

(a) 6 (b) $6\frac{1}{3}$ (c) $6\frac{2}{3}$ (d) $7\frac{2}{3}$

18. Rakesh can complete a piece of work in 40 days. He can complete same piece of work with help of Ranjan in 30 days. If they are paid ₹ 1200 for the work then what is share of Ranjan

(a) 900 (b) 300 (c) 600 (d) 400

19. A and B can do a piece of work in 20 days and 45 days respectively. If the total of ₹ 910 is paid for the work then what is the share of B?

(a) 420 (b) 390 (c) 280 (d) 260

20. A alone would take 8 days more to complete the job than if both A and B would together. If B worked alone, he took $4\frac{1}{2}$ days more to complete the job than A and B worked together. What time would they take if both A and B worked together?

(a) 7 days (b) 5 days (c) 4 days (d) 6 days

21. A contractor undertook to do a piece of work in 9 days. He employed certain number of labourers but 6 of them were absent from the very first day and the rest could finish the work in only 15 days. Find the number of men originally employed .

(a) 15 (b) 6 (c) 13 (d) 9

22. A can do a job in 3 days less time than B. A works at it alone for 4 days and then B takes over and completes it. If altogether 14 days were required to finish the job, then in how many days would each of them take alone to finish it?

(a) 17 days, 20 days (b) 12 days, 15 days

(c) 13 days, 16 days (d) None of these

23. Two workers A and B working together completed a job in 5 days. If A worked twice as efficiently as he actually did and B worked $\frac{1}{3}$ as efficiently as he actually did, the work would have completed in 3 days. Find the time for A to complete the job alone.

(a) $6\frac{1}{4}$ days (b) $5\frac{3}{4}$ days

(c) 5 days (d) None of these

24. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be:

(a) 4 days (b) 5 days (c) 6 days (d) 7 days

25. If 5 men and 8 boys can complete a piece of work in 8 days and 3 men and 12 boys can complete same work in 10 days then what is respective ratio of the work efficiency of a man to that of a boy? **[SSC CGL Tier I 2017]**

(a) 5 : 24 (b) 5 : 28 (c) 28 : 5 (d) 14 : 5

26. A and B can finish a work in 10 days while B and C can do it in 18 days. A started the job, worked for 5 days, then B worked for 10 days and the remaining work was finished by C in 15 days. In how many days could C alone have finished the whole work ?

(a) 30 days (b) 15 days

(c) 45 days (d) 24 days

27. One hundred men in 10 days do onethird of a piece of work. The work is then required to be completed in another 23 days. On the next day (the eleventh day) 50 more men are employed, and on the day after that, another 50. How many men must be discharged at the end of the 18th day so that the rest of the men, working for the remaining time, will just complete the work ?

(a) 100 (b) 105 (c) 110 (d) 115

GP_3386

28. The work done by a man, a woman and a child is in the ratio of 3 : 2 : 1. There are 20 men, 30 women and 36 children in a factory. Their weekly wages amount to ₹ 780, which is divided in the ratio of work done by the men, women and children. What will be the wages of 15 men, 21 women and 30 children for 2 weeks?
(a) ₹585 (b) ₹292.5 (c) ₹1170 (d) ₹900

29. A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the three are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern ?
(a) 10 min (b) 12 min
(c) 15 min (d) None of these

30. A pump can be operated both for filling a tank and for emptying it. The capacity of tank is 2400 m^3. The emptying capacity of the pump is 10 m^3 per minute higher than its filling capacity. Consequently, the pump needs 8 minutes less to empty the tank to fill it. Find the filling capacity of pump.
(a) 50 m^3/min (b) 60 m^3/min
(c) 58 m^3/min (d) None of these

31. A cistern has three pipes, A, B and C. The pipes A and B can fill it in 4 and 5 hours respectively and C can empty it in 2 hours. If the pipes are opened in order at 1, 2 and 3 a.m. respectively, when will the cistern be empty ?
(a) 3 p.m. (b) 4 p.m. (c) 5 p.m. (d) 6 p.m.

32. Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. In how much time, will the tank be full?
(a) 12 hrs (b) 13 hrs (c) 16 hrs (d) 18 hrs

33. Mayank can do 50% more work than Shishu in the same time. Shishu alone can do a piece of work in 30 hours. Shishu starts working and he had already worked for 6 hours when Mayank joins him. How many hours should Shishu and Mayank work together to complete the remaining work ?
(a) 6 (b) 12 (c) 4.8 (d) 9.6

34. A can do a piece of work in 90 days, B in 40 days and C in 12 days. They work for a day each in turn, i.e., first day A does it alone, second day B does it alone and 3rd day C does it alone. After that the cycle is repeated till the work is finished. They get ₹ 240 for this job. If the wages are divided in proportion to the work each had done. Find the amount A will get?
(a) 14 (b) 24 (c) 34 (d) 36

35. Manoj and Harvinder can complete a piece of work in 16 days, which Harvinder can complete in 32 days. If Manoj and Harvinder work on alternate days, starting with Harvinder, in how much time would the work be completed?
(a) 17.25 days (b) 17.66 days
(c) 20 days (d) 32 days

36. Kareena can do a piece of work in 9 days and Karishma can do the same work in 18 days. They started the work. After 3 days Shahid joined them, who can complete alone the same whole work in 3 days. What is the total number of days in which they had completed the work?
(a) 12 (b) 8 (c) 4 (d) 6

37. 33 men can do a job in 30 days. If 44 men started the job together and after everyday of the work, one person leaves. What is the minimum number of days required to complete the whole work ?
(a) 21 (b) 42
(c) 45 (d) None of these

38. Three pipes A, B and C can fill a tank in 6 hours. After running them together for two hours the tap C is closed and A and B fills it completely in 7 hours. How much time would C take to fill the tank alone?
(a) 7 hours (b) 10.5 hours
(c) 14 hours (d) 21 hours

39. Three taps A, B and C can fill a tank respectively in 12, 15 and 20 hours. If tap A is open through out and taps B & C are opened alternatively for one hour then in how much time will the tank full?
(a) 6 hours (b) $6\frac{1}{2}$ hours
(c) 7 hours (d) $7\frac{1}{2}$ hours

40. Two pipes can fill an empty tank in 24 min and 40 min respectively and a third pipe can empty 30 gallon water per minute. If all three pipes are open then the empty tank fills in 1 hour. The capacity of the tank (in gallon) is
(a) 800 (b) 600 (c) 500 (d) 400

41. A contractor undertakes to built a walls in 50 days. He employs 50 peoples for the same.

However after 25 days he finds that only 40% of the work is complete. How many more man need to be employed to complete the work in time?
(a) 25 (b) 30 (c) 35 (d) 20

42. A tyre has two punctures. The first puncture along would have made the tyre flat in 9 minutes and the second alone would have done it in 6 minutes. If air leaks out at a constant rate, how long does it take both the punctures together to make it flat ?

(a) $1\frac{1}{2}$ minutes (b) $3\frac{1}{2}$ minutes

(c) $3\frac{3}{5}$ minutes (d) $4\frac{1}{4}$ minutes

43. A can do a piece of work in 10 days, while B alone can do it in 15 days. They work together for 5 days and the rest of the work is done by C in 2 days. If they get ₹ 450 for the whole work, how should they divide the money ?
(a) ₹225, ₹150, ₹75 (b) ₹250, ₹100, ₹100
(c) ₹200, ₹150, ₹100 (d) ₹175, ₹175, ₹100

44. 12 men can complete a piece of work in 4 days, while 15 women can complete the same work in 4 days. 6 men start working on the job and after working for 2 days, all of them stopped working. How many women should be put on the job to complete the remaining work, If it is to be completed in 3 days ?
(a) 15 (b) 18
(c) 22 (d) Data inadequate

45. One hundred men in 10 days do a third of a piece of work. The work is then required to be completed in another 13 days. On the next day (the eleventh day) 50 more men are employed, and on the day after that, another 50. How many men must be discharged at the end of the 18th day so that the rest of the men, working for the remaining time, will just complete the work ?
(a) 100 (b) 105 (c) 110 (d) 115

46. A pipe can fill a tank in 15 minutes and another one in 10 minutes. A third pipe can empty the tank in 5 minutes. The first two pipes are kept open for 4 minutes in the beginning and then the third pipe is also opened. In what time will the tank be empited ?
(a) 35 min (b) 15 min
(c) 20 min (d) Cannot be emptied

47. Two fill taps A and B can separately fill a cistern in 45 and 40 minutes respectively. They started to fill a cistern together but tap A is turned off after few minutes and tap B fills the rest part of cistern in 23 minutes. After how many minutes, was tap A turned-off ?
(a) 9 min (b) 10 min
(c) 12 min (d) None of these

48. A pump can be operated both for filling a tank and for emptying it. The capacity of tank is 2400 m^3. The emptying capacity of the pump is 10 m^3 per minute higher than its filling capacity. Consequently, the pump needs 8 minutes less to empty the tank to fill it. Find the filling capacity of pump.
(a) 50 m^3/min (b) 60 m^3/min
(c) 58 m^3/min (d) None of these

49. A ship 55 kms. from the shore springs a leak which admits 2 tones of water in 6 min ; 80 tones would suffer to sink her, but the pumps can throw out 12 tones an hour. Find the average rate of sailing that she may just reach the shore as she begins to sink.
(a) 5.5 km/h (b) 6.5 km/h
(c) 7.5 km/h (d) 8.5 km/h

50. The total number of men, women and children working in a factory is 18. They earn ₹ 4000 in a day. If the sum of the wages of all men, all women and all children is in the ratio of 18 : 10 : 12 and if the wages of an individual man, woman and child is in the ratio 6 : 5 : 3, then how much a woman earn in a day?
(a) ₹400 (b) ₹250
(c) ₹150 (d) ₹120

GP_3386

Hints and Solutions

Level 1

1. (a) $(A+B)$'s 1 day's work $= \frac{1}{12}$th part of whole work.

B's 1 day's work $= \frac{1}{28}$th part of whole work.

$\therefore$ A's 1 day's work $= \frac{1}{12} - \frac{1}{28} = \frac{1}{21}$th part of whole work.

$\therefore$ A alone can finish the work in 21 days

2. (b) Ratio of times taken by A and $B = 100 : 130 = 10 : 13$.

Suppose B takes x days to do the work.

Then, $10 : 13 :: 23 : x$

$$\Rightarrow x = \left(\frac{23\times 13}{10}\right) \Rightarrow x = \frac{299}{10}.$$

A's 1 day's work $= \frac{1}{23}$; B's 1 days work $= \frac{10}{299}$.

$(A+B)$'s 1 day's work

$$= \left(\frac{1}{23} + \frac{10}{299}\right) = \frac{23}{299} = \frac{1}{13}.$$

$\therefore$ A and B together can complete the job in 13 days.

3. (d) Use Shortcut Approach-1

4. (a) A's 1 day's work $= \frac{1}{18}$ and B's 1 day's work $= \frac{1}{9}$.

$\therefore$ $(A+B)$'s 1 day's work $= \left(\frac{1}{18} + \frac{1}{9}\right) = \frac{1}{6}$.

5. (a) Given 6 BSF = 10 CRPF $\Rightarrow$ 4 BSF + 9 CRPF

$$= 4 + (9 \times 6/10)\text{ BSF} = \frac{94}{10}\text{ BSF}$$

Now work $= 6 \times 2$ BSF days $= \frac{94}{10} \times X$ BSF days

We have $6 \times 2 = \frac{94}{10} \times X \Rightarrow X = 1.27$ days

6. (d) Let required number of binders be 'x'

Less books, less binders (direct)

More days, less binders (indirect)

$$\left.\begin{matrix}\text{Books } 900 & : 660 \\ \text{Days } 12 & : 10\end{matrix}\right\} :: 18 : x$$

$900 \times 12 \times x = 660 \times 10 \times 18$

$$x = \frac{660\times 10\times 18}{900\times 12} = 11$$

7. (c) $(X+Y)$'s one day work $= \frac{1}{72}$

$(Y+Z)$'s one day work $= \frac{1}{120}$

$(Z+X)$'s one day work $= \frac{1}{90}$

$\therefore$ $2(X+Y+Z)$'s one day work $= \frac{1}{72} + \frac{1}{120} + \frac{1}{90}$

$$= \frac{5+3+4}{360} = \frac{12}{360} = \frac{1}{30}$$

$\therefore$ $(X+Y+Z)$'s one day work $= \frac{1}{2} \times \frac{1}{30} = \frac{1}{60}$

$\therefore$ They will complete the work in 60 days.

8. (c) Use Shortcut Approach-3

9. (b) x men do the work in 60 days and $(x+8)$ men do the work in $(60-10=)$ 50 days.

Then by "basic formula", $60x = 50(x+8)$

$$\therefore x = \frac{50\times 8}{10} = 40 \text{ men.}$$

10. (a) Use Shortcut Approach -7

11. (b) $(A+B)$'s 5 days' work

$$= 5\left(\frac{1}{25} + \frac{1}{20}\right) = \frac{45}{100} = \frac{9}{20}$$

Remaining work $= \left(1 - \frac{9}{20}\right) = \frac{11}{20}$

$\frac{11}{20}$ of the work would be finished by B in

$$\frac{\frac{11}{20}}{\frac{1}{20}} = 11 \text{ days.}$$

12. (c) Use Shortcut Approach-2

13. (d) In 1 day, work done by 12 men $= \frac{1}{18}$

In 6 days, work done by 12 men $= \frac{6}{18} = \frac{1}{3}$

Remaining work $= \frac{2}{3}$

Now, $m_1 \times d_1 \times w_2 = m_2 \times d_2 \times w_1$

or $12 \times 18 \times \frac{2}{3} = 16 \times d_2 \times 1$

or $d_2 = \frac{4 \times 18 \times 2}{16} = 9$ days

14. (b) Use Shortcut Approach-10

15. (a) Let 1 man's 1 day's work $= x$
1 boy's 1 day's work $= y$

$12x + 16y = \frac{1}{5}$ ....(1)

$13x + 24y = \frac{1}{4}$ ....(2)

After solving (1) and (2)

$x = \frac{1}{100}$ and $y = \frac{1}{200}$

Thus, required ratio 2 : 1.

16. (c) $M_1 \times D_1 = M_2 \times D_2$
$m \times r = (m + n) \times D_2$

$D_2 = \frac{mr}{(m+n)}$

17. (d) After 27 days, food left = 4 × 200 = 800 soldier days worth of food. Since, now there are only 80 soldiers, this food would last for 800/80 = 10 days. Number of extra days for which the food lasts = 10 – 4 = 6 days.

18. (b) Use Shortcut Approach-14

19. (b) Sambhu requires 16 days to do the work while Kalu requires 18 days to do the work.
$(1/16 + 1/18) \times n = 1$
$\Rightarrow n = 288/34 = 144/17$

20. (d) Use Shortcut Approach-11

21. (b) A's 1 day's work $= \frac{1}{10}$ and B's 1 day's work $= \frac{1}{15}$

$\therefore$ (A + B)'s 1 day's work $= \left(\frac{1}{10} + \frac{1}{15}\right) = \frac{1}{6}$.

So, both together will finish the work in 6 days.

22. (d) (Man + Son)'s one day's work $= \frac{1}{8}$

Man's one day's work $= \frac{1}{10}$

$\Rightarrow$ Son's one day's work $= \frac{1}{8} - \frac{1}{10} = \frac{1}{40}$

$\therefore$ Son can do it in 40 days.

23. (b) X's one day's work $= \frac{1}{25}$th part of whole work.

Y's one day's work $= \frac{1}{30}$th part of whole work.

Their one day's work $= \frac{1}{25} + \frac{1}{30} = \frac{11}{150}$th part of whole work.

Now, work is done in 5 days

$= \frac{11}{150} \times 5 = \frac{11}{30}$th of whole work

$\therefore$ Remaining work $= 1 - \frac{11}{30} = \frac{19}{30}$th of whole work

Now, $\frac{1}{30}$th work is done by Y in one day.

$\therefore \frac{19}{30}$th work is done by Y in

$\frac{1}{1/30} \times \frac{19}{30} = 19$ days

24. (b) B alone can do a work in 20 hours.

$\therefore$ A alone can do $\frac{3}{2}$ of the work in 20 hours.

i.e., A alone can do the same work in $\frac{40}{3}$ hours

$\therefore$ (A + B)'s one hour's work

$= \frac{3}{40} + \frac{1}{20} = \frac{5}{40} = \frac{1}{8}$

$\Rightarrow$ A and B together can finish the whole work in 8 hours.

25. (a) $15\,W = 10\,M$

Now, $5W + 4M$

$= 5W + \frac{4\times15}{10}\,W = 5W + 6W = 11\,W$

If 15 women can complete the project in 55 days, 11 women can complete the same project in

$\frac{55\times15}{11} = 75$ days

26. (b) $m_1 \times d_1 \times t_1 \times w_2 = m_2 \times d_2 \times t_2 \times w_1$

$24 \times 10 \times 8 \times 1 = m_2 \times 6 \times 10 \times 1$

$\Rightarrow m_2 = \frac{24\times10\times8}{6\times10} = 32$ men

27. (d) A's one day's work $= \frac{1}{32}$

B's one day's work $= \frac{1}{20}$

(B + C)'s one day's work $= \frac{1}{12}$

$\therefore$ C's one day's work $= \frac{1}{12} - \frac{1}{20} = \frac{1}{30}$

D's one day's work $= \frac{1}{24}$

$\therefore$ (A + B + C + D)'s one day's work

$= \frac{1}{32} + \frac{1}{20} + \frac{1}{30} + \frac{1}{24} = \frac{75+120+80+100}{2400}$

$= \frac{375}{2400} = \frac{15}{96} = \frac{5}{32}$

$\therefore$ Out of $\frac{5}{32}$ of work done, $\frac{1}{30}$ of the work is done by C.

$\Rightarrow$ Out of ₹ 25 paid for the work, C will receive

₹ $\frac{1/30}{5/32}\times25$, i.e. $\frac{1}{30}\times\frac{32}{5}\times25$, i.e. ₹ $\frac{16}{3}$

28. (b) Use Shortcut Approach-4

29. (b) X's one day's work $= \frac{1}{15}$th work.

Y's one day's work

$= \frac{1}{15} + 50\%$ of $\frac{1}{15} = \frac{1}{10}$th work

$\therefore$ (X + Y)'s one day's work

$= \frac{1}{15} + \frac{1}{10} = \frac{1}{6}$th work

Hence, they together finish the work in 6 days.

30. (a) Ratio of time taken by A and B = 160 : 100 = 8 : 5

Suppose, B alone takes x days to do the job then, $8 : 5 :: 12 : x$

$8x = 5 \times 12$

$x = \frac{5\times12}{8} = 7\frac{1}{2}$ days.

31. (a) If x complete a work in x days. y will do the same task in $3x$ days.

$3x - x = 40 \Rightarrow x = 20$

y will finish the task in 60 days

$(x+y)$'s 1 day's work $= \frac{1}{20} + \frac{1}{60} = \frac{1}{15}$

Both of them will complete the work in 15 days.

32. (c) Net part filled in 1 hour

$= \left(\frac{1}{5} + \frac{1}{6} - \frac{1}{12}\right) = \frac{17}{60}$.

$\therefore$ The tank will be full in $\frac{60}{17}$ hrs i.e., $3\frac{9}{17}$ hrs.

33. (a) Use Shortcut Approach-17

34. (c) Let B can fill the cistern in x min. Then,

then A can fill the cistern in $\frac{x}{3}$ min

Given $x - \frac{x}{3} = 10 \Rightarrow x = 15$ min

35. (a) Let cistern will be full in x min. Then, part filled by B in x min + part filled by A in $(x-4)$ min = 1

$\Rightarrow \frac{x}{16} + \frac{x-4}{12} = 1 \Rightarrow x = \frac{64}{7} = 9\frac{1}{7}$ hours.

36. (a) Let cistern will be full in x min. Then, part filled by A in x min + part filled by B in $(x-1)$ min + part filled by C in $(x-2)$ min = 1

$\Rightarrow \frac{x}{3} + \frac{x-1}{4} + \frac{x-2}{6} = 1$

$\Rightarrow 9x = 19 \Rightarrow x = \frac{19}{9} = 2\frac{1}{9}$ min

37. (d) Pipe 1 (Hot) $\rightarrow 3 + X$, $X \rightarrow$ Pipe 2 (cold)

Together $\frac{X(X+3)}{2X+3} = 6\frac{2}{3}$ min.

$= \frac{X(X+3)}{2X+3} = 6\frac{2}{3}\text{min.} = \frac{20}{3}$

$40X + 60 = 3X(X+3)$

$\Rightarrow 40X + 60 = 3X^2 + 9X$

$\Rightarrow 3X^2 - 31X - 60 = 0$

$\Rightarrow X = 12$ minutes

38. (b) Pranesh takes 30 days to finish the work.
Since Sagar is 20% more efficient than Pranesh, he will take 16.66% less time.
So Sagar takes = 25 days
So work done by both of them in one day

$= (1/30) + (1/25) = \frac{11}{150}$

No of days to complete the work

$= \frac{150}{11} = 13\frac{7}{11}$

or $\left(13 + \frac{7}{11}\right)$ days

39. (a) Efficiency of Chandni = 11.11%
Efficiency of Divakar = 8.33%
They do 19.44% work in 2 days
$\therefore$ They need 10 days to do 97.22% work,
Now the rest work (2.78) was done by Chandni in

$\frac{2.78}{11.11} = \frac{1}{4}$ day

Therefore total number of days required

$= 10 + \frac{1}{4} = 10\frac{1}{4}$ days

Alternatively :

Chandni's one day's work $= \frac{1}{9}$

Divakar's one day's work $= \frac{1}{12}$

Chandni's and Divakar's (1 + 1) = 2 day's work $= \frac{1}{9} + \frac{1}{12} = \frac{7}{36}$

So, in 10 days they do $= \frac{7 \times 5}{36} = \frac{35}{36}$ work

So, the remaining $\frac{1}{36}\left(= 1 - \frac{35}{36}\right)$ work will be done by

Chandni $= \frac{1/36}{1/9} = \frac{1}{4}$ day

Thus total number of required days

$= 10 + \frac{1}{4} = 10\frac{1}{4}$ days

Hint :

Day	1	2	3	4	5	6	7	8	9	10	11
Turn	C	D	C	D	C	D	C	D	C	D	C
Work	1/9	1/12	1/9	1/12	1/9	1/12	1/9	1/12	1/9	1/12	1/36
	7/36		7/36		7/36		7/36		7/36		
	35/36										1/36

40. (d) 3 days before the completion of the work Raja left the work means in last 3 days only Rani has worked alone.
So, in last 3 days work done by Rani

$= 3 \times \frac{1}{21} = \frac{1}{7}$

So, the rest $\left(1 - \frac{1}{7}\right) = \frac{6}{7}$ work was done by Raja and Rani both.
Number of days in which Raja and Rani worked together

$= \frac{6/7}{5/42} = \frac{36}{5}$

Total time $= \left(\frac{36}{5} + 3\right) = \left(\frac{36+15}{5}\right)$

$= \frac{51}{5} = 10\frac{1}{5}$

41. (a) Use Shortcut Approach-6

42. (c) Use Shortcut Approach-12

43. (c) Efficiency of A $= \frac{100}{12} = 8.33\%$

Efficiency of B $= \frac{100}{15} = 6.66\%$

Combined efficiency of A and B
= 8.33 + 6.66 = 15%
Number of days taken by A and B, when worked together

$= \frac{100}{15} = 6\frac{10}{15} = 6\frac{2}{3}$ Days

44. (c) Use shortcut approach-22

45. (d) Use Shortcut Approach-23

46. (a) 6C + 2M = 6 days

GP_3386

$\Rightarrow$ 36 C + 12 M = 1 day

Again 1M = 2 C

$\therefore$ (36 + 12 × 2) C = 1 day

60 children can do the work in 1 day

Now, 5 men = 10 children

$\therefore$ 10 children can do the work in 6 days.

47. (b) Efficiency (per minute) of Modi = 4 pages/min

Efficiency of Modi and Xerox together = 10 pages/min

$\therefore$ Efficiency of Xerox alone = 10 – 4 = 6 pages / min

$\therefore$ Mr. Xerox needs 6 minutes to copy 36 pages.

48. (b) Let x kg of oil is used for eating purpose, daily, then

$(x + 11) \times 50 = (x + 15) \times 45 = x = 25$

$\therefore$ Total quantity of oil = $(25 + 11) \times 50 = 1800$

$\therefore$ Required number of days

$$= \frac{1800}{25} = 72 \text{ days}$$

49. (c) Work done $= \frac{2}{3}$

Remaining work $= \frac{1}{3}$, which is half of $\frac{2}{3}$

$\therefore$ $\frac{1}{2} \times (20 \times 32) = 8 \times x$

$\Rightarrow$ x = 40 men

Therefore, 20 more men were required.

50. (d) Efficiency of A = 8.33%

Effiective efficiency = 6.66%, when there is leakage

$\therefore$ Efficiency of leakage = 1.66% = (8.33 – 6.66)

It means due to leakage a full tank will be empty in 60 hours.

51. (b) Area $= \pi r^2 = \pi\left(\frac{d}{2}\right)^2$

So, after doubling the diameter the cross-section area is increased 4 times.

So, to empty the tank it will take $\frac{40}{4}$ = 10 min by the pipe.

52. (b) Clearly, 5 men × 6 = 10 women × 5

or, 1 man $= \frac{5}{3}$ women

$\therefore$ M_2 = 5 women + 3 men = 10 women

from the question, 10 women complete the work in 5 days.

53. (b) $\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}, \frac{20 \times 20}{\frac{1}{3}} = \frac{(20 + x) \times 25}{\frac{2}{3}}$

or, $2 \times 20 \times 20 = (20 + x) \times 25$

or, x = 12

54. (b) Emptied part by the third pipe in1 hour

$$= \frac{1}{2} + \frac{1}{3} - \frac{7}{12}$$

$$= \frac{6 + 4 - 7}{12} = \frac{3}{12} = \frac{1}{4} \text{part}$$

$\therefore$ So, time taken by the third pipe to empty the tank = 4 hours.

55. (a) Use Shortcut Approach-25

56. (c) Use Shortcut Approach-26

57. (b) Work efficieny of B = 160% of A's work efficiency

$= \frac{8}{5}$ A's work efficiency

$\therefore$ Time taken by B = $24 \times \frac{5}{8} = 15$ days

58. (a) In an hour, George and Sonia together can copy $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of a 50-page manuscript.

i.e. In an hour they together can copy $\frac{7}{48}$ of the 100-page manuscript.

i.e. They together can copy a 100-page manuscript in $\frac{48}{7}$ hours, i.e. $6\frac{6}{7}$ hours.

59. (b) Let the man alone do the work in x days.

Then, the woman alone do the work in 2x days.

Their one day's work = $\frac{1}{8}$ th part of whole work

i.e. $\frac{1}{x} + \frac{1}{2x} = \frac{1}{8}$

$\Rightarrow$ x = 12 days

$\therefore$ man takes 12 days and woman 2x = 24 days.

60. **(b)** In one min, (A + B) fill the cistern

$= \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$th

In 3 min, (A + B) fill the cistern $= \frac{3}{6} = \frac{1}{2}$th

Remaining part $= 1 - \frac{1}{2} = \frac{1}{2}$

$\because$ $\frac{1}{10}$th part filled by A in one min.

$\therefore$ $\frac{1}{2}$nd part filled by A in $10 \times \frac{1}{2} = 5$ min.

$\therefore$ Total time $= 3 + 5 = 8$ min.

61. **(b)** Use Shortcut Approach-24

62. **(c)** Let Pratibha can finish the work in x days then, Sonia can finish the same work in 3x days

According to question

$3x - x = 60$

$2x = 60 \Rightarrow x = 30$

Pratibha and Sonia can individually complete the work in 30 days and 90 days respectively.

63. **(c)**

Level 2

1. **(c)** In 8 days, Anil does $= \frac{1}{3}$rd work.

$\therefore$ in 1 day, he does $= \frac{1}{24}$th work.

$\therefore$ Rakesh's one day's work = 60% of $\frac{1}{24}$

$= \frac{1}{40}$th work.

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$

(Anil and Rakesh)'s one day's work

$= \frac{1}{24} + \frac{1}{40} = \frac{1}{15}$th work

Now, $\frac{1}{15}$th work is done by them in one day

$\therefore$ $\frac{2}{3}$rd work is done by them in $15 \times \frac{2}{3} = 10$ days

2. **(b)** Use Shortcut Approach-10

3. **(a)** Use Shortcut Approach-13

4. **(a)** Let 1 man's 1 days' work = x and 1 boy's 1 day's work = y

Then, $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}$

Solving, we get : $x = \frac{7}{200}$ and $y = \frac{1}{100}$

$\therefore$ (2 men + 1 boy)'s 1 day's work

$= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$

So, 2 men and 1 boy together can finish the work in $12\frac{1}{2}$ days.

5. **(c)** Part of the cistern filled by first pipe in 1 minute $= \frac{1}{6}$

Part of the cistern filled by second pipe in 1 minute $= \frac{1}{7}$

Part of the cistern filled in first 2 minutes

$= \frac{1}{6} + \frac{1}{7} = \frac{13}{42}$

Part of the cistern filled in 6 minutes

$= \frac{3 \times 13}{42} = \frac{39}{42}$

Remaining part $= 1 - \frac{39}{42} = \frac{3}{42} = \frac{1}{14}$

$\therefore$ Time taken to fill $\frac{1}{14}$ parts $= \frac{6}{14} = \frac{3}{7}$

$\therefore$ Total time $= 6 + \frac{3}{7} = 6\frac{3}{7}$ minutes

6. **(a)** Use Shortcut Approach-18

7. **(a)** Part filled in 7 min. $= 7 \times \left(\frac{1}{36} + \frac{1}{45}\right) = \frac{7}{20}$

Remaining part $= \left(1 - \frac{7}{20}\right) = \frac{13}{20}$

Part filled by $(A + B + C)$ in 1 min.

GP_3386

$= \left(\frac{1}{36}+\frac{1}{45}-\frac{1}{30}\right) = \frac{1}{60}$.

Now, $\frac{1}{60}$ part is filled in 1 min.

$\therefore$ $\frac{13}{20}$ part is filled in $\left(60\times\frac{13}{20}\right) = 39$ min

$\therefore$ Total time taken to fill the tank $= 39+7 = 46$ min.

8. **(a)** Use Shortcut Approach - 19

9. **(c)** Use Shortcut Approach-20

10. **(b)** Use Shortcut Approach-21

11. **(c)** A's one day's work $= \frac{1}{16}$ th work

B's one day's work $= \frac{1}{12}$ th work

Let the number of days B has worked alone $= x$ days. Then,

A's amount of work + B's amount of work = 1

$\Rightarrow 4\left(\frac{1}{16}\right)+(x+4)\left(\frac{1}{12}\right)=1$

$\Rightarrow \frac{1}{4}+\frac{x+4}{12}=1 \Rightarrow x=\frac{3}{4}\times 12-4$

$\Rightarrow x = 5$ days

12. **(a)** The per day digging of all three combined is 54 meters. Hence, their average should be 18. This means that the first should be $18-x$, the second 18 and the third $18+x$.

The required conditions are met if we take the values as 15, 18, 21 metres for the first, second and third diggers respectively.

13. **(c)** 1 minute's work of both the punctures

$= \left(\frac{1}{9}+\frac{1}{6}\right)=\frac{5}{18}$.

So, both the punctures will make the tyre flat in $\frac{18}{5} = 3\frac{3}{5}$ min.

14. **(c)** Use Shortcut Approach-8

15. **(a)** Use Shortcut Approach-9

16. **(a)** 50 men complete 0.4 work in 25 days.

Applying the work rule,

$m_1 \times d_1 \times w_2 = m_2 \times d_2 \times w_1$

we have,

$50\times 25\times 0.6 = m_2 \times 25\times 0.4$

or $m_2 = \frac{50\times 25\times 0.6}{25\times 0.4} = 75$ men

Number of additional men required $= (75-50) = 25$

17. **(c)** 10 men's 1 day's work $= \frac{1}{15}$;

15 women's 1 day's work $= \frac{1}{12}$.

(10 men + 15 women)'s 1 day's work

$= \left(\frac{1}{15}+\frac{1}{12}\right)=\frac{9}{60}=\frac{3}{20}$.

$\therefore$ 10 men and 15 women will complete the work in $\frac{20}{3} = 6\frac{2}{3}$ days.

18. **(b)** Use Shortcut Approach-16

19. **(c)** Use Shortcut Approach-15

20. **(d)** Let if both A and B work together, they take x days.

$\therefore$ (A + B)'s 1 days's work $= \frac{1}{x}$th work.

A's 1 day's work $= \frac{1}{x+8}$th work.

B's 1 day's work $= \frac{1}{x+9/2}$th work.

Now, $\frac{1}{x+8}+\frac{2}{2x+9}=\frac{1}{x}$

$\Rightarrow x(2x+9+2x+16) = (x+8)(2x+9)$

$\Rightarrow 4x^2+25x = 2x^2+25x+72$

$\Rightarrow x^2 = 36 \Rightarrow x = 6$ days

21. **(a)** Let the number of men originally employed be x.

$9x = 15(x-6)$

or $x = 15$

22. **(b)** Let B can finish the work in x days.

Then A can finish the work in $(x-3)$ days.

B's one day's work $= \frac{1}{x}$th work

A's one day's work $= \frac{1}{x-3}$th work

A's 4 days' work $= \frac{4}{x-3}$th work

Remaining work $= 1 - \frac{4}{x-3}$

$= \frac{x-7}{x-3}$th work

The remaining work done by B in 14 – 4 = 10 days.
Now, in 10 days, work done by B

$= \frac{x-7}{x-3}$th work

$\therefore$ In 1 day, work done by B $= \frac{1}{10}\left(\frac{x-7}{x-3}\right)$th work

and $\frac{1}{10}\left(\frac{x-7}{x-3}\right) = \frac{1}{x} \quad \Rightarrow x = 15$ days

$\therefore$ B will finish in 15 days and A will finish in 12 days

23. (a) (A + B)'s one day's work $= \frac{1}{5}$th work

Let A can do job in x days. Then,

A's one day's work $= \frac{1}{x}$th work

and B's one day's work

$= \frac{1}{5} - \frac{1}{x} = \frac{x-5}{5x}$th work

Now, (2A)'s work + $\left(\frac{1}{3}\right)$ B's work $= \frac{1}{3}$ rd work

$\Rightarrow \frac{2}{x} + \frac{1}{3}\left(\frac{x-5}{5x}\right) = \frac{1}{3} \Rightarrow x = \frac{25}{4} = 6\frac{1}{4}$ days

24. (a) Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y.

Then, $6x + 8y = \frac{1}{10}$ and $26x + 48y = \frac{1}{2}$.

Solving these two equations, we get :

$x = \frac{1}{100}$ and $y = \frac{1}{200}$.

$\therefore$ (15 men + 20 boys)'s 1 day's work

$= \left(\frac{15}{100} + \frac{20}{200}\right) = \frac{1}{4}$.

$\therefore$ 15 men and 20 boys can do the work in 4 days.

25. (c) Use Shortcut Approach-5

26. (c) Let C completes the work in x days.

Work done by (A + B) in 1 day $= \frac{1}{10}$

Work done by (B + C) in 1 day $= \frac{1}{18}$

A's 5 days' work + B's 10 days' work + C's 15 days' work = 1
or (A + B)'s 5 days' work + (B + C)'s 5 days' work + C's 10 days' work = 1

or $\frac{5}{10} + \frac{5}{18} + \frac{10}{x} = 1$

or $x = 45$ days

27. (c) Suppose that X men must be discharged at the end of the 18th day.

$100 \times 10 + 150 \times 1 + 200 \times 7 + (200 - X) \times 5 = 100 \times 30$

$5X = 550 \Rightarrow X = 110$ men

28. (c)

	Men	Women	Children
Work	3	2	1
Numbers	20	30	36

Ratio of wages = $(3 \times 20) : (2 \times 30) : (1 \times 36)$
$= 5 : 5 : 3$

Total wages of men $= \frac{5}{13} \times 780 = ₹300$

$\therefore$ Wages of a man = ₹ 15
Similarly, wages of woman = ₹ 10
and wages of child = ₹ 5
Total wages of 15 men, 21 women and 30 children
$= 15 \times 15 + 21 \times 10 + 30 \times 5 = 585$
Total wages for 2 weeks = ₹ 1170

29. (a) Work done by the waste pipe in 1 minutes

$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right) = -\frac{1}{10}$ [–ve sign means emptying]

$\therefore$ Waste pipe will empty the full cistern in 10 minutes.

30. (a) Let the filling capacity of pump be x m^3/min.
Then, emptying capacity of pump
$= (x + 10)$ m^3 /min.

GP_3386

$$\therefore \frac{2400}{x}-\frac{2400}{x+10}=8$$

$\Rightarrow x^2+10x-3000=0$

$\Rightarrow (x-50)(x+60)=0 \Rightarrow x=50\ m^3/min.$

31. (c) Let the time be t hours after 1 a.m.

$$\therefore \frac{t}{4}+\frac{(t-1)}{5}-\frac{(t-2)}{2}=0 \quad \Rightarrow t=16$$

16 hours after 1 a.m., the time will be 5 p.m.

32. (a) Part filled in 10 hours

$$=10\left(\frac{1}{15}+\frac{1}{20}-\frac{1}{25}\right)=\frac{23}{30}.$$

$$\text{Remaining part}=\left(1-\frac{23}{30}\right)=\frac{7}{30}.$$

$$(A+B)\text{'s 1 hour's work}=\left(\frac{1}{15}+\frac{1}{20}\right)=\frac{7}{60}.$$

$$\frac{7}{60}:\frac{7}{30}::1:x \text{ or } x=\left(\frac{7}{30}\times 1\times\frac{60}{7}\right)$$

= 2 hours.

$\therefore$ The tank will be full in (10 + 2) hrs = 12 hrs.

33. (d) Ratio of efficiency of Mayank and Shishu = 3/2

So ratio of time taken by Mayank and Shishu = 2/3

So if Shishu takes 30 hours, then Mayank will take 20 hours

Shishu 6 hours work = $1/5^{th}$ work,

Remaining work = $1 - 1/5 = 4/5^{th}$ work

Shishu and Mayank 1 day work together

$$=\frac{1}{20}+\frac{1}{30}=\frac{1}{12}$$

$$\text{So required time}=\frac{4/5}{1/12}=9.6 \text{ hours}$$

34. (b) Let the total amount of work = 180 unit

A does = 2 units/day

B does = 4.5 units/day

C does = 15 units/day

To finish 180 units of work, everybody has to work for 9 days.

In 9 days A will do = $9 \times 2 = 18$ units work.

$$\text{So A's contribution}=\ldots.=\frac{18}{180}\ldots.\frac{1}{10}$$

so out of ₹ 240 A's share = $240 \times 1/10 = 24$

35. (d) Let's assume the total amount of work = 32 units

So Manoj and Harvinder do = 2 unit/2day

Harvinder does = 1 unit/day

So Manoj does = 1 unit/day

So 32 units of work will be completed in 32 days.

36. (c) Efficiency of Kareena and Karishma = 11.11 + 5.55 = 16.66%

Work done in 3 days = $3 \times 16.66 = 50\%$

Rest work done by Kareena, Karishma and Shahid

$$=\frac{50}{50}=1 \text{ day}$$

(Since efficiency of Shahid = 33.33%)

Thus in 4 (= 3 + 1) days they have completed the work.

37. (d) Total work = $33 \times 30 = 990$. man - days

$$\text{First day's work}=\frac{1}{990}\times 44$$

$$\text{Second day's work}=\frac{1}{990}\times 43$$

$$\text{Third day's work}=\frac{1}{990}\times 42 \text{ and so on}$$

So, the total work in 44 days

$$=\frac{1}{990}(44+43+42+.....)$$

$$=\frac{1}{990}\times\frac{44\times 45}{2}=1$$

Hence in 44 days total work will be completed.

38. (c) $$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}=\frac{1}{6} \quad \ldots(i)$$

and $$\frac{9}{A}+\frac{9}{B}+\frac{2}{C}=1 \quad \ldots(ii)$$

or, $$9\left(\frac{1}{A}+\frac{1}{B}\right)+\frac{2}{C}=1$$

or, $$9\left(\frac{1}{6}-\frac{1}{C}\right)+\frac{2}{C}=1 \text{ (from eq. (i))}$$

or, $$\frac{3}{2}-\frac{7}{C}=1 \text{ or, } -\frac{7}{C}=1-\frac{3}{2}=\frac{-1}{2}$$

or, C = 14 hours

39. (c) In first 2 hours

$\frac{2}{12}+\frac{1}{15}+\frac{1}{20}=\frac{10+4+3}{60}=\frac{17}{60}$ part of the tank will be filled.

Clearly, after 6 hours $\frac{3\times17}{60}=\frac{17}{20}$ part will be filled

Now, $1-\frac{17}{20}=\frac{3}{20}$ part of the tank is empty

In next 1 hour, $\frac{1}{12}+\frac{1}{15}=\frac{5+4}{60}=\frac{9}{60}=\frac{3}{20}$ part will be filled

So, the tank will be filled in 7 hours

40. (b) Required time = inverse of $\frac{1}{24}+\frac{1}{40}-\frac{1}{60}$

$=$ inverse of $\frac{5+3-2}{120}=$ inverse of $\frac{6}{120}=20$

$\therefore$ Capacity of the tank $=20\times30=600$ gallon

41. (a) 50 men complete 0.4 work in 25 days.

Applying the work rule,

$m_1\times d_1\times w_2=m_2\times d_2\times w_1$

we have,

$50\times25\times0.6=m_2\times25\times0.4$

or $m_2=\frac{50\times25\times0.6}{25\times0.4}=75$ men

Number of additional men required $=(75-50)=25$

42. (c) 1 minute's work of both the punctures

$=\left(\frac{1}{9}+\frac{1}{6}\right)=\frac{5}{18}$.

So, both the punctures will make the tyre flat in $\frac{18}{5}=3\frac{3}{5}$ min.

43. (a) Work done by A and B in 5 days

$=\left(\frac{1}{10}+\frac{1}{15}\right)\times5=\frac{5}{6}$

Work remaining $=1-\frac{5}{6}=\frac{1}{6}$

$\therefore$ C alone can do the work in $6\times2=12$ days

Ratio of their share work

$=\frac{5}{10}:\frac{5}{15}:\frac{2}{12}=3:2:1$

Share of wages = ₹225, ₹150, ₹75.

44. (a) 1 man's 1 day's work $=\frac{1}{48}$;

1 woman's 1 day's work $=\frac{1}{60}$.

6 men's 2 day's work $=\left(\frac{6}{48}\times2\right)=\frac{1}{4}$.

Remaining work $=\left(1-\frac{1}{4}\right)=\frac{3}{4}$.

Now, $\frac{1}{60}$ work is done in 1 day by 1 woman.

So, $\frac{3}{4}$ work will be done in 3 days by

$\left(60\times\frac{3}{4}\times\frac{1}{3}\right)=15$ women.

45. (c) Suppose that X men must be discharged at the end of the 18th day.

$100\times10+150\times1+200\times7+(200-X)\times5$

$=100\times30$

$5X=550\Rightarrow X=110$ men

46. (c) Proportion of the volume of the tank filled by both the pipes in 4 min $=4\left(\frac{1}{15}+\frac{1}{10}\right)$

$=\frac{2}{3}$ rd of the tank.

Volume of the tank filled by all the pipes working together $=\frac{1}{15}+\frac{1}{10}-\frac{1}{5}=\frac{-1}{30}$

i.e. $\frac{1}{30}$ tank is emptied in 1 min.

$\therefore$ $\frac{2}{3}$ rd of the tank can be emptied in $\frac{2\times30}{3}=20$ min

47. (a) Let A was turned off after x min. Then,

cistern filled by A in x min + cistern filled by B in $(x+23)$ min $=1$

$\Rightarrow\frac{x}{45}+\frac{x+23}{40}=1$

$\Rightarrow17x+207=360\Rightarrow x=9$ min.

GP_3386

48. (a) Let the filling capacity of pump be x m^3/min.
Then, emptying capacity of pump $= (x + 10)\ m^3$/min.

$$\therefore \frac{2400}{x} - \frac{2400}{x+10} = 8$$

$$\Rightarrow x^2 + 10x - 3000 = 0$$

$$\Rightarrow (x-50)(x+60) = 0 \Rightarrow x = 50\ m^3/\text{min}.$$

49. (a) Rate of admission of water

$$= \frac{2}{6} \text{ tonnes / min} = \frac{1}{3} \text{ tonnes/ min}$$

Rate of pumping out of water

$$= \frac{12}{60} \text{ tonnes/min} = \frac{1}{5} \text{ tonnes/ min.}$$

$$\text{Rate of accumulation} = \left(\frac{2}{6} - \frac{12}{60}\right) \text{ tonnes/min.}$$

Time to accumulate 80 tonnes of water

$$= \frac{\text{Amount of water}}{\text{Accumulation rate}} = \frac{80}{\left(\frac{1}{3} - \frac{1}{5}\right)} = 600 \text{ min.}$$

$= 10$ hours

$\therefore$ Average sailing rate so as avoid sinking

$$= \frac{\text{Distance}}{\text{Time}} = \frac{55}{10} \text{ km/h} = 5.5 \text{ km/h}$$

50. (b) Ratio of number of men, women and children

$$= \frac{18}{6} : \frac{10}{5} : \frac{12}{3} = 3x : 2x : 4x$$

$$\therefore (3x + 2x + 4x) = 18$$

$$\therefore x = 2$$

Therefore, number of women $= 4$

$$\text{Share of all women} = \frac{10}{40} \times 4000 = ₹1000$$

$(\because 18 + 10 + 12 = 40)$

$$\therefore \text{Share of each woman} = \frac{1000}{4} ₹250$$

CHAPTER

16

Time, Speed and Distance
(Including Trains, Boats and Stream, Circular Motion, Races and Games)

MOTION OR MOVEMENT

The relation between speed (S), distance (D) and time (T) is given below :

Distance = Speed × Time i.e. $D = S \times T$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \text{ i.e. } S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \text{ i.e. } T = \frac{D}{S}$$

Conversion of KMPH (Kilometer per Hour) to M/S (Metre per Second) and Vice-versa

$$1 \text{ kmph or } 1 \text{ km/h} = \frac{1 \text{ km}}{1 \text{ hr}}$$

$$= \frac{1000 \text{ m}}{60 \times 60 \text{ sec}} = \frac{5 \text{ m}}{18 \text{ sec}} = \frac{5}{18} \text{ m/s}$$

$$\Rightarrow x \text{ kmph} = \frac{5x}{18} \text{ m/s and vice-versa}$$

$$x \text{ m/s} = \frac{18x}{5} \text{ kmph} \quad \text{or} \quad \frac{18x}{5} \text{ km/h}$$

i.e. to convert km/hr to m/sec, multiply by $\frac{5}{18}$ and to convert m/sec to km/hr multiply by $\frac{18}{5}$.

Illustration – 1

Convert 90 km/h into m/s.

Solution : $90 \text{ km/h} = 90 \times \frac{5}{18} = 25 \text{ m/s}.$

Illustration – 2

The driver of a Maruti car driving at the speed of 68 km/h locates a bus 40 metres ahead of him. After 10 seconds, the bus is 60 metres behind. The speed of the bus is

(a) 30 km/h **(b) 32 km/h**
(c) 25 km/h **(d) 38 km/h**

Solution : (b) Let speed of bus = S_B km/h.

Now, in 10 sec., car covers the relative distance = (60 + 40) m = 100 m

$$\therefore \text{ Relative speed of car} = \frac{100}{10} = 10 \text{ m/s}$$

$$= 10 \times \frac{18}{5} = 36 \text{ km/h}$$

$$\therefore 68 - S_B = 36 \Rightarrow S = 32 \text{ km/h}$$

AVERAGE SPEED

Average speed is defined as the ratio of total distance covered to the total time taken by an object *i.e.*

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

If an object travels $d_1, d_2, d_3, \ldots, d_n$ distances with different speeds $s_1, s_2, s_3, \ldots, s_n$ in time $t_1, t_2, t_3, \ldots, t_n$ respectively; then average speed (S_a) is given by

$$\boxed{S_a = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{t_1 + t_2 + t_3 + \ldots + t_n}} \quad \ldots (1)$$

Since, Distance = Speed × Time

$\therefore \quad d_1 = s_1 t_1, \quad d_2 = s_2 t_2, \quad d_3 = s_3 t_3, \ldots, d_n = s_n t_n$

Hence from (1),

$$\boxed{S_a = \frac{s_1 t_1 + s_2 t_2 + s_3 t_3 + \ldots + s_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n}}$$

GP_3386

Since Time $= \dfrac{\text{Distance}}{\text{Speed}}$

$\therefore t_1 = \dfrac{d_1}{s_1},\ t_2 = \dfrac{d_2}{s_2},\ t_3 = \dfrac{d_3}{s_3}, \ldots, t_n = \dfrac{d_n}{s_n}$

Hence from (1),

$$S_a = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots + \frac{d_n}{s_n}}$$

Special Cases

(i) If with two different speeds s_1 and s_2 the same distance d is covered, then

$$\text{Average speed} = \frac{2s_1 \cdot s_2}{s_1 + s_2}$$

(ii) If with three different speeds s_1, s_2 and s_3 the same distance d is covered, then

$$\text{Average speed} = \frac{3s_1 \cdot s_2 \cdot s_3}{s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1}$$

Illustration – 3

A car moves 300 km at a speed of 45 km/h and then it increases its speed to 60 km/h to travel another 500 km. Find average speed of car.

Solution : Average speed

$$= \frac{d_1 + d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}} = \frac{300 + 500}{\frac{300}{45} + \frac{500}{60}} = \frac{800}{\frac{45}{3}} = \frac{160}{3} = 53\frac{1}{3} \text{ km/h}$$

Illustration – 4

A covers 1/3rd of the journey at the speed of 10 km/h and half of the rest at the speed of 20 km/h and rest at the speed of 30 km/h. What is the average speed of A ?

Solution : Distance covered at 10 km/h = 1/3rd of the whole journey

Distance covered at 20 km/h

$= \left(\left(1 - \frac{1}{3}\right) \times \frac{1}{2} = \frac{1}{3}\right)$ rd of the whole journey

Distance covered at 30 km/h

$= \left(1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}\right)$ rd of the whole journey

Since the distances covered with each of the three given speeds are the same, therefore

$$\text{Average speed} = \frac{3s_1 \cdot s_2 \cdot s_3}{s_1 \cdot s_2 + s_2 \cdot s_3 + s_3 \cdot s_1}$$

$$\frac{3 \times 10 \times 20 \times 30}{10 \times 20 + 20 \times 30 + 30 \times 10} = 16\frac{4}{11} = \text{km/h}.$$

Illustration – 5

A man makes his upward journey at 16 km/h and downward journey at 28 km/h. What is his average speed?

(a) 32 km/h **(b) 56 km/h**

(c) 20.36 km/h **(d) 22 km/h**

Solution :(c) Let the distance travelled during both upward and down-ward journey be x km.

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{x + x}{\frac{x}{16} + \frac{x}{28}} = \frac{2}{\frac{28+16}{28 \times 16}} = \frac{2 \times 28 \times 16}{44} = 20.36 \text{ km/h}$$

RELATIVE SPEED

Generally, when we talk about the speed of a body (A), we mean the speed of the body with respect to a stationary point (or object). In many cases, we need to determine the speed of a body (A) with respect to an independent moving body (B).

The speed of a body 'A' with respect to the independent moving body 'B' is called relative speed of the body A with respect to the body 'B'.

Formulae of Relative Speed

(i) If two bodies are moving in opposite directions at speeds s_1 and s_2 respectively, then relative speed of any one body with respect to other body is $(s_1 + s_2)$.

(ii) If two bodies are moving in the same direction at speeds s_1 and s_2 respectively, then relative speed of any one body with respect to other body is given by $s_1 - s_2$, when s_1 is greater than s_2 and $s_2 - s_1$ when s_2 is greater than s_1.

Illustration – 6

A car X starts from Delhi and another car Y starts from Moradabad at the same time to meet each other. Speed of car X is 40 km/h while speed of car Y is 50 km/h. If the distance between Delhi and Moradabad is 210 kms, when will they meet ?

Solution :

Effective speed = Relative speed = 40 + 50

= 90 km/h

Time taken = $\frac{210}{90} = 2\frac{1}{3}$ hrs.

Concept Related to Motion of Trains

Different situations in which into be find out		Formula of time taken in different situations
(i)	Time taken by a train to cross a pole, a tree or a man standing on a plateform	$= \frac{\text{Length of the train}}{\text{Speed of the train}}$
(ii)	Time taken by a train to cross a man moving in the opposite direction	$= \frac{\text{Length of the train}}{(\text{Speed of the train}) + (\text{Speed of the man})}$
(iii)	Time taken by a train to cross a man moving in the same direction	$= \frac{\text{Length of the train}}{(\text{Speed of the train}) - (\text{Speed of the man})}$
(iv)	Time taken by a train to cross a platform, bridge or tunnel	$= \frac{(\text{Length of the plateform bridge}) + (\text{Length of the train})}{\text{Speed of the train}}$
(v)	Time taken by two trains to cross each other travelling in opposite direction	$= \frac{\text{Sum of length of two trains}}{\text{Sum of speed of two trains}}$
(vi)	Time taken by a faster train to cross a slower train travelling in the same direction	$= \frac{\text{Sum of length of two trains}}{(\text{Speed of faster trains}) - (\text{Speed of slower train})}$

In the case of a train crossing a man, tree or a pole, the length of the man, tree or pole is actually its diameter (or width) which is generally considered as negligible.

Illustration – 7

A train passes an electric pole in 10 seconds and a platform 120 m long in 18 seconds. Find the length of the train.

Solution : Let the length of the train be = x m

Speed = $\frac{x}{10} = \frac{120 + x}{18}$

$\Rightarrow x = 150$

Hence length of the train = 150 m.

Illustration – 8

A train of length 100 m takes 1/6 min. to pass over another train 150 m long coming from the opposite direction. If the speed of first train is 60 km/h, then find speed of the second train.

Solution : Let speed of the second train be x km/h.

Relative speed = Sum of speed of two trains

$= (60 + x)$ km/h $= (60 + x)\frac{5}{18}$ m/s

Time = $\frac{\text{Sum of length of two trains}}{\text{Relative speed}}$

$\Rightarrow 10 = \frac{250 \times 18}{(60 + x) \times 5} \Rightarrow x = 30$ km/h.

Illustration – 9

Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

GP_3386

(a) 10 sec (b) 12 sec
(c) 14 sec (d) cannot be determined

Solution :(b) Relative speed of the trains
= (42 + 48) kmph = 90 kmph

$= \left(90 \times \frac{5}{18}\right)$ m/sec = 25 m/sec.

Time taken by the trains to pass each other
= Time taken to cover (137 + 163) m at 25 m/sec

$= \left(\frac{300}{25}\right)$ sec = 12 seconds.

Illustration – 10

A train 110 m in length travels at 60 km/h. How much time does the train take in passing a man walking at 6 km/h against the train?

(a) 6 s (b) 12 s (c) 10 s (d) 18 s

Solution :(a) Relative speeds of the train and the man

$= (60 + 6) = 66 \text{ km/h} = \frac{66 \times 5}{18}$ m/s

Distance = 110 m
Therefore, time taken in passing the men

$= \frac{110 \times 18}{66 \times 5} = 6\text{s}$

BOATS AND STREAMS

In still water, a boat moves with its own speed which is called **speed of the boat in still water (S_B).**
When the boat is moving against the flow of the water or with the flow of the water, the speed of the boat depends on the speed of flow of water [*i.e.* speed of stream (S_S)].
Speed of the boat moving against the flow of water (*i.e.*moving in upstream)
= Speed of boat in still water – Speed of stream
$= S_B - S_S$
Speed of the boat moving with flow of water (*i.e.* moving in downstream)
= Speed of boat in still water + Speed of stream
$= S_B + S_S$

$$S_s = \frac{\text{Speed in down stream} - \text{Speed in up stream}}{2}$$

Illustration – 11

A man can row a boat in downstream at 12 km/h and in upstream at 8 km/h. Find the speed of the boat that the man can row in still water.

Solution : $S_B + S_S = 12$ ...(1)
$S_B - S_S = 8$ ...(2)
On adding (1) and (2), we get
$2S_B = 20 \Rightarrow S_B = 10$
Hence speed of boat in still water = 10 km/h.

Illustration – 12

A boat covers 48 km in upstream and 72 km in downstream in 12 hours, while it covers 72 km in upstream and 48 km in downstream in 13 hours. Find the speed of the stream.

Solution : $\frac{48}{S_B - S_S} + \frac{72}{S_B + S_S} = 12$

$\frac{72}{S_B - S_S} + \frac{48}{S_B + S_S} = 13$

Let $\frac{1}{S_B - S_S} = x$ and $\frac{1}{S_B + S_S} = y$

Then $48x + 72y = 12$ ...(1)
and $72x + 48y = 13$ ...(2)
On adding (1) and (2),

$120x + 120y = 25 \Rightarrow x + y = \frac{5}{24}$ ...(3)

On subtracting (2) from (1),

$24y - 24x = -1 \Rightarrow x - y = \frac{1}{24}$ ...(4)

On adding (3) and (4),

$2x = \frac{1}{4} \Rightarrow x = \frac{1}{8} \Rightarrow S_B - S_S = 8$ ...(5)

On subtracting (4) from (3),

$2y = \frac{1}{6} \Rightarrow y = \frac{1}{12} \Rightarrow S_B + S_S = 12$ ...(6)

Subtracting (5) from (6),
$2S_S = 4 \Rightarrow S_S = 2$
Hence speed of stream = 2 km /h.

Illustration – 13

A motor boat takes 12 hours to go downstream and it takes 24 hours to return the same distance. Find the ratio of the speed of boat in still water to the speed of stream.

Solution : Distance = Speed × Time

Distance travelled in downstream

= Distance travelled in upstream

$(S_B + S_S) \times 12 = (S_B - S_S) \times 24$

$\Rightarrow S_B + S_S = 2S_B - 2S_S$

$\Rightarrow 3S_S = S_B \Rightarrow \frac{S_B}{S_S} = \frac{3}{1} \Rightarrow S_B : S_S = 3 : 1$

Hence required ratio = 3 : 1.

CIRCULAR MOTION

When two bodies start moving from a place on a circular track simultaneously in the same direction, the faster body keeps increasing the distance by which the slower body is behind the faster body. When this distance becomes equal to the circumference of the track, the faster body meets the slower body first time i.e. faster body comes in line with the slower body.

(i) When two bodies are moving in the opposite directions, their relative speed is equal to the sum of their individual speeds.

(ii) When two bodies are moving in the same direction, their relative speed is equal to the difference of the speeds of the two bodies.

First Meeting

If A and B are two runners running in opposite direction from the same point, then time taken by A and B to meet for the first time

$$= \frac{\text{Circumference of the circular track}}{\text{Relative speed}}$$

Second Meeting

If A and B are two runners running in opposite direction, then time taken by A and B to meet for the second time after first meeting

$$= \frac{\text{Circumference of the circular track}}{\text{Relative speed}}$$

Illustration – 14

The jogging track in a sports complex is 726 metres in circumference. Pradeep and his wife start from the same point and walk in opposite directions at 4.5 km/h and 3.75 km/h, respectively. They will meet for the first time in

(a) 5.5 min **(b) 6.0 min**

(c) 5.28 min **(d) 4.9 min**

Solution : (c) Let the husband and the wife meet after x minutes 4500 metres are covered by Pradeep in 60 minutes.

In x minutes, he will cover $\frac{4500}{60}x$ metres.

Similarly,

In x minutes, his wife will cover $\frac{3750}{60}x$ m.

Now, $\frac{4500}{60}x + \frac{3750}{60}x = 726$

$\Rightarrow x = \frac{726 \times 60}{8250} = 5.28 \text{ min}$

Illustration – 15

A and B run on a circular track of circumference 800 m in the opposite direction. Speeds of A and B are 50 m/s and 30 m/s respectively. Initially A and B are diametrically opposite to each other.

(i) When will they meet for the first time ?

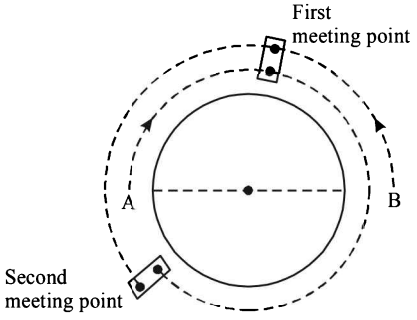
(ii) What is the time taken to meet second time after first meeting?

Solution : (i) Relative speed of A with respect to B = 50 + 30 = 80 m/s

Initially A and B are diametrically opposite to each other means B is 400 m ahead of A in the race.

Time taken by A to meet B first time = $\frac{400}{80} = 5s$

(ii) To meet second time A and B have to cover 800 m after first meeting.



Hence time taken to meet second time after first meeting = $\frac{800}{80} = 10$ seconds

GP_3386

BASIC TERMINOLOGY RELATED TO RACES

1. Startup or Head Start

When a runner allows to another runner to stay ahead in the same race, then it is said that there is a startup in the race.

For example : If A allows B to go ahead before starting the race, then it is said that A gives startup to B and B has the startup. If before starting the race B goes ahead of x metre, then we can say A gives x metre startup to B or B has startup (or headstart) of x metre.

2. Dead Heat

If all the runners reach the finishing line (or the final post) exactly at the same time, then it is said that these runners finish (or end) the race in dead heat.

Some Useful Concepts

(i) When it is said that A can give B a startup x metre in y metre race, then it means in y metre race B runs x metre less than A in the same time.

(ii) When A beats B by t second in a race of y metre then it means B is the loser and A is the winner and when A reaches the finishing line, B is still some distance back to A, from which B takes t seconds to cover the remaining distance. Hence we can calculate the speed of loser B.

(iii) The ratio of speed of the runners is always maintained throughout the race.

Illustration – 16

In 2 km race A gives a startup of 300 m to B. Despite this, A wins the race by 400 m. Find the ratio of speed of A and B.

Solution : A and B covers 2000 m and 1300 m respectively in same time intervals.

Since time period for both runners A and B are the same, hence ratio of speeds of A and B

$= 2000 : 1300 = 20 : 13$

Illustration – 17

In a 2 km race A wins over B by 200 m or 20 seconds. B can give a startup 500 m to C in 2 km race. Find out by how much time A will win over C?

Solution :

Ratio of speeds of A and B = 2000 : 1800 = 10 : 9

Ratio of speeds of B and C = 2000 : 1500 = 4 : 3

Ratio of speeds of A, B and C = 2000 : 1800 : 1350

Speed of B $= \frac{200}{20} = 10$ m/s

$\therefore$ Speed of C = 7.5 m/s [$\because$ Ratio of speed of B to C = 4 : 3]

Now C has to cover 650 m in extra time. Therefore, the time taken by C to cover the remaining distance = $\frac{650}{7.5} = 86.7$ seconds.

Hence, required time = 1 minute 27 seconds (approx)

SHORTCUTS

Shortcut Approach - 1

A man takes x hours to walk to a certain place and ride back. However, if he walks both ways he needs t hours more, then the time taken by him to ride both ways is $(x - t)$ hours.

Illustration – 18

A man takes 8 hrs to walk to a certain place and ride back. However, if he walks both ways he needs 1 hour more. How long would he have taken to ride both ways?

(a) 9 hrs **(b) 7 hrs**
(c) 6 hrs **(d) None of these**

Shortcut Solution : (b)

Time taken to ride both ways
$= (x - t)$ hours.
$= (8 - 1)$ hours. $= 7$ hours.

Shortcut Approach - 2

A person goes to a destination at a speed of x km/hr and returns to his place at a speed of y km/hr. If he takes T hours in all, the distance between his place and destination is $\left(\frac{xy}{x+y} \times T\right)$ km. In other words,

Required distance

$$= \text{Total time taken} \times \frac{\text{Product of the two speeds}}{\text{Addition of the two speeds}}$$

Illustration – 19

A boy goes to school with the speed of 3 km an hour and returns with a speed of 2 km/hr. If he takes 5 hours in all, the distance in km between the village and the school is

(a) 6 **(b) 7** **(c) 8** **(d) 9**

Shortcut Solution : (a)

$$\text{Total distance} = \frac{3\times 2}{3+2}\times 5 = 6\text{ km}$$

Shortcut Approach - 2

If a certain distance is covered at x km/hr and the same distance is covered at y km/hr, then the average speed during the whole journey is $\frac{2xy}{x+y}$ km/hr.

Illustration – 20

A man covers a certain distance by car driving at 15 km/hr and he returns back to the starting point riding on a scooter at 35 km/hr. Find his average speed for the whole journey.

(a) 20 km/hr (b) 18 km/hr
(c) 21 km/hr (d) 24 km/hr

Shortcut Solution : (c)

$$\therefore \text{Speed} = \frac{2\times 15\times 35}{(15+35)} = \frac{1050}{50} = 21\text{ km/hr}$$

Shortcut Approach - 2

If a person does a journey in T hours and the first half at S_1 km/hr and the second half at S_2 km/hr, then the distance = $\frac{2\times T\times S_1\times S_2}{S_1+S_2}$

Illustration – 21

A motor car does a journey in 6 hrs, the first half at 10 km/hr and the second half at 20 km/hr. Find the distance.

(a) 90 km (b) 80 km
(c) 60 km (d) None of these

Shortcut Solution : (b)

$$\therefore \text{Distance} = \frac{2T\times S_1\times S_2}{S_1+S_2} = \frac{2\times 6\times 10\times 20}{10+20} = 80\text{ km.}$$

Shortcut Approach - 2

A person covers a certain distance between two points. Having an average speed of x km/hr, he is late by x_1 hours. However, with a speed of y km/hr he reaches his destination y_1 hours earlier. The distance between the two points is given by $\left[\frac{xy((x_1+y_1)}{(y-x)}\right]km.$

Illustration – 22

Ram travels at the rate of 3 km/hr and he reaches 15 minutes late. If he travels at the rate of 4 km/hr, he reaches 15 minutes earlier. The distance Ram has to travel is

(a) 1 km (b) 6 km (c) 7 km (d) 12 km

Shortcut Solution : (b)

$$\therefore \text{Distance } \left[\frac{xy(x_1\times y_1)}{(y-x)}\right]\text{ km.}$$

$$= \left[\frac{3\times 4(15+15)}{(4-3)\times 60}\right]\text{ km} = \left[\frac{12\times 30}{60}\right]\text{ km} = 6\text{ km.}$$

Shortcut Approach - 2

A person walking at a speed of x km/hr reaches his destination x_1 hrs late. Next time he increases his speed by y km/hr, but still he is late by y_1 hrs. The distance of his destination from his house is given by $\left[(x_1-y_1)(x+y)\frac{x}{y}\right]$ km.

Illustration – 23

A boy walking at a speed of 45 km/hr reaches his school 10 minutes late. Next time he increases his speed 15 km/hr, but still he is late by 5 minutes. Find the distance of his school from his house.

(a) 10 km (b) 12 km (c) 25 km (d) 15 km

Shortcut Solution : (b)

$$\therefore \text{Distance} = \left[(x_1-y_1)(x+y)\frac{x}{y}\right]\text{km}$$

$$= \left[\left(\frac{10}{60}-\frac{5}{60}\right)(45+15)\frac{45}{15}\right]\text{ km} = \left[\frac{5\times 60\times 3}{60}\right]\text{ km}$$

$$= 15\text{ km}$$

Shortcut Approach - 1

If a train travelling x km an hour leaves a place and t hours later another train travelling y km an hour, where $y > x$, in the same direction, then they will be together after travelling $\left[\frac{t(xy)}{y-x}\right]$ km from the starting place.

GP_3386

Illustration – 24

A train leaves Calcutta at 7.30 am and travels 40 km an hour, another train leaves Calcutta at noon and travels 64 km an hour, when and where will the second train overtake the first?
(a) 480 km, 7.30 pm (b) 480 km, 2.30 pm
(c) 840 km, 7.30 pm (d) 480 km, 6.30 pm

Shortcut Solution : (a)

t = 12 noon – 7.30 am = $4\frac{1}{2} = \frac{9}{2}$ hrs

Now applying the given rule, we have

$$\frac{64 \times 40}{64 - 40} \times \frac{9}{2} = 480 \text{ km}$$

$$\text{Required time} = \frac{480}{64} = \frac{15}{2} = 7\frac{1}{2} \text{ hrs}$$

$\therefore$ required answer = 12 noon + $7\frac{1}{2}$ hrs = 7.30 pm.

Shortcut Approach - 2

A person has to cover a distance of x km in t hours. If he covers n^{th} part of the journey in m^{th} of the total time, then his speed should be $\left(\frac{x}{t} \times \frac{1-n}{1-m}\right)$ km/hr to cover the remaining distance in the remaining time.

Illustration – 25

A person has to cover a distance of 60 km in 15 hrs. If he covers $\frac{1}{4}$ of the journey in $\frac{2}{5}$th time, what should be his speed to cover the remaining distance in the time left?
(a) 5 km/hr (b) 9 km/hr
(c) 6 km/hr (d) 12 km/hr

Shortcut Solution : (a)

$$\therefore \text{Speed} = \left[\frac{x}{t} \times \frac{1-n}{1-m}\right] \text{ km/hr}$$

$$= \left[\frac{60}{15} \times \frac{\left(1-\frac{1}{4}\right)}{\left(1-\frac{2}{5}\right)}\right] \text{ km/hr} = 5 \text{ km/hr}$$

Shortcut Approach - 2

A thief is spotted by a policeman from a distance of d km. When the policeman starts the chase, the thief also starts running. Assuming the speed of the thief x kilometres an hour, and that of the policeman y kilometres an hour, then the thief will run before he is overtaken $= d\left[\frac{x}{y-x}\right]$ km.

Illustration – 26

A thief is spotted by a policeman from a distance of 300 metres. When the policeman starts the chase, the thief also starts running. Assuming the speed of the thief 12 kilometres an hour and that of the policeman 15 kilometres an hour, how far will have the thief run before he is overtaken?
(a) 1 km (b) 1.2 km (c) 1.5 km (d) 2 km

Shortcut Solution : (b)

$$\therefore \text{Distance} = d\left[\frac{x}{y-x}\right] \text{ km.}$$

$$= \left[\frac{300}{1000}\left[\frac{12}{15-12}\right]\right] \text{km} = \left[\frac{3}{10} \times \frac{12}{3}\right] \text{km} = 1.2 \text{ km}$$

Shortcut Approach - 10

A thief goes away with a car at a speed of x km/hr. If the theft has been discovered after 't' hours and the owner sets off in another car at y km/hr, then the owner will overtake the thief from the start in $\left(\frac{x}{y-x} \times t\right)$ hours, and the distance $\left(\frac{xyt}{y-x}\right)$ km

Illustration – 27

A thief goes away with a Maruti car at a speed of 20 km/hr. The theft has been discovered after 15 minutes and the owner sets off in another car at 30 km/hr. When will the owner overtake the thief from the start?
(a) 30 minutes (b) 20 minutes
(c) 45 minutes (d) None of these

Shortcut Solution : (a)

$$\therefore \text{Time} = \left[\frac{x}{y-x} \times t\right] \text{ hours.}$$

$$= \left[\frac{20 \times \frac{15}{60}}{30-20}\right] \text{hours} = \left[\frac{300}{600}\right] \text{hours}$$

$$= \left[\frac{300}{600}\right] \times 60 \text{ minutes, Time} = 30 \text{ minutes}$$

Shortcut Approach - 10

A person covers a certain distance on scooter. Had he moved x_1 km/hr faster, he would have taken t_1 hours less. If he had moved x_2 km/hr slower, he would have taken t_2 hours more, then the original speed (S) is $\left[\frac{x_1x_2(t_1+t_2)}{t_2x_1-t_1x_2}\right]$ km/hr and the distance is given by $\left[\frac{t_1\times S(S+x_1)}{x_1}\right]$ km.

Illustration – 28

A man covers a certain distance on scooter. Had he moved 8 km/hr faster, he would have taken 30 minutes less. If he had moved 6 km/hr slower, he would have taken 15 minutes more. Find the distance (in km) and original speed.

(a) 36 km/hr (b) 24 km/hr
(c) 18 km/hr (d) 30 km/hr

Shortcut Solution : (a)

$$\therefore \text{Speed} = \left[\frac{x_1x_2(t_1+t_2)}{t_2x_1-t_1x_2}\right] \text{ km/hr}$$

$$= \left[\frac{8\times 6\left[\frac{30}{60}+\frac{15}{60}\right]}{\frac{15}{60}\times 8-\frac{30}{60}\times 6}\right] \text{ km/hr} = -36 \text{ km/hr}$$

$$\therefore \text{Distance} = \left[\frac{t_1\times s(s+x_1)}{x_1}\right] \text{km}$$

$$= \left[\frac{\frac{30}{60}\times 36(36+8)}{8}\right] \text{ km} = 99 \text{ km.}$$

Shortcut Approach - 10

One aeroplane starts t hours later than the scheduled time from a place D km away from its destination. To reach the destination at the scheduled time the pilot has to increase the speed by 'p' km/hr. Then the plane takes $\left[\frac{\sqrt{t^2+\frac{4Dt}{p}}+t}{2}\right]$ hours in the normal case. And the normal speed of the aeroplane is $\left[\frac{2D}{\sqrt{t^2+\frac{4Dt}{p}}+t}\right]$ km/hr.

Illustration – 29

One aeroplane started 1 hour later than the scheduled time from a place 3000 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed by 500 km/hr. What was the speed of the aeroplane per hour during the journey?

(a) 1500 km/hr (b) 1000 km/hr
(c) 850 km/hr (d) None of these

Shortcut Solution : (b)

$$\therefore \text{Speed} = \left[\frac{2D}{\sqrt{t^2+\frac{4Dt}{p}}+t}\right] \text{ km/hr}$$

$$= \left[\frac{2\times 3000}{\sqrt{(1)^2+\frac{4\times 3000\times 1}{500}}+1}\right] \text{ km/hr}$$

$$= \left[\frac{6000}{\sqrt{1+24+1}}\right] = \left[\frac{6000}{6}\right] \text{ km/hr} = 1000 \text{ km/hr}$$

Shortcut Approach - 10

A goods train and a passenger train are running on parallel tracks in the same or in the opposite direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in T_1 seconds. Whereas a passenger on the passenger train marks that he crosses the goods train in T_2 seconds. If the speeds of the trains be in the ratio of a : b, then the ratio of their lengths is given by $\left[\frac{T_2}{T_1-T_2}\right]$.

Illustration – 30

A goods train and a passenger train are running on parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in 40 seconds. Whereas a passenger on

GP_3386

the passenger train marks that he crosses the goods train in 30 seconds. If the speeds of the trains be in the ratio of 1 : 2, find the ratio of their lengths.
(a) 3 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 2

Shortcut Solution : (a)

$\therefore$ Ratio of their lengths

$$=\left[\frac{T_2}{T_1-T_2}\right]=\left[\frac{30}{40-30}\right]=\frac{3}{1}=3:1$$

Shortcut Approach - 10

Two trains start at the same time from A and B and proceed towards each other at the rate of x km/hr and y km/hr respectively. When they meet it is found that one train has travelled d km more than the other. Then the distance between A and B is $\left[\frac{x+y}{x-y}\right]d$ km.

Illustration – 31

Two trains start at the same time from Ranchi and Rourkela and proceed towards each other at the rate of 85 km and 63 km per hour respectively. When they meet, it is found that one train has travelled 11 km more than the other. Find the distance between Rourkela and Ranchi.
(a) 84 km (b) 148 km (c) 74 km (d) None of these

Shortcut Solution : (c)

$\therefore$ Distance between A and B

$$=\left[\left[\frac{x+y}{x-y}\right]d\right]\text{km}=\left[\frac{85+63}{85-63}\text{d}\right]\times 11=\frac{148\times 11}{22}=74\text{km}.$$

Shortcut Approach - 10

A train meets with an accident 't_1' hours after starting, which detains it for 't' hours, after which it proceeds at $\frac{x}{y}$ of its original speed. It arrives at the destination 't_2' hours late. Had the accident taken place 'd' km farther along the railway line, the train would have arrived only 't_3' hours late. The original speed of the train is given by $\left[\frac{d\left(1-\frac{x}{y}\right)}{\frac{x}{y}(t_2-t_3)}\right]$ km/hr and the length of the trip is given by $\left(\frac{d}{t_2-t_3}\right)\left[t_2+t_1\left(\frac{y}{x}-1\right)\right]$ km.

Illustration – 32

A train meets with an accident 4 hours after starting, which detains it for 2 hours, after which it proceeds at 60% of its original speed. It arrives at the destination 5 hours late. Had the accident taken place 180 km farther along the railway line, the train would have arrived only 4 hours late. Find the length of the trip and the original speed of the train.
(a) 120 km/hr, 1830 km
(b) 120 km/hr, 1380 km
(c) 100 km/hr, 1530 km
(d) 100 km/hr, 1280 km

Shortcut Solution : (b)

$$\therefore \text{Speed} = \left[\frac{\text{d}\left[1-\frac{\text{x}}{\text{y}}\right]}{\frac{\text{x}}{\text{y}}(\text{t}_2-\text{t}_3)}\right]\text{km/hr}=\frac{180\left[1-\frac{60}{100}\right]}{\frac{60}{100}(5-4)}$$

$$=\frac{180\times 40}{60}\ 120\,\text{km/hr}$$

$$\therefore \text{Distance}=\left(\frac{d}{t_2-t_3}\right)\left[t_2+t_1\left(\frac{y}{x}-1\right)\right]\text{km}$$

$$\text{Distance}=\frac{180}{(5-4)}\times\left[5+4\left(\frac{100}{60}-1\right)\right]\text{km}$$

$$=\frac{180}{1}\left[5+4\times\frac{40}{60}\right]\text{km}=\frac{180\times 23}{3}=1380\ \text{km}$$

Shortcut Approach - 10

Two trains of the length l_1 m and l_2 m respectively with different speeds pass a static pole in t_1 seconds and t_2 seconds respectively. When they are moving in the opposite direction they will cross each other in $\left[\frac{(l_1+l_2)t_1t_2}{t_2l_1+t_1l_2)}\right]$ seconds.

Illustration – 33

Two trains of the length 150 m and 350 m respectively with different speeds pass a static pole

in 5 min and 9 min respectively. In what time will they cross each other when they are moving in the opposite direction?

(a) $\frac{225}{31}$ min (b) $\frac{235}{31}$ min

(c) $\frac{225}{21}$ min (d) None of these

Shortcut Solution : (a)

$$\therefore \text{Time} = \left[\frac{(l_1+l_2)t_1t_2}{t_2l_1+t_1l_2}\right]$$

$$= \left[\frac{(150+350)5\times 9}{9\times 150+5\times 350}\right] = \frac{22500}{3100} = \frac{225}{31} \text{ minutes}$$

Shortcut Approach - 2

Two trains of the length l_1 and l_2 m respectively with different speeds pass a static pole in t_1 seconds and t_2 seconds respectively. When they are moving in the same direction, they will cross each other in $\left[\frac{(l_1+l_2)t_1t_2}{t_2l_1-t_1l_2}\right]$ seconds.

Illustration – 34

Two trains of the length 125 m and 150 m respectively with different speeds pass a static pole in 6 seconds and 8 seconds respectively. In what time will they cross each other when they are moving in the same direction?

(a) 130 sec (b) 128 sec

(c) 132 sec (d) 136 sec

Shortcut Solution : (c)

$$\therefore \text{Time} = \left[\frac{(l_1+l_2)t_1t_2}{t_2l_1-t_1l_2}\right] = \left[\frac{(125+150)\times 6\times 8}{8\times 125-6\times 150}\right]$$

= 132 sec.

Shortcut Approach - 10

A train after travelling d_1 km meets with an accident and then proceeds at $\frac{x}{y}$ of its former speed and arrives at its destination t_1 hours late. Had the accident occurred d_2 km further, it would have reached the destination only t_2 hours late. The speed of the train is $\left[\frac{d_2\left(1-\frac{x}{y}\right)}{\frac{x}{y}(t_1-t_2)}\right]$ km/hr and the distance which train travels is $\left(d_1+\frac{d_2t_1}{t_1-t_2}\right)$ km.

Illustration – 35

A train after travelling 60 km meets with an accident and then proceeds at $\frac{2}{3}$ of its former speed and arrives at its destination 40 minutes late. Had the accident occurred 30 km further, it would have reached the destination only 20 minutes late. Find the speed of the train and the distance which the train travels.

(a) 45 km/hr, 90 km (b) 60 km/hr, 120 km

(c) 45 km/hr, 120 km (d) None of these

Shortcut Solution : (c)

$$\therefore \text{Speed} = \left[\frac{d_2\left(1-\frac{x}{y}\right)}{\frac{x}{y}(t_1-t_2)}\right] \text{ km/hr}$$

$$= \left[\frac{30\left(1-\frac{2}{3}\right)}{\frac{2}{3}\left(\frac{40}{60}-\frac{20}{60}\right)}\right] = \left[\frac{30\times\frac{1}{3}}{\frac{2}{3}\times\frac{20}{60}}\right] = \frac{10}{\frac{20}{90}} = \frac{90\times 10}{20} = 45 \text{ km/hr}$$

$$\therefore \text{Distance} = \left[d_1+\frac{d_2t_1}{t_1-t_2}\right] \text{ km.}$$

$$= \left[60+\frac{30\times 40}{40-20}\right] = 120 \text{ km.}$$

Shortcut Approach - 10

A train covers a distance between station A and B in T_1 hours. If the speed is reduced by x km/hr, it will cover the same distance in T_2 hours, then the distance between the two stations A and B is $\left(\frac{xT_1T_2}{T_2-T_1}\right)$ km and the speed of the train is given by $\left(\frac{xT_2}{T_2-T_1}\right)$ km/hr.

GP_3386

Illustration – 36

A train covers a distance between stations A and B in 3 hours. If the speed is reduced by 8 km/hr, it will cover the same distance in 6 hours. What is the distance between the two stations A and B (in km)? Also, find the speed of the train.

(a) 36 km, 12 km/hr (b) 40 km, 15 km/hr
(c) 48 km, 16 km/hr (d) Data inadequate

Shortcut Solution : (c)

$$\therefore \text{Distance} = \left(\frac{xT_1T_2}{T_2 - T_1}\right) \text{ km.}$$

$$= \left(\frac{8\times3\times6}{6-3}\right) \text{km} = 48 \text{ km.}$$

$$\therefore \text{Speed} = \left(\frac{xT_2}{T_2 - T_1}\right) = \left(\frac{8\times6}{6-3}\right) = 16 \text{ km/hr}$$

Shortcut Approach - 10

Two trains *A* and *B* start from *P* and *Q* towards *Q* and *P* respectively. After passing each other they take T_1 hours and T_2 hours to reach *Q* and *P* respectively. If the train from P is moving x km/hr, then the speed of the other train is $\left(x\sqrt{\frac{T_1}{T_2}}\right)$ km/hr.

Illustration – 37

Two trains *A* and *B* start from Lucknow and Delhi towards Delhi and Lucknow respectively. After passing each other they take 4 hours and 9 hours to reach Delhi and Lucknow respectively. If the train from Lucknow is moving at 60 km/hr then find the speed of the other train.

(a) 40 km/hr (b) 30 km/hr
(c) 35 km/hr (d) 50 km/hr

Shortcut Solution : (a)

$$\therefore \text{Speed} = \left(x.\sqrt{\frac{T_1}{T_2}}\right) \text{ km/hr}$$

$$= \left(60\times\sqrt{\frac{4}{9}}\right) \text{km/hr} = \left(60\times\frac{2}{3}\right) = 40 \text{ km/hr}$$

Shortcut Approach - 10

A man rows a certain distance downstream in *x* hours and returns the same distance in *y* hrs. If the stream flows at the rate of *z* km/hr then the speed of the man in still water is given by $\left[\frac{z(x+y)}{y-x}\right]$.

Illustration – 38

Rohit can row a certain distance downstream in 8 hours and return the same distance in 12 hours. If the stream flows at the rate of 5 km per hour find the speed of Rohit in still water.

(a) 20 km/hr (b) 30 km/hr
(c) 15 km/hr (d) 25 km/hr

Shortcut Solution : (d)

$$\therefore \text{Speed} = \left[\frac{z(x+y)}{y-x}\right] \text{ km/hr.}$$

$$= \left[\frac{5(8+12)}{(12-8)}\right] \text{km/hr.} = \left[\frac{5\times20}{4}\right] = 25 \text{ km/hr.}$$

Shortcut Approach - 10

If x km be the rate of stream and a man takes *n* times as long to row up as to row down the river, then the rate of the man in still water is given by

$$x\left[\frac{n+1}{n-1}\right] \text{ km/hr.}$$

Illustration – 39

A man can row 4 km/hr in still water and he finds that it takes him twice as long to row up as to row down the river. Find the rate of stream.

(a) 1.3 km/hr (b) 2 km/hr
(c) 1 km/hr (d) 1.5 km/hr

Shortcut Solution : (a)

$$\therefore \text{Speed} = x\left[\frac{n+1}{n-1}\right] \text{ km/hr}$$

$$4 \text{ km/hr} = x\left[\frac{2+1}{2-1}\right] = x = \frac{4}{3} \text{ km/hr.} = 1.3 \text{ km/hr.}$$

Shortcut Approach - 10

A man can row x km/hr in still water. If in a stream which is flowing at y km/hr, it takes him z hrs to row to a place and back, the distance between the two places is $\left(\frac{z(x^2-y^2)}{2x}\right)$.

Illustration – 40

A man can row 5 km per hour in still water. If the river is running at 1 km an hour, it takes him 75 minutes to row to a place and back. The place is at a distance of ____ km from the starting point.

(a) 3 km (b) 4 km (c) 5 km (d) 2 km

Shortcut Solution : (a)

$$\therefore \text{Distance} = \left(\frac{z(x^2-y^2)}{2x}\right) \text{km}$$

$$= \left(\frac{75}{60}\frac{(15)^2-(1)^2}{2\times 5}\right) = \frac{75\times 24}{60\times 10} = 3 \text{ km}$$

Shortcut Approach - 10

If a man can row at a speed of x km/hr in still water to a certain upstream point and back to the starting point in a river which flows at y km/hr, then the average speed for total journey (up + down) is given by $\left(\frac{(x+y)(x-y)}{x}\right)$ km/hr.

Illustration – 41

A man row at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river which flows at 4 km/hr. Find his average speed for total journey.

(a) 8 km/hr (b) 6 km/hr
(c) 4 km/hr (d) 10 km/hr

Shortcut Solution : (b)

$$\therefore \text{Speed} = \left[\frac{(x+y)(x-y)}{x}\right] \text{km/hr}$$

$$= \left[\frac{(8+4)(8-4)}{8}\right] \text{km/hr} = 6 \text{ km/hr}$$

Shortcut Approach - 10

In 'd' metre race, A runs at 'x' km/hr. A gives B a start of y metres and still beats him by 't' seconds, then the speed of B is given by $\frac{18x(d-y)}{18d+5xt}$ km/hr.

Illustration – 42

In a km race, A runs at 5 km per hour. If A gives B a start of 90 metres and still beats him by 8 seconds, what is the speed of B?

(a) 4 km/hr (b) 5 km/hr
(c) 5.5 km/hr (d) 4.5 km/hr

Shortcut Solution : (d)

$$\therefore \text{Speed} = \left[\frac{18x(d-y)}{18d+5xt}\right] \text{km/hr}$$

$$= \left[\frac{18\times 5(1000-90)}{18\times 1000+5\times 5\times 8}\right] \text{km/hr}$$

$$= \frac{90\times 910}{18000+200} = \frac{9}{2} = 4.5 \text{ km/hr}$$

GP_3386

LEVEL - 1

1. A certain distance is covered by a train with a certain speed. If half the distance is covered in double time, then the ratio of this speed to that of the original one is
(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1

2. A salesman travels a distance of 50 km in 2 hours and 30 minutes. How much faster, in kilometres per hour, on an average, must he travel to make such a trip in $\frac{5}{6}$ hour less time?
(a) 10 (b) 20
(c) 30 (d) None of these

3. If a man travels at 30 km/h, he reaches his destination late by 10 minutes but if he travels at 42 km/h then he reaches 10 minutes earlier. The distance travelled by him is
(a) 30km (b) 35km (c) 45km (d) 36km

4. Two trains starting at the same time from two stations, 200 km apart and going in opposite directions, cross each other at a distance of 110 km from one of them. What is the ratio of their speeds?
(a) 11 : 20 (b) 9 : 20
(c) 11 : 9 (d) 19 : 20

5. Two runner start running together for a certain distance, one at 8 km/h and another at 5 km/h. The former arrives one and half an hour, before the latter. The distance (in km) is:
(a) 12 (b) 20
(c) 25 (d) 36

6. A person travels equal distances with speeds of 3km/hr, 4 km/hr and 5km/hr and takes a total time of 47 minutes. The total distance (in km) is:
(a) 2 (b) 3 (c) 4 (d) 5

7. A and B travel the same distance at 9 km/h and 10 km/h respectively. If A takes 20 minutes longer than B, the distance travelled by each is:
(a) 16 (b) 20
(c) 30 (d) None of these

8. A gun is fired at a distance of 3.32 km from Chauhan. He hears its sound 10 seconds later. Find the speed of the sound.
(a) 301 m/s (b) 302 m/s
(c) 332 m/s (d) 340 m/s

9. Excluding stoppages, the speed of a train is 45 km/h and including stoppages, it is 36 km/h. For how many minutes does the train stop per hour ?
(a) 10 min. (b) 12 min.
(c) 15 min. (d) 18 min.

10. A train 300 m long is running at a speed of 90 km/hr. How many seconds will it take to cross a 200 m long train running in the opposite direction at a speed of 60 km/hr?
(a) $7\frac{1}{5}$ (b) 60 (c) 12 (d) 20

11. A boat goes 24 km upstream and 28 km downstream in 6 hours. It goes 30km upstream and 21 km downstream in 6 hours and 30 minutes. The speed of the boat in still water is :
(a) 10 km/h (b) 4 km/h (c) 14 km/h (d) 6 km/h

12. A clock gains 15 minutes per day. It is set right at 12 noon. What time will it show at 4.00 am, the next day?
(a) 4 : 10 am (b) 4 : 45 am
(c) 4 : 20 am (d) 5 : 00 am

13. A travels from B to C, a distance of 250 km, in 5.5 hours. He returns to B in 4 hours 40 minutes. His average speed is
(a) 44 (b) 46 (c) 48 (d) 50

14. A train covers 180 km distance in 4 hours. Another train covers the same distance in 1 hour less. What is the difference in the distances covered by these trains in one hour ?
(a) 45 km (b) 9 km
(c) 40 km (d) None of these

15. A person travels from P to Q at a speed of 40 kmph and returns by increasing his speed by 50%. What is his average speed for both the trips?
(a) 36 kmph (b) 45 kmph
(c) 48 kmph (d) 50 kmph

16. A constable is 114 meters behind a thief. The constable runs 21 meters per minute and the thief runs 15 meters in a minute. In what time will the constable catch the thief ?

(a) 19 minutes (b) 18 minutes
(c) 17 minutes (d) 16 minutes

17. A dog starts chasing to a cat 2 hours later. It takes 2 hours to dog to catch the cat. If the speed of the dog is 30 km/h, what is the speed of cat?
(a) 10 km/h (b) 15 km/h
(c) 20 km/h (d) Can't be determined

18. A man can row 4.5 km/hr in still water and he finds that it takes him twice as long to row up as to row down the river. Find the rate of the stream.
(a) 1.5 km/hr (b) 2 km/hr
(c) 2.5 km/hr (d) 1.75 km/hr

19. An accurate clock shows 8 o'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 o'clock in the afternoon?
(a) 144° (b) 150°
(c) 168° (d) 180°

20. A man walks a certain distance and rides back in $6\frac{1}{4}$ h. He can walk both ways in $7\frac{3}{4}$ h. How long it would take to ride both ways ?
(a) 5 hours (b) $4\frac{1}{2}$ hours
(c) $4\frac{3}{4}$ hours (d) 6 hours

21. I walk a certain distance and ride back and take $6\frac{1}{2}$ hours altogether. I could walk both ways in $7\frac{3}{4}$ hours. How long would it take me to ride both ways? **(SSC CGL Tirer I-2014)**
(a) 4 hr 25 min (b) 5 hr 25 min
(c) 4 hr 15 min (d) 5 hr 15 min

22. A man rode out a certain distance by train at the rate of 25 km per hour and walked back at the rate of 4 km per hour. The whole journey took 5 hours 48 minutes. What distance did he ride? **(RBI 2013)**
(a) 20 km (b) 29 km (c) 24 km (d) 25 km

23. Sahil covers a certain distance by car driving at 35 km/h and he returns back to the starting point riding on a scooter with a speed of 25 km/hr. Find the average speed for the whole journey. **(FCI 2013)**
(a) 57.1 km/hr (b) 16.41 km/hr
(c) 29.16 km/hr (d) 23.14 km/hr

24. Shantanu covers a certain distance in 84 min by covering $\frac{2}{3}$rd of distance at 4 km/hr and the rest at 5 km/hr. Find the total distance. **(SSC CHSL 2012)**
(a) 6 km (b) 8 km
(c) 9 km (d) 15 km

25. If I walk at 4 km/h, I miss the bus by 10 minutes. If I walk at 5 km/h, I reach 5 minutes before the arrival of the bus. How far I walk to reach the bus stand ?
(a) 5 km (b) 4.5 km
(c) $5\frac{1}{4}$ km / h (d) Cannot be determined

26. Two rifles are fired from the same place at a difference of 11 minutes 45 seconds. But a man who is coming towards the place in a train hears the second sound after 11 minutes. Find the speed of train.
(a) 72 km/h (b) 36 km/h
(c) 81 km/h (d) 108 km/h

27. A and B can run 200 m in 22 seconds and 25 seconds, respectively. How far is B from the finishing line when A reaches in ?
(a) 8m (b) 12m (c) 16m (d) 24m

28. A is twice as fast as B and B is trice as fast as C is the journey covered by C in 1½ hours will be covered by A in
(a) 15 minutes (b) 2 minutes
(c) 30 minutes (d) 1 hour

29. The speed of a boat in still water is 15 km/h and the rate of stream is 5 km/h. The distance travelled downstream in 24 minutes is
(a) 4 km (b) 8 km
(c) 6 km (d) 16 km

30. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water (in litres) will fall into the sea in a minute?
(a) 4,00,000 (b) 40,00,000
(c) 40,000 (d) 4,000

GP_3386

31. A man makes his upward journey at 16 km/h and downward journey at 28 km/h. What is his average speed ?
(a) 32 km/h (b) 56 km/h
(c) 20.36 km/h (d) 22 km/h

32. Walking $\frac{6}{5}$ of his usual speed, a person takes 10 min less to reach his office. His usual time taken to reach the office is
(a) 50 min (b) 55 min
(c) 60 min (d) None of these

33. A car travels 25 km an hour faster than a bus for a journey of 500 km. If the bus takes 10 hours more than the car, then the speeds of the bus and the car are
(a) 25 km/h and 40 km/h respectively
(b) 25 km/h and 60 km/h respectively
(c) 25 km/h and 50 km/h respectively
(d) None of these

34. A bus covers a certain distance in 16 hours. It covers half the distance at 40 km/hr and the rest at 60 km/hr. Find the length of the journey.
(CDS 2010)
(a) 528 km (b) 408 km
(c) 314 km (d) 768 km

35. A man riding a bicycle from his house at 10 km/ hr and reaches his office late by 6 min. He increases his speed by 2 km/hr and reaches 6 min before. How far is the office from his house?
[SSC CGL 2012]
(a) 6 km (b) 7 km
(c) 12 km (d) 16 km

36. If a train runs at 40 km/hr, it reaches its destination late by 11 min, but if it runs at 50 km/hr, it is late by 5 min only. Find the correct time for the train to complete its journey. **[SSC FCI 2012]**
(a) 19 min (b) 20 min
(c) 21 min (d) 18 min

37. Amit walks at a uniform speed of 4 km/hr and 4 hr after his start Brijesh cycles after him at the uniform rate of 20 km/hr. How far from the starting point will Brijesh catch Amit?
[SSC CHSL 2010]
(a) 15 km (b) 18 km
(c) 13 km (d) 20 km

38. If a man walks to his office at 3/4 of his usual rate, he reaches office 1/3 of an hour later than usual. What is his usual time to reach office.
(a) $\frac{1}{2}$ hr (b) 1 hr
(c) $\frac{3}{4}$ hr (d) None of these

39. R and S start walking each other at 10 am at the speeds of 3 km/h and 4 km/h respectively. They were initially 17.5 km apart. At what time do they meet?
(a) 2 : 30 pm (b) 11 : 30 am
(c) 1 : 30 pm (d) 12 : 30 pm

40. A man is walking at a speed of 10 km per hour. After every kilometre, he takes rest for 5 minutes. How much time will he take to cover a distance of 5 kilometres?
(a) 48 min. (b) 50 min.
(c) 45 min. (d) 55 min.

41. Cars C_1 and C_2 travel to a place at a speed of 30 and 45 km/h respectively. If car C_2 takes $2\frac{1}{2}$ hours less time than C_1 for the journey, the distance of the place is
(a) 300 km (b) 400 km
(c) 350 km (d) 225 km

42. A man walking at the rate of 5 km/h crosses a bridge in 15 minutes. The length of the bridge (in metres) is :
(a) 600 (b) 750 (c) 1000 (d) 1250

43. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/h of the cyclist ?
(a) 18 km/h (b) 15 km/h
(c) 20 km/h (d) None of these

44. A 200 m long train passes a 350 m long platform in 5s. If a man is walking at a speed of 4 m/s along the track and the train is 100 m away from him, how much time will it take to reach the man?
(a) Less than 1s (b) 1.04 s
(c) More than 2s (d) Data insufficient

45. A man can row against the current three-fourth of a kilometer in 15 minutes and returns the same distance in 10 minutes. The ratio of his speed to that of current is
(a) 3 : 5 (b) 5 : 3 (c) 1 : 5 (d) 5 : 1

46. A motor boat runs at a speed of 36 km/h in still water. It goes 56 km upstream in 1 hour 45 min. How much time will it take to cover same distance downstream?
(a) 2 hours 25 min (b) 3 hours
(c) 1 hour 24 min (d) 2 hours 21 min

47. A sailor goes 2 km upstream in one hour and 1 km downstream in 10 minutes . In how much time will he go 5 km in still water?
(a) 60 min (b) 50 min
(c) 75 min (d) 90 min

48. A person can reach a certain place in 30 hours. If the reduces his speed by $\frac{1}{15}$ he goes 10 km less in that time. What is speed in km/h?
(a) 6 km/h (b) 5.5 km/h
(c) 4 km/h (d) 5 km/h

49. A beats B by 100 m in a race of 1200 m and B beats C by 200 m in a race of 1600 m. Approximately by how many metres can A beat C in a race of 9600 m?
(a) 1600 m (b) 1800 m
(c) 1900 m (d) 2400 m

50. Mallah can row 40 km upstream and 55 km downstream in 13 hours and 30 km upstream and 44 km downstream in 10 hours. What is the speed of Mallah in still water?
(a) 6 km/h (b) 12 km/h (c) 3 km/h (d) 8 km/h

51. If Deepesh had walked 20 km/h faster he would have saved 1 hour in the distance of 600 km. What is the usual speed of Deepesh?
(a) 100 (b) 120
(c) 150 (d) None of these

52. A bullockcart has to cover a distance of 80 km in 10 hr. If it covers half of the journey in $\frac{3}{5}$ th time, what should be its speed to cover the remaining distance in the left time? **[SSC FCI 2010]**
(a) 5 km (b) 10 km
(c) 15 km (d) 20 km

53. A thief is spotted by a policeman from a distance of 200 m. When the policeman starts chasing, the thief also starts running. If the speed of the thief be 16 km/hr and that of the policeman be 20 km/hr. How far the thief will have run before he is overtaken? **[CDS 2011]**
(a) 800 m (b) 900 m
(c) 1000 m (d) 700 m

54. A train covered a certain distance at a uniform speed. If the train had been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, the train would have taken 6 hours more than the scheduled time. The length of the journey is: **[SSC CGL Tier-II 2013]**
(a) 700 km (b) 720 km
(c) 740 km (d) 760 km

55. From stations M and N, two trains start moving towards each other at speed 125 km/hr and 75 km/hr, respectively. When the two trains meet each other, it is found that one train covers 50 km more than another. Find the distance between *M* and *N*. **[RBI 2010]**
(a) 190 km (b) 200 km
(c) 145 km (d) 225 km

56. A goods train and a passenger train are running on the parallel tracks in the same direction. The driver of the goods train observes that the passenger train coming from behind overtakes and crosses his train completely in 1 min whereas a passenger on the passenger train marks that he crosses the goods train in $\frac{2}{3}$ min. If the speeds of the train is in the ratio of 1 : 2, then find the ratio of their lengths. **[SSC CGL 2010]**
(a) 4 : 1 (b) 3 : 1 (c) 1 : 4 (d) 2 : 1

LEVEL - 2

1. Two trains of equal length running in opposite directions, pass a student standing by the side of railway line in 18 sec, and 12 sec respectively. The time that the two trains take to cross each other, is **[SSC CGL Tier I 2013]**
(a) 7.2 sec (b) 9.6 sec
(c) 10.8 sec (d) 14.4 sec

2. A man standing on a railway platform notices that a train going in one direction takes 6 seconds to pass him and other train of the same length takes 9 seconds to pass him. Find the time taken by the two trains to cross each other when they are running in the same direction. **[BANK PO 2014]**

(a) 12 seconds (b) 24 seconds
(c) 36 seconds (d) 48 seconds

3. A train left station A for station B at a certain speed. After travelling for 100 km, the train meets with an accident and could travel at $\frac{4}{5}$th of the original speed and reaches 45 minutes late at station B. Had the accident taken plain 50 km further on, it would have reached 30 minutes late at station B. What is the distance between station A and B? **[BANK PO 2015]**
(a) 100km (b) 150km
(c) 200km (d) 250km

4. One aeroplane started 30 minutes later than the scheduled time from a place 1500 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed by 250 km/hr. What was the speed of the aeroplane per hour during the journey? **[BANK PO 2016]**
(a) 750km/hr (b) 500km/hr
(c) 350km/hr (d) 1000km/hr

5. Two trains each of 120 m in length, run in opposite directions with a velocity of 40 m/s and 20 m/s respectively. How long will it take for the tail ends of the two trains to meet each other during the course of their journey?
(a) 20 s (b) 3 s (c) 4 s (d) 5 s

6. A car driver travels from the plains to a hill station, which are 200 km apart at an average speed of 40 km/h. In the return trip he covers the same distance at an average speed of 20 km/h. The average speed of the car over the entire distance of 400 km is
(a) 16.56km/h (b) 17.89km/h
(c) 26.67km/h (d) 35km/h

7. In a 800 m race around a stadium having the circumference of 200 m, the top runner meets the last runner on the 5th minute of the race. If the top runner runs at twice the speed of the last runner, what is the time taken by the top runner to finish the race ?
(a) 20min (b) 15min (c) 10min (d) 5min

8. If a train runs at 40 kmph, it reaches its destination late by 11 minutes but if it runs at 50 kmph, it is late by 5 minutes only. The correct time for the train to complete its journey is:
(a) 13min. (b) 15min. (c) 19min. (d) 21min.

9. A thief goes away with a Maruti car at a speed of 40 km/h. The theft has been discovered after half an hour and the owner sets off in another car at 50 km/h. When will the owner overtake the thief from the start.
(a) $2\frac{1}{2}$ hours
(b) 2hr 20min
(c) 1hr 45min
(d) Cannot be determined

10. Two trains are moving in opposite directions at speeds of 60 km/hour and 90 km/hour. Their lengths are 1.10 km and 0.9 km respectively. The time taken by the slower train to cross the faster train in seconds is
(a) 36 (b) 49 (c) 45 (d) 48

11. A man takes 5 hour 45 min. in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. The time he would take to walk both ways, is
(a) 3 hrs 45 min (b) 7 hrs 30 min
(c) 7 hrs 45 min (d) 11 hrs 45 min

12. A boatman rows to a place 45 km distant and back in 20 hours. He finds that he can row 12 km with the stream in same time as 4 km against the stream . Find the speed of the stream.
(a) 3 km/hr
(b) 2.5 km/hr
(c) 4 km/hr
(d) Cannot be determined

13. A student rides on a bicycle at 8 km/h and reaches his school 2.5 minutes late. The next day he increases his speed to 10 km/h and reaches the school 5 minutes early. How far is the school from his house?
(a) 1.25km (b) 8km (c) 5km (d) 10km

14. A passenger sitting in a train of length 100 m, which is running with speed of 60 km/h passing through two bridges, notices that he crosses the first bridge and the second bridge in time intervals which are in the ratio of 7 : 4 respectively. If the length of first bridge be 280 m, then the length of second bridge is:
(a) 490m

(b) 220m
(c) 160m
(d) Can't be determined

15. A dog sees a cat. It estimates that the cat is 25 leaps away. The cat sees the dog and starts running with the dog in hot pursuit. If in every minute, the dog makes 5 leaps and the cat makes 6 leaps and one leap of the dog is equal to 2 leaps of the cat. Find the time in which the cat is caught by the dog (assume an open field with no trees)
(a) 12 minutes (b) 15 minutes
(c) 12.5 minutes (d) None of these

16. A man covers a certain distance on a toy train. If the train moved 4 km/h faster, it would take 30 minutes less. If it moved 2 km/h slower, it would have taken 20 minutes more. Find the distance.
(a) 60km (b) 58km
(c) 55km (d) 50km

17. Two trains, 130 m and 110 m long, are going in the same direction. The faster train takes one minute to pass the other completely. If they are moving in opposite directions, they pass each other completely in 3 seconds. Find the speed of each train.
(a) 38 m/sec, 36 m/sec (b) 42 m/sec, 38 m/sec
(c) 36 m/sec, 42 m/sec (d) None of these

18. Two people A and B start from P and Q (distance = D) at the same time towards each other. They meet at a point R, which is at a distance 0.4 D from P. They continue to move to and fro between the two points. Find the distance from point P at which the fourth meeting takes place.
(a) 0.8D (b) 0.6D (c) 0.3D (d) 0.4D

19. The metro service has a train going from Mumbai to Pune and Pune to Mumbai every hour, the first one at 6 a.m. The trip from one city to other takes 4½ hours, and all trains travel at the same speed. How many trains will you pass while going from Mumbai to Pune if you start at 12 noon?
(a) 8 (b) 10 (c) 9 (d) 13

20. It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the speed of the car is :
(a) 4 : 3 (b) 3 : 4 (c) 3 : 2 (d) 2 : 3

21. Points A and B are 70 km apart on a highway. One car starts form A and the another one from B at the same time. If they travel in the same direction, they meet in 7 hours. But if they travel towards each other, they meet in one hour. The speeds of the two cars are, respectively.
(a) 45 and 25 km/h (b) 70 and 10 km/h
(c) 40 and 30 km/h (d) 60 and 40 km/h

22. A train M leaves Meerut at 5 am and reaches Delhi at 9 am Another train N leaves Delhi at 7 a.m. and reaches Meerut at 10 : 30 am. At what time do the two trains cross each other?
(a) 7:36am (b) 7 : 56am
(c) 8am (d) 8 : 26am

23. In a stream, running at 2 km/hr, a motorboat goes 10 km upstream and back again to the starting point in 55 minutes. Find the speed of the motorboat in still water. **[IBPS Clerk 2011]**
(a) 22 km/hr (b) 12 km/hr
(c) 6 km/hr (d) 16 km/hr

24. Ramesh can row a certain distance downstream in 6 hours and return the same distance in 9 hours. If the speed of Ramesh in still water be 12 km/hr, find the speed of the stream. **[RBI 2011]**
(a) 2.2 km/hr (b) 3 km/hr
(c) 2.4 km/hr (d) 3.2 km/hr

25. Speed of a boat is 5 km/hr in still water and the speed of the stream is 3 km/hr. If the boat takes 3 hours to go to a place and come back, the distance of the place is: **[SSC CHSL 2011]**
(a) 3.75km (b) 4km
(c) 4.8km (d) 4.25km

26. In a 400 m race, A runs at a speed of 16 m/s. If A gives B a start of 16 m and still beats him by 40s, what will be the speed of B? **[CDS 2012]**
(a) 6 m/s (b) 8 m/s
(c) 15 m/s (d) 15.9 m/s

27. A train of 300 m is travelling with the speed of 45 km/h when it passes point A completely. At the same time, a motorbike starts from point A with the speed of 70 km/h. When it exactly reaches the middle point of the train, the train

GP_3386

increases its speed to 60 km/h and motorbike reduces its speed to 65 km/h. How much distance will the motorbike travel while passing the train completely?
(a) 2.52km (b) 2.37km
(c) 2km (d) None of these

28. Two riders on the horseback with a gun and a bullet proof shield were moving towards each other at a constant speed of 20 km/h and 5 km/h respectively. When they were 100 km apart, they started firing bullets at each other at the speed of 10 km/h. When a bullet of rider 1 hits the shield of rider 2, rider 2 fires a bullet and the process continues vice-versa. Neglecting the time lag at the instant when the bullet hits the shield and the rider fires the shot, find the total distance covered by all the bullets shot by both the riders.
(a) 50km (b) 40km
(c) 25km (d) None of these

Directions (29 and 30) : There is a race between Sagar and Sapna. Both of them Delhi. Sagar started on a bike with a speed of 40 km/h and Sapna has started in a car with a speed of 60 km/h from Mumbai to Delhi. After $3\frac{1}{2}$ hours of the journey there was a snag in the car. She tried to repair the car but in vain. After half an hour she got a lift for another 500 km in a truck, which was travelling with a speed of 45 km/h. From there Delhi was at a distance of 200 km on road, instead, Sapna took a shorter route which was only 100 km away from Delhi. She started running at the speed of 30 km/h on the shorter route to reach Delhi.

29. If there was no sang in the car, by how much distance Sapna would have defeated Sagar?
(a) 252km (b) 264km
(c) 303km (d) 321km

30. How much time did Sapna take to reach Delhi?
(a) 15.5h (b) 16.5h
(c) 17.5h (d) None of these

31. Two places A and B are 100 km a part on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at constant speed, they meet in 5 hours. If the cars travel towards each other, they meet in one hour. What is the speed of the car running faster?
(a) 60km/h (b) 50km/h
(c) 40km/h (d) 32km/h

32. Two cities A and B are at a distance of 500 km. A train departs from A to B at 8 O'clock in the morning at a speed of 70 km/h. An another train runs from B to A at 10 O' clock at a speed of 110 km/h. When will both the train meet?
(a) 1 O ' clock afternoon
(b) 12 O' clock noon
(c) 12.30 O ' clock afternoon
(d) 1.30 O' clock afternoon

33. A boat covers 48 km upstream and 72 km downstream in 12 hours, while it covers 72 km upstream and 48 km downstream in 13 hours. The speed of stream is :
(a) 2km/h (b) 2.2km/h
(c) 2.5km/h (d) 4km/h

34. A gives both B and C a start of 60 m in a 1500 m race. However, while B finishes with him, C is 15 m behind them when A and B cross the finishing line. How much start can B give C for the 1500 m race course?
(a) $7\frac{6}{23}$m (b) $15\frac{5}{8}$m
(c) $7\frac{11}{16}$m (d) $5\frac{5}{24}$m

35. Two trains whose respective lengths are 200 m and 250 m cross each other in 18 s, when they are travelling in opposite direction and in 1 min, when they are travelling in the same direction. What is the speed of the faster train (in km/h)?
(a) 38.5 (b) 48.5 (c) 54 (d) 58.5

36. The driver of an ambulance sees a school bus 40 m ahead of him. After 20 second, the school bus is 60 metre behind. If the speed of the ambulance is 30 km/h, what is the speed of the school bus?
(a) 10km/h (b) 12km/h
(c) 15km/h (d) 22km/h

Directions (37 – 42) : A person P is at X and another person Q is at Y. The distance between X and Y is 100 km. The speed of P is 20 km/h. While the speed of Q is 60 km/h?

37. If they first time meet at point Z somewhere between X and Y. then the distance between X and Z is :
(a) 20km (b) 40km (c) 25km (d) 30km

38. If they continue to move to and fro between X

and Y, then what is the distance covered by P when they meet second time?
(a) 105 km (b) 100 km
(c) 80 km (d) 75 km

39. If they continue to move to and fro between X and Y, then what is the distance travelled by Q, when they meet each other for the third time?
(a) 375 km (b) 225 km
(c) 350 km (d) 445 km

40. If P and Q continue to move between X and Y in the given manner and if they meet for the fourth time at a place M somewhere between X and Y, then the distance between X and M is :
(a) 10 km (b) 90 km
(c) 75 km (d) 25 km

41. If P and Q continue to move between X and Y, then the ratio of distances covered by P and Q when they meet for the 5th time?
(a) 1 : 4 (b) 1 : 3 (c) 2 : 3 (d) 3 : 4

42. If P and Q continue to move between X and Y, then the distance covered by P and Q together between any two consecutive meeting?
(a) 100
(b) 300
(c) 200
(d) can't be determined

43. A boat goes 6 km an hour in still water. It takes thrice as much time going upstream as much time going downstream. The speed of the current is:
[SSC CGL Tier I 2013]
(a) 4 km/hr (b) 5 km/hr
(c) 3 km/hr (d) 2 km/hr

44. Two trains *x* and *y* start from Mumbai and Delhi towards Delhi and Mumbai, respectively. After passing each other, they take 12 hours 30 minutes and 8 hours to reach Delhi and Mumbai, respectively. If the train from Mumbai is moving at 60 km/hr, then find the speed of the other train. **[CDS – 2015]**
(a) 75 km/hr (b) 68 km/hr
(c) 72 km/hr (d) 62 km/hr

45. A train left station A for station B at a certain speed. After travelling for 100 km, the train meets with an accident and could travel at $\frac{4}{5}$th of the original speed and reaches 45 minutes late at station B. Had the accident taken place 50 km further on, it would have reached 30 minutes late at station B. What is the distance between station A and B? **[SSC CGL Tier-I-2014]**
(a) 200 km (b) 250 km
(c) 300 km (d) 350 km

46. A train can complete its journey from P to Q in 6 hrs with its uniform speed. If speed of train is reduced by 20 km/h it takes 2 hrs more to reach from P to Q. What is distance between P and Q? Also find the speed of train?
[SSC CGL Tier-I-2017]
(a) 420 km, 70 km/h (b) 360 km, 60 km/h
(c) 240 km, 40 km/h (d) 480 km, 80 km/h

GP_3386

Hints and Solutions

Level 1

1. (a) Let a distance x be covered in time t.

$$\text{Required ratio} = \frac{x/2}{\frac{2t}{t}} = \frac{1}{4} = 1:4$$

2. (a) Time required = (2 hrs 30 min – 50 min)

= 1hr 40 min = $1\frac{2}{3}$ hrs

$\therefore$ Required speed = $\left(50 \times \frac{3}{5}\right)$ km/hr

= 30 km/hr.

Original speed = $\left(50 \times \frac{2}{5}\right)$ km/hr = 20 km/hr.

$\therefore$ Difference in speed = (30 – 20) km/hr

= 10 km/hr.

3. (b) Let the distance travelled be x km.

Then, the correct time at a speed of 30 km/h

$= \frac{x}{30} - \frac{10}{60}$ and the correct time at a speed

of 42 km/h = $\frac{x}{42} + \frac{10}{60}$

Now, $\frac{x}{30} - \frac{10}{60} = \frac{x}{42} + \frac{10}{60}$

or $\frac{x}{30} - \frac{x}{42} = \frac{2}{6}$ or $\frac{12x}{1260} = \frac{2}{6}$ or $x = 35$ km

4. (c) Let the speed of trains be x km/h and y km/h, respectively.

When the trains cross each other, time taken by both the trains will be equal.

i.e. $\frac{110}{x} = \frac{90}{y} \Rightarrow \frac{x}{y} = \frac{110}{90} \Rightarrow x : y = 11:9$

5. (b) Required distance = $\frac{S_1 S_2}{(S_1 - S_2)} \times$ Time difference

$= \frac{8 \times 5}{3} \times \frac{3}{2} = 20$ km

6. (b) Let the total distance be $3x$ km.

Then, $\frac{x}{3} + \frac{x}{4} + \frac{x}{5} = \frac{47}{60} \Leftrightarrow \frac{47x}{60} = \frac{47}{60} \Leftrightarrow x = 1$.

$\therefore$ Total distance = (3 × 1) km = 3 km.

7. (c) $\frac{x}{9} - \frac{x}{10} = \frac{20}{60}$

or, $\frac{10x - 9x}{90} = \frac{20}{60} \Rightarrow x = 30$ km

8. (c) Distance between Chauhan and the gun

= 3.32 × 1000 = 3320 m

Time taken = 10 s

$\Rightarrow$ Speed = $\frac{3320}{10} = 332$ m/s

9. (b) Due to stoppges the train travels (45 – 36) = 9 km less in an hour than it could have travelled without stoppages.

Thus train stops per hour for

$\frac{9}{45} \times 60 = 12$ min.

10. (c) Relative speed = 90 + 60 = 150 km/hr.

Total distance to be covered

= 300 + 200 = 500 m

Time required = $\frac{500}{150 \times 1000} \times 3600 = 12$ sec.

11. (a) Let speed of the boat in still water be x km/h and speed of the current be y km/h.

Then, upstream speed = $(x - y)$ km/h

and downstream speed = $(x + y)$ km/h

Now, $\frac{24}{(x-y)} + \frac{28}{(x+y)} = 6$ ...(1)

and $\frac{30}{(x-y)} + \frac{21}{(x+y)} = \frac{13}{2}$ ...(2)

Solving (1) and (2), we have

$x = 10$ km/h and $y = 4$ km/h

12. (a) The clock gains 15 min in 24 hours.

Therefore, in 16 hours, it will gain 10 minutes.

Hence, the time shown by the clock will be 4.10 am.

13. (d) Total distance = 250 × 2 = 500 km

Total time = $5\frac{1}{2}$ hrs + $4\frac{2}{3}$ hrs = $10\frac{1}{6}$ hrs

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$= \frac{500}{10\frac{1}{6}} = \frac{3000}{61}$ km/h

= 49.18 km/h $\approx$ 50 km/h (approx.)

14. (d) Required difference $= \frac{180}{3} - \frac{180}{4} = 15$ km

15. (c) Speed on return trip = 150% of 40 = 60 kmph.

$\therefore$ Average speed $= \left(\frac{2 \times 40 \times 60}{40+60}\right)$ km/hr

$= \left(\frac{4800}{100}\right)$ km/hr = 48 km/hr.

16. (a) $V_{relative} = (21 - 15)$ m/min = 6 m/min

Time taken to catch the thief

$= \frac{114}{6}$ min = 19 minutes

17. (b) $\text{Time} = \frac{\text{Distance advanced}}{\text{Relative speed}}$

$2 = \frac{2 \times x}{(30 - x)} \Rightarrow x = 15$ km/h

18. (a) If the rate of the stream is x, then

$\Rightarrow 2(4.5 - x) = 4.5 + x$

$\Rightarrow 9 - 2x = 4.5 + x \Rightarrow 3x = 4.5$

$\Rightarrow x = 1.5$ km/hr

19. (d) Time difference between 8 am and 2 pm = 6 hrs.

Angle traced by the hour hand in 6 hours

$= \left(\frac{360}{12} \times 6\right)^{\circ} = 180^{\circ}$

20. (c) We know that, the relation in time taken with two different modes of transport is

$t_{\text{walk both}} + t_{\text{ride both}} = 2\,(t_{\text{walk}} + t_{\text{ride}})$

$\frac{31}{4} + t_{\text{ride both}} = 2 \times \frac{25}{4}$

$\Rightarrow t_{\text{ride both}} = \frac{25}{2} - \frac{31}{4} = \frac{19}{4} = 4\frac{3}{4}$ hrs

21. (d) Use Shortcut Approach-1

22. (a) Use Shortcut Approach-2

23. (c) Use Shortcut Approach-3

24. (a) Use Shortcut Approach-4

25. (a) $d = \text{product of speed}\left[\frac{\text{difference of time}}{\text{difference of speed}}\right]$

$d = \frac{4 \times 5}{60}\left[\frac{10 - (-5)}{5 - 4}\right]$

[Here, –ve sign indicates before the schedule time]

$\Rightarrow d = 5$ km

26. (c) If we assume the speed of the sound as 330 m/s, we can see that the distance traveled by the sound in 45 seconds is the distance traveled by the train in 11 minutes.

$330 \times 45 = 660 \times s \rightarrow s = 22.5$ m/s = 81 kmph

27. (d) When A covers 200 metres, B covers

$200 \times \frac{22}{25} = 176$ m

So, B is (200 – 176) = 24 m far away from the end point when A reaches in.

28. (a) According to question,

A : B : C

2 : 1 :

3 : 1

A : B : C

6 : 3 : 1 $\rightarrow$ Ratio of speed

$\frac{1}{6} : \frac{1}{3} : \frac{1}{1}$ $\rightarrow$ Ratio of time

Since, time $\propto \frac{1}{\text{speed}}$

$\Rightarrow 1 : 2 : 6$

$\downarrow \frac{1}{4}$

$\frac{3}{2}$

$\therefore$ Time taken by A

= 1 ratio $= 1 \times \frac{1}{4} = 15$ minutes

29. (b) Downstream speed = 15 + 5 = 20 km/h.

$\therefore$ Required distance $= 20 \times \frac{24}{60} = 8$km.

30. (b) Volume of water flowed in an hour

$= 2000 \times 40 \times 3$ cubic metre $= 240000$ cubic metre

$\therefore$ volume of water flowed in 1 minute

$= \frac{240000}{60} = 4000$ cubic metre

= 40,00,000 litre

31. (c) Let the distance travelled during both upward and downward journey be x km.

$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$

$= \frac{x + x}{\frac{x}{16} + \frac{x}{28}} = \frac{2}{\frac{28 + 16}{28 \times 16}}$

$= \frac{2 \times 28 \times 16}{44} = 20.36$ km/h

GP_3386

32. (c) Usual time $= \dfrac{-10}{\left(\frac{5}{6}-1\right)} = 60$ min

33. (c) Let the speed of the bus be x km / h.
then speed of the car = (x + 25) km / h

$\therefore \quad \dfrac{500}{x} = \dfrac{500}{x+25} + 10$

$\Rightarrow x^2 + 25x - 1250 = 0 \Rightarrow x = 25$

Thus speed of the bus = 25 km/h

Speed of the car = 50 km/h

Alternative:

Difference in speeds 25 km / hr is in only option (c).

34. (d) Use Shortcut Approach-4

35. (c) Use Shortcut Approach-5

36. (a) Use Shortcut Approach-6

37. (d) Use Shortcdut Approach-7

38. (b) If new speed is $\dfrac{a}{b}$ of original speed, then

usual time $\times \left(\dfrac{b}{a}-1\right)$ = change in time

$\therefore$ usual time $\times \left(\dfrac{4}{3}-1\right) = \dfrac{1}{3}$

$\Rightarrow$ usual time $= \dfrac{1}{3} \times 3 = 1$ hr

39. (d) Let after t hours they meet then,

$3t + 4t = 17.5 \Rightarrow t = 2.5$

$\therefore$ Time = 10 am + 2.5 h = 12 : 30 pm

40. (b) Rest time = Number of rest × Time for each rest = 4 × 5 = 20 minutes

Total time to cover 5 km

$= \left(\dfrac{5}{10} \times 60\right)$ minutes + 20 minutes = 50 minutes.

41. (d) Let C_1 takes t hrs. Then,

$\because$ Distance is same.

$\therefore \quad 30t = 45\left(t - \dfrac{5}{2}\right) \Rightarrow t = \dfrac{15}{2}$ hrs

$\therefore$ Distance $= 30 \times \dfrac{15}{2} = 225$ km

42. (d) Speed $= \left(5 \times \dfrac{5}{18}\right)$ m / sec $= \dfrac{25}{18}$ m / sec.

Distance covered in 15 minutes

$= \left(\dfrac{25}{18} \times 15 \times 60\right)$ m $= 1250$ m.

43. (a) Speed $= \left(\dfrac{750}{150}\right)$ m / sec $= 5$ m / sec

$= \left(5 \times \dfrac{18}{5}\right)$ km / hr $= 18$ km / hr.

44. (a) The train can cover (200 + 350) m distance in five seconds which means the speed of the train is 110 m/s. Relative speed of man and train is 114 m/s. To over the distance of 100 metre, it will take less than one second.

45. (d) If the speed of the boat in still water and the speed of the stream is x km/h and y km/h respectively then

$\dfrac{750}{x-y} = 15$ and $\dfrac{750}{x+y} = 10$

or, 15 (x – y) = 10 (x + y)

or, 3x – 3y = 2x + 2y

or, x = 5y or, x : y = 5 : 1

46. (c) Let the speed of the stream = x km / h then

$\dfrac{56}{\frac{7}{4}} = 36 - x \Rightarrow \dfrac{56 \times 4}{7} = 36 - x \Rightarrow 32 = 36 - x$

$\therefore \quad x = 36 - 32 = 4$ km/h

$\therefore$ Time taken to go downstream

$= \dfrac{56}{(36+4)} = 1.4$ hours

= 1 hour 24 min

47. (c) The speed downstream = 6 km/h

$\therefore$ Speed in still water $= \dfrac{6+2}{2} = 4$km / h

So, time taken to go 5 km in still water

$= \dfrac{5}{4} \times 60$ min $= 75$ min

48. (d) Let original speed
= v km/h then s = 30v ...(i)

and $s - 10 = 30\left(1 - \dfrac{1}{15}\right)v = 28v$ ...(ii)

after solving (i) and (ii) v = 5 km/h

49. (c) Ratio of speeds of A : B = 12 : 11
and ratio of speeds of B : C = 8 : 7
Therefore ratio of speeds of A : B : C = 96 : 88 : 77
So in 9600 m race A will beat C by 1900 m.

50. (d) $\dfrac{40}{(B-S)} + \dfrac{55}{(B+S)} = 13$ ... (i)

$$\frac{30}{(B-S)}+\frac{44}{(B+S)}=10 \quad ...(ii)$$

After solving (i) and (ii),

$B-S=5$

$B+S=11$

$\therefore$ Speed in still water $=\frac{5+11}{2}=8$ km/h

51. (a) Let the original speed be x km/h, then

$$\frac{600}{x}-\frac{600}{(x+20)}=1$$

$$\Rightarrow \quad 600\left(\frac{x+20-x}{x(x+20)}\right)=1$$

$\Rightarrow x^2+20x-12000=0$

$\Rightarrow (x+120)(x-100)=0$

$\Rightarrow x=100$ and $x=-120$

$\therefore$ Original speed = 100 km/h

Alternatively :

$$S_1 \times S_2 = \text{Distance} \times \frac{\text{Difference in speed}}{\text{Difference in time}}$$

$$=600\times\frac{20}{1}=600\times20=12000$$

$\Rightarrow S_1 \times S_2 = 12000$

$\therefore S_1 = 100$ and $S_2 = 120$

(Factorise 12000 in such a way that the difference be 20) S_1 is the original speed and S_2 is the changed speed.

52. (b) Use Shortcut Approach-8

53. (a) Use Shortcut Approach-9

54. (b) Use Shortcut Approach-11

55. (b) Use Shortcut Approach-14

56. (d) Use Shortcut Approach-13

Level 2

1. (d) Use Shortcut Approach-16

2. (c) Use Shortcut Approach-17

3. (d) Use Shortcut Approach-18

4. (a) Use Shortcut Approach-12

5. (c) Relative speed of the trains

$=(40+20)=60$ m/s

Distance $=(120+120)=240$ m

Time taken by trains to cross each other completely

$$=\frac{240}{60}=4 \text{ s}$$

6. (c) Average speed $=\frac{\text{Total distance covered}}{\text{Total time taken}}$

$$=\frac{2\times200}{\frac{200}{40}+\frac{200}{20}}=\frac{2\times40\times20}{40+20}$$

$$=\frac{2\times40\times20}{60}=\frac{80}{3}=26.67 \text{ km/h.}$$

7. (c) After 5 minutes (before meeting), the top runner covers 2 rounds i.e., 400 m and the last runner covers 1 round *i.e.,* 200 m.

$\therefore$ Top runner covers 800 m race in 10 minutes.

8. (c) Let the correct time to complete the journey be x min. Distance covered in $(x+11)$ min. at 40 kmph

= Distance covered in $(x+5)$ min. at 50 kmph

$$\therefore \frac{(x+11)}{60}\times40=\frac{(x+5)}{60}\times50 \Leftrightarrow x=19 \text{ min.}$$

9. (a) Distance to be covered by the thief and by the owner is same.

Let after time 't', owner catches the thief.

$$\therefore 40\times t=50\left(t-\frac{1}{2}\right)$$

$$\Rightarrow 10t=25 \Rightarrow t=\frac{5}{2}\text{hr}=2\frac{1}{2}\text{hr}$$

10. (d) Relative speed of both trains

$=60+90=150$ km/hr

Total distance $=1.10+0.9=2$ km

$$\therefore \text{Required time}=\frac{2\times60\times60}{150}=48 \text{ seconds.}$$

11. (c) Let the distance be x km. Then,

(Time taken to walk x km) + (Time taken to ride x km)

$$=\frac{23}{4}\text{ hrs.}$$

$\Rightarrow$ (Time taken to walk $2x$ km) + (Time taken to ride $2x$ km) $=\frac{23}{2}$ hrs.

But, time taken to ride $2x$ km $=\frac{15}{4}$ hrs.

$$\therefore \text{Time taken to walk } 2x \text{ km}=\left(\frac{23}{2}-\frac{15}{4}\right)\text{ hrs}$$

$$=\frac{31}{4}\text{ hrs}=7\text{ hrs }45\text{ min.}$$

12. (a) Let the speed of the boatman be x km/hr and that of stream by y km/hr. Then

$$\frac{12}{x+y}=\frac{4}{x-y}$$

$\Rightarrow 12x-12y=4x+4y$

$\Rightarrow 8x=16y \Rightarrow x=2y$

Now $\frac{45}{x+y}+\frac{45}{x-y}=20$

GP_3386

$\Rightarrow 45 + 135 = 60y \Rightarrow 180 = 60y$

$\Rightarrow y = 3$km/hr.

13. (c) Let the distance between the school and the home be x km.

Then, $\frac{x}{8} - \frac{2.5}{60} = \frac{x}{10} + \frac{5}{60}$

or $\frac{x}{8} - \frac{x}{10} = \frac{5}{60} + \frac{2.5}{60}$

or $\frac{2x}{80} = \frac{7.5}{60}$ or $x = \frac{7.5 \times 80}{2 \times 60} = 5$ km

14. (c) Note here the length of the train in which passenger is travelling is not considered since we are concerned with the passenger instead of train. So, the length of the bridge will be directly proportional to the time taken by the passenger respectively.

$t \rightarrow$ Time

$l \rightarrow$ Length of bridge

Therefore. $\frac{t_1}{t_2} = \frac{l_1}{l_2}$

$\frac{7}{4} = \frac{280}{x}$

$\Rightarrow x = 160$ m

15. (c) Initial distance = 25 dog leaps.

Per minute $\rightarrow$ dog makes 5 dog leaps

Per minute $\rightarrow$ cat makes 6 cat leaps = 3 dog leaps.

Relative speed = 2 dog leaps/minutes.

An initial distance of 25 dog leaps would get covered in 12.5 minutes.

16. (a) Let the distance be x km. Let speed of train be y km/h. Then by question, we have

$$\frac{x}{y+4} = \frac{x}{y} - \frac{30}{60} \quad ...(1)$$

and $$\frac{x}{y-2} = \frac{x}{y} + \frac{20}{60} \quad ...(2)$$

On solving (1) and (2), we get $x = 3y$

Put $x = 3y$ in (1) we get

$$\frac{3y}{y+4} = 3 - \frac{1}{2} \Rightarrow y = 20$$

Hence, distance = $20 \times 3 = 60$ km.

17. (b) Let the Speed of faster train be x and speed of slower train be y.

Now, when both the train move in same direction their relative speed = $x - y$

Now, total distance covered = 130 + 110 = 240

Now, distance = speed × time

$\therefore 240 = (x - y) \times 60$ ($\because 1$ min = 60 sec)

$\Rightarrow x - y = 4$ ...(1)

When the trains move in opposite direction then their relative speed = $x + y$

$\therefore 240 = (x + y) \times 3$

$\Rightarrow 80 = x + y$ ...(2)

on solving eqs (1) and (2), we get $x = 42$ m/sec and $y = 38$ m/sec

18. (a) The ratio of speeds of A to B would be 2 : 3.

P R Q

0.4 D means ratio of speeds = 2:3

The 4th meeting would occur after a combined movement of $D + 6D = 7D$. 2/5th of this distance would be covered by A and 3/5th of this distance would be the distance covered by B. Thus, distance covered by A would be 2/5th of $7D$ __: distance covered by A = $2.8D$ – which means that the 4th meeting occurs at a distance of 0.8 D from P.

19. (c) If you start at 12 noon, you would reach at 4 : 30 PM. You would be able to meet the train which left Pune at 8 AM, 9 AM, 10 AM, 11 AM, 12 Noon, 1 PM, 2 PM, 3 PM and 4 PM – a total of 9 trains.

20. (b) Let the speed of the train and the car be x km/h and y km/h, respectively.

Now, $\frac{120}{x} + \frac{480}{y} = 8$ ...(i)

and $\frac{200}{x} + \frac{400}{y} = \frac{25}{3}$ ...(ii)

From (i), $120y + 480x = 8xy$ and ...(iii)

From (ii), $200y + 400x = \frac{25}{3}xy$ ...(iv)

From (iii) and (iv),

$$\frac{120y + 480x}{8} = \frac{3(200y + 400x)}{25}$$

or $15y + 60x = 24y + 48x$

or $12x = 9y$ or $\frac{x}{y} = \frac{3}{4}$

21. (c) Let the speed of the cars be x km/h and y km/h, respectively.

Their relative speeds when they are moving in same direction = (x – y) km/h.

Their relative speeds when they are in opposite directions = (x + y) km/h.

Now, $\frac{70}{x+y}=1$ or $x+y=70$ .....(i)

and $\frac{70}{(x-y)}=7$ or $x-y=10$ .....(ii)

Solving (i) and (ii), we have
$x=40$ km/h and $y=30$ km/h.

22. (b) Let the distance between Meerut and Delhi be x km and let the trains meet y hours after 7 a.m.
Clearly, M covers x km in 4 hrs and N covers x km in (7/2) hrs.

$\therefore$ Speed of M $=\frac{x}{4}$ kmph,

Speed of N $=\frac{2x}{7}$ kmph.

(Distance covered by M in $(y+2)$ hrs) + (Distance covered in y hrs.) = x.

$$\therefore \frac{x}{4}(y+2)+\frac{2x}{7}\times y=x \Rightarrow \frac{(y+2)}{4}+\frac{2y}{7}=1$$

$$\Rightarrow y=\frac{14}{15}\text{ hrs}=\left(\frac{14}{15}\times 60\right)\text{min.}=56\text{ min.}$$

Hence, the trains meet at 7 : 56 a.m..

23. (a) Use Shortcut Approach-23

24. (c) Use Shortcut Approach-21

25. (c) Use Shortcut Approach-24

26. (d) Use Shortcut Approach-25

27. (b) Speed of train while passing point
$A=45\times(5/18)$ m/s $=V_1$
Speed of bike initially $=70\times(5/18)$ m/s $=V_2$
Time taken by the bike to reach at the mid-point of the train $=150/(V_2-V_1)$

$$\frac{150\times 18}{125}=\frac{108}{5}\text{ sec.}$$

Again time taken by the bike to cover the rest 150 m $=\frac{150\times 18}{25}=108$ second.

$\therefore$ The distance travel by bike

$$=\frac{108}{5}\times\frac{70\times 5}{18}+\frac{108\times 65\times 5}{18}$$

$=2370\text{m}=2.37\text{km}$.

28. (b) This question has actually got nothing to do with bullets initially. We can see that it takes them 4 hours to reach each other. And this is the same time for which bullets will cover some distance.
So, the total distance covered by the bullet $=4\times 10=40$ km.

29. (c) Total distance from Mumbai to Delhi = 910 km
Time taken by Sapna to cover this distance, had there been no snag = 910/60
Since their speeds are in the ratio 3 : 2, Manoj would have covered
$[910-(910\times 40)/60]$ km less than Sapna.

30. (d) Required time $=3\frac{1}{2}+\frac{1}{2}+\frac{500}{45}+\frac{100}{30}$
$=18.44$

31. (a) If the speed of both the cars be x km/h and y km/h respectively then going in same direction $5(x-y)=100$
And going in opposite direction $1.(x+y)=100$
After solving $x=60$, $y=40$

32. (b) Distance travelled by the train departing from A in 2 hours
$=2\times 70=140$ km
$\therefore$ Rest distance $=500-140=360$ km
and Relative speed $=70+110=180$ km/h

$\therefore$ Time taken to meet $=\frac{360}{180}=2$ hours

So, they will meet each other at 12 O' clock

33. (a) Let the downstream speed be D and upstream speed be U, then

$$\frac{48}{U}+\frac{72}{D}=12$$

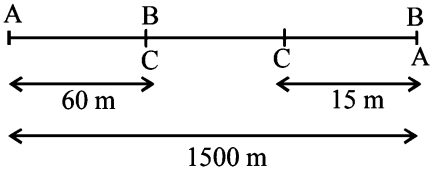
and $\frac{48}{D}+\frac{72}{U}=13$

$\therefore$ $48m+72n=12$ ...(i)
and $48n+72m=13$ ...(ii)
Solving Eqs. (i) and (ii), we get

$m+n=\frac{5}{24}$ and $m-n=\frac{1}{24}$

$\therefore$ D = 12 km/h and U = 8 km/h
$\therefore$ Speed of current = 2 km/h

34. (b)



In the same time, when A covers 1500 m, B covers 1440 m and C covers 1425 m.
So, in 1440 m race B can give a start of 15 m.
$\therefore$ In 1500 m race B will give a start of

$$\frac{15}{1440}\times 1500=15\frac{5}{8}\text{m}$$

35. (d) Let the speed of the faster train be f_s and slower train be s_s, then

$$f_S + s_S = \frac{200+250}{18} = 25\text{m/s}$$

and $f_S - s_S = \frac{200+250}{60} = \frac{450}{60} = 7.5\text{m/s}$

$\therefore$ $f_s = 16.25$ m/s

$$= 16.25 \times \frac{18}{5} = 58.5 \text{ km/h}$$

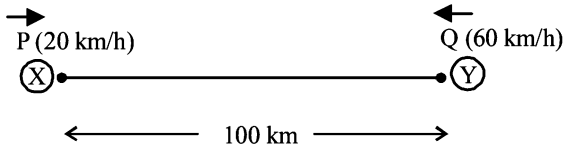
36. (b) Relative speed $= \frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{60+40}{20} = 5 \text{ m/s}$$

$\therefore$ $5 \text{ m/s} = 5 \times \frac{18}{5} = 18 \text{ km/h}$

Now, relative speed = 18 km/h
= Speed of ambulance – Speed of school bus
18 = 30 – speed of school bus
Speed of school bus = 12 km/h

37. (c)



To meet each other they have to cover 100 km distance together and the ratio of distances covered by each one is directly proportional to the ratio of their speeds, respectively. Since the time taken by each one is same (i.e, constant).

Hence, $\frac{XZ}{YZ} = \frac{20}{60} = \frac{1}{3}$

$\therefore$ $XZ = \frac{1}{4} \times 100 = 25 \text{ km}$

38. (d) To meet the second time they have to cover 300 km distance together [for n^{th} time distance = (2n – 1) d]
Time taken by them to meet each other, for the second time

$$= \frac{300}{80} = 3\frac{3}{4} \text{ hour}$$

Distance covered by P $= 20 \times 3\frac{3}{4} = 75$ km

Alternatively :
The ratio of distance covered = Ratio of their speeds.

$\therefore$ Distance covered by P $= \frac{1}{4} \times 300 = 75$ km

39. (a) The ratio of distances covered by each of them
= Ratio of their respective speeds

$\therefore$ Distance covered by Q for the third meeting

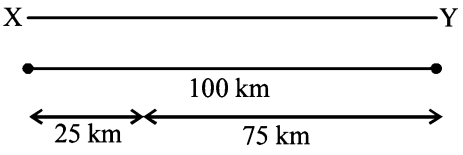
$$= \frac{3}{4} \times 500 = 375 \text{ km}$$

40. (d) Consider the distance travelled by any one of them, then find the distance between X and M, where they meet.

$\therefore$ The distance travelled by P for the fourth meeting

$$= \frac{1}{4} \times 700 = 175 \text{ km}$$

Therefore P will be 75 km from Y. It means P will be 25 km away from X.



41. (b) This is constant for any number of meeting and is equal to the ratio of their speeds. Hence, 1 : 3.

42. (c) It is always twice the length of race course. Hence, between any two consecutive meeting they have to cover total 200 km distance to meet each other for the next meeting.

43. (c) Use Shortcut Approach-22

44. (a) Use Shortcut Approach-20

45. (b) Use Shortcut Approach-15

46. (d) Use Shortcut Approach-19

CHAPTER

17

Clock and Calendar

CLOCKS

The concept of clocks is the same as the concept of circular motion. Just like in circular motion, in case of clocks, the hour hand and the minute hand are continuously racing against each other around a circle. The hour hand travels 5 minutes or 30° in 1 hour. Similarly, the minute hand travels 60 minutes or 360° in 1 hour. Therefore, in one hour, the minute hand moves 55 minutes or 330° more than the hour hand. This can also be called the relative speed of minute hand with respect to the hour hand.

Minute and Hour hand of a clock			
Actual Time	Movement		Relative speed of minute hand with respect to hour hand
	Minute Hand	Hour Hand	
60 minutes	60 minutes or 360°	5 minutes or 30°	55 minutes or 330°

Other Important Concepts

(i) The two hands of a clock coincide once every hour, but in 12 hours the coincide only 11 times. This is because the coincide position at 12 is counted in both 11 to 12 and 12 to 1.

(ii) The hands are at right angles twice in every hour, but in 12 hours, they are at right angles only 22 times. This is because the right-angle position at 3 p.m. is common to both 2 p.m. to 3 p.m. and 3 p.m. to 4 p.m. Similarly, the right-angled position at 9 p.m. is common to both 8 p.m. to 9 p.m. and 9 p.m. to 10 p.m.

(iii) The hands point in the opposite direction once in every hour, but in 12 hours they are opposite only 11 times. This is because, the opposite position of at 6 p.m. is counted in both 5 p.m. to 6 p.m. and 6 p.m. to 7 p.m.

(iv) The hands are said to be in same straight line whenever they are coincident or opposite to each other.

(v) If the two hands of a clock will coincide again when the minute hand takes a lead of 60 minutes on the hour hand. The minute hand takes a lead of 55 minutes in 60 minutes of actual time.

Concept of Incorrect Clocks

Two hands of every correct clock coincide after every $\left(65+\frac{5}{11}\right)$ minutes.

An incorrect clock can either be a fast clock which gains time or a slow clock which loses time.

Fast clock : In fast clock, two hands of a clock coincide every x minutes, where x is less than $\left(65+\frac{5}{11}\right)$ minutes.

$$\text{Time gained per minute} = \frac{65+\frac{5}{11}-x}{x}$$

Slow clock : In slow clock, two hands of a clock coincide every x minutes, where x is greater than $\left(65+\frac{5}{11}\right)$ minutes.

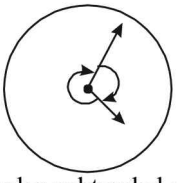
$$\text{Time loss per minute} = \frac{x-\left(65+\frac{5}{11}\right)}{x}$$

Illustration – 1

Mrs. Veena Gupta goes for marketing between 5 P.M. and 6 P.M. When she comes back, she finds that the hour hand and the minute hand have inter-changed their positions. For how much time was she out of her house ?

GP_3386

Solution : Since two hands are interchange their positions, so sum of the angles subtended at the centre by hour hand and minute hand = 360°
Let us suppose that she was out of house for 't' minutes.



So, the sum of the angles subtended at the centre by the hour hand and minute hand = $(0.5 \times t)° + (6t)°$

$\because \quad 0.5t + 6t = 360$

$\Rightarrow \quad 6.5t = 360 \Rightarrow t = 55.4$ (app.)

Hence required time = 55.4 minutes.

CALENDARS

The topic of calendar includes concepts such as odd days, leap year and finding the day of the day week for a given date.

Leap Year

An end of the century year (the last year of the century, e.g. 1500, 1800, 2000 etc.) is a leap year only if divisible by 400. For all the other years, check the divisibility by 4, and if the year is divisible by 4 it is said to be a *leap year* and will have 366 days. This would mean that the year 1500 and 1800 inspite of being divisible by 4 not a leap year.

Concept of Odd Days : When a given number of days converted into week(s) by dividing 7, then the remaining days which are not covered in week(s) are called *odd days*.

(i) An ordinary year has 365 days, that is 52 weeks and one odd day. This means out of the 365 days in an ordinary year, 364 days will get converted into 52 weeks and one day will remain. This one day is referred to 1 odd day. This means that when we proceed from ordinary year to the next year, the name of the first day of the next year will be the name of the day which will come one day after the first day of previous (ordinary) year.
For example, if 2 October 2015 is a Friday, then 2 October 2016 is a Sunday, that is a shift of two days because 2016 is a leap year.

(ii) A leap year has 366 days, that is 52 weeks and two odd days. This means that when we proceed from leap year to the next year, the name of the first day of the next year will be the name of the day which will come two days after the first day of previous (leap) year.
For example, if 26th January 2016 is a Saturday, then 26th January 2017 would be a Monday, that is a shift of two days because 2016 is a leap year.

(iii) If name of the day of a particular date is given and we have to find the name of the day of an other particular date, then we need to look at not only the years but also the date from which we are moving to the other given date. If the 29th of February falls between the two dates, there will be a shift of two days, otherwise there will be a shift of one day.

Counting Odd Days

1st January AD from where our calendar started was a Monday. That's why our week starting on a Monday. Saturday and Sunday are called **weekends.**
Therefore, if we are calculating from 1st January AD and after converting into weeks, whenever we have 1 odd day left, it would be a Monday. If there are 2 odd days left, then the first one would be a Monday, then second a Tuesday and so on.

First odd day = Monday
Second odd day = Tuesday
Third odd day = Wednesday
Fourth odd day = Thursday
Fifth odd day = Friday
Sixth odd day = Saturday
Seventh or Zeroth odd day = Sunday

In ordinary year,
1 year = 365 days = 52 weeks + 1 day
(i.e. one odd day)

In leap year,
1 year = 366 days = 52 weeks + 2 days
(i.e. two odd days)

Concept of Total Odd Days in 100, 200, 300 and 400 Years

If we take 100 consecutive years from 1st AD, there will be 24 leap years (remember the 100th year will not be a leap year) and 76 ordinary years.
24 leap years = 24 × 2 odd days = 48 odd days
Now 48 odd days = 6 weeks + 6 odd days

Hence 24 leap years have 6 odd days

76 ordinary years = 76 × 1 odd days = 76 odd days
Now 76 odd days = 10 weeks and 6 odd days

Hence 76 ordinary years have 6 odd days

Now 6 odd days + 6 odd days = 1 week + 5 odd days
Therefore, 100 consecutive years from 1st January AD have 5 odd days.

Similarly,

- 200 consecutive years contain 10 odd days i.e. 3 odd days.
- 300 consecutive years contain 15 odd days i.e. 1 odd day.
- But 400 consecutive years contain (20 + 1) odd days i.e. 0 odd day (this is because 400th year will be a leap year and contribute 1 extra day.)

Note that the last day of a century be one day out of Friday, Wednesday, Monday or Saturday. First day of a new century will be a Saturday, Thursday, Tuesday or Monday.

Illustration – 2

What day of the week was 15th August 1949?

Solution : 15th August 1949 means
1948 complete years + first 7 months of the year 1949 + 15 days of August.
1600 years give no odd days.
300 years give 1 odd day.
48 years give {48 + 12} = 60 = 4 odd days.
[∵ For ordinary years → 48 odd days and for leap year 1 more day (48 ÷ 4) = 12 odd days; 60 = 7 × 8 + 4]
From 1st January to 15th August 1949
Odd days :
January – 3, February – 0, March – 3, April – 2, May – 3, June – 2, July – 3, August – 1.
17 ⇒ 3 odd days.
∴ 15th August 1949 → 1 + 4 + 3 = 8 = 1 odd day.
This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

Illustration – 3

How many times does the 29th day of the month occur in 400 consecutive years?

Solution : In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day 97 times and the remaining eleven months have the 29th day
400 × 1100 = 4400 times
∴ The 29th day of the month occurs (4400 + 97) or 4497 times.

Illustration – 4

Today is 5th February. The day of the week is Tuesday. This is a leap year. What will be the day of the week on this date after 5 years?

Solution : This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year give 1 odd day. Therefore (3 + 2 + 1) = 6 odd days will be there. Hence the day of the week will be 6 odd days beyond Tuesday, i.e., it will be Monday.

Illustration – 5

What day of the week was 20th June 1837 ?

Solution : 20th June 1837 means 1836 complete years + first 5 months of the year 1837 + 20 days of June.
1600 years give no odd days.
200 years give 3 odd days.
36 years give (36 + 9) or 3 odd days.
1836 years give 6 odd days.
From 1st January to 20th June there are 3 odd days.

Odd Days	
Jan	3
Feb	0
Mar	3
Apr	2
May	3
Jun	6
Total	**17**

Therefore, the total number of odd days = (6 + 3) or 2 odd days.
This means that the 20th of June fell on the 2nd day commencing from Monday. Therefore, the required day was Tuesday.

SHORTCUTS

Shortcut Approach - 1

Between x and $(x + 1)$ o'clock the two hands are at right angle at $(5x \pm 15) \times \frac{12}{11}$ minutes past x.

Illustration – 6

At what time between 4'o clock and 5'o clock will the hands of a clock be at right angles?

(a) $43\frac{5}{11}$ **min past 4** **(b)** $5\frac{5}{11}$ **min past 4**
(c) 40 min past 4 **(d) 45 min past 4**

Shortcut Solution : (b)

The hands of a clock will be at right angles twice between 4'O clock and 5'O clock

GP_3386

$\therefore \left(5x \pm 15 \times \frac{12}{11}\right)$ minutes past x

$= \left(5 \times 4 \pm 15 \times \frac{12}{11}\right)$ minutes past 4

$= 38\frac{2}{11}$ min past 4 and $= 5\frac{5}{11}$ min past 4.

Shortcut Approach - 1

Between x and $(x+1)$ O' clock the two hands are in the same straight line

Case I : (a) When they are in opposite directions at

$(5x-30)\frac{12}{11}$ minutes past x, [where $x > 6$]

(b) $(5x+30)\frac{12}{11}$ minutes past x. [where $x < 6$]

Note : At 6 O' clock two hands will be in opposite direction.

Case II : When they coincide (or come together), at

$5x\left(\frac{12}{11}\right)$ minutes past x.

Illustration – 7

Find at what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together.

(a) $10\frac{10}{11}$ **min past 8** **(b)** $10\frac{9}{11}$ **min past 8**

(c) $11\frac{10}{11}$ **min past 8** **(d) None of these**

Shortcut Solution : (a)

Required answer $= (5 \times 8 - 30)\frac{12}{11} = \frac{120}{11}$ min

$= 10\frac{10}{11}$ min past 8

Shortcut Approach - 1

The minute hand of a clock overtakes the hour hand at certain intervals (given in minutes) of correct time. The clock lose or gain in a day is given by

$$\left[\frac{720}{11} - \text{given interval in minute}\right] \times \left[\frac{60 \times 24}{\text{given interval in minutes}}\right]$$

according as the sign in +ve or –ve.

Illustration – 8

How much does a watch gain or lose per day, if its hands coincide every 64 minutes?

(a) $32\frac{8}{11}$ **min** **(b)** $31\frac{8}{11}$ **min**

(c) $32\frac{3}{11}$ **min** **(d) None of these**

Shortcut Solution : (a)

Required answer

$= \left(\frac{720}{11} - 64\right) \times \left(\frac{60 \times 24}{64}\right)$

$= \frac{16}{11} \times \frac{60 \times 24}{64}$

$= \frac{360}{11} = 32\frac{8}{11}$ min.

LEVEL - 1

1. If the two hands in a clock are 3 minutes divisions apart, then the angle between them is
(a) 3° (b) 18° (c) 24° (d) 60°

2. What will be the acute angle between hands of a clock at 2 : 30?
(a) 105° (b) 115° (c) 95° (d) 135°

3. If a clock strikes 12 in 33 seconds, it will strike 6 in how many seconds?
(a) $\frac{33}{2}$ (b) 15 (c) 12 (d) 22

4. The reflex angle between the hands of a clock at 10 : 25 is ?
(a) 180° (b) $192\frac{1}{2}°$ (c) 195° (d) $197\frac{1}{2}°$

5. At what angle the hands of a clock are inclined at 15 minutes past 5 ?
(a) $72\frac{1}{2}°$ (b) 64° (c) $58\frac{1}{2}°$ (d) $67\frac{1}{2}°$

6. If 09/12/2001 happens to be Sunday, then 09/12/1971 would have been at
(a) Wednesday (b) Tuesday
(c) Saturday (d) Thursday

7. At what time between 8 and 9 o'clock will the hands of a watch be in straight line but not together?
(a) $10\frac{11}{10}$ min. past 8 (b) $10\frac{10}{11}$ min. past 8
(c) $11\frac{10}{11}$ min. past 8 (d) $12\frac{10}{11}$ min. past 8

8. At what time between 9'O clock and 10'O clock will the hands of a clock point in the opposite directions?
(a) $16\frac{4}{11}$ min past 9 (b) $16\frac{1}{11}$ min past 9
(c) $16\frac{3}{11}$ min past 9 (d) $16\frac{2}{11}$ min past 9

9. At what time between 3o' clock and 4o' clock will the hands of a clock be in opposite directions?
(a) $48\frac{2}{11}$ min past 3 (b) $49\frac{1}{11}$ min past 3
(c) $50\frac{1}{11}$ min past 4 (d) $50\frac{2}{11}$ min past 4

10. Find the exact time between 7 am and 8 am when the two hands of a watch meet ?
(a) 7 hrs 35 min (b) 7 hrs 36.99 min
(c) 7 hrs 38.18 min (d) 7 hrs 42.6 min

11. The year next to 1991 having the same calendar as that of 1990 is –
(a) 1998 (b) 2001 (c) 2002 (d) 2003

12. What is the angle between the 2 hands of the clock at 8:24 pm?
(a) 100° (b) 107° (c) 106° (d) 108°

13. In 16 minutes, the minute hand gains over the hour hand by
(a) 16° (b) 80° (c) 88° (d) 96°

14. At what time between 9'O clock and 10'O clock will the hands of a clock point in the opposite directions?
(a) $16\frac{4}{11}$ minutes past 9
(b) $16\frac{4}{11}$ minutes past 8
(c) $55\frac{5}{61}$ minutes past 7
(d) $55\frac{5}{61}$ minutes to 8

15. At what time between 4'o clock and 5'o clock, will the hands of a clock be together?
(a) $18\frac{7}{11}$ min past 4 (b) $15\frac{1}{11}$ min past 4
(c) $21\frac{9}{11}$ min past 4 (d) $11\frac{9}{10}$ min past 4

16. A clock gains 15 minutes per day. It is set right at 12 noon. What time will it show at 4.00 am, the next day?
(a) 4 : 10 am (b) 4 : 45 am
(c) 4 : 20 am (d) 5 : 00 am

GP_3386

17. An accurate clock shows 8 O'clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 O'clock in the afternoon?
(a) 144° (b) 150° (c) 168° (d) 180°

18. At what time between 7 and 8 O'clock will the hands of a clock be in the same straight line but, not together?
(a) 5 min. past 7 (b) $5\frac{2}{11}$ min. past 7
(c) $5\frac{3}{11}$ min. past 7 (d) $5\frac{5}{11}$ min. past 7

19. On January 12, 1980, it was Saturday. The day of the week on January 12, 1979 was –
(a) Saturday (b) Friday
(c) Sunday (d) Thursday

20. At what time between 5.30 and 6 will the hands of a clock be at right angles?
(a) $43\frac{5}{11}$ min. past 5 (b) $43\frac{7}{11}$ min. past 5
(c) 40 min. past 5 (d) 45 min. past 5

21. The minute hand of a clock overtakes the hour hand at the intervals of 63 min of the correct time. How much does a clock gain or lose in a day?
(a) $56\frac{8}{77}$ min (b) $44\frac{1}{11}$ min
(c) $77\frac{4}{23}$ min (d) $91\frac{1}{11}$ min

22. The calendar for the year 2005 is the same as for the year :
(a) 2010 (b) 2011 (c) 2012 (d) 2013

23. At what time between 8 and 9 O' clock will the hours and minute hand make an angle of 72° with each other?
(a) 8 h 306/11 (b) 8 h 568 / 11
(c) a or b (d) None of these

24. In a day how many times the minute and hour hands coincide (or overlap or come together)?
(a) 12 (b) 20 (c) 22 (d) 24

25. In a day how many times the minutes hand and hour hand make right angle between them?
(a) 12 (b) 20 (c) 22 (d) 44

26. In a day how many times the minute hand and hour hand are opposite to each other?
(a) 12 (b) 24 (c) 11 (d) 22

27. When do the hands of a clock coincide between 5 and 6 ?
(a) 5 : 30 (b) 5 : 27 : 16
(c) 5 : 32 : 16 (d) 5 : 28 : 56

28. What is the angle between hour hand and minute hand at 2 : 25 pm?
(a) 77.5° (b) 68°
(c) 67.5° (d) None of these

29. When do the two hands of a clock of just after 3 pm make 30° angle between them?
(a) 3 : 15 : 00 (b) 3 : 10 : 54
(c) 3 : 01 : 59 (d) None of these

30. At what time are the hands of clock together between 7 pm and 8 pm?
(a) 7 : 45 : 54 (b) 7 : 36 : 27
(c) 7 : 37 : 49 (d) 7 : 38 : 11

31. What is the angle between the hands of a clock when the time is 15 minutes past 6 ?
(a) 97.5° (b) 87.5°
(c) 77° (d) None of these

32. What are the possible times when a clock shows 35° angle between two hands between 3 pm and 4 pm?
(a) 20 min 25 sec (b) 25 min 20 sec
(c) 22 min 43 sec (d) None of these

33. At what time between 3 O'clock and 4 O' clock will the hands of a clock be together?
(a) $16\frac{3}{11}$ min past 3 (b) $14\frac{3}{11}$ min past 3
(c) $13\frac{2}{11}$ min past 3 (d) $16\frac{4}{11}$ min past 3

34. At what point of time after 3 O' clock, hour hand and the minute hand are at right angles for the first time?
(a) 9 O' clock (b) 4 h $37\frac{1}{6}$ min
(c) 3 h $30\frac{8}{11}$ min (d) 3 h $32\frac{8}{11}$ min

35. At what time between 2 O' clock and 3' O clock will the hands of a clock point in opposite directions?

(a) $\frac{480}{11}$ min past 2 (b) $\frac{485}{11}$ min past 2

(c) $\frac{580}{11}$ min past 2 (d) $\frac{520}{11}$ min past 2

36. A clock is set right at 6 am. The clock losses 16 min in 24 h. What will be true time, if the clock indicates 11 pm on 4th day?

(a) 12 O' clock (b) 11 pm

(c) 9 pm (d) 2 pm

37. If 5th March, 1999 was Friday, what day of the week was it on 9th March 2000?

(a) Wednesday (b) Saturday

(c) Friday (d) Thursday

38. January 3, 2007 was Wednesday. What day of the week fell on January 3, 2008?

(a) Tuesday (b) Friday

(c) Thursday (d) Saturday

39. January 3, 2008 was Friday. What day of the week fell on January 3, 2009?

(a) Sunday (b) Monday

(c) Tuesday (d) Saturday

40. What was the day of the week on 2nd Jan, 2010, if it was Sunday on 1st Jan, 2006?

(a) Saturday (b) Thursday

(c) Sunday (d) Friday

41. On 6th March, 2005, Monday falls. What was the day of the week on 7th March, 2004?

(a) Tuesday (b) Monday

(c) Friday (d) Sunday

42. What was the day of the week on 17th August, 2010?

(a) Sunday (b) Wednesday

(c) Tuesday (d) Friday

LEVEL - 2

1. At what approximate time between 4 and 5 am will the hands of a clock be at right angle?

(a) 4 : 40 am (b) 4 : 38 am

(c) 4 : 35 am (d) 4 : 39 am

2. What was the day of the week on 15th August, 1947?

(a) Wednesday (b) Tuesday

(c) Friday (d) Thursday

3. Find the day of the week on 16th July, 1776.

(a) Tuesday (b) Wednesday

(c) Monday (d) Thursday

4. The first Republic Day of India was celebrated on 26th January, 1950. It was :

(a) Tuesday (b) Wednesday

(c) Thursday (d) Friday

5. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 O'clock, the true time is

(a) 4 p.m. (b) $59\frac{7}{12}$ min past 3

(c) $58\frac{7}{11}$ min past 3 (d) $2\frac{3}{11}$ min past 4

6. A clock gains 5 minutes. in 24 hours. It was set right at 10 a.m. on Monday. What will be the true time when the clock indicates 10:30 a.m. on the next Sunday ?

(a) 10 a.m. (b) 11 a.m.

(c) 25 min past 10 a.m. (d) 5 min to 11 a.m.

7. At what time between 3 and 4'O clock, the hands of a clock coincide?

(a) $16\frac{4}{11}$ min past 3 (b) $15\frac{5}{61}$ min past 3

(c) $15\frac{5}{60}$ min to 2 (d) $16\frac{4}{11}$ min to 4

8. A clock is set right at 1 p.m. If it gains one minute in an hour, then what is the true time when the clock indicates 6 p.m. in the same day?

(a) $55\frac{5}{61}$ min past 5 (b) 5 min past 6

(c) 5 min to 6 (d) $59\frac{1}{64}$ min past 5

GP_3386

9. Two clocks were set right at noon on Sunday. One gains 2 min and the other loses 3 min in 24 hours. What will be the true time when the first clock indicates 3 pm on Wednesday?

(a) 2:38 pm (b) 2:54 pm

(c) 2:23 pm (d) 2:48 pm

10. The relative speed of minute-hand with respect to hour – hand is :

(a) $\left(5\frac{1}{2}\right)^{\circ}$ per min (b) $\left(\frac{11}{12}\right)$ min per min

(c) 6° per min (d) both (a) and (b)

11. At what time between 9 O' clock and 10 O' clock, will the hands of a clock be in the same straight line but not together?

(a) $16\frac{2}{11}$ min past 9 (b) $16\frac{2}{11}$ min past 10

(c) $16\frac{4}{11}$ min past 9 (d) $16\frac{4}{11}$ min past 10

12. At what time between 5 O' clock and 6' O clock in the morning, will the both hands of a clock be at right angle?

(a) $43\frac{7}{11}$ min past 5 and $1\frac{10}{11}$ min past 6

(b) $43\frac{7}{11}$ min past 5 and $10\frac{10}{11}$ min past 5

(c) $42\frac{7}{11}$ min past 5 and $11\frac{10}{11}$ min past 5

(d) $47\frac{7}{11}$ min past 5 and $1\frac{10}{11}$ min past 6

13. How much does a watch gain per day. If its hands coincide every 64 min?

(a) 37 min (b) $32\frac{8}{11}$

(c) 31 min (d) None of these

14. The last day of a century cannot be

(a) Thursday (b) Wednesday

(c) Friday (d) Monday

15. 4th April, 1988 was Monday. What day of the week was it, on 6th November 1987?

(a) Tuesday (b) Friday

(c) Sunday (d) Saturday

16. What was the day of the week on 17th July, 1776?

(a) Wednesday (b) Thursday

(c) Monday (d) Saturday

17. On what dates of March, 2005 did Saturday fall?

(a) 4th, 5th, 9th, 12th

(b) 5th, 12th, 19th, 26th

(c) 3rd, 10th, 17th, 25th

(d) 4th, 11th, 18th, 25th

Hints and Solutions

Level 1

1. (b) In a clock, each minute makes 6°

$\therefore$ 3 minutes will make $6 \times 3 = 18°$

2. (a) At 2'O Clock, Minute Hand will be $10 \times 6 = 60°$ behind the Hour Hand.

In 30 minutes, Minute Hand will gain

$$\left(5\frac{1}{2}\right)^\circ \times 30 = 150 + 15 = 165°$$

$\therefore$ Angle between Hour Hand and Minute Hand = 165 – 60 = 105°

3. (b) In order to strike 12, there are 11 intervals

of equal time $= \frac{33}{11} = 3$ seconds each

Therefore, to strike 6 it has 5 equal intervals, it requires $5 \times 3 = 15$ sec.

4. (d) Angle traced by hour hand in $\frac{125}{12}$ hrs.

$$= \left(\frac{360}{12} \times \frac{125}{12}\right)^\circ = 312\frac{1}{2}^\circ$$

Angle traced by minute hand in 25 min.

$$= \left(\frac{360}{60} \times 25\right)^\circ = 150°$$

$\therefore$ Reflex angle

$$= 360 - \left(312\frac{1}{2} - 150\right)^\circ = 197\frac{1}{2}^\circ$$

5. (d) At 15 minutes past 5, the minute hand is at 3 and hour hand slightly advanced from 5. Angle between their 3rd and 5th position.

Angle through which hour hand shifts in 15 minutes is $\left(15 \times \frac{1}{2}\right)^\circ = 7\frac{1}{2}^\circ$

$\therefore$ Required angle $= \left(60 + 7\frac{1}{2}\right) = 67\frac{1}{2}^\circ$

6. (d) 09/12/2001—— Sunday

No. of days between 9/ 12/ 71 & 9 / 12/ 2001

we know every year has 1 odd days

we know leap year has 2 odd days

Here, No. of normal years = 22

And no. of leap years = 8

So odd days = 22 + 16 = 38 i.e. 3odd day

(remainder when 38 is divided by 7, i.e. 3)

7. (b) At 8 o'clock, the hands of the watch are 20 min. spaces apart.

To be in straight line but not together they will be 30 min. space apart.

$\therefore$ Minute hand will have to gain 10 min. spaces

55 min. spaces are gained in 60 min.

10 min. spaces will be gained in

$$\left(\frac{60}{55} \times 10\right) \text{ min or } 10\frac{10}{11} \text{ min}$$

$\therefore$ Required time $= 10\frac{10}{11}$ min. past 8

8. (a) Use Shortcut Approach-1

9. (b) Use Shortcut Approach -2

10. (c) 55 min spaces are gained in 60 min

$\Rightarrow$ 35 min spaces will be gained in 38.18 min

$\Rightarrow$ Answer = 7 hrs + 38.18 min

11. (c) We go on counting the odd days from 1991 onwards till the sum is divisible by 7. The number of such days are 14 upto the year 2001. So, the calendar for 1991 will be repeated in the year 2002.

12. (d) Required angle = 240 – 24 × (11/2)

= 240 –132 = 108°.

13. (c) In 1 hour, the minute hand gains 330° over the hour hand.

i.e. in 60 minute, the minute hand gains

GP_3386

330° over the hour hand.

∴ In 16 minutes, the minute hand gains over the hour hand by $\frac{330^\circ}{60}\times 16^\circ = 88^\circ$

14. (a) At 9'O clock, the Minute Hand is ahead of Hour Hand by 15 minutes. The hands will be opposite to each other when there is a space of 30 minutes between them.

This will happen when the Minute Hand gains 15 minutes' space over Hour Hand.

Time taken by Minutes Hand to gain 15 minutes

$$= 15\times\left(1+\frac{1}{11}\right) = 15+\frac{15}{11} = 15+1\frac{4}{11}$$

$$= 16\frac{4}{11} \text{ minutes.}$$

Hence the hands are opposite to each other at $16\frac{4}{11}$ minutes past 9.

15. (c) Use Shortcut Approach-2

16. (a) The clock gains 15 min in 24 hours.

Therefore, in 16 hours, it will gain 10 minutes.

Hence, the time shown by the clock will be 4:10 am.

17. (d) Angle traced by the hour hand in 6 hours

$$= \left(\frac{360}{12}\times 6\right)^\circ = 180^\circ$$

18. (d) When the hands of the clock are in the same straight line but not together, they are 30 minute spaecs apart.

At 7 o'clock, they are 25 min spaces apart.

∴ Minute hand will have to gain only 5 min spaces.

55 min spaces are gained in 60 min.

5 min spaces are gained in

$$\left(\frac{60}{55}\times 5\right)\text{min} = 5\frac{5}{11}\text{min.}$$

∴ Required time = $5\frac{5}{11}$ min past 7

19. (b) The year 1979 being an ordinary year, it has 1 odd day.

So, the day on 12th January 1980 is one day beyond on the day on 12th January, 1979.

But, January 12, 1980 being Saturday.

∴ January 12, 1979 was Friday.

20. (b) At 5 O'clock, the hands are 25 min. spaces apart.

To be at right angles and that too between 5.30 and 6, the minute hand has to gain (25 + 15) = 40 min. spaces.

55 min. spaces are gained in 60 min.

40 min. spaces are gained in

$$\left(\frac{60}{55}\times 40\right)\text{min.} = 43\frac{7}{11}\text{min.}$$

∴ Required time = $43\frac{7}{11}$ min past 5

21. (a) Use Shortcut Approach -3

22. (c) Count the number of days from 2005 onwards to get 0 odd day.

Year	2005	2006	2007	2008	2009	2010
Odd days	1	1	1	2	1	1

= 7 or 0 odd day.

∴ Calendar for the year 2005 is the same as that for the year 2011.

23. (c)

24. (c) Since between 11 am and 1 pm and 11 pm and 1 am two hands of a clock coincide only once, each time.

25. (d) In every 12 hours, the hands of clock are at right angles 22 times.

26. (d) Since between 5 am and 7 am (5 pm and 7 pm) this happens only once. In rest each of the hours it happens one time.

27. (b) $\frac{5\times30}{11/2}=\frac{300}{11}=27\frac{3}{11}\text{min}=27\text{ min }16\text{s}$

Therefore, required time = 5 : 27 : 16

28. (a) The angle made by minute-hand

$=5\times30=150°$

The angle made by hour-hand

$=2\times30+25\times\frac{1}{2}=72.5°$

Hence, required angle = $150-72.5=77.5°$

29. (b) $\frac{90°-30°}{5.5}=\frac{60}{11}\times2=\frac{120}{11}=10\frac{10}{11}$ minute

= 10 min 54 s

$\therefore$ Required time = 3 : 10 : 54

30. (d) $\frac{210}{5.5}=\frac{210}{11}\times2=\frac{420}{11}=38\frac{2}{11}\text{min}$

= 38 min 11s.

Therefore, required time = 7 : 38 : 11

31. (a) The angle made by minute-hand = 90°

The angle made by hour-hand

$=6\times30+15\times\frac{1}{2}=187.5°$

$\therefore$ Required difference = 97.5°

32. (c) $\frac{90-35}{5.5}=\frac{55}{11}\times2=10\text{min}$

So, the required time = 3 : 10 : 00

Again $\frac{90+35}{5.5}=\frac{125}{11}\times2=\frac{250}{11}=22\frac{8}{11}\text{min}$

= 22 min 43 s

33. (d) At 3 O' clock, the hour hand is at 3 and the minute hand is at 12. It means they are 15 min spaces apart.

To be together, the minute hand must gain 15 min over the hour-hand.

As we know,

55 min is gained by minute hand in 60 min

$\therefore$ 15 min will be gained in $\left(\frac{60}{55}\times15\right)\text{min}$

$=\frac{60\times3}{11}=\frac{180}{11}\text{min}=16\frac{4}{11}\text{min}$

Hence, the hands will coincide at $16\frac{4}{11}$ min past 3.

Fast Track Method :

Here, n = 3 and (n + 1) = 4

$\therefore$ Two hands will coincide at

$\frac{60n}{11}$ min past n.

$\Rightarrow$ $\frac{60\times3}{11}$ min past 3

$\Rightarrow$ $\frac{180}{11}$ min past 3

$\Rightarrow$ $16\frac{4}{11}$ min past 3.

34. (d) Clock will make right angle at

$(5n+15)\times\frac{12}{11}$ min past n.

Given that, n = 3

$\therefore$ $(5\times3+15)\times\frac{12}{11}$ min past 3.

$=30\times\frac{12}{11}$ min past 3

$=32\frac{8}{11}$ min past 3

i.e., 3 h and $32\frac{8}{11}$ min.

35. (a) Given that n = 2 and (n + 1) = 3 (n < 6)

The hands will be at the same straight line at

$\Rightarrow$ $(5n+30)\times\frac{12}{11}$ min past n

GP_3386

$\Rightarrow \quad (5\times 2+30)\times\frac{12}{11}$ min past 2

$\Rightarrow \quad (10+30)\times\frac{12}{11}$ min past 2

$\Rightarrow \quad \frac{40\times 12}{11}$ min past 2 $\Rightarrow \frac{480}{11}$ min past 2.

36. (a) Time from 6 am on a day to 11 pm on 4th day = 89 h

Now, 23 h 44 min of this clock = 24 h of correct clock

89 h of this clock $=\left(24\times\frac{15}{356}\times 89\right)$h of correct clock = 90 h of correct clock.

$\therefore$ Correct time will be 12 O' clock (mid – night).

37. (d) 5 th March, 1999 is Friday.

Then, 5th March 2000

= Friday + 2 = Sunday.

{$\because$ 2000 is a leap year and it crosses 29th Feb 2000, so 2 is taken as odd day}

$\therefore$ 5th March 2000 = Sunday.

Then, 9th March 2000 = Thursday.

38. (c) The year 2007 is an ordinary year, so it has 1 odd day.

3rd day of the year 2007 was Wednesday.

$\therefore$ 3rd day of the year 2008 will be one day beyond the Wednesday.

Hence, it will be Thursday.

39. (a) The year 2008 is a leap year, so it has 2 odd days, 3rd day of the year 2008 was Friday.

$\therefore$ 3rd day of the year 2009 will be 2 days beyond the Friday.

Hence, the required answer is Sunday.

40. (a) On 31st December, 2005, it was Saturday. Number of odd days from the year 2006 to the year 2009

= 1 odd day of 2006 + 1 odd day of 2007 + 2 odd days of 2008 (leap year) + 1 odd day of 2009

= 1 + 1 + 2 + 1 = 5

$\therefore$ On 31st December 2009, it was Thursday.

Thus, on 2nd January 2010, it was Saturday.

41. (b) 6th March 2005 = Monday

Then, 6th March 2004

= Monday – 1 day = Sunday.

{$\because$ 2004 is a leap year but it does not cross 29th February of 2004, so only 1 is taken as odd day}.

6th March 2004 = Sunday

7th March 2004 = Monday.

42. (c) Period upto 17th August, 2010

= (2009 yr + Period from 1.1.2010 to 17.8.2010)

Counting of odd days

Odd days in 1600 yr = 0

Odd day in 400 yr = 0

9 yr = (2 leap years + 7 ordinary years)

= (2 × 2 + 7 × 1) = 1 week + 4 days

= 4 odd days

Number of days between 1.1.2010 to 17.8.2010

January + February + March + April + May + June + July + August.

= (31 + 28 + 31 + 30 + 31 + 30 + 31 + 17 days)

= 229 days = 32 weeks + 5 odd days

Total number of odd days

= (0 + 0 + 4 + 5) days

= 9 days = 1 week + 2 odd days

Hence, the required day is Tuesday.

Level 1

1. (b) Here $H \times 30 = 4 \times 30 = 120°$.

(Since initially the hour hand is at 4.

$\therefore$ H = 4).

Required angle A = 90° and since,

$H \times 30 > A°$ so, there will be two timings.

Required time $T = \frac{2}{11}(H \times 30 \pm A)$ min past H.

$\therefore$ One timing $= \frac{2}{11}(4 \times 30 + 90)$ min past 4

$= 38\frac{2}{11}$ minutes past 4 Or 4 : 38 approx.

2. (c) 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th Aug., 1947)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

46 years = (11 leap years + 35 ordinary years)

= [(11 × 2) + (35 × 1)] = 57 odd days

$\Rightarrow$ 1 odd day.

Jan.	Feb.	March	April	May
31	28	31	30	31
June	July	Aug.		
30	31	15		

= 227 days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = (0 + 1 + 1 + 3) = 5

Hence, the required day was 'Friday'.

3. (a) 16th July, 1776 mean (1775 years + 6 months + 16 days)

Now, 1600 years have 0 odd days.

100 years have 5 odd days

75 years contain 18 leap years and 57 ordinary years and therefore (36 + 57) or 93 or 2 odd days.

$\therefore$ 1775 years have 0 + 5 + 2 = 7 and so 0 odd days.

Also number of days from 1st Jan. 1776 to 16th July, 1776

Jan. Feb. March April May June July

31 + 29 + 31 + 30 + 31 + 30 + 16

= 198 days = 28 weeks + 2 days = 2 odd days

$\therefore$ Total number of odd days = 0 + 2 = 2.

Hence the day on 16th July, 1776 was 'Tuesday'.

4. (c) 26th Jan., 1950 = (1949 years + Period from 1st Jan., 1950 to 26th Jan., 1950)

1600 years have 0 odd day. 300 years have 1 odd day.

49 years = (12 leap years + 37 ordinary years)

= [(12 × 2) + (37 × 1)] odd days = 61 odd days

= 5 odd days.

Number of days from 1st Jan. to 26th Jan = 26

= 5 odd days

Total number of odd days = (0 + 1 + 5 + 5) = 11

= 4 odd days

$\therefore$ The required day was 'Thursday'

5. (a) Time from 7 a.m. to quarter past 4

= 9 hours 15 min = 555 min

Now, $\frac{37}{12}$ min of this watch = 3 min of the correct watch.

555 min of this watch $= \left(\frac{3 \times 12}{37} \times 555\right)$ min

$= \left(\frac{3 \times 12}{37} \times \frac{555}{60}\right)$ hrs. = 9 hrs of the correct watch.

Correct time is 9 hours after 7 a.m. i.e., 4 p.m.

6. (a) Time between 10 a.m. on Monday to 10:30 a.m. on

Sunday $= 144\frac{1}{2}$ hours.

GP_3386

$24\frac{1}{12}$ hours of incorrect clock = 24 hours of correct time.

$\therefore$ $144\frac{1}{2}$ hours of incorrect clock = x hours of correct time.

$\therefore$ $x = \dfrac{144\frac{1}{2} \times 24}{24\frac{1}{12}} = 144$ hours i.e.,

The true time is 10 a.m. on Sunday.

7. (a) Since, in one hour, two hands of a clock coincide only once, so, there will be value.

$$\text{Required time } T = \frac{2}{11}(H \times 30 + A°) \text{ min past H.}$$

Here H = initial position of hour hand = 3 (Since 3 O'clock)

A° = required angle = 0°(Since it coincides)

$$T = \frac{2}{11}(3 \times 30 + 0) \text{ min past 3}$$

$$= 16\frac{4}{11} \text{ minutes past 3.}$$

8. (a) Time interval indicated by incorrect clock = 6 p.m – 1 p.m = 5 hrs.

Time gained by incorrect clock in one hour

$$= +1 \text{ min} = +\frac{1}{60} \text{ hr.}$$

Using the formula,

$$\frac{\text{True time interval}}{\text{Time interval in incorrect clock}}$$

$$= \frac{1}{1 + \text{hour gained in 1 hour by incorrect clock}}$$

$$\Rightarrow \frac{\text{True time interval}}{5} = \frac{1}{1+\frac{1}{60}}$$

$$\Rightarrow \text{True time interval} = \frac{5 \times 60}{61} = 4\frac{56}{61}$$

$\therefore$ True time = 1 p.m. + $4\frac{56}{61}$ hrs.

$$= 5 \text{ p.m.} + \frac{56}{61} \text{ hrs.} = 5 \text{ p.m.} + \frac{56}{61} \times 60 \text{ min.}$$

$$= 55\frac{5}{61} \text{ minutes past 5.}$$

9. (b) Time from noon on Sunday to 3 pm on Wednesday = 75 hours.

24 hours 2 minutes of the first clock

= 24 hours of the correct one.

$\Rightarrow$ 1 hour of the first clock = 24 × (30/721) hours of correct one.

$\Rightarrow$ 75 hours of the first clock

= 24 × 30 × (75/721) hours of correct one

= 54000/721 hours = 74 hours 53.7 min.

Hence the answer is 2:54 pm.

10. (d) Both (a) and (b) are correct.

Relative speed = Speed of minute hand- Speed of hour hand

$$= 6° - \frac{1}{2}° = 5\frac{1}{2}° \text{ and } 1 \text{ min} - \frac{1}{12} \text{ min} = \frac{11}{12} \text{ min}$$

11. (c) At 9 O' clock, the hour-hand is at 9 and the minute hand is at 12. It means that the two hands are 15 min spaces apart. To be in the same straight line (but not together), they will be 30 min space apart.

$\therefore$ The minute hand will have to gain (30 – 15) = 15 min spaces over the hour hand.

As we know,

55 min spaces are gained in 60 min.

$\therefore$ 15 min will be gained in

$$\left(\frac{60}{55} \times 15\right) \text{min} = \frac{180}{11} = 16\frac{4}{11} \text{ min}$$

Hence, the hands will be in the same

straight line but not together at $16\frac{4}{11}$ min past 9.

Fast Track Method :

Here, n = 9 and n + 1 = 10 (n > 6)

The hands will be in the same straight line at

$(5n-30)\times\frac{12}{11}$ min past n

$\Rightarrow (5\times9-30)\frac{12}{11}$ min past 9

$\Rightarrow \frac{15\times12}{11}$ min past 9.

$\Rightarrow \frac{180}{11}$ min past 9. $\Rightarrow 16\frac{4}{11}$ min past 9.

12. (b) Given that, n = 5 and n + 1 = 6

The hands will make right angle at

$(5n\pm15)\times\frac{12}{11}$ min past 5

$\Rightarrow (5\times5\pm15)\times\frac{12}{11}$ min past 5

$\Rightarrow (25+15)\times\frac{12}{11}$ min past 5 and

$(25-15)\times\frac{12}{11}$ min past 5.

$\Rightarrow 40\times\frac{12}{11}$ min past 5 and $\frac{10\times12}{11}$ min past 5

$\Rightarrow \frac{480}{11}$ min past 5 and $\frac{120}{11}$ min past 5.

$\Rightarrow 43\frac{7}{11}$ min past 5 and $10\frac{10}{11}$ min past 5.

13. (b) If the minute hand of a clock overtakes the hour hand at interval x min of the correct time, then clock losses or gains by

$\left(\frac{720}{11}-x\right)\times\left(\frac{60\times24}{x}\right)$ min

Positive for gain and negative for loss

Here, $x = 64$

Putting the value of x in above formula,

$\left(\frac{720}{11}-64\right)\times\left(\frac{60\times24}{64}\right)=\frac{360}{11}=32\frac{8}{11}$ min

14. (a) 100 years have 5 odd days.

$\therefore$ Last day of 1st century is Friday.

200 years have

(5 × 2) = 1 week + 3 odd days = 3 odd days

$\therefore$ Last day of 2nd century is Wednesday,

300 years have (5 × 3) = 2 week + 1 odd day = 1 odd day

$\therefore$ last day of 3rd century is Monday

400 years have 0 odd day.

$\therefore$ Last day of 4th century is Sunday.

This cycle is repeated.

$\therefore$ Last day of a century cannot be Tuesday or Thursday or Saturday.

15. (b) Number of odd days between 6th November, 1987 and 4th April, 1988.

= (30 – 6) days of November 1987 + 31 days of December 1987 + 31 days of January 1988 + 29 days of February 1988 + 31 days of March 1988 + 4 days of April 1988

= {(30 – 6) + 31} + (31 + 29 + 31 + 4)

= 24 + 31 + 31 + 29 + 31 + 4 = 150

= 21 weeks + 3 days = 3 odd days

$\therefore$ 6th November, 1987 will be three days backward from Monday.

$\therefore$ Required day is Friday.

16. (a) Period upto 17th July, 1776

= (1775 yr + Period from 1st January 1776 to 17th July, 1776)

Counting of odd days

In 1600 yr = 0

In 100 yr = 5

75 yr = 18 leap years + 57 ordinary years

= (18 × 2 + 57 × 1) odd days = 93 odd days

GP_3386

= (13 weeks + 2 days) = 2 odd days

∴ 1775 yr have (0 + 5 + 2) 7 odd days

= 7 odd days = 1 week + 0 odd day

= 0 odd day

Number of days between 1.1.1776 to 17.7.1776

January + February + March + April + May + June + July

31 + 29 + 31 + 30 + 31 + 30 + 17

= 199 days = 28 weeks + 3 odd days

∴ Total number of odd days = (3 + 0) = 3

Hence, the required day is Wednesday.

17. (b) First, we find out the day on 1.03.2005.

Period upto 1.3.2005

= (2004 yr + Period from 1.1.2005 to 1.3.2005)

∴ Odd day in 1600 yr = 0

Odd day in 400 yr = 0

4 yr = (1 leap year + 3 ordinary years)

= (1 × 2 + 3 ×1) = 5 odd days

Number of days between 1.1.2005 and 1.3.2005,

January + February + March = 31 + 28 + 1

= 60 days = (8 weeks + 4 days) = 4 odd days

Total number of odd days = (0 + 0 + 5 + 4) = 9

=1 week + 2 odd days = 2 odd days

∴ 1.3.2005 was Tuesday.

So, Saturday will fall on 5.3.2005.

Hence, Saturday lies on 5th, 12th, 19th and 26th of March 2005.

CHAPTER

18

Permutation & Combination and Probability

PERMUTATION & COMBINATION

Permutation is the different arrangement of a given number of things by taking some or all at a time and on the other hand **combinations** are the selection which can be done by taking some or all from a number of objects.

FUNDAMENTAL PRINCIPLE OF COUNTING

Multiplication Principle

If a certain task can be done in 'm' different ways; followed by an another task in 'n' different ways, then the two operations in succession can be performed in $m \times n$ ways. This can be extended to any finite number of tasks.

Illustration – 1

A person wants to go from station *P* to station *R* via station *Q*. There are 4 routes from *P* to *Q* and 5 routes from *Q* to *R*. In how many ways can he travel from *P* to *R* via Q?

Solution : He can go from P to Q in 4 ways and Q to R in 5 ways. So number of ways of travel from P to R via Q is $4 \times 5 = 20$.

Illustration – 2

A college offers 6 courses in the morning and 4 in the evening. Find the possible number of choices with the student if he wants to study one course in the morning and one in the evening.

Solution : The college has 6 courses in the morning out of which the student can select one course in 6 ways.
In the evening the college has 4 courses out of which the student can select one in 4 ways.
Hence the required number of ways = $6 \times 4 = 24$.

Illustration – 3

In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes ?

Solution : First prize may be given to any one of the 4 boys, hence first prize can be distributed in 4 ways.
Similarly every one of second, third, fourth and fifth prizes can also be given in 4 ways.
$\therefore$ The number of ways of their distribution
$= 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 1024$

Addition Principle

If a certain task can be done in 'm' different ways and an another task, which is independent of the first task, can be done in 'n' different ways. Then either of the two operations can be performed in $(m + n)$ ways. This can be extended to any finite number of independent operations.

Illustration – 4

A college offers 6 courses in the morning and 4 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.

Solution : The college has 6 courses in the morning out of which the student can select one course in 6 ways.
In the evening the college has 4 courses out of which the student can select one in 4 ways.
Hence the required number of ways = $6 + 4 = 10$.

GP_3386

Illustration – 5

A person wants to leave station *Q*. There are 4 routes from station *Q* to *P* and 5 routes from *Q* to *R*. In how many ways can he travel from the station *Q* ?

Solution : He can go from Q to P in 4 ways and Q to R in 5 ways. To go from Q to P and Q to R are independent to each other. Hence the person can leave station Q in $4 + 5 = 9$ ways.

Note that whenever the concept in the question is the same as 'and' we will use multiplication and whenever the concept is same as 'or' we will use addition.

FACTORIALS

If n is a natural number then the product of all natural numbers upto n is called **factorial** n and it is denoted by n ! or

Thus, $n! = n(n-1)(n-2) \ldots\ldots 3.2.1$

Note that $0! = 1 = 1!$

$n! = n(n-1)!$

$= n(n-1)(n-2)!$

$= n(n-1)(n-2)(n-3)!$, etc.

For example : $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

But $4! = 4 \times 3 \times 2 \times 1$

$\therefore$ $6! = 6 \times 5 \times 4!$ or $6 \times 5 \times 4 \times 3!$

Remember that

$0! = 1, 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720$, etc.

MEANING OF PERMUTATION AND COMBINATION

Each of the different arrangements which can be made by taking some or all of a number of things is called a **permutation**.

Note that in an arrangement, the order in which the things arranged is considerable i.e., arrangement *AB* and *BA* of two letters *A* and *B* are different because in *AB*, *A* is at the first place and *B* is at the second place from left whereas in *BA*, *B* is at the first place and *A* is at the second place.

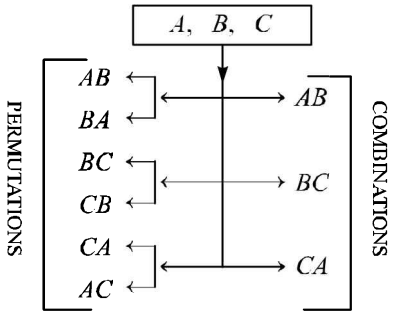
The all different arrangements of three letters *A*, *B* and *C* are *ABC*, *ACB*, *BCA*, *BAC*, *CAB* and *CBA*.

Here each of the different arrangements *ABC*, *ACB*, *BCA*, *BAC*, *CAB* and *CBA* is a *permutation* and number of different arrangement i.e. 6 is the *number of permutations*.

ABC, *ACB*, *BCA*, *BAC*, *CAB* and *CBA* are different arrangements of three letters *A*, *B* and *C*, because in each arrangement, order in which the letters arranged, is considered. But if the order in which the things are arranged is not considered; then *ABC*, *ACB*, *BCA*, *BAC*, *CAB* and *CBA* are not different but the same.

As in arrangements, order in which things are arranged is considered. Hence all arrangements of two letters out of the three letters *A*, *B* and *C* are *AB*, *BA*, *BC*, *CB*, *CA* and *AB*.

$\therefore$ Number of permutations (or arrangements) of two letters out of three letters *A*, *B* and *C* = 6.



Each of the different selections or groups which can be made by some or all of a number of given things without consideration of the order of things in any selection or group is called a **combination**.

As in selection order in which things are selected is not considered; hence, selections of two letters *AB* and *BA* out of three letters *A*, *B* and *C* are the same. Similarly selections of *BC* and *CB* are the same. Also selections of *CA* and *AC* are the same.

Hence selection of two letters out of the three letters *A*, *B* and *C* can be made as *AB*, *BC* and *CA* only.

Number of combinations (or groups) of two letters out of three letters *A*, *B* and *C* = 3.

You can easily identify the given problem is of permutation or combination (where the word permutation or combination is not mentioned) using the following classifications of problems:

Problems of Permutations

(i) Problems based on arrangements
(ii) Problems based on standing in a line
(iii) Problems based on seated in a row
(iv) Problems based on digits
(v) Problems based on arrangement letters of a word
(vi) Problems based on rank of a word (in a dictionary)

Problems of Combinations

(i) Problems based on selections or choose
(ii) Problems based on groups or committee
(iii) Problems based on geometry

If in any problem, it is neither mentioned that the

problem is of permutation or combination nor does the problem fall in the categories mentioned above for the problems of permutations or problems of combinations, then you think whether arrangement (i.e. order) is considerable or not? If arrangement (i.e., order) is considerable in the given problem, then the problem is of permutation otherwise it is of combination.

COUNTING FORMULA FOR LINEAR PERMUTATIONS

Without Repetition

1. Number of permutations of n different things, taking r at a time is denoted by ${}^nP_r$ or $P(n, r)$, which is given by

$$ {}^nP_r = \frac{n!}{(n-r)!} (0 \le r \le n), $$

where n is a natural number and r is a whole number.

2. Number of arrangements of n different objects taken all at a time is ${}^nP_n = n$!

Note : ${}^nP_1 = n$ and ${}^nP_n = {}^nP_{n-1}$

Illustration – 6

Find the number of ways in which four persons can sit on six chairs.

Solution : ${}^6P_4 = 6.5.4.3 = 360$

With Repetition

1. Number of permutations of n things taken all at a time, if out of n things p are alike of one kind, q are alike of second kind, r are alike of a third kind and the rest $n - (p + q + r)$ are all different is $\frac{n!}{p!q!r!}$

2. Number of permutations of n different things taken r at a time when each thing may be repeated any number of times is n^r.

Illustration – 7

Find the number of words that can be formed out of the letters of the word COMMITTEE taken all at a time.

Solution : There are 9 letters in the given word in which two T's, two M's and two E's are identical. Hence the required number of

$$ \text{words} = \frac{9!}{2!2!2!} = \frac{9!}{(2!)^3} = \frac{9!}{8} = 45360 $$

Number of Linear Permutations under Certain Conditions

1. Number of permutations of n different things taken all together when r particular things are to be placed at some r given places $= {}^{n-r}P_{n-r} = (n - r)!$
2. Number of permutations of n different things, taken r at a time, when a particular thing is to be always included in each arrangement, is $r.\ {}^{n-1}P_{r-1}$.

Illustration – 8

How many different words can be formed with the letters of the word 'JAIPUR' which start with '*A*' and end with '*I*'?

Solution : After putting A and I at their respective places (only in one way) we shall arrange the remaining 4 different letters at 4 places in 4! ways. Hence the required number = $1 \times 4! = 24$.

Illustration – 9

How many different 3 letter words can be formed with the letters of word 'JAIPUR' when *A* and *I* are always to be excluded?

Solution : After leaving A and I, we are remained with 4 different letters which are to be used for forming 3 letters words.
Hence the required number
$= {}^4P_3 = 4 \times 3 \times 2 = 24$.

CIRCULAR PERMUTATIONS

Total number of ways in which n different things can be arranged in a circle = $(n - 1)!$
For example: If six persons are to be seated on a circular table for dinner, the number of ways of doing so is (6–1)! = 5! = 5×4×3×2×1 = 120 ways.

COUNTING FORMULA FOR COMBINATION

1. **Selection of Objects Without Repetition**
The number of combinations or selections of n different things taken r at a time is denoted

GP_3386

by ${}^nC_r$ or $C(n, r)$ or $C\binom{n}{r}$

where ${}^nC_r = \frac{n!}{r!(n-r)!}$; $(0 \le r \le n)$,

where n is a natural number and r is a whole number.

Some Important Results

(i) ${}^nC_n = 1$ **(ii)** ${}^nC_0 = 1$ **(iii)** ${}^nC_r = {}^nC_{n-r}$

Illustration – 10

How many different 4-letter words can be formed with the letters of the word 'JAIPUR' when *A* and *I* are always to be included ?

Solution : Since *A* and *I* are always to be included, so first we select 2 letters from the remaining 4, which can be done in ${}^4C_2 = 6$ ways. Now these 4 letters can be arranged in 4! = 24 ways, So the required number $= 6 \times 24 = 144$.

Illustration – 11

How many combinations of 4 letters can be made of the letters of the word 'JAIPUR' ?

Solution : Here 4 things are to be selected out of 6 different things. ∴ number of combinations $= {}^6C_4$

$= \frac{6.5.4.3}{4.3.2.1} = 15$

2. Selection of Objects with Repetition

The total number of selections of r things from n different things when each thing may be repeated any number of times is ${}^{n+r-1}C_r$

3. Restricted Selection

(i) Number of combinations of n different things taken r at a time when k particular things always occur is ${}^{n-k}C_{r-k}$.

(ii) Number of combinations of n different things taken r at a time when k particular things never occur is ${}^{n-k}C_r$.

4. Selection From Distinct Objects

Number of ways of selecting at least one thing from n different things is

${}^nC_1 + {}^nC_2 + {}^nC_3 + ...+ {}^nC_n = 2^n - 1$.

This can also be stated as the total number of combination of n different things is $2^n - 1$.

Illustration – 12

Ramesh has 6 friends. In how many ways can he invite one or more of them at a dinner ?

Solution : He can invite one, two, three, four, five or six friends at the dinner. So total number of ways of his invitation

$= {}^6C_1 + {}^6C_2 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 2^6 - 1 = 63$

DIVISION AND DISTRIBUTION OF OBJECTS

1. The number of ways in which $(m + n)$ different things can be divided into two groups which contain m and n things respectively is

$${}^{m+n}C_m \, {}^nC_n = \frac{(m+n)!}{m!n!}, \; m \neq n$$

Particular case:

When $m = n$, then total number of ways is $\frac{(2m)!}{(m!)^2}$, when order of groups is considered and $\frac{(2m)!}{2!(m!)^2}$, when order of groups is not considered.

2. The number of ways in which $(m + n + p)$ different things can be divided into three groups which contain m, n and p things respectively is

$${}^{m+n+p}C_m \cdot {}^{n+p}C_p \cdot {}^pC_p = \frac{(m+n+p)!}{m!\,n!\,p!}, \; m \neq n \neq p$$

Particular case:

When $m = n = p$, then total number of ways is $\frac{(3m)!}{(m!)^3}$, when order of groups is considered and $\frac{(3m)!}{3!(m!)^3}$, when order of groups is not considered.

3. (i) Total number of ways to divide n identical things among r person is ${}^{n+r-1}C_{r-1}$

(ii) Also total number of ways to divide n identical things among r persons so that each gets atleast one is ${}^{n-1}C_{r-1}$.

Illustration – 13

In how many ways 20 identical mangoes may be divided among 4 persons if each person is to be given at least one mango?

Solution : If each person is to be given at least one mango, then number of ways will be

${}^{20-1}C_{4-1} = {}^{19}C_3 = 969$.

IMPORTANT RESULTS ABOUT POINTS

1. If there are n points in a plane of which m ($< n$) are collinear, then
   (i) Total number of different straight lines obtained by joining these n points is ${}^nC_2 - {}^mC_2 + 1$.
   (ii) Total number of different triangles formed by joining these n points is ${}^nC_3 - {}^mC_3$
2. Number of diagonals of a polygon of n sides is ${}^nC_2 - n$ *i.e.*, $\frac{n(n-3)}{2}$
3. Given n points on the circumference of a circle, then
   (i) Number of straight lines obtained by joining these n points = ${}^nC_2$
   (ii) Number of triangles obtained by joining these n points = ${}^nC_3$
   (iii) Number of quadrilaterals obtained by joining these n points = ${}^nC_4$

FINDING THE RANK OF A WORD

We can find the rank of a word out of all the words with or without meaning formed by arranging all the letters of a given word in all possible ways when these words are listed as in a dictionary. You can easily understand the method to find the above mentioned rank by the following illustrations.

Illustration – 14

If the letters of the word RACHIT are arranged in all possible ways and these words (with or without meaning) are written as in a dictionary, then find the rank of this word RACHIT.

Solution : The order of the alphabet of RACHIT is A, C, H, I, R, T.
The number of words beginning with A (i.e. the number of words in which A comes at first place) is ${}^5P_5 = 5!$.
Similarly, number of words beginning with C is 5!, beginning with H is 5! and beginning with I is also 5!.
So before R, four letters A, C, H, I can occur in $4 \times (5!) = 480$ ways.
Now the word RACHIT happens to be the first word beginning with R.
$\therefore$ the rank of this word RACHIT = 480 + 1 = 481.

Illustration – 15

The letters of the word MODESTY are written in all possible orders and these words (with or without meaning) are listed as in a dictionary then find the rank of the word MODESTY.

Solution : The order of the alphabet of MODESTY is D, E, M, O, S, T, Y.
Number of words beginning with D is ${}^6P_6 = 6!$
Number of words beginning with E is ${}^6P_6 = 6!$
Number of words beginning with MD is ${}^5P_5 = 5!$
Number of words beginning with ME is ${}^5P_5 = 5!$
Now the first word start with MO is MODESTY.
Hence rank of the word MODESTY
$= 6! + 6! + 5! + 5! + 5! + 1$
$= 720 + 720 + 120 + 120 + 1 = 1681$.

PROBABILITY

Probability is the most important concept that we use in our day to day life. A mathematically measure of uncertainty is known as **probability**.
In probability, concept of permutation and combination are used, therefore students would need to master the concepts of combination and permutation before starting probability.

CONCEPT OF PROBABILITY

If you go to buy 10 kg of sugar at ₹ 40 per kg, you can easily find the exact price of your purchase is ₹ 400. On the other hand, the shopkeeper may have a good estimate of the number of kg of sugar that will be sold during the day, but it is impossible to predict the exact amount, because the number of kg of sugar that the consumers will purchase during a day is random.
There are various phenomenon in nature, leading to an outcome, which cannot be predicted in advance. **For example**, we cannot exactly predict that
(i) a head will occur on tossing a coin,
(ii) India will win the cricket match against Pakistan, etc. But we can measure the amount of certainty of occurrence of an outcome of a phenomenon. This amount of certainty of occurrence of an outcome of a phenomenon is called **probability**. **For example**, on tossing a coin certainty of occurrence of each of a head and a tail are the same. Hence amount certainty of occurrence of each of a head and a tail is 50% i.e., $\frac{50}{100} = \frac{1}{2}$. Therefore $\frac{1}{2}$ is the amount of certainty of

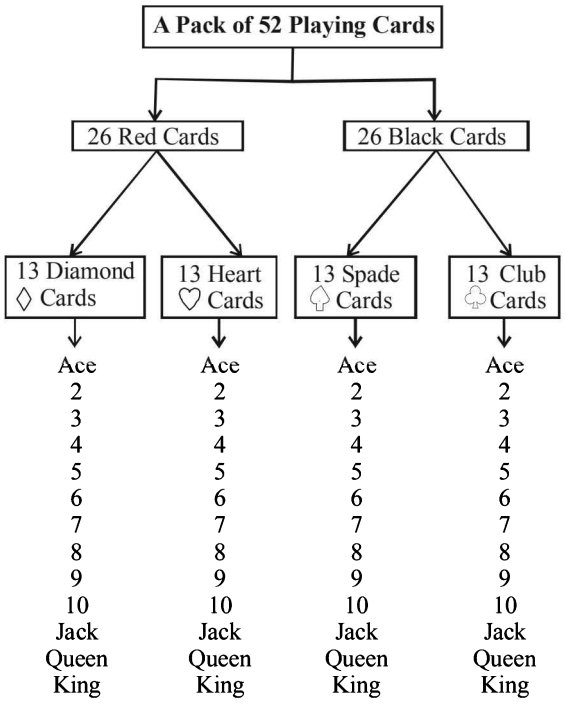
GP_3386

occurrence of a head (or a tail) on tossing a coin and hence $\frac{1}{2}$ is the probability of occurrence of a head (or a tail) on tossing a coin.

On throwing a dice (a dice is a cuboid having one of the numbers 1, 2, 3, 4, 5 and 6 on each of its six faces), certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 and 6 on its top face are the same. Therefore certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 and 6 is $\frac{1}{6}$. Therefore $\frac{1}{6}$ is the amount of certainty of occurrence of each of the numbers 1, 2, 3, 4, 5 or 6 on the top face of the dice on throwing the dice and hence $\frac{1}{6}$ is the probability of occurrence of each of the numbers 1, 2, 3, 4, 5 or 6 on the top face of the dice.

Basic Terms

1. **An Experiment:** An action or operation resulting in two or more outcomes is called an **experiment**.
   For example:
   (i) Tossing of a coin is an experiment because there are two possible outcomes head and tail.
   (ii) Drawing a card from a pack of 52 cards is an experiment because there are 52 possible outcomes.
2. **Sample Space:** The set of all possible outcomes of an experiment is called the **sample space**, denoted by S. An element of S is called a **sample point**.
   For example :
   (i) In the experiment of tossing a coin, the sample space has two points corresponding to head (H) and Tail (T) i.e., $S\{H, T\}$.
   (ii) When we throw a dice then any one of the numbers 1, 2, 3, 4, 5 and 6 will come up. So the sample space,
   $S = \{1, 2, 3, 4, 5, 6\}$
   (iii) When we throw two coins simultaneously, then sample space, S = {HT, TH, TT, HH}
   (iv) When we throw three coins simultaneously, then sample space, S = {HHT, HTH, THH, HTT, THT, TTH, TTT, HHH}
   (v) When we throw two dice simultaneously then sample space, S = {(1, 1), (1, 2), (1, 3), ..., (1, 6), (2, 1), (2, 2), (2, 3), ..., (2, 6), (3, 1), (3, 2), ..., (3, 6), (4, 1), ..., (4, 6), (5, 1), ..., (5, 6), (6, 1), ..., (6, 6)}



- Diamond, Heart, Spade and Club are called **four suits**.
- Number of Ace cards = 4 (1 Diamond, 1 Heart, 1 Spade, 1 Club)
- Jack, Queen and King are called **Face cards**. There are 4 Jacks, 4 Queens and 4 Kings

$\therefore$ Number of face cards = 4 + 4 + 4 = 12

Mathematical Definition of Probability

Probability of occuring an event E of an experiment, P(E)

$$= \frac{\text{Number of outcomes favourable to the event E}}{\text{Total number of outcomes}}$$

OR

$$P(E) = \frac{n(E)}{n(S)}$$

Here, n(E) is the number of outcomes favourable to the event and n(S) is the total number of outcomes i.e. number of outcomes in the sample space of the experiment.

- Probability of not occuring an event E = 1 – P(E)

Illustration – 16

Two dice are thrown at a time. Find the probability of the following :

(i) the numbers shown are equal

(ii) the difference of numbers shown is 1

Solution : The sample space in a throw of two dice

$S = \{(1,1),(1,2),...,(1,6),(2,1),(2,2),...,(2,6),(3,1),...,(3,6),(4,1),...,(4,6),(5,1),...,(5,6),(6,1),...,(6,6)\}$

$\therefore$ total no. of outcomes, $n(S) = 36$

(i) Here E_1 = the event of showing equal number on both dice

Favourable outcomes = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6}

$$\therefore\ n(E_1) = 6 \Rightarrow P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(ii) Here E_2 = the event of showing numbers whose difference is 1.

Favourable outcomes t = {(1, 2) (2, 1) (2, 3) (3, 2) (3, 4) (4, 3) (4, 5) (5, 4) (5, 6) (6, 5)}

$$\therefore\quad n(E_2) = 10 \Rightarrow P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{36} = \frac{5}{18}.$$

Illustration – 17

If three cards are drawn from a pack of 52 cards, what is the chance that all will be queen?

Solution : If the sample space be S, then $n(S)$ = the total number of ways of drawing 3 cards out of 52 cards = $^{52}C_3$. Now, if A = the event of drawing three queens, then $n(A) = {}^4C_3$

$$\therefore\ P(E) = \frac{n(A)}{n(S)} = \frac{{}^4C_3}{{}^{52}C_3} = \frac{4}{\frac{52\times51\times50}{3\times2}} = \frac{1}{5525}$$

Range of Probability of any Event

As the number of favourable outcomes cannot be less than zero (0), $0 \le P(E)$ ...(i)

Also the number of favourable outcomes cannot be greater than the total outcomes

$\therefore\quad P(E) \ge 1$ ...(ii)

From (i) and (ii), $\boxed{0 \le P(E) \le 1}$

Odds Against and Odds in Favour of an Event

(i) Odds in favour of an event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

(ii) Odds against an event

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

Illustration – 18

In a single toss of two dice, find the odds against drawing 7 as the sum of numbers on two dice.

Solution : Total number of outcomes = $(6)^2 = 36$

Favourable outcomes = (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

No. of favourable outcomes = 6

No. of unfavourable outcomes = 36 – 6 = 30

Illustration – 19

Find the odds in favours of getting a king when a card is drawn from a well shuffled pack of 52 cards.

Solution : Required probability $= \frac{4}{48} = \frac{1}{12}$.

Mutually Exclusive Events

Any two events are said to be mutually exclusive events if they cannot occur together.

For any two mutually exclusive events A and B

P (A or B) = P(A) + P(B)

and if events A and B are not mutually exclusive, then

P (A or B) = P(A) + P(B) – P(A and B)

In these expressions, P (A or B) represents the probability of either event A happening or event B happening.

Illustration – 20

A bag contains 6 white, 5 black and 4 red balls. Find the probability of getting either a white or a black ball in a single draw.

Solution : Let A = Event that we get a white ball.

B = Event that we get a black ball.

Two events A and B are mutually exclusive.

So, $P(A \text{ or } B) = P(A) + P(B) = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$.

Illustration – 21

One digit is selected from first 20 positive integers. What is the probability that it is divisible by 3 or 4?

Solution : Let A = Event that the selected number is divisible by 3

B = Event that the selected number is divisible by 4

Here, the events A and B are not mutually exclusive because 12 is divisible by both 3 and 4.

$\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

GP_3386

$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20} = \frac{10}{20} = \frac{1}{2}$.

Illustration – 22

The probability that at least one of the events A and B occurs is 0.7 and they occur simultaneously with probability 0.2. Then $P(\bar{A}) + P(\bar{B}) =$

(a) 1.8 (b) 0.6 (c) 1.1 (d) 0.4

Solution : (c)

We have $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.2$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = 0.9 \Rightarrow 1 - P(\bar{A}) + 1 - P(\bar{B}) = 0.9$

$\Rightarrow P(\bar{A}) + P(\bar{B}) = 1.1$

Independent Events

Two or more events are said to be *independent* if occurrence or non-occurrence of any of them does not influence the probability of occurrence or non-occurrence of other events.

For example: When two cards are drawn one by one from a pack of 52 playing cards with replacement (i.e., the first card drawn is put back in the pack and then the second card is drawn), then the event of occurrence of a king in the first draw and the event of occurrence of a king in the second draw are *independent events* because the occurrence or non-occurrence of a king in first draw does not influence the probability of occurrence or non-occurrence of the king in second draw.

But if the second card is drawn without replacement of first card, then the two events are not independent, because in this case probability of drawing a king in the second draw depends on weather a king is drawn in first draw or not. If a king is drawn in first draw, then probability of drawing a king in second draw will be $\frac{3}{51}$ but if a king is not drawn in first draw, then the probability of drawing a king in second draw will be $\frac{4}{51}$.

- For any two independent events A and B P(A and B) = P(A) × P(B). The above result will be true for any number of independent events
- For any three independent events A, B and C P(A, B and C) = P(A) × P(B) × P(C)

Illustration – 23

A fair coin is tossed repeatedly. If the tail appears on first four tosses, then the probability of the head appearing on the fifth toss equals

(a) $\frac{1}{2}$ (b) $\frac{1}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{5}$

Solution : (a) The event that the fifth toss results a head is independent of the event that the first four tosses results tails.

$\therefore$ Probability of the required event = 1/2.

Geometrical Applications

The following statements are axiomatic:

1. If a point is taken at random on a given line segment AB, the probability that it fails on a particular segment PQ of the line segment is PQ/AB

   i.e. probability = $\frac{\text{favourable area}}{\text{total area}}$

2. If a point is taken at random on the area S which includes an area P, the probability that the point falls on P is P/S.

   i.e. probability = $\frac{\text{favourable area}}{\text{total area}}$

Probability Regarding n Letters and Their Envelopes

If n letters corresponding to n envelopes are placed in the envelopes at random, then

Probability that all letters are in right envelopes = $\frac{1}{n!}$

Probability that no letters is in right envelopes

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}$

Probability that exactly r letters are in right envelopes

$= \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^{n-r} \frac{1}{(n-r)!}\right]$

Illustration – 24

There are four letters and four envelopes, the letters are placed into the envelopes at random, find the probability that all letters are placed in the wrong envelopes.

Solution : Since all letters are to be placed in wrong envelopes, hence required probability

$= \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right] = \frac{1}{2} - \frac{1}{6} + \frac{1}{24} = \frac{3}{8}$.

LEVEL - 1

1. Two dice are thrown simultaneously. The probability of obtaining a total score of seven is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{7}$ (d) $\frac{5}{6}$

2. If A and B are two independent events with $P(A) = 0.6$, $P(B) = 0.3$, then $P(A' \cap B')$ is equal to :

(a) 0.18 (b) 0.28 (c) 0.82 (d) 0.72

3. Anil can kill a bird once in 3 shots. On the assumption that he fires 3 shots, find the probability that the bird is killed.

(a) $\frac{1}{3}$ (b) $\left(\frac{1}{3}\right)^3$ (c) $\frac{19}{27}$ (d) $\frac{8}{9}$

4. A pair of dice is thrown thrice. The probability of throwing doublets at least once is

(a) $\frac{1}{36}$ (b) $\frac{25}{216}$

(c) $\frac{125}{216}$ (d) None of these

5. A die is thrown once. What is the probability of occurrence of an odd number on the upper face?

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

6. A die is thrown once. Find the probability that 3 or greater than 3 turns up.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{3}$

7. The probability that the two digit number formed by digits 1, 2, 3, 4, 5 is divisible by 4 is

(a) $\frac{1}{30}$ (b) $\frac{1}{20}$

(c) $\frac{1}{5}$ (d) None of these

8. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square, is

(a) $\frac{1}{10}$ (b) $\frac{1}{100}$ (c) $\frac{9}{10}$ (d) $\frac{90}{100}$

9. The probability that a leap year will have 53 Friday or 53 Saturday, is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{7}$

10. What is the probability that in a family of 4 children there will be at least one boy?

(a) $\frac{15}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{16}$ (d) $\frac{7}{8}$

11. In tossing three coins at a time, what is the probability of getting at most one head?

(a) $\frac{3}{8}$ (b) $\frac{7}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

12. In a single throw with four dice, the probability of throwing seven is

(a) $\frac{4}{6^4}$ (b) $\frac{8}{6^4}$ (c) $\frac{16}{6^4}$ (d) $\frac{20}{6^4}$

13. The probability that the birth days of six different persons will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$

(c) ${}^{12}C_2 \times \frac{2^6 - 1}{12^6}$ (d) $\frac{341}{12^5}$

14. Three integers are chosen at random from the first 20 integers. The probability that their product is even, is

(a) $\frac{2}{19}$ (b) $\frac{3}{29}$ (c) $\frac{17}{19}$ (d) $\frac{4}{29}$

15. The probability of getting 10 in a single throw of three fair dice is :

(a) $\frac{1}{6}$ (b) $\frac{1}{8}$ (c) $\frac{1}{9}$ (d) $\frac{1}{5}$

16. The probability that when 12 balls are distributed among three boxes, the first will contain three balls is,

(a) $\frac{2^9}{3^{12}}$ (b) $\frac{{}^{12}C_3 . 2^9}{3^{12}}$

(c) $\frac{{}^{12}C_3 . 2^{12}}{3^{12}}$ (d) None of these

17. If three vertices of a regular hexagon are chosen at random, then the chance that they form an equilateral triangle is:

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{2}$

GP_3386

18. A positive integer N is selected such that $100 < N < 200$. The probability that it is divisible by either 4 or 7 is :

(a) $\frac{38}{99}$ (b) $\frac{24}{99}$ (c) $\frac{34}{99}$ (d) $\frac{14}{99}$

19. Two squares are chosen at random on a chessboard, the probability that they have a side in common is :

(a) $\frac{3}{32}$ (b) $\frac{1}{32}$

(c) $\frac{1}{18}$ (d) None of these

20. Three squares of a chessboard are chosen at random, the probability that two are of one colour and one of another is :

(a) $\frac{67}{992}$ (b) $\frac{16}{21}$

(c) $\frac{31}{32}$ (d) None of these

21. Four numbers are multiplied together. Then the probability that the product will be divisible by 5 or 10 is :

(a) $\frac{169}{625}$ (b) $\frac{369}{625}$

(c) $\frac{169}{1626}$ (d) None of these

22. An old person forgets the last two digits of a telephone number, remembering only that these are different dialled at random. The probability that the number is dialled correctly is :

(a) 1/90 (b) 81/91
(c) 2/99 (d) None of these

23. Three fair coins are tossed. Find the probability that they are all tails, if one of the coins shows a tail .

(a) $\frac{2}{7}$ (b) $\frac{5}{14}$

(c) $\frac{1}{7}$ (d) None of these

24. A speaks truth in 75% of the cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

(a) 15% (b) 20% (c) 5% (d) 35%

25. How many words beginning with vowels can be formed with the letters of the word EQUATION?

(a) 25200 (b) 15200 (c) 25300 (d) 35200

26. The number of words that can be formed out of the letters of the word COMMITTEE is

(a) $\frac{9!}{(2!)^3}$ (b) $\frac{9!}{(2!)^2}$ (c) $\frac{9!}{2!}$ (d) 9!

27. How many different letter arrangements can be made from the letter of the word EXTRA in such a way that the vowels are always together?

(a) 48 (b) 60 (c) 40 (d) 30

28. Every body in a room shakes hands with every else. If total number of hand-shaken is 66, then number of persons in the room is

(a) 11 (b) 12 (c) 13 (d) 14

29. The number of words from the letters of the words BHARAT in which *B* and *H* will never come together, is

(a) 360 (b) 240
(c) 120 (d) None of these

30. How many different ways are possible to arrange the letters of the word "MACHINE" so that the vowels may occupy only the odd positions?

(a) 800 (b) 125 (c) 348 (d) 576

31. In how many ways a hockey team of eleven can be elected from 16 players?

(a) 4368 (b) 4267 (c) 5368 (d) 4166

32. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected?

(a) 110 (b) 55 (c) 22 (d) 11

33. In a company, each employee gives a gift to every other employee. If the number of gifts is 61, then the number of employees in the company is :

(a) 11 (b) 13 (c) 12 (d) 8

34. Number of ways in which the letters of word GARDEN can be arranged with vowels in alphabetical order, is

(a) 360 (b) 240 (c) 120 (d) 480

35. The number of ways in which a couple can sit around a table with 6 guests if the couple take consecutive seat is

(a) 1440 (b) 720
(c) 5040 (d) None of these

36. How many different words beginning with O and ending with E can be formed with the letters of the word ORDINATE, so that the words are beginning with O and ending with E?

(a) 8! (b) 6! (c) 7! (d) 7!/2!

37. If two dices are tossed simultaneously, the number of elements in the resulting sample space is

(a) 6 (b) 8 (c) 36 (d) 24

38. In a football championship 153 matches were played. Every team played one match with each other team. How many teams participated in the championship?
(a) 21 (b) 18 (c) 17 (d) 15

39. Letters of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.
(a) 4320 (b) 2720 (c) 2160 (d) 1120

40. 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated?
(a) 1200 (b) 7200 (c) 148 (d) 240

41. The total number of ways in which letters of the word ACCOST can be arranged so that the two C's never come together will be
(a) 120 (b) 360 (c) 240 (d) $6!-2!$

42. How many new words are possible from the letters of the word PERMUTATION?
(a) 11!/2! (b) (11!/2!)−1
(c) 11!−1 (d) None of these

43. A,B,C and D are four towns any three of which are non-colinear. Then the number of ways to construct three roads each joining a pair of towns so that the roads do not form a triangle is
(a) 7 (b) 8
(c) 9 (d) More than 9

44. India and Pakistan play a 5 match test series of hockey, the probability that India wins at least three matches is
(a) $\frac{1}{2}$ (b) $\frac{3}{5}$
(c) $\frac{4}{5}$ (d) None of these

45. The alphabets of word ALLAHABAD are arranged at random. The probability that in the words so formed, all identical alphabets are found together, is
(a) 1/63 (b) 16/17
(c) 5!/9! (d) None of these

46. The probability that in the random arrangement of the letters of the word 'UNIVERSITY', the two I's does not come together is
(a) $\frac{4}{5}$ (b) 1/5 (c) 1/10 (d) 9/10

47. Among 15 players, 8 are batsmen and 7 are bowlers. Find the probability that a team is chosen of 6 batsmen and 5 bowlers:
(a) $\frac{{}^{8}C_6 \times {}^{7}C_5}{{}^{15}C_{11}}$ (b) $\frac{28}{15}$
(c) $\frac{15}{28}$ (d) None of these

48. Each of A and B tosses two coins. What is the probability that they get equal number of heads?
(a) $\frac{3}{16}$ (b) $\frac{5}{16}$ (c) $\frac{4}{16}$ (d) $\frac{6}{16}$

49. The chance of winning the race of the horse A is 1/5 and that of horse B is 1/6. What is the probability that the race will be won by A or B?
(a) 1/30 (b) 1/3 (c) 11/30 (d) 1/15

50. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is $\frac{3}{4}$. If both attempt the problem, what is the probability that the problem gets solved?
(a) $\frac{11}{12}$ (b) $\frac{7}{12}$ (c) $\frac{5}{12}$ (d) $\frac{9}{12}$

51. A bag contains 5 white and 7 black balls and a man draws 4 balls at random. The odds against these being all black is
(a) 7:92 (b) 92:7 (c) 92:99 (d) 99:92

52. There are 5 pairs of shoes in a cupboard from which 4 shoes are picked at random. The probability that there is at least one pair is
(a) $\frac{8}{21}$ (b) $\frac{11}{21}$ (c) $\frac{13}{21}$ (d) $\frac{12}{31}$

53. Two dice are thrown simultaneously. What is the probability of obtaining a multiple of 2 on one of them and a multiple of 3 on the other
(a) $\frac{5}{36}$ (b) $\frac{11}{36}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

GP_3386

LEVEL - 2

1. Four balls are drawn at random from a bag containing 5 white, 4 green and 3 black balls. The probability that exactly two of them are white is

(a) $\frac{14}{33}$ (b) $\frac{7}{16}$ (c) $\frac{18}{33}$ (d) $\frac{9}{16}$

2. From eighty cards numbered 1 to 80, two cards are selected randomly. The probability that both the cards have the numbers divisible by 4 is given by

(a) $\frac{21}{316}$ (b) $\frac{19}{316}$

(c) $\frac{1}{4}$ (d) None of these

3. The probability of getting sum more than 7 when a pair of dice are thrown is

(a) $\frac{7}{36}$ (b) $\frac{5}{12}$

(c) $\frac{7}{12}$ (d) None of these

4. The probability of guessing a correct answer is $\frac{x}{12}$. If the probability of not guessing the correct answer is $\frac{2}{3}$, then what is x equal to?

(a) 2 (b) 3 (c) 4 (d) 6

5. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is

(a) $\frac{1}{25}$ (b) $\frac{24}{25}$

(c) $\frac{2}{25}$ (d) None of these

6. A book has 999 pages. If a page is opened at random the probability that the sum of the digits in its number is 9, is

(a) $\frac{55}{999}$ (b) $\frac{22}{333}$ (c) $\frac{117}{999}$ (d) $\frac{4}{37}$

7. Fifteen persons, among whom are A and B sit down at random at a round table. The probability that there are 4 persons between A and B is

(a) $\frac{9!}{14!}$ (b) $\frac{10!}{14!}$ (c) $\frac{9!}{15!}$ (d) $\frac{1}{7}$

8. If two squares are chosen at random on a chess board, the probability that they have a side in common is

(a) $\frac{1}{9}$ (b) $\frac{2}{7}$ (c) $\frac{1}{18}$ (d) $\frac{2}{9}$

9. I forgot the last digit of a 7-digit telephone number. If I randomly dial the final 3 digits after correctly dialling the first four, then what is the chance of dialling the correct number ?

(a) $\frac{1}{1001}$ (b) $\frac{1}{990}$ (c) $\frac{1}{999}$ (d) $\frac{1}{1000}$

10. Two integers x and y are chosen with replacement out of the set {0, 1, 2, 3, ...10}. Then the probability that $|x-y|>5$ is

(a) $\frac{81}{121}$ (b) $\frac{30}{121}$ (c) $\frac{25}{121}$ (d) $\frac{20}{121}$

11. Nine squares are chosen at random on a chessboard. What is the probability that they form a square of size 3 × 3?

(a) $\frac{9}{{}^{64}C_9}$ (b) $\frac{36}{{}^{64}C_9}$

(c) $\frac{6}{{}^{64}C_9}$ (d) None of these

Directions (12–15) : Ajay and his wife Kajol appear in an interview for two vaccancies in the same post. The probability of Ajay's selection is $\frac{1}{7}$ and that of his wife Kajol's selection is $\frac{1}{5}$. What is the probability that

12. Both of them will be selected?

(a) $\frac{1}{12}$ (b) $\frac{1}{35}$ (c) $\frac{13}{35}$ (d) $\frac{12}{35}$

13. Only one of them will be selected?

(a) $\frac{5}{7}$ (b) $\frac{1}{5}$ (c) $\frac{2}{7}$ (d) $\frac{2}{35}$

14. None of them will be selected?

(a) $\frac{12}{35}$ (b) $\frac{6}{35}$

(c) $\frac{24}{35}$ (d) None of these

15. Atleast one of them will be selected?

(a) $\frac{11}{35}$ (b) $\frac{24}{35}$ (c) $\frac{2}{7}$ (d) $\frac{1}{35}$

Directions (16 – 17) : The odds against a husband who is 50 years old, living till he is 70 are 7 : 5 and the odds against his wife who is now 40, living till she is 60 are 5 : 3. Find the probability that

16. The couple will be alive 20 years hence.

(a) $\frac{21}{32}$ (b) $\frac{5}{32}$ (c) $\frac{15}{32}$ (d) $\frac{12}{35}$

17. Atleast one of them will be alive 20 years hence.

(a) $\frac{61}{96}$ (b) $\frac{31}{96}$

(c) $\frac{41}{70}$ (d) None of these

18. If A and B are two events such that $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.8$. Find $P\left(\frac{A}{B}\right)$.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) None of these

19. If A and B are two events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P\left(\frac{B}{A}\right) = 0.6$, find $P(A \cup B)$:

(a) 0.24 (b) 0.96

(c) 0.04 (d) None of these

Directions (20 – 23) : A bag contains 3 red and 4 black balls and another bag has 4 red and 2 black balls. One bag is selected at random and from the selected bag a ball is drawn. Let E be the event that the first bag is selected, F be the event that the second bag is selected, G be the event that ball drawn is red.

20. Find P (E).

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{5}{8}$

21. Find P (F).

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

22. Find $P\left(\frac{G}{E}\right)$.

(a) $\frac{5}{6}$ (b) $\frac{5}{14}$

(c) $\frac{3}{7}$ (d) None of these

23. Find $P\left(\frac{G}{F}\right)$.

(a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) $\frac{5}{9}$ (d) $\frac{4}{5}$

24. Only one bomb is needed to destroy a bridge and there are three bombers engaged in an attempt to destroy the bridge. The probabilities of the bombers hitting the bridge are respectively 0.3, 0.4 and 0.7. What is probability of the bridge being destroyed?

(a) 0.874 (b) 0.126 (c) 0.916 (d) 0.084

25. Triangles are formed by joining vertices of an octagon. Any one of those triangle is selected at random. What is the probability that the selected triangle has no side common with the octagon?

(a) 3/7 (b) 2/7 (c) 5/7 (d) 1/7

Directions (26 –28) : Three mountaineers Amit, Vinit and Nishit are climbing up a mountain with their respective probabilities of reaching the summit being $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{4}$ respectively. What is the probability that

26. All of them reach the summit?

(a) $\frac{3}{60}$ (b) $\frac{1}{60}$ (c) 1 (d) 0

27. At least one of them reaches the summit?

(a) $\frac{59}{60}$ (b) $\frac{36}{60}$ (c) $\frac{17}{60}$ (d) $\frac{1}{60}$

28. Exactly one of them reaches the summit?

(a) $\frac{13}{30}$ (b) $\frac{17}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

29. The probability that a man will live 10 more years is $\frac{1}{4}$ and the probability that his wife will live 10 more years is $\frac{1}{3}$. Then the probability that neither will be alive in 10 years is

GP_3386

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$ (c) $\frac{1}{2}$ (d) $\frac{11}{12}$

30. The sum of all the four digit even numbers which can be formed by using the digits 0, 1, 2, 3, 4 and 5 if repetition of digits is allowed is
(a) 1765980 (b) 1756980
(c) 1769580 (d) 1759680

31. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. The number of at most 6 balls containing balls of all the colours is
(a) 36(4!) (b) $2^6 \times 4!$
(c) $(2^6 - 1)(4!)$ (d) None of these

32. To fill a number of vacancies, an employer must hire 3 programmers from among 6 applicants, and 2 managers from among 4 applicants. What is the total number of ways in which she can make her selection?
(a) 1,490 (b) 132 (c) 120 (d) 60

33. A father has 2 apples and 3 pears. Each weekday (Monday through Friday) he gives one of the fruits to his daughter. In how many ways can this be done?
(a) 120 (b) 10 (c) 24 (d) 12

34. On a railway route there are 20 stations. What is the number of different tickets required in order that it may be possible to travel from every station to every other station?
(a) 40 (b) 380 (c) 400 (d) 420

35. A man has 3 shirts, 4 trousers and 6 ties. What are the number of ways in which he can dress himself with a combination of all the three?
(a) 13 (b) 72
(c) 13!/3!·4!·6! (d) 3!·4!·6!

36. There are 20 people among whom two are sisters. Find the number of ways in which we can arrange them around a circle so that there is exactly one person between the two sisters.
(a) 18! (b) 2!19!
(c) 19! (d) None of these

37. There are three rooms in a hotel: one single, one double and one for four persons. How many ways are there to house seven persons in these rooms?
(a) 7!/1!2!4! (b) 7! (c) 7!/3 (d) 7!/3!

38. In how many ways can the eight directors, the vice-chairman and the chairman of a firm be seated at a round-table, if the chairman has to sit between the vice-chairman and the director?
(a) $9! \times 2$ (b) $2 \times 8!$
(c) $2 \times 7!$ (d) None of these

39. 5 men and 6 women have to be seated in a straight row so that no two women are together. Find the number of ways this can be done.
(a) 48400 (b) 39600 (c) 9900 (d) 86400

40. The number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two person, is
(a) 60(5!) (b) 15(4!) × (5!)
(c) $4! \times 5!$ (d) None of these

41. From 6 boys and 7 girls a committee of 5 is to be formed so as to include atleast one girl. The number of ways this can be done is
(a) ${}^{13}C_4$ (b) ${}^{6}C_4 \cdot {}^{7}C_1$
(c) $7 \cdot {}^{6}C_4$ (d) ${}^{13}C_5 - {}^{6}C_1$

42. If a team of four persons is to be selected from 8 males and 8 females, then in how many ways can the selections be made to include at least one male.
(a) 1550 (b) 1675 (c) 1725 (d) 1750

43. In how many ways can 7 Englishmen and 7 Americans sit down at a round table, no 2 Americans being in consecutive positions?
(a) 3628800 (b) 2628800
(c) 3628000 (d) 3328800

44. 5 Indian and 5 American couples meet at a party & shake hands . If no wife shakes hands with her husband and no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is
(a) 95 (b) 110 (c) 135 (d) 150

45. In how many ways can a term of 11 cricketers be chosen from 6 bowlers. 4 wicket keepers and 11 batsmen to give a majority of bastemen if at least 4 bowlers are to be included and there is one wicket keeper?
(a) 27730 (b) 27720
(c) 17720 (d) 26720

46. All the words that can be formed using alphabets A, H, L, U and R are written as in a dictionary (no alphabet is repeated). Rank of the word RAHUL is
(a) 71 (b) 72 (c) 73 (d) 74

47. There are five boys and three girls who are sitting together to discuss a management problem at a round table. In how many ways can they sit around the table so that no two girls are together?
(a) 1220 (b) 1400 (c) 1420 (d) 1440

48. A class photograph has to be taken. The front row consists of 6 girls who are sitting. 20 boys are standing behind. The two corner positions are reserved for the 2 tallest boys. In how many

ways can the students be arranged?
(a) $18! \times 1440$ (b) $6! \times 1440$
(c) $18! \times 2! \times 1440$ (d) None of these

49. Find the number of numbers between 300 and 3000 that can be formed with the digits 0, 1, 2, 3, 4 and 5, no digit being repeated.
(a) 120 (b) 160
(c) 240 (d) None of these

50. Of a total of 600 bolts, 20% are too large and 10% are too small. The remainder are eonsidered to be suitable. If a bolt is selected at random, the probability that it will be suitable is

(a) $\left(\frac{1}{5}\right)$ (b) $\left(\frac{7}{10}\right)$ (c) $\left(\frac{1}{10}\right)$ (d) $\left(\frac{3}{10}\right)$

51. The probability that Krishna will be alive 10 years hence, is $\frac{7}{15}$ and that Hari will be alive is $\frac{7}{10}$. What is the probability that both Krishna and Hari will be dead 10 years hence ?

(a) $\frac{21}{150}$ (b) $\frac{24}{150}$ (c) $\frac{49}{150}$ (d) $\frac{56}{150}$

52. An aircraft has three engines *A*, *B* and *C*. The aircraft crashes if all the three engines fail. The probabilities of failure are 0.03, 0.02 and 0.05 for engines *A*, *B* and *C* respectively. What is the probability that the aircraft will not crash?
(a) 0.00003 (b) 0.90
(c) 0.99997 (d) 0.90307

53. The probability that a student passes in mathematics is 4/9 and that he passes in physics is 2/5. Assuming that passing in mathematics and physics are independent of each other, what is the probability that he passes in mathematics but fails in physics?

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$ (c) $\frac{26}{45}$ (d) $\frac{19}{45}$

54. From a pack of 52 cards, two cards are drawn, the first being replaced before the second is drawn. What is the probability that the first is a diamond and the second is a king?

(a) $\frac{1}{4}$ (b) $\frac{4}{13}$ (c) $\frac{1}{52}$ (d) $\frac{4}{15}$

55. In a lottery, 16 tickets are sold and 4 prizes are awarded. If a person buys 4 tickets,what is the probability of his winning a prize?

(a) $\frac{4}{16^4}$ (b) $\frac{175}{256}$ (c) $\frac{1}{4}$ (d) $\frac{81}{256}$

56. Two cards are selected at random from a deck of 52 playing cards. The probability that both the cards are greater than 2 but less than 9 is

(a) $\frac{46}{221}$ (b) $\frac{63}{221}$ (c) $\frac{81}{221}$ (d) $\frac{93}{221}$

57. Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

(a) $\frac{11}{21}$ (b) $\frac{9}{21}$
(c) $\frac{10}{21}$ (d) None of these

58. A bag contains 5 white and 3 black balls, and 4 are successively drawn out and not replaced. What's the chance of getting different colours alternatively?

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{7}$

59. A class consists of 80 students, 25 of them are girls and 55 are boys. If 10 of them are rich and the remaining poor and also 20 of them are intelligent then the probability of selecting an intelligent rich girl is

(a) $\frac{5}{128}$ (b) $\frac{25}{128}$
(c) $\frac{5}{512}$ (d) None of these

60. One bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, determine the probability that one ball is white and another is black.
(a) 6/24 (b) 5/24 (c) 7/24 (d) 13/24

GP_3386

Hints and Solutions

Level 1

1. (a) When two dice are thrown then there are 6×6 exhaustive cases $\therefore n = 36$.

Let A denote the event "total score of 7" when 2 dice are thrown then

$A = [(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)]$.

Thus there are 6 favourable cases. $\therefore m = 6$

By definition $P(A) = \frac{m}{n}$

$\therefore P(A) = \frac{6}{36} = \frac{1}{6}$.

2. (b) Since, A and B are independent events

$\therefore A'$ and B' are also independent events

$\Rightarrow P(A' \cap B') = P(A').P(B')$

$= (0.4)(0.7) = 0.28$

$\left[\because P(A') = 1 - P(A),\ P(B') = 1 - P(B)\right]$

3. (c) $P(A) = \frac{1}{3}$; $P(\bar{A}) = \frac{2}{3}$

P (bird killed) $= 1 - P$ (none of 3 shots hit)

$= 1 - \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{19}{27}$.

4. (d) Doublets occur when the numbers thrown are (1, 1), (2, 2), ..., (6, 6). Therefore the probability of a doublet occurring in single throw $= \frac{6}{36} = \frac{1}{6}$.

The probability of a doublet not occurring at all in three throws $= \left(\frac{5}{6}\right)^3 = \frac{125}{216}$.

Required probability $= 1 - \frac{125}{216} = \frac{91}{216}$.

5. (b) Any of the six numbers 1, 2, 3, 4, 5, 6 may appear on the upper face. ($\therefore n = 6$)

Number of odd numbers = 3, since the odd numbers are 1, 3, 5. ($\therefore m = 3$).

$\therefore$ The required probability

$= \frac{\text{number of favourable cases}}{\text{number of all cases}} = \frac{m}{n} = \frac{3}{6}$

$= \frac{1}{2}$

6. (d) n = Number of all cases = 6

m = Number of favourable cases = 4 (since the numbers that appear are 3, 4, 5, 6)

$\therefore$ The required probability $= p = \frac{m}{n}$

$= \frac{4}{6} = \frac{2}{3}$

7. (c) Given digits are 1, 2, 3, 4, 5

Total no. of 2 digits numbers formed $= (5)^2 = 25$

Favourable cases are 12, 24, 32, 44, 52

No. of favourable cases = 5

$\therefore$ Required probability $= \frac{5}{25} = \frac{1}{5}$

8. (a) $n(S) = 100$

E = square of terms lies between 1 to 100.

$= 1, 4, 9, 16, 25, 36, 49, 64, 81, 100$

$n(E) = 10$

$\therefore$ Required probability $= \frac{n(E)}{n(S)}$

$= \frac{10}{100} = \frac{1}{10}$

9. (b) In a leap year there are 366 days in which 52 weeks and two days. The combination of 2 days may be : Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun.

P (53 Fri) $= \frac{2}{7}$; P (53 Sat) $= \frac{2}{7}$ and

P (53 Fri and 53 Sat) $= \frac{1}{7}$

$\therefore P$ (53 Fri or Sat) $= P$ (53 Fri) $+ P$ (53 Sat) $- P$ (53 Fri and Sat)

$= \frac{2}{7} + \frac{2}{7} - \frac{1}{7} = \frac{3}{7}$

10. (a) Total possibility of 4 children, either girl or boy is $2^4 = 16$. Out of these there is one possibility in which there will be no boy and only girls. So, total possibility of at least one boy is $16 - 1 = 15$

$\Rightarrow$ P (at least one boy) $= \frac{15}{16}$.

11. (c) Possible samples are as follows
{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT}
Let A be the event of getting one head.
Let B be the event of getting no head.
Favourable outcome for

$$A = \{TTH, THT, HTT\}$$

Favourable outcome for $B = \{TTT\}$

Total no. of outcomes = 8

$$\therefore \quad P(A) = \frac{3}{8},\ P(B) = \frac{1}{8}$$

$\therefore$ Required probability = Probability of getting one head + Probability of getting no head, $= P(A) + P(B) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

12. (d) Total of seven can be obtained in the following ways

$1, 1, 1, 4$ in $\frac{4!}{3!} = 4$ ways

[there are four objects, three repeated]
Similarly,

$1, 1, 2, 3$ in $\frac{4!}{2!} = 12$ ways

$1, 2, 2, 2$ in $\frac{4!}{3!} = 4$ ways

Hence, required probability

$$= \frac{4+12+4}{6^4} = \frac{20}{6^4}$$

[$\because$ Exhaustive no. of cases $= 6 \times 6 \times 6 \times 6 = 6^4$]

13. (d) Exhaustive number of cases = 12
Favourable cases = ${}^{12}C_2\,(2^6 - 2)$

$$\therefore \text{ Probabiltity} = \frac{{}^{12}C_2(2^6 - 2)}{12^6} = \frac{341}{12^5}$$

14. (c) The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}C_3$.
The product of three integers will be even if at least one of the integers is even. Therfore, the required probability
= 1 – Prob.
So that none of the three integers is even

$$= 1 - \frac{{}^{10}C_3}{{}^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}.$$

[Three odd integers can be chosen in ${}^{10}C_3$ ways as there are 10 even and 10 odd integers.]

15. (b) Exhaustive no. of cases = 6^3
10 can appear on three dice either as distinct number as following (1, 3, 6) ; (1, 4, 5); (2, 3, 5) and each can occur in 3! ways. Or 10 can appear on three dice as repeated digits as following (2, 2, 6), (2, 4, 4), (3, 3, 4) and each can occur in $\frac{3!}{2!}$ ways.

$\therefore$ No. of favourable cases

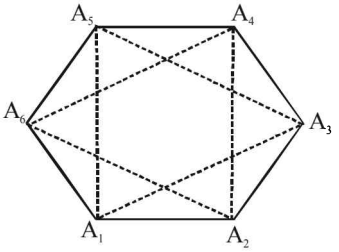
$$= 3 \times 3! + 3 \times \frac{3!}{2!} = 27$$

Hence, the required probability $= \frac{27}{6^3} = \frac{1}{8}$

16. (b) Since each ball can be put into any one of the three boxes. So, the total number of ways in which 12 balls can be put into three boxes is 3^{12}.
Out of 12 balls, 3 balls can be chosen in ${}^{12}C_3$ ways. Now, remaining 9 balls can be put in the remaining 2 boxes in 2^9 ways. So, the total number or ways in which 3 balls are put in the first box and the remaining in other two boxes is ${}^{12}C_3 \times 2^9$.

Hence, required probability $= \frac{{}^{12}C_3 . 2^9}{3^{12}}$

17. (c) Three vertices can be selected in ${}^6C_3$ ways.



The only equilateral triangles possible are $A_1A_3A_5$ and $A_2A_4A_6$

$$p = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$$

GP_3386

18. **(c)** Numbers divisible by 4 are 104, 108.., 196; 24 in number. Numbers divisible by 7 are 105, 112, ...196; 14 in number. Numbers divisible by both, i.e.divisible by 28 are 112, 140, 168, 196; 4 in number. Hence, required probability

$$=\frac{24}{99}+\frac{14}{99}-\frac{4}{99}=\frac{34}{99}$$

19. **(c)** Two different squares can be chosen in 64×63 ways. For each of the four corner squares, the favourable number of cases is 2.
For each of the 24 non-corner squares on all the four sides of the chessboard, the favourable number of cases is 3.
For each of the 36 remaining squares, the favourable number of cases is 4.
Thus, the total number of favourable cases
$=4\times2+24\times3+36\times4=224$
Hence, the required probability

$$=\frac{224}{64\times63}=\frac{1}{18}$$

20. **(b)** 3 squares on a chessboard can be chosen in ${}^{64}C_3$ ways.
Two squares of one colour and third square of different colour can be chosen in two mutually exclusive way.
(i) 2 white and one black (ii) 2 black and one white
Thus the favourable number of cases
$={}^{32}C_2\times{}^{32}C_1+{}^{32}C_1\times{}^{32}C_2$
$=2({}^{32}C_2\times{}^{32}C_1)$
Hence, the required probability

$$=\frac{2\left({}^{32}C_2\times{}^{32}C_1\right)}{{}^{64}C_3}=\frac{16}{21}$$

21. **(b)** The divisibility of the product of four number depends upon the value of the last digit of each number. The last digit of a number can be any of the 10 digits 0, 1 , 2, ... 9. So the total number of ways of selecting last digits of four number is $10\times10\times10\times10=10^4$.
If the product of the 4 numbers is not divisible by 5 or 10.
Then the number of choices for the last digit of each number is 8 (excluding 0 or 5).
So, favourable number of ways
$=8\times8\times8\times8=8^4$
$\therefore$ The probability that the product is not divisible by 5 or 10

$$=\frac{8^4}{10^4}=\left(\frac{8}{10}\right)^4$$

Hence, required probability

$$=1-\left(\frac{8}{10}\right)^4=\frac{369}{625}$$

22. **(a)** The last two digits can be dialled in ${}^{10}P_2=90$ ways.
Out of these 90 cases only one case is favourable.
Hence, the required probability $=\frac{1}{90}$.

23. **(c)** Here
S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
Let A be the event that one of the coins shows a. tail
$\therefore$ A = {HHT, HTH, THH, HTT, THT, TTH, TTT}

$$\therefore\quad P(A)=\frac{7}{8}$$

Now, let B be the events that they are all tails

$$\therefore\quad B=\{T\,T\,T\}\qquad \therefore\ P(B)=\frac{1}{8}$$

$$\therefore\quad A\cap B=\{T\,T\,T\}\Rightarrow P(A\cap B)=\frac{1}{8}$$

$$\therefore\quad P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{1/8}{7/8}=\frac{1}{7}$$

24. **(d)** Contradiction can occur if A speaks truth and B lies, B speaks truth and A lies.
So, required probability
$=0.75\times0.2+0.8\times0.25=0.35$
Req% $=0.35\times100$
$=35\%$

25. **(a)** There are 8 letters in the word EQUATION.

A/E/I/O/U								
5 ways	${}^7P_7=7!=5040$							

$\therefore$ Reqd. no. $=5\times5040=25200$

26. **(a)** There are 9 letters in the given word in which two T's, two M's and two E's are identical.
Hence the required number of words

$$=\frac{9!}{2!2!2!}=\frac{9!}{(2!)^3}$$

27. (a) Considering the two vowels E and A as one letter, the total no. of letters in the word 'EXTRA' is 4 which can be arranged in ${}^4P_4$, i.e. 4! ways and the two vowels can be arranged among themselves in 2! ways.
$\therefore$ reqd. no. $= 4! \times 2!$
$= 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 48$

28. (b) If number of persons be n, then total number of hand-shaken $= {}^nC_2 = 66$
$\Rightarrow n(n-1) = 132 \Rightarrow (n+11)(n-12) = 0$
$\therefore n = 12 \quad (\because n \neq -11)$

29. (b) There are 6 letters in the word BHARAT, 2 of them are identical.
Hence total number of words with these letter $= 360$
Also the number of words in which B and H come together $= 120$
$\therefore$ The required number of words $= 360 - 120 = 240$

30. (d) MACHINE has 4 consonants and 3 vowels. The vowels can be placed in position no. 1, 3, 5, 7
$\Rightarrow$ Total number of ways possible $= {}^4P_3 = 24$.
For each of these 24 ways the 4 consonants can occupy the other 4 places in ${}^4P_4$ ways
$\Rightarrow$ Total $= 24 \times 24 = 576$

31. (a) Total number of ways $= {}^{16}C_{11} = \dfrac{16!}{11! \times 5!}$

$$= \frac{16 \times 15 \times 14 \times 13 \times 12}{5 \times 4 \times 3 \times 2 \times 1} = 4368.$$

32. (b) Selection of 2 members out of 11 has ${}^{11}C_2$ number of ways ${}^{11}C_2 = 55$

33. (c) Let the total number of employees in the company be n.
Total number of gifts

$$= {}^nC_2 = \frac{n(n-1)}{2} = 66 \Rightarrow n^2 - n - 132 = 0$$

or $(n+11)(n-12) = 0$ or $n = 12$
[-11 is rejected]

34. (a) Order of vowels is fixed

$\therefore$ Required number of ways are $\dfrac{6!}{2!} = 360$

35. (a) A couple and 6 guests can be arranged in $(7-1)!$ ways. But in two people forming the couple can be arranged among themselves in 2! ways.
$\therefore$ the required number of ways $= 6! \times 2! = 1440$

36. (b) 6! ways, O fixed 1st and E fixed in last.

37. (c) Number of elements in the sample space $= 6 \times 6 = 36$

38. (b) Let total no. of team participated in a championship be n.
Since, every team played one match with each other team.

$$\therefore {}^nC_2 = 153 \Rightarrow \frac{n!}{2!(n-2)!} = 153$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2!(n-2)!} = 153$$

$$\Rightarrow \frac{n(n-1)}{2} = 153 \Rightarrow n^2 - n - 306 = 0$$

$\Rightarrow n(n-18) + 17(n-18) = 0 \Rightarrow n = 18, -17$
n cannot be negative
$\therefore n \neq -17 \Rightarrow n = 18$

39. (c) Taking all vowels (IEO) as a single letter (since they come together) there are six letters among which there are two R.

Hence no. of arrangements $= \dfrac{6!}{2!} \times 3! = 2160$

[Three vowels can be arranged in 3! ways among themselves, hence multiplied with 3!.]

40. (d) Assume the 2 given students to be together (i.e. one). Now these are five students.
Possible ways of arranging them are $= 5! = 120$
Now they (two girls) can arrange themselves in 2! ways.
Hence total ways $= 120 \times 2 = 240$

41. (c) Total number of ways to permute 6 alphabets 2 of which are common
$= 6!/2! = 360$.
(1) Treat the two C's as one
$\Rightarrow$ Number of possible ways $= {}^5P_5 = 120$
(b) Number of ways = Total arrangements – Number of arrangements in which they always come together
$= 360 - 120 = 240$.

42. (b) Number of 11 letter words formed from the letter P, E, R, M, U, T, A, I, O, N = 11!/2!.
Number of new words formed = total words $- 1 = (11!/2!) - 1$.

43. (d) To construct 2 roads, three towns can be selected out of 4 in $4 \times 3 \times 2 = 24$ ways.
Now if the third road goes from the third town to the first town, a triangle is formed, and if it goes to the fourth town, a triangle is

GP_3386

not formed. So, there are 24 ways to form a triangle and 24 ways of avoiding a triangle.

44. (a) India win atleast three matches

$$= {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_4\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \left(\frac{1}{2}\right)^5 (16) = \left(\frac{1}{2}\right)$$

45. (a) (AAAA), (LL), HBD

$$P = \frac{5!}{\frac{9!}{4!\,2!}} = \frac{5! \times 4! \times 2!}{9!}$$

$$= \frac{24 \times 2}{9 \times 8 \times 7 \times 6} = \frac{1}{63}$$

46. (a) Total no. of arrangements of the letters of the word UNIVERSITY is $\frac{10!}{2!}$.

No. of arrangements when both I's are together = 9!

So. the no. of ways in which 2 I's do not together $= \frac{10!}{2!} - 9!$

$\therefore$ Required probability

$$= \frac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}} = \frac{10! - 9!\,2!}{10!} = \frac{4}{5}$$

47. (a) Total no. of players = 15
Total no. of batsmen = 8
Total no. of bowlers = 7
Total no. of players in the team = 11

$\therefore$ No. of ways to choose a team = ${}^{15}C_{11}$

$\therefore$ No. of way to choose 6 batsmen and 5 bowler $= {}^8C_6 \times {}^7C_5$

$\therefore$ Required probability $= \frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$

48. (a) For two coins, sample space will be {HH, HT, TH, TT}
The probability that both A and B get equal number of heads $= \frac{3}{4 \times 4} = \frac{3}{16}$

49. (c) Let $P(A)$ be the probability that the race will be won by A and $P(B)$ be the probability that the race will be won by B.

$\therefore P(A) = \frac{1}{5}$ and $P(B) = \frac{1}{6}$

$\therefore$ Probability that the race will be won by

A or $B = P(A) + P(B) = \frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

50. (a) The probability that A cannot solve the problem $= 1 - \frac{2}{3} = \frac{1}{3}$

The probability that B cannot solve the problem $= 1 - \frac{3}{4} = \frac{1}{4}$

The probability that both A and B cannot solve the problem $= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$\therefore$ The probability that at least one of A and B can solve the problem $= 1 - \frac{1}{12} = \frac{11}{12}$

$\therefore$ The probability that the problem is solved $= \frac{11}{12}$

51. (b) There are 7 + 5 = 12 balls in the bag and the number of ways in which 4 balls can be drawn is ${}^{12}C_4$ and the number of ways of drawing 4 black balls (out of seven) is ${}^7C_4$.
Hence, P (4 black balls)

$$= \frac{{}^7C_4}{{}^{12}C_4} = \frac{7.6.5.4}{1.2.3.4} \times \frac{1.2.3.4}{12.11.10.9} = \frac{7}{99}$$

Thus the odds against the event 'all black balls' are

$$(1 - \frac{7}{99}) : \frac{7}{99} : \text{i.e.,}\ \frac{92}{99} : \frac{7}{99} \text{ or } 92 : 7.$$

52. (c) There are 5 pairs of shoes and 4 shoes can be picked in $10 \times 9 \times 8 \times 7$ ways. Number of ways in which 4 shoes can be picked such that no two are alike $= 10 \times 8 \times 6 \times 4$.
The required probability

$$= 1 - \frac{10 \times 8 \times 6 \times 4}{10 \times 9 \times 8 \times 7} = \frac{13}{21}.$$

53. (b) Favourable cases for one are there *i.e.,* 2, 4 and 6 and for other are two i.e., 3, 6.
Hence required probability

$$= \left[\left(\frac{3\times 2}{36}\right)2 - \frac{1}{36}\right] = \frac{11}{36}$$

[As same way happen when dice changes numbers among themselves]

Level 1

1. (a) $\because$ No of ways of drawing 2 white balls from 5 white balls $= {}^5C_2$.

Also, no. of ways of drawing 2 other from remaining 7 balls $= {}^7C_2$

Total number of balls = 12

$\therefore$ Required probability $= \dfrac{{}^5C_2 \times {}^7C_2}{{}^{12}C_4} = \dfrac{14}{33}$

2. (b) Total no. of divisible by 4 between 1 to 80

$80 = 4 + (n-1)4$

$80 = 4n \Rightarrow n = 20$

$\therefore$ Required probability $= \dfrac{{}^{20}C_2}{{}^{80}C_2} = \dfrac{19}{316}$

3. (b) Here $n(S) = 6^2 = 36$

Let E be the event "getting sum more than 7" *i.e.* sum of pair of dice = 8, 9, 10, 11, 12 *i.e.*, E

$$= \begin{Bmatrix} (2,6) & (3,5) & (4,4) & (5,3) & (6,2) \\ (3,6) & (4,5) & (5,4) & (6,3) & \\ (4,6) & (5,5) & (6,4) & & \\ (5,6) & (6,5) & (6,6) & & \end{Bmatrix}$$

$\therefore$ $n(E) = 15$

$\therefore$ Required prob $= \dfrac{n(E)}{n(S)} = \dfrac{15}{36} = \dfrac{5}{12}$

4. (c) Given probability of guessing a correct answer $= \dfrac{x}{12}$ and probability of not guessing the correct answer $= \dfrac{2}{3}$

As we know

P (occurence of an event) + P (non-occurence of an event) = 1

$\therefore$ $\dfrac{x}{12} + \dfrac{2}{3} = 1 \Rightarrow \dfrac{x+8}{12} = 1$

$\Rightarrow$ $x = 12 - 8 = 4$

5. (b) Total number of possibilities $= 25 \times 25$

Favourable cases for their winning = 25

$\therefore$ P (they win a prize) $= \dfrac{25}{25\times 25} = \dfrac{1}{25}$

$\therefore$ P (they will not win a prize)

$= 1 - \dfrac{1}{25} = \dfrac{24}{25}$

6. (a) Total no. of cases = 999

Favourable cases.

The page number can be with all identical digits,

i.e. $333 \longrightarrow 1$

The identical and one different like 009, 117, 225 and 441 $\longrightarrow 4 \times \dfrac{3!}{2!} = 12$

All different digits like

$018, 027, 036, 045, 126, 135, 234 \longrightarrow 7\times 3! = 42$

So favourable cases = 55

Desired probability $= \dfrac{55}{999}$

7. (d) Exhaustive number of cases $= (15-1)! = 14!$

Favourable cases

${}^{13}C_4 (10-1)! \times 2! \times 4! = 2! \times 13!$

[A, B and four persons are treated as one, A and B can interchange positions]

$\therefore$ Probability $= \dfrac{2\times 13!}{14!} = \dfrac{1}{7}$

8. (c) Two squares out of 64 can be selected in

${}^{64}C_2 = \dfrac{64\times 63}{2} = 32\times 63$ ways

The number of ways of selecting those pairs which have a side in common

$= \left(\dfrac{1}{2}\right)(4\times 2 + 24\times 3 + 36\times 4) = 112$

[Since each of the corner squares has two neighbours each of 24 squares in border rows, other than corner ones has three neighbours and each of the remaining 36 squares have four neighours and in this computation, each pair of squares has been considered twice].

Hence required probability $= \dfrac{112}{32\times 63} = \dfrac{1}{18}$.

GP_3386

9. (d) It is given that last 3 digits are randomly dialled. Then each of the digits can be selected out of 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in 10 ways. Hence the required

$$\text{probability} = \left(\frac{1}{10}\right)^3 = \frac{1}{1000}$$

10. (b) Total number of selections $= 11 \times 11$

Now $|x - y| > 5$ clearly $x \neq 5$

If $x = 0$, then $y > 5 \Rightarrow 5$ favourable cases

If $x = 1$, then $y > 6 \Rightarrow 4$ favourable cases

If $x = 2$, then $y > 7 \Rightarrow 3$ favourable cases

If $x = 3$, then $y > 8 \Rightarrow 2$ favourable cases

If $x = 4$, then $y > 9 \Rightarrow 1$ favourable cases

Symmetrical cases will be obtained for $x = 6, 7, 8, 9, 10$

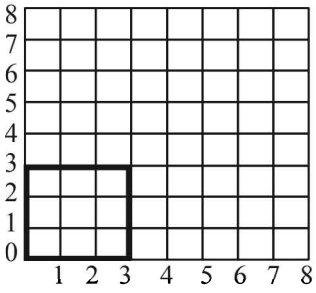
$\therefore$ Favourable cases $= 30$

$$\therefore \text{Desired probability} = \frac{30}{121}$$

11. (b) We can choose 9 squares out of 64 squares in ${}^{64}C_9$ ways.

Hence, exhaustive number of cases $= {}^{64}C_9$

From the figure it is clear that the given square of size 3×3



can be formed by using four consecutive horizontal and 4 consecutive vertical lines, which can be done in

${}^6C_1 \times {}^6C_1 = 36$ ways

Basically you can make 6 squares of size 3×3 in vertical direction and 6 squares of the size 3×3 in horizontal direction, Hence total $6 \times 6 = 36$ squares can be chosen.

$$\therefore \text{The required probability} = \frac{36}{{}^{64}C_9}$$

Solution (12 to 15) :

Let E = the event that Ajay is selected

and F = the event that Kajol is selected

Clearly, E and F are independent events

$$\text{Now, } P(E) = \frac{1}{7} \text{ and } P(F) = \frac{1}{5}$$

$$\therefore P(\bar{E}) = \frac{6}{7} \text{ and } P(\bar{F}) = \frac{4}{5}$$

12. (b) P (Both of them will be selected)

$= P(E \text{ and } F) = P(E \cap F)$

$$= P(E)P(F) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$$

13. (c) P (only one of them will be selected)

$= P[(E \text{ and not } F) \text{ or } (F \text{ and not } E)]$

$$= P\left[(E \cap \bar{F}) \cup (F \cap \bar{E})\right]$$

$$= P(E \cap \bar{F}) + P(F \cap \bar{E})$$

$$= P(E)P(\bar{F}) + P(F)P(\bar{E})$$

$$= \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = \frac{2}{7}$$

14. (c) P (none of them will be selected)

$= P[(\text{not } E) \text{ and } (\text{not } F)]$

$$= P(\bar{E} \cap \bar{F}) = P(\bar{E}) \times P(\bar{F}) = \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$$

15. (a) P (atleast one of them will be selected)

$= 1 - P$ (none will be selected)

$$= 1 - \frac{24}{35} = \frac{11}{35}$$

Solution (16 and 17) : Let E = the event that the husband will be alive 20 years hence and F = the event that the wife will be alive 20 years hence.

$$\text{Then } P(E) = \frac{5}{12} \text{ and } P(F) = \frac{3}{8}$$

$$\therefore \quad P(\bar{E}) = \frac{7}{12} \text{ and } P(\bar{F}) = \frac{5}{8}$$

Clearly, E and F are independent events.

16. (b) P (Couple will be alive 20 years hence)

$= P(E \text{ and } F) = P(E \cap F)$

$$= P(E).P(F) = \frac{5}{12} \times \frac{3}{8} = \frac{5}{32}$$

17. (a) P (atleast one of them will be alive 20 years hence)

$= 1 - P$ (none will be alive 20 years hence)

$$= 1 - P(\bar{E} \cap \bar{F})$$

$$= 1 - P(\bar{E})P(\bar{F})$$

($\because \bar{E}$ and $\bar{F}$ are independent)

$= 1 - \left(\frac{7}{12} \times \frac{5}{8}\right) = \frac{61}{96}$

18. (b) $\because P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.5 + 0.6 - 0.8 = 0.3$

$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.6} = \frac{1}{2}$

19. (b) $\because P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{0.4} = 0.6$

$\Rightarrow P(A \cap B) = 0.24$
$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.4 + 0.8 - 0.24 = 0.96$

20. (a) $P(E) = \frac{1}{2}$

21. (b) $P(F) = \frac{1}{2}$

22. (c) $P\left(\frac{G}{E}\right)$ = Probability of drawing a red ball when first bag is selected
= Probability of drawing a red ball from first bag $= \frac{3}{7}$

23. (a) $P\left(\frac{G}{F}\right)$ = Probability of drawing a red ball from second bag $= \frac{4}{6} = \frac{2}{3}$

24. (c) It is nothing but [1 – probability of bridge not getting destroyed] = 1 – 0.084 = 0.916

25. (b) Total number of triangles formed = ${}^8C_3 = 56$
Triangles having two sides common = 8
Triangles having one side common
$= {}^8C_1 \times {}^4C_1 = 32$
Triangles having three sides common = 0
Triangles having no side common
= 56 – 40 = 16
So, probability = 16/56 = 2/7.

26. (b) All of them reach the summit
= 1/3 × 1/5 × 1/4 = 1/60

27. (b) To find out "At least one of them reaches the summit", we will find the probability of nobody reaching the summit = 2/3×4/5×3/4 = 2/5. Hence, the probability of at least one of them reaching the summit
= 1 – 2/5 = 3/5

28. (a) This is same as : Amit reaches the summit, but Vinit and Nishit do not reach the summit. Or, Vinit reaches the summit, but Amit and Nishit do not reach the summit. Or, Nishit reaches the summit, but Vinit and Amit do not reach the summit.
Amit reaches the summit, but Vinit and Nishit do not reach the summit
= 1/3 × 4/5 × 3/4 = 1/5 and similarly we will find all the other possibilities which will be

$\frac{1}{10}$ and $\frac{2}{15}$.

So, required probability

$= \frac{1}{5} + \frac{1}{10} + \frac{2}{15} = \frac{13}{30}$

29. (c) The probability that a man will not live 10 more years = 3/4 and the probability that his wife will not live 10 more years = 2/3. Then the probability that neither will be alive in 10 years = 3/4 × 2/3 = $\frac{1}{2}$.

30. (c) $\square\square\square\square$

Last place can be filled by 0, 2, 4
So total sum = 5 × 6 × 6 (0 + 2 + 4) + 5 × 6 × 3 × 10 (0 + 1 + 2 + 3 + 4 + 5) + 5 × 6 × 3 × 100 (0 + 1 + 2 + 3 + 4 + 5) + 6 × 6 × 3 × 1000 (0 + 1 + 2 + 3 + 4 + 5)
$= 180 \times 6 + 900 \times 15 + 9000 \times 15 + 108000 \times 15$
$= 1080 + 13500 + 135000 + 1620000 = 1769580$

31. (a) The required number of selections
$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 36 \times 4!$

32. (c) Required no. of the ways = ${}^6C_3 \times {}^4C_2$
$= 20 \times 6 = 120$

33. (b) Required number of ways = $\frac{5!}{2!3!} = 10$.

34. (b) From each railway station, there are 19 different tickets to be issued. There are 20 railway station
So, total number of tickets = 20 × 19 = 380.

35. (b) ${}^3C_1 \times {}^4C_1 \times {}^6C_1 = 72$

GP_3386

36. (d) First arrange the two sisters around a circle in such a way that there will be one seat vacant between them.

[This can be done in 2! ways.]

Then, the other 18 people can be arranged on 18 seats in 18! ways.

37. (a) Choose 1 person for the single room & from the remaining choose 2 people for the double room & from the remaining choose 4 people for the 4 persons room

$\Rightarrow {}^{7}C_1 \times {}^{6}C_2 \times {}^{4}C_4$.

38. (b) Let the vice-chairman and the chairman form 1 unit along with the eight directors, we now have to arrange 9 different units in a circle. This can be done in 8! ways.

At the same time, the vice-chairman & the chairman can be arranged in two different ways. Therefore, the total number of ways $= 2 \times 8!$.

39. (d) Total seats = 5 + 6 = 11.

Arrangement will be : *WMWMWMW MWMW*

$\Rightarrow$ Total possible arrangements will be : ${}^{6}P_6 \times {}^{5}P_5 = 86400$.

40. (a) The number of 4 persons including $A, B = {}^{6}C_2$

Considering these four as a group, number of arrangements with the other four = 5!

But in each group the number of arrangements = 2! × 2!

$\therefore$ The required number of ways

$= {}^{6}C_2 \times 5! \times 2! \times 2!$

41. (d) From total 13 members 5 can be selected as ${}^{13}C_5$.

For at least one girl in the committee, number of ways are ${}^{13}C_5 - {}^{6}C_1$

42. (d) $1\,m + 3f = {}^{8}C_1 \times {}^{8}C_3 = 8 \times 56 = 448$

$2\,m + 2f = {}^{8}C_2 \times {}^{8}C_2 = 28 \times 28 = 784$

$3\,m + 1f = {}^{8}C_3 \times {}^{8}C_1 = 56 \times 8 = 448$

$4\,m + 0f = {}^{8}C_4 \times {}^{8}C_0 = 70 \times 1 = 70$

Total = 1750

43. (a) Putting 1 Englishman in a fixed position, the remaining 6 can be arranged in 6! 720 ways,

For each such arrangement, there are 7 positions for the 7 Americans and they can be arranged in 7! ways. Total number of arrangements = 7! × 6! = 3628800

44. (c) Total number of hand shakes = ${}^{20}C_2$ of those no Indian female shakes hand with male

$\Rightarrow 5 \times 10 = 50$ hand shakes

No American wife shakes hand with her husband = 5 × 1 = 5 hand shakes

$\Rightarrow$ total number of hand shakes occurred

$= {}^{20}C_2 - (50 + 5) = 190 - 55 = 135$

45. (b) 1 wicket keeper from 4 can be selected in

${}^{4}C_1 = \dfrac{4!}{3!.1!} = 4$ ways

If 4 bowlers are chosen then remaining 6 batsmen - can be chosen in ${}^{11}C_6$.

${}^{6}C_4 . {}^{11}C_6 = \dfrac{6!}{4!.2!} \times \dfrac{11!}{6!5!}$

$= \dfrac{5 \times 6}{2} \times \dfrac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2}$

$= 15 \times 14 \times 33 = 6930$

If we choose 5 bowlers then we have to choose 5 batsmen

If we choose 6 bowlers then we have to choose 4 batsmen. There is no majority.

$\therefore$ Total number of ways = 4 × 6930 = 27720.

46. (d) No. of words starting with *A* are 4 ! = 24

No. of words starting with *H* are 4 ! = 24

No. of words starting with *L* are 4 ! = 24

These account for 72 words

Next word is RAHLU and the 74th word RAHUL

47. (d) We have no girls together, let us first arrange the 5 boys and after that we can arrange the girls in the space between the boys.

Number of ways of arranging the boys around a circle = [5 – 1]! = 24.

Number of ways of arranging the girls would be by placing them in the 5 spaces that are formed between the boys. This can be done in ${}^{5}P_3$ ways = 60 ways.

Total arrangements = 24 × 60 = 1440.

48. **(a)** Two tallest boys can be arranged in 2! ways. Rest 18 can be arranged in 18! ways.
Girls can be arranged in 6! ways.
Total number of ways of arrangement
$= 2! \times 18! \times 6! = 18! \times 2 \times 720 = 18! \times 1440$

49. **(d)** 3 5 4
(3 or 4 or 5)
$3 \times 5 \times 4 = 60$
2 5 4 3
(1 or 2)
$2 \times 5 \times 4 \times 3 = 120$
Total = 120 + 60 = 180

50. **(b)** Total number of bolts = 600
Number of too large bolts = 20% of 600

$$= \frac{20 \times 600}{100} = 120$$

Number of too small bolts = 10% of 600 = 60
Number of suitable bolts = 600 – 120 – 60
= 420

Thus required probability $= \frac{420}{600} = \frac{7}{10}$

51. **(b)** The probability that Krishna will be alive 10 years hence, is $\frac{7}{15}$

So, probability that Krishna will be dead 10 years

$$= 1 - \frac{7}{15} = \frac{8}{15}$$

Also, probability that Hari will be alive 10 years hence is $\frac{7}{10}$

So, the probability that Hari will be dead 10 years $= 1 - \frac{7}{10} = \frac{3}{10}$

So, the probability that both Krishna and Hari will be dead 10 years hence

$$= \frac{8}{15} \times \frac{3}{10} = \frac{24}{150}$$

52. **(c)** Since, probabilities of failure for engines A, B and C $P(A)$, $P(B)$ and $P(C)$ are 0.03, 0.02 and 0.05 respectively.
The aircraft will crash only when all the three engine fail. So, probability that it crashes
$= P(A).P(B).P(C)$
$= 0.03 \times 0.02 \times 0.05 = 0.00003$
Hence, the probability that the aircraft will not crash, $= 1 - 0.00003 = 0.99997$

53. **(a)** Probability of passing in mathematics $= \frac{4}{9}$

Probability of passing in physics $= \frac{2}{5}$

Probability of failure in physics $= 1 - \frac{2}{5} = \frac{3}{5}$

Given that both the events are independent.

Required probability $= \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$

54. **(c)** Probability of getting a diamond, $P(D)$

$$= \frac{13}{52} = \frac{1}{4}$$

and probability to king, $P(K) = \frac{4}{52} = \frac{1}{13}$

So, required probability $= P(D).P(K)$

$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

55. **(c)** 16 tickets are sold and 4 prizes are awarded. A person buys 4 tickets, then required probability $= \frac{4}{16} = \frac{1}{4}$

56. **(a)** The cards are of four colours and the number of cards of given description is 24.

The probability $= \frac{24}{52} \cdot \frac{23}{51} = \frac{46}{221}$.

GP_3386

57. (c) Total number of ways in which 4 persons can be selected out of 3 + 2 + 4 = 9 persons $= {}^9C_4 = 126$

Number of ways in which a selection of 4 contains exactly 2 children $= {}^4C_2 \times {}^5C_2 = 60$

$\therefore$ reqd. prob. $= \frac{60}{126} = \frac{10}{21}$

58. (d) Total number of balls = 8. Let the first drawn ball is white, so required probability

$$= \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} = \frac{1}{14}.$$

But here we had started with a white ball. When we start with a black ball, the required probability

$$= \frac{3}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{4}{5} = \frac{1}{14}.$$

Since these two cases are mutually exclusive.

Total probability $= \frac{1}{14} + \frac{1}{14} = \frac{2}{14} = \frac{1}{7}$.

59. (c) Total 80, Girls = 25, Boys = 55

10 *R*, 70 *P*, 20 *I*

$$= \frac{1}{4} \times \frac{1}{8} \times \frac{25}{80} = \frac{5}{512}$$

60. (d) Probability that first ball is white and second black = (4/6) × (5/8) = 5/12

Probability that first ball is black and second white = (2/6) × (3/8) = 1/8

These are mutually exclusive events hence the required probability

$$P = \frac{5}{12} + \frac{1}{8} = \frac{13}{24}.$$

CHAPTER

19

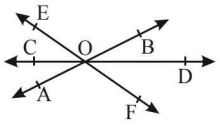
Geometry-I

Lines & Angles, Quadrilateral, Polygons, Circles (Including Chords, Secant and Tangent)

LINES AND ANGLES

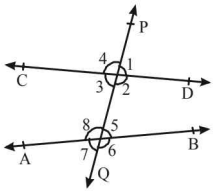
Lines

Concurrent Lines: If three or more lines pass through a point, then they are called **concurrent lines** and the point through which these all lines pass is called **point of concurrence**.



In the figure, *AB*, *CD* and *EF* are concurrent lines and point *O* is the point of concurrent.

Transversal Line: A line which intersects two or more given lines at distinct points is called a **transversal** of the given lines.



In figure straight lines *AB* and *CD* are intersected by a transversal *PQ*.

(i) **Corresponding angles:** In the figure $\angle 1$ and $\angle 5$, $\angle 4$ and $\angle 8$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ are four pairs of corresponding angles.

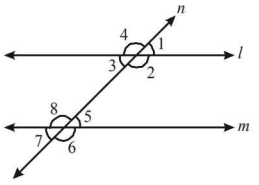
(ii) **Alternate interior angles:** $\angle 3$ and $\angle 5$, $\angle 2$ and $\angle 8$, are two pairs of alternate interior angles.

(iii) **Alternate exterior angles:** $\angle 1$ and $\angle 7$, $\angle 4$ and $\angle 6$ are two pairs of alternate exterior angles.

(iv) **Consecutive interior angles:** In the figure, $\angle 2$ and $\angle 5$, $\angle 5$ and $\angle 8$, $\angle 8$ and $\angle 3$, $\angle 3$ and $\angle 2$ are four pairs of consecutive interior angles.

Interior angles on the same side of a transversal are called cointerior angles. In the fig. $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$ are two pairs of cointerior angles.

When a transversal intersects two parallel lines:



In the figure two parallel lines *l* and *m* are intersected by a transversal line *n*, then

(a) Two angles of each pair of corresponding angles are equal i.e. $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$; $\angle 3 = \angle 7$

(b) Two angles of each pair of alternate interior angles are equal i.e. $\angle 2 = \angle 8$; $\angle 3 = \angle 5$

(c) Two angles of each pair of alternate exterior angles are equal i.e. $\angle 1 = \angle 7$; $\angle 4 = \angle 6$

(d) Any two consecutive interior angles are supplementary. i.e. their sum is 180°.
Hence $\angle 2 + \angle 5 = 180°$; $\angle 5 + \angle 8 = 180°$; $\angle 8 + \angle 3 = 180°$; $\angle 3 + \angle 2 = 180°$

Note that

(i) If two angles of any pair of corresponding angles are equal, then the two lines are parallel.

(ii) If two angles of any pair of alternate interior angles are equal, then the two lines are parallel.

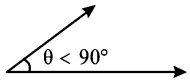
(iii) If two angles of any pair of alternate exterior angles are equal, then the two lines are parallel.

(iv) If any two consecutive interior angles are supplementary (i.e. their sum is 180°), then the two lines are parallel.

GP_3386

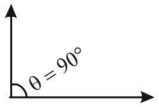
ANGLES

Acute Angle: An angle is said to be acute angle if it is less than 90°.



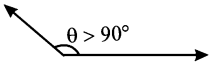
Here $0° < \theta < 90°$, hence θ is acute angle.

Right Angle: An angle is said to be right angle if it is of 90°.



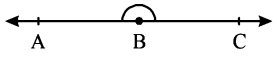
Here θ is right angle.

Obtuse Angle : An angle is said to be obtuse angle if it is greater than 90°.



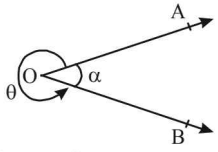
Here θ is obtuse angle.

Straight Angle: An angle is said to be straight angle if it is of 180°.



Here θ is a straight angle.

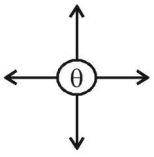
Reflex Angle : An angle is said to be reflex angle if it is greater than 180°.



Here θ is the reflex angle.
Reflex angle θ is written as
$\theta = 360° - \angle AOB$ (or $360° - \alpha$)
Here $\angle AOB$ or α is less than 180°

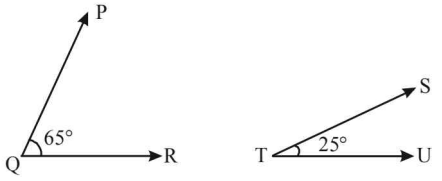
Complete Angle

An angle of measure 360° is said to be complete angle.

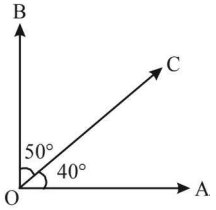


Here θ is a complete angle.

Complementary Angles: Two angles, the sum of whose measures is 90°, are called the complementary angles.

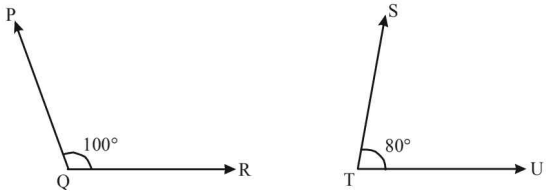


$\angle PQR$ and $\angle STU$ are complementary angles.

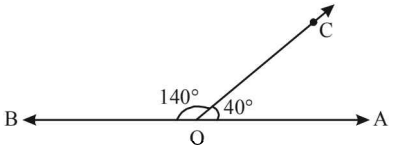


In figure, $\angle AOC$ and $\angle BOC$ are also complementary angles.

Supplementary Angles : Two angles, the sum of whose measures is 180°, are called the supplementary angles.



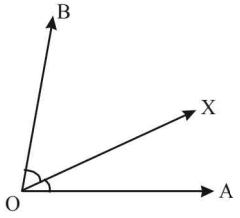
In figure, $\angle PQR$ and $\angle STU$ are supplementary angles.



In figure, ∠AOC and ∠BOC are also supplementary angles.

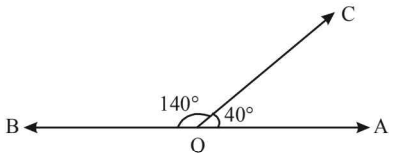
Adjacent Angles: Two angles are called adjacent angles, if

(i) they have the same vertex
(ii) they have a common arm and
(iii) non-common arms are on either side of the common arm.



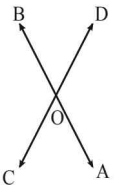
In figure, $\angle AOX$ and $\angle BOX$ are adjacent angles because O is the common vertex, OX is common arm, non-common arm OA and OB are on either side of OX.

Linear Pair of Angles : Two adjacent angles are said to form a linear pair of angles, if their non common arms are two opposite rays. In other words, if the sum of two adjacent angles is 180°, then they are said to form a linear pair of angles.



In figure, $\angle AOC$ and $\angle BOC$ are linear pair angles.

Vertically Opposite Angles : Two angles are called a pair of vertically opposite angles, if their arms form two intersecting lines.



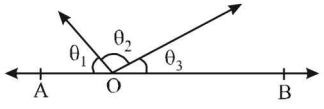
In figure, $\angle AOC$ and $\angle BOD$ form a pair of vertically opposite angles. Also $\angle AOD$ and $\angle BOC$ from a pair of vertically opposite angles.

The two angles in a pair of vertically opposite angles are equal.

$\therefore \angle AOD = \angle BOC$

and $\angle AOC = \angle BOD$

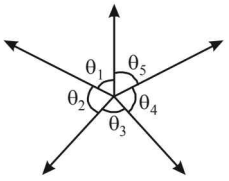
Angles on one side of a line at a point on the line: Sum of all the angles on any one side of a line at a point on the line is always 180°.



Here AOB is a straight line, hence in figure,

$\theta_1 + \theta_2 + \theta_3 = 180°$.

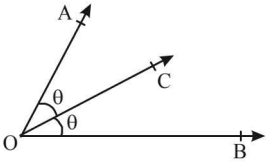
Angle around a point: Sum of all the angles around a point is always 360°.



Here θ_1, θ_2, θ_3, θ_4 and θ_5 are the angles around a point.

Hence $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360°$

Angle bisector: An angle bisector is a ray which bisects the angle whose initial point be the vertex of the angle.



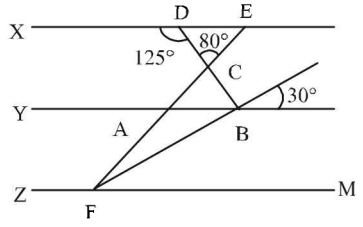
Since $\angle AOC = \angle BOC = \theta$

Hence ray OC is the bisector of $\angle AOB$.

Illustration – 1

Three straight lines, X, Y and Z are parallel and the angles are as shown in the figure above. What is $\angle AFB$ equal to?

(a) 20° (b) 15° (c) 30° (d) 10°



Solution : (b) $\angle CDE = 180° - 125° = 55°$

In ΔDCE,

$\angle CED = 180° - 55° - 80° = 45°$

and $\angle ABF = 30°$ (vertically opposite)

Also, $\angle ABF = \angle BFM = 30°$ (alternate angle)

and, $\angle DEF = \angle EFM$ (alternate angle)

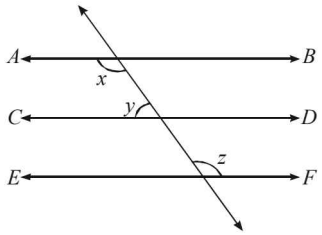
$\angle EFM = 45°$

$\Rightarrow \angle EFB + \angle BFM = 45° \Rightarrow \angle EFB = 45° - 30°$

$\Rightarrow \angle AFB = 15°$

Illustration – 2

In figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, $x = ?$



(a) 112° **(b) 116°** **(c) 96°** **(d) 126°**

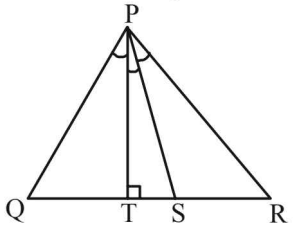
Solution : (d) As $y + z = 180°$, $\therefore y = 54°$

$x + y = 180°$

$x = 180 - 54 = 126°$

Illustration – 3

In the ΔPQR, PS is the bisector of $\angle P$ and $PT \perp QR$, then $\angle TPS$ is equal to



(a) $\angle Q + \angle R$ **(b)** $90° + \frac{1}{2}\angle Q$

(c) $90° - \frac{1}{2}\angle R$ **(d)** $\frac{1}{2}(\angle Q - \angle R)$

Solution : (d) PS is the bisector of $\angle QPR$

$\therefore \quad \angle 1 + \angle 2 = \angle 3 \quad ...(1)$

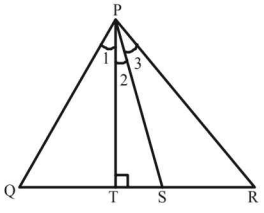
$\Rightarrow \quad \angle Q = 90° - \angle 1$

$\angle R = 90° - \angle 2 - \angle 3$

So, $\angle Q - \angle R = [90° - \angle 1] - [90° - \angle 2 - \angle 3]$

$\Rightarrow \angle Q - \angle R = \angle 2 + \angle 3 - \angle 1$

$= \angle 2 + (\angle 1 + \angle 2) - \angle 1$ [From Eq. (1)]



$\Rightarrow \angle Q - \angle R = 2\angle 2 \Rightarrow \frac{1}{2}(\angle Q - \angle R) = \angle TPS$

POLYGONS

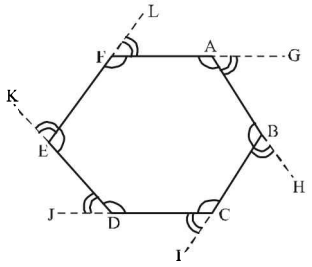
Polygons are closed plane figures formed by series of line segments.

Interior and Exterior Angles of a Regular Polygon

An angle inside a polygon between any two adjacent sides at a vertex of the polygon is called an **interior angle** of the polygon. An angle outside a polygon made by a side of the polygon with the its adjacent side produced is called an **exterior angle** of the polygon.

In the figure $ABCDEF$ is a polygon.

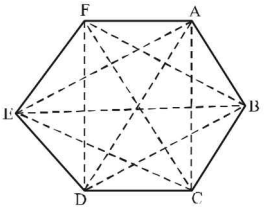
$\angle FAB$, $\angle ABC$, $\angle BCD$, $\angle CDE$, $\angle DEF$ and $\angle EFA$ are interior angles of the polygon $ABCDEF$.



$\angle BAG$, $\angle CBH$, $\angle DCI$, $\angle EDJ$, $\angle FEK$ and $\angle AFL$ are exterior angles of the polygon $ABCDEF$.

Diagonals of a Regular Polygon

A diagonal of a polygon is a line segment connecting two non-consecutive vertices of the polygon.

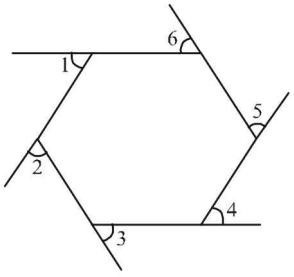


In the figure, diagonals are drawn by dotted line segments.

Properties of a Regular Polygons

(i) Sum of all the interior angles of a polygon with 'n' sides = $(n - 2) \times 180°$

(ii) Sum of all the exterior angles of a polygon $= 360°$ or $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$



(iii) Perimeter of a regular polygon with a side length of $a = n \times a$

(iv) No. of sides of a regular polygon

$$= \frac{360°}{\text{An exterior angle}}$$

(v) Number of diagonals of a polygon with n sides

$$= \frac{n(n-3)}{2}$$

Illustration – 4

An interior angle of a regular polygon is 135°. Find the number of sides of the polygon.

Solution : Since interior angle of the regular polygon = 135°, hence exterior angle = 180° – 135° = 45°

$$\therefore \quad \text{No. of sides} = \frac{360°}{\text{An exterior angle}} = \frac{360°}{45°} = 8$$

$\therefore$ No. of sides = 8

Illustration – 5

An interior angle of a regular polygon is 100° more than its an exterior angle. Find the number of sides the polygon.

Solution : Let measure of each exterior angle be $x°$.

Then measure of each interior angle = $(x + 100)$

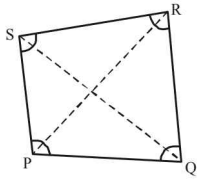
Now $x + (x + 100) = 180°$

$\Rightarrow 2x = 80 \Rightarrow x = 40$

$$\text{Now, number of sides} = \frac{360°}{\text{An exterior angle}} = \frac{360°}{40°} = 9.$$

QUADRILATERALS

Quadrilateral is a plane figure bounded by four straight lines. The line segment which joins the opposite vertices of a quadrilateral is called **diagonal** of the quadrilateral. In figure, $PQRS$ is a quadrilateral and PR, QS are its two diagonals.

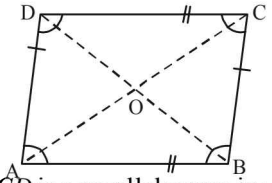


Sum of angles of a quadrilateral = 360°

i.e. $\angle P + \angle Q + \angle R + \angle S = 360°$

Types of Quadrilaterals

1. Parallelogram: A parallelogram is a quadrilateral with opposite sides parallel and equal.



In figure, $ABCD$ is a parallelogram in which AC and BD as diagonals which intersect each other at O.

Properties:

(i) Opposite sides are equal i.e. $AB = DC$, $AD = BC$

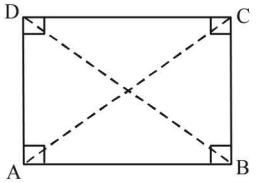
(ii) Opposite sides are parallel i.e. $AB \parallel DC$ and $AD \parallel BC$

(iii) Opposite angles are equal i.e. $\angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$

(iv) Diagonals bisect each other, i.e. $OA = OC$, $OB = OD$

(v) Sum of pair of consecutive angles is 180° i.e., $\angle A + \angle B = 180°$, $\angle B + \angle C = 180°$, $\angle C + \angle D = 180°$, $\angle D + \angle A = 180°$.

2. Rectangle: A rectangle is a parallelogram with all angles equal to 90°.



In figure, $\angle A = \angle B = \angle C = \angle D = 90°$

Properties:

(i) In a rectangle the length of diagonal, are equal i.e.

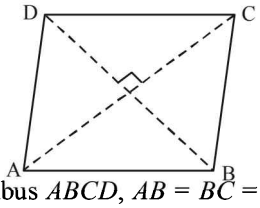
$$AC = \sqrt{AB^2 + BC^2} = BD$$

GP_3386

(ii) In a rectangle diagonals bisect each other.

(iii) All rectangles are parallelogram but all parallelograms are not rectangles.

3. **Rhombus:** A parallelogram is a rhombus if its all sides are equal.



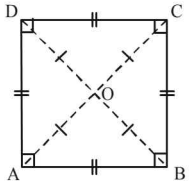
In rhombus $ABCD$, $AB = BC = CD = DA$

Properties:

(i) In a rhombus diagonals bisect each other at right angles i.e. angle between AC and DB is 90°.

(ii) All rhombus are parallelogram but all parallelograms are not rhombus.

4. **Square:** A parallelogram is a square if all the four sides are equal and also all the four angles are equal (i.e. 90°).



In figure, $ABCD$ is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$

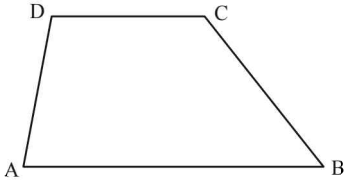
Properties:

(i) In a square diagonals are equal i.e. $AC = BD$

(ii) In a square diagonals bisect each other at right angle,
i.e. $OA = OC$, $OB = OD$
and $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

(iii) All square are rhombus but rhombus may or may not be a square.

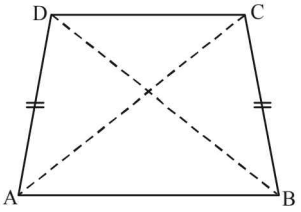
5. **Trpaezium:** A quadrilateral is a trapezium if one pair of opposite sides are parallel.
In trapezium $ABCD$, $AB \parallel DC$.



If lateral sides (i.e. non-parallel sides) of a trapezium are equal, then it is called **isosceles trapezium**.

Properties of isosceles trapezium

In the figure ABCD is an isosceles trapezium, then



(i) $AB \parallel DC$

(ii) $AD = BC$

(iii) Diagonals are equal i.e. AC = BD

Diagonal Properties of all Parallelograms

Sr. No.	Diagonal Properties	Type of Parallelogram			
		Parallelogram	Rectangle	Rhombus	Square
1	Diagonals bisect each other	√	√	√	√
2	Diagonals are equal	×	√	×	√
3	Diagonals are at 90° to each other	×	×	√	√

ANGLES

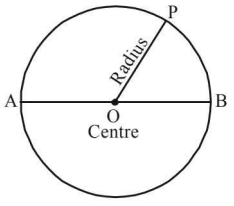
A circle is a set of all points in a plane equidistant from a fixed point.

In figure, APB is a circle and 'O' is the fixed point called the **centre** of the circle and the constant distance OP is called **radius** of the circle.

A diameter is a line segment passing through the **centre** and joins the two different points on the circle.

In the figure, AB is the diameter as it passes through the centre and joins the two points on the circle.

or Diameter = 2 × radius.

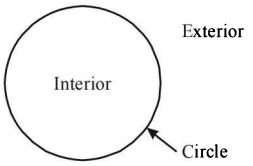


A circle divides the plane in which it lies into three parts.

(i) Inside the circle, called interior of the circle

(ii) The circle

(iii) Outside the circle, called the exterior of the circle.



The circle and its interior make up the circular region.

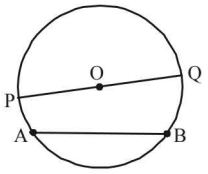
Circumference

Length of a complete circle is called its **circumference**.

Circumference = $2\pi r$

where $\pi = \frac{22}{7}$ or 3.142 and r = radius

Chord: A line segment joining any two points on the circle is called *chord of the circle*. A chord which passes through the centre is the diameter of the circle.



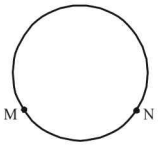
In the figure, O is the centre of the circle.

AB and PQ both are chords.

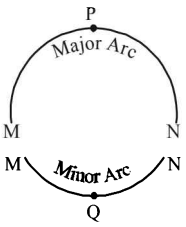
But PQ is the diameter (longest chord) also.

Arcs: A piece of a circle between two points is called an *arc*.

Consider two points M and N on the circle. We find that there are two pieces of circle between M and N. One is longer and other is smaller.



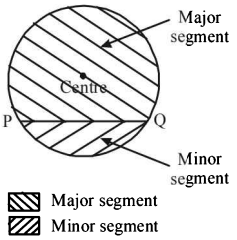
The longer piece is called **major arc** and smaller piece is called **minor arc**



Major arc is denoted by $\overset{\frown}{MPN}$ and minor arc is denoted by $\overset{\frown}{MQN}$. When M and N are ends of a diameter then both the arcs are equal and both are called **semicircle**.

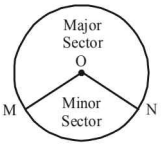
Segment: The region between a chord and an arc of a circle is called a *segment*.

There are two segments corresponding to two arcs, major segment and minor segment. Major segment is the segment enclosed by major arc. Centre of the circle lies in the major segment. Minor segment is the segment enclosed by minor arc. Centre of the circle does not lie in the minor segment.



If two arcs are equal, then both segments are semi-circles.

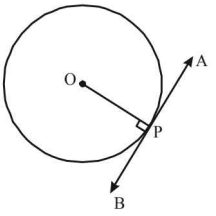
Sector: The region between an arc and the two radii joining the centre to the end point of the arc is called a sector. There are two sectors Minor and Major Sectors.



The sector which is larger than semicircular region is called **major sector** and the region less than the semicircular region is called **minor sector**.

If both sectors are equal, then each sector is a semi-circle.

Tangent: A tangent is a straight line which touches the circumference of a circle at only one point. A tangent does not intersect the circumference, if produced infinitely on either sides.



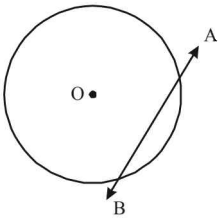
In figure, AB is tangent to circle, which touches the circle at point P.

P is called the **point of contact** of tangent to the circle.

GP_3386

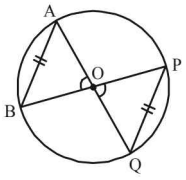
Radius OP through the point of contact is always perpendicular to the tangent at the point of contact i.e. $OP \perp AB$.

Secant: A secant is a straight line of infinite length which intersects the circumference of a circle at two different points. In figure, AB is a secant.



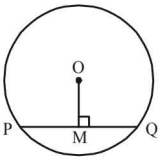
Basic Properties of a Circle

1. Equal chord of a circle subtend equal angles at the centre.



If $AB = PQ$, then $\angle AOB = \angle POQ$
The converse is also true.

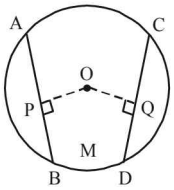
2. The perpendicular from the centre of a circle to a chord of the circle bisects the chord.



In figure, PQ is chord of a circle with centre 'O', OM is perpendicular to PQ therefore $PM = MQ$. The converse is also true.

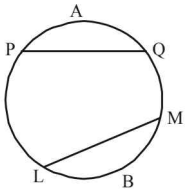
3. One and only one circle can pass through given three non-collinear points.
If three or more points lie on a line, then they are called **collinear points** otherwise called **non-collinear points**.

4. Equal chords of a circle are equidistant from the centre of the circle.
In the figure, if $AB = CD$, then $OP = OQ$



The converse is also true.

5. Two equal chords have equal corresponding arcs.

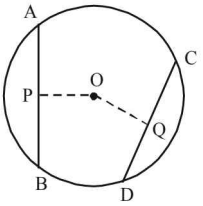


If $PQ = LM$ then

(a) $\overset{\frown}{PAQ} = \overset{\frown}{LBM}$ (Minor Arc)

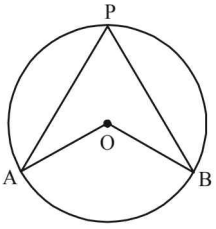
(b) $\overset{\frown}{PBQ} = \overset{\frown}{LAM}$ (Major Arc)

6. The greater of the two chords is nearer to the centre.



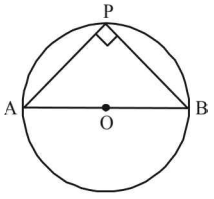
If $AB > CD$, then $OP < OQ$

7. The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



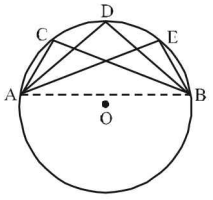
Minor arc $\overset{\frown}{AB}$ subtend $\angle AOB$ at the centre O and also subtend $\angle APB$ at point P (situated on remaining part of circle). So $\angle AOB = 2\ \angle APB$

8. Angle in a semicircle is a right angle.



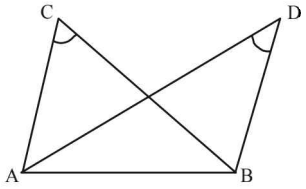
In figure, AOB is a diameter, hence $AOBPA$ is a semicircle, therefore $\angle APB = 90°$.

9. Angles in the same segment of a circle are equal. $\angle ACB$, $\angle ADB$, $\angle AEB$ are in the same segment $ACDEBA$ of the circle.



$\therefore \quad \angle ACB = \angle ADB = \angle AEB.$

10. If in a plane a line segment joining two points subtends equal angles at two other points lying on the same side of a line containing the line segment, the four points lie on a circle i.e. they are concyclic.

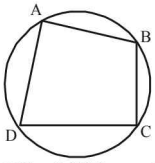


In figure, if $\angle ACB = \angle ADB$, then points A, B, D, C lie on a circle.

11. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

A **cyclic quadrilateral** is the quadrilateral whose four vertices are concyclic i.e. the four vertices lie on a circle.

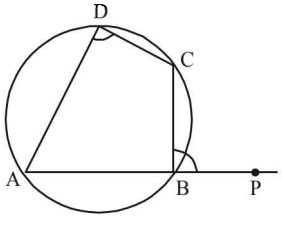
In figure, $ABCD$ is a cyclic quadrilateral,



$\therefore \quad \angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$

The converse is also true.

12. If a side of a cyclic quadrilateral is produced the exterior angle so formed is equal to the interior opposite angle.

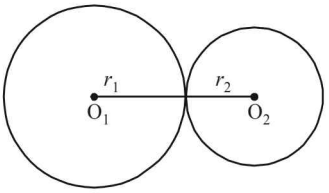


In figure, $ABCD$ is a cyclic quadrilateral,

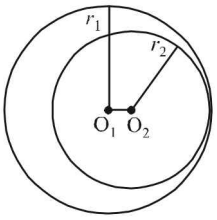
$\therefore \quad \angle CBP = \angle CDA$

13. Two circles C_1 with centre O_1, radius r_1 and C_2 with centre O_2, radius r_2 will touch

(a) Externally, if and only if $O_1O_2 = r_1 + r_2$



(b) Internally, if and only if $O_1O_2 = |r_1 - r_2|$



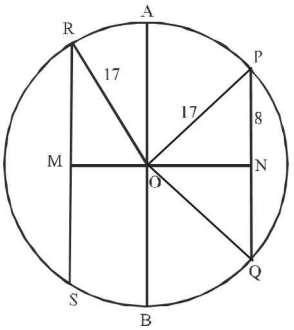
14. Two circles are congruent if their radii are equal.

Illustration – 6

In a circle of radius 17 cm, two parallel chords are drawn on opposite sides of a diameter. The distance between the chords is 23 cm. If length of one chord is 16 cm, then the length of the other one is :

(a) 15 cm (b) 23 cm (c) 30 cm (d) 34 cm

Solution : (c) Let PQ and RS be two parallel chords of the circle on the opposite sides of the diameter (AB)



Now, $PN = 8$ (Since ON is the perpendicular bisector)

In ΔPON

$ON^2 = OP^2 - PN^2$

$= (17)^2 - (8)^2 = 289 - 64 = 225$

or $ON = 15$

So, $OM = 23 - 15 = 8$

In ΔORM,

$RM^2 = OR^2 - OM^2 = 17^2 - 8^2 = 289 - 64 = 225$

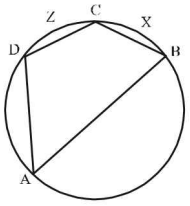
or $RM = 15 \Rightarrow RS = 15 \times 2 = 30$ cm

GP_3386

Illustration – 7

In the cyclic quadrilateral $ABCD$, $\angle BCD = 120°$, m (arc DZC) = 70°, find DAB and m (arc CXB).

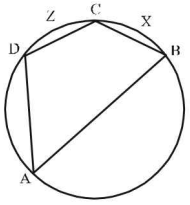
(a) 60°, 70° **(b) 60°, 40°**
(c) 60°, 50° **(d) 60°, 60°**



Solution : (c) $\overline{\text{m}}\angle DAB + 180° - 120° = 60°$ .....

(Opposite angles of a cyclic quadrilateral)

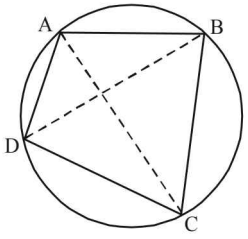
m (arc BCD) = 2m $\angle DAB = 120°$.



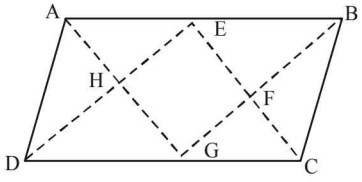
$\therefore$ m (arc CXB) = m (BCD) – m (arc DZC)
= 120° – 70° = 50°.

Important Points

1. In a cyclic quadrilateral, product of the diagonals is equal to the sum of the products to the opposite sides,
$AC \times BD = (AD \times BC) + (AB \times CD)$

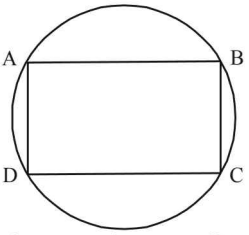


2. Bisectors of the angles of a parallelogram or a rectangle form a rectangle.

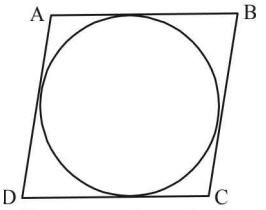


In parallelogram $ABCD$, AG, BG, CE and DE are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively. Hence in the figures $EFGH$ is a rectangle.

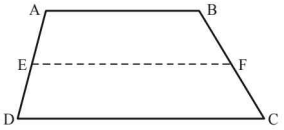
3. A parallelogram inscribed in a circle is a rectangle.
In figure, $ABCD$ is a rectangle.



4. A parallelogram circumscribed a circle is a rhombus. In figure, $ABCD$ is a rhombus.



5. Median of a trapezium is the line segment joining mid-points of non-parallel sides of the trapezium.
In the figure E and F are the mid points of non-parallel sides AB and CD respectively. Hence EF is the median of trapezium $ABCD$.

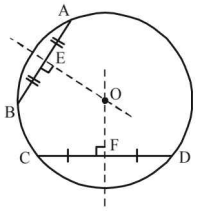


$$EF = \frac{1}{2}(AB + CD)$$

Also $EF = \dfrac{a \times (\text{AB}) + b \times (\text{DC})}{\text{AD}}$,

where $AE = a$ and $ED = b$

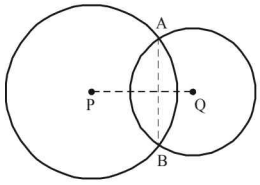
6. Perpendicular bisectors of two chords of a cricle intersect at its centre of the circle.



In figure, OE and OF are perpendicular bisectors of chords AB and CD, OE and OF meet at point O. Hence O is the centre of the circle.

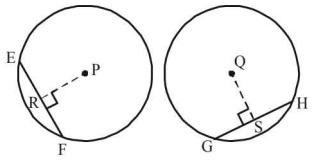
7. If two circles intersect each other at two points then the line through the centres is the perpendicular bisectors of the common chord.

In figure, two circles with centre P and Q intersect each other at two points A and B. Hence AB is the common chord of the two circles.



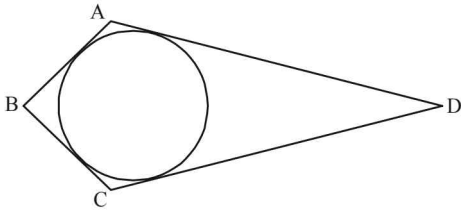
Therefore, PQ is the perpendicular bisector of common chord AB.

8. Equal chords of congruent circles are equidistant from the centre.



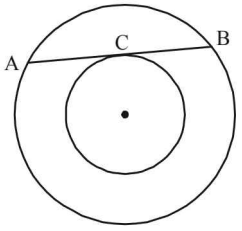
In figure two circles are congruent i.e. their radii are equal. EF and GH are two equal chords. Hence their perpendicular distances from centre P and Q respectively are equal.

9. If a circle touches all the four sides of a quadrilateral then the sum of the two opposite sides is equal to the sum of other two.



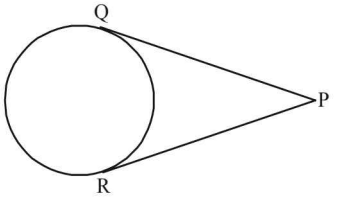
$AB + DC = AD + BC$

10. In two concentric circles, if a chord of the larger circle is also tangent to the smaller circle, then the chord is bisected at the point of contact.



Hence in the figure, $AC = CB$

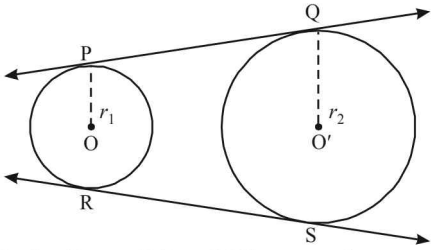
11. Length of two tangents from an exterior point to a circle are equal.



In figure PQ and PR are two tangents drawn from an exterior point to a circle.

$\therefore \quad PQ = PR$

12. **Direct common tangent:** A tangent to two circles are such that the two circles lies on the same side of the tangent, then the tangent is called direct tangent to the two circles.



In the figure, PQ and RS are two direct common tangent to the same two circles. Length of these two common tangents to the same two circles are equal.

i.e. $PQ = RS$

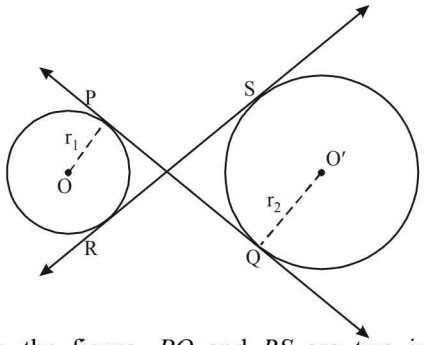
Also $PQ = RS = \sqrt{(OO')^2 + (r_2 - r_1)^2}$

Here O, O' are the centres and r_1, r_2 are the radii of the two circles respectively. Also $r_2 > r_1$.

13. **Indirect or Transverse Common Tangent:** If a tangent to two circles is such that the two circles lie on opposite sides of the tangent, then the tangent is called indirect tangent.

Length of two indirect tangents to two circles is equal.

GP_3386



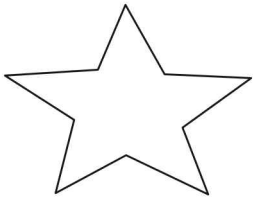
In the figure, PQ and RS are two indirect common tangents to the same two circles.

$\therefore \quad PQ = RS$

Also $PQ = RS = \sqrt{(OO')^2 - (r_1 + r_2)^2}$

Here O, O' are centres r_1, r_2 are radii of the two circles respectively.

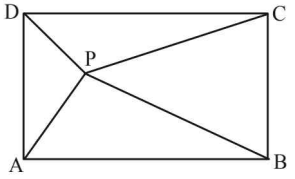
14. Star: A star has a shape like given in the figure.



If a star has n sides, then sum of its all interior angles = $(n - 4) \times 180°$.

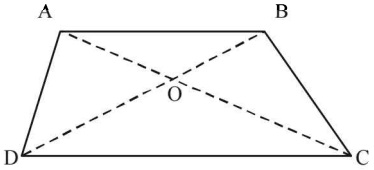
15. In the figure given below, if P is any point inside the rectangle $ABCD$, then

$PA^2 + PC^2 = PB^2 + PD^2$



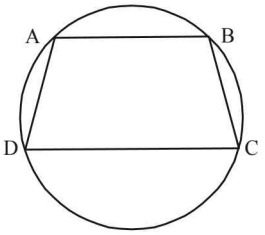
16. Diagonals of a trapezium divide each other in the ratio of the parallel sides of the trapezium.

In trapezium $ABCD$, $AB \parallel DC$



$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD} = \frac{AB}{CD}$.

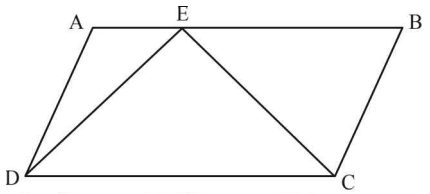
17. If a trapezium is inscribed inside a circle, then the trapezium is an isosceles trapezium i.e. its non-parallel sides are equal.



In the figure, $ABCD$ is a trapezium in which $AB \parallel CD$

$\therefore \quad AD = BC$

18. If a parallelogram and a triangle are on the same base and lie between the same pair of parallel lines, then area of the parallelogram is twice the area of the triangle.

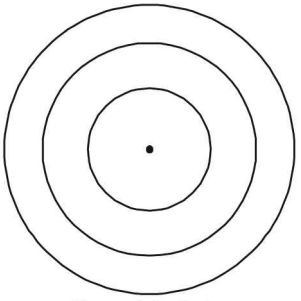


In the figure, $ABCD$ a parallelogram and EDC a triangle are on the same base and lie between the same pair of parallel lines AB and CD.

$\therefore$ area of parallelogram $ABCD$

$= 2 \times$ (area of ΔEDC).

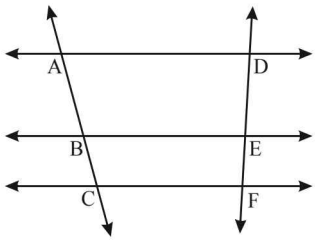
19. Concentric circles: Two or more circles in a plane arc said to be concentric, if they have the same centre.



Concentric circles

20. Intercepts made by three or more parallel lines on two or more lines are in the same ratios.

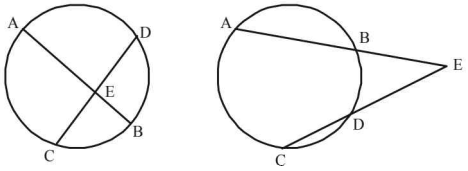
In the figure three parallel lines AD, BE and CF made intercepts AB, BC and DE, EF on two lines AC and DF respectively.



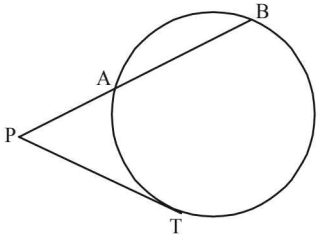
$\therefore \quad \frac{AB}{BC} = \frac{DE}{EF}$

21. A parallelogram is a rectangle if its diagonals are equal.

22. If two chords AB and CD of a circle intersect inside a circle (or outside a circle when produced) at point E, then
$AE \times EB = CE \times ED$.

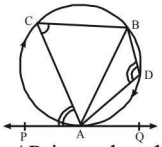


23. If PB is a secant which intersects the circle at A and B and PT is a tangent at T to the circle, then



$PA \times PB = PT^2$

24. Angles in the alternate segment:

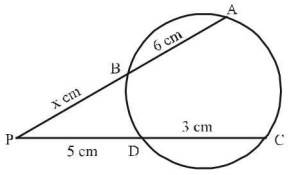


In the figure, AB is a chord of a circle. PQ is a tangent at an end point A of the chord to the circle. C is any point on arc AB and D is any point on arc BA.
$\angle BAQ$ and $\angle ACB$ are angles in the alternate segments
$\angle BAP$ and $\angle ADB$ are angles in the alternate segments.

Angles in the alternate segments of a circle are equal i.e.
$\angle BAQ = \angle ACB$ and $\angle BAP = \angle ADB$

Illustration – 8

In the given figure, chords AB and CD of a circle intersect externally at P. If $AB = 6$ cm, $CD = 3$ cm and $PD = 5$ cm, then $PB = ?$



(a) 5 cm (b) 6.25 cm (c) 6 cm (d) 4 cm

Solution : (d) $PA \times PB = PC \times PD$ (According to property of circle)

$\Rightarrow \quad (x+6) \times x = 8 \times 5$

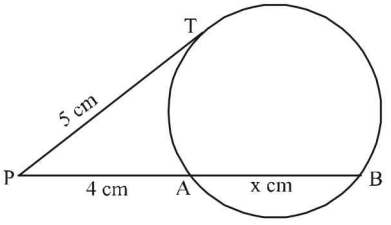
$\Rightarrow \quad x^2 + 6x - 40 = 0$

$\Rightarrow \quad (x+10)(x-4) = 0 \Rightarrow x = 4$

$\therefore \quad PB = 4$ cm

Illustration – 9

In the given figure, PAB is a secant and PT is a tangent to the circle from P. If $PT = 5$ cm, $PA = 4$ cm and $AB = x$ cm, then x is equal to



(a) 2.5 cm (b) 2.6 cm (c) 2.25 cm (d) 2.75 cm

Solution : (c)

$PA \times PB = PT^2 \Rightarrow 4 \times (4+x) = 25$

$\Rightarrow \quad 4 + x = \frac{25}{4} = 6.25 \Rightarrow x = 2.25$ cm

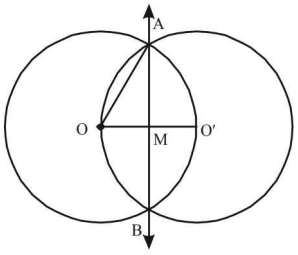
Illustration – 10

Two equal circles pass through each other's centre. If the radius of each circle is 5 cm, what is the length of the common chord?

(a) $5\sqrt{3}$ (b) $10\sqrt{3}$ (c) $\frac{5\sqrt{3}}{2}$ (d) 5

GP_3386

Solution : (a)



Given, distance between the centres of two circle = 5 cm

$$OO' = 5\,\text{cm}$$

$$\therefore \quad OM = \frac{5}{2}\,\text{cm}$$

In ΔOAM,

$$OA^2 = OM^2 + AM^2$$

$$(5)^2 = \left(\frac{5}{2}\right)^2 + AM^2$$

$$AM = \sqrt{25 - \frac{25}{4}} = \frac{5\sqrt{3}}{2}\,\text{cm}$$

$\therefore$ The length of common chord, $AB = 2 \times AM$

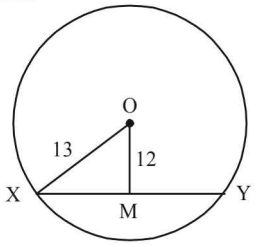
$$= 2 \times \frac{5\sqrt{3}}{2} = 5\sqrt{3}\,\text{cm}$$

Illustration – 11

The radius of a circle is 13 cm and *xy* is a chord which is at a distance of 12 cm from the centre. The length of the chord is

(a) 12 cm (b) 10 cm (c) 20 cm (d) 15 cm

Solution : (b) From figure,



$$XM = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = 5$$

$\therefore$ Length of the chord = $2 \times XM$

$= 2 \times 5 = 10$ cm

Illustration – 12

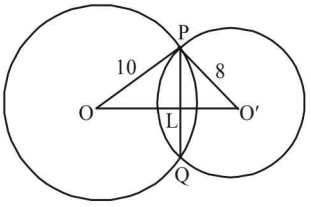
Two circles of radii 10 cm and 8 cm. intersect and length of the common chord is 12 cm. Find the distance between their centres.

(a) 13.8 cm (b) 13.29 cm

(c) 13.2 cm (d) 12.19 cm

Solution : (b)

Here, $OP = 10$ cm; $O'P = 8$ cm



$PQ = 12\,\text{cm}$

$\therefore PL = 1/2\ PQ \Rightarrow PL = \frac{1}{2} \times 12 \Rightarrow PL = 6\,\text{cm}$

In ΔOLP, $OP^2 = OL^2 + LP^2$

(using Pythagoras theorem)

$\Rightarrow \quad (10)^2 = OL^2 + (6)^2 \Rightarrow OL^2 = 64;\ OL = 8$

In $\Delta O'LP$, $(O'L)^2 = O'P^2 - LP^2 = 64 - 36 = 28$

$$O'L^2 = 28 \Rightarrow O'L = \sqrt{28}$$

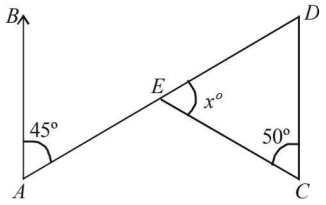
$$O'L = 5.29\,\text{cm}$$

$\therefore \quad OO' = OL + O'L = 8 + 5.29$

$$OO' = 13.29\,\text{cm}$$

LEVEL - 1

1. In the given figure, $AB \parallel CD$, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x$, then find the value of x.

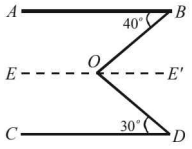


(a) 85° (b) 95° (c) 60° (d) 20°

2. How many sides a regular polygon has with its sum of interior angles eight times its sum of exterior angles?

(a) 16 (b) 24 (c) 18 (d) 30

3. In the given figure, $AB \parallel CD$, $\angle ABO = 40°$ and $\angle CDO = 30°$. If $\angle DOB = x$, then find the value of x.



(a) 10° (b) 70° (c) 110° (d) 20°

4. Angle between the hour and minute hand of a clock at 11:15 is –

(a) 90° (b) 112.5° (c) 120° (d) 150°

5. Each interior angle of a regular polygon exceeds its exterior angle by 132°. How many sides does the polygon have?

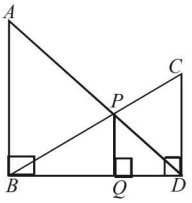
(a) 9 (b) 15

(c) 12 (d) None of these

6. The sum of the interior angles of a polygon is 1620°. The number of sides of the polygon are :

(a) 9 (b) 11 (c) 15 (d) 12

7. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90°$. If $AB : CD = 3 : 1$, the ratio of $CD : PQ$ is



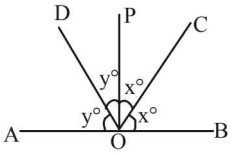
(a) 1 : 0.69 (b) 1 : 0.75

(c) 1 : 072 (d) None of these

8. What is the sum of all the angles of a 9 pointed star (i.e., $\angle 1 + \angle 2 + \angle 3 + ... \angle 8 + \angle 9$)?

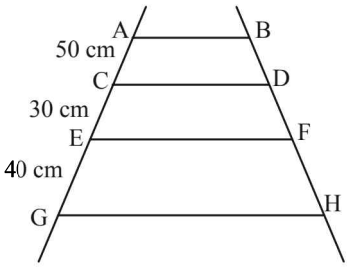
(a) 909° (b) 900° (c) 720° (d) 540°

9. In the following figure $\angle BOP = 2x°$, $\angle AOP = 2y°$, OC and OD are angle bisectors of $\angle BOP$ and $\angle AOP$ respectively. Find the value of $\angle COD$:



(a) 75° (b) 90° (c) 100° (d) 120°

10. In the given figure AB || CD || EF || GH and BH = 100 cm. Find the value of DF :

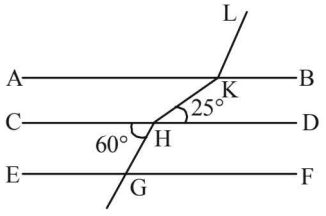


(a) 26 cm (b) 40 cm (c) 25 cm (d) 24 cm

11. Maximum number of points of intersection of four lines on a plane is :

(a) 4 (b) 6 (c) 8 (d) 5

12. In the given figure AB || CD || EF and GH || KL. Find m $\angle$HKL :



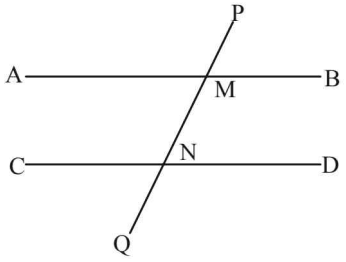
(a) 85° (b) 145° (c) 120° (d) 95°

13. The measures of the angles of a quadrilateral taken in order are proportional to 1 : 2 : 3 : 4, then the quadrilateral is :
(a) parallelogram (b) trapezium
(c) rectangle (d) rhombus

14. Find the measure of largest angle of a quadrilateral if the measures of its interior angles are in the ratio of 3 : 4 : 5 : 6.
(a) 60° (b) 120°
(c) 90° (d) can't be determined

15. In a parallelogram ABCD, bisectors of consecutive angles A and B intersect at P. Find the measure of ∠APB :
(a) 90° (b) 60°
(c) 120° (d) data insufficient

16. AB and CD are two parallel lines and a transversal PQ intersects AB and CD at M and N respectively. The bisector of the interior angles form a quadrilateral :



(a) rectangle (b) square
(c) parallelogram (d) None of these

17. If ABCD is a parallelogram in which P and Q are the centroids of ΔABD and ΔBCD, then PQ equals :
(a) AQ (b) AP (c) BP (d) DQ

18. Diagonals of a parallelogram are 8 cm and 6 cm respectively. If one of side is 5 cm, then the area of parallelogram is :
(a) 18 cm^2 (b) 30 cm^2 (c) 24 cm^2 (d) 48 cm^2

19. Each interior angle of a regular polygon is 140°. The number of sides is:
(a) 10 (b) 8 (c) 6 (d) 9

20. The sum of the interior angles of a polygon is 1260°. The number of sides of the polygon is :
(a) 6 (b) 7 (c) 8 (d) 9

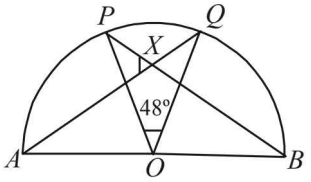
21. A polygon has 54 diagonals. The number of sides in the polygon is :
(a) 7 (b) 9
(c) 12 (d) None of these

22. The sum of all the interior angles of a regular polygon is four times the sum of its exterior angles. The polygon is :
(a) hexagon (b) triangle
(c) decagon (d) nonagon

23. The ratio of the measure of an angle of a regular nonagon to the measure of its exterior angle is :
(a) 3 : 5 (b) 5 : 2 (c) 7 : 2 (d) 4 : 5

24. If each interior angle of a regular polygon is 3 times its exterior angle, the number of sides of the polygon is :
(a) 4 (b) 5 (c) 6 (d) 8

25. In the figure given below, *AB* is a diametre of the semicircle *APQB*, centre *O*, ∠*POQ* = 48° cuts *BP* at *X*, calculate ∠*AXP*.

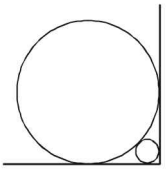


(a) 50° (b) 55°
(c) 66° (d) 40°

26. If four sides of a quadrilateral *ABCD* are tangential to a circle, then.
(a) AC + AD = BD + CD
(b) AD + BC = AB + CD
(c) AB + CD = AC + BC
(d) AC + AD = BC + DB

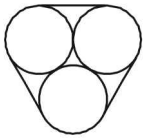
27. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is
(a) 1/2 (b) 2/3 (c) 1/4 (d) 3/4

28. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



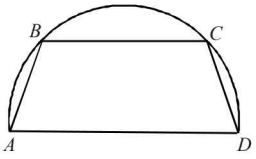
(a) $3-2\sqrt{2}$ (b) $4-2\sqrt{2}$
(c) $7-4\sqrt{2}$ (d) $6-4\sqrt{2}$

29. The diagram below represents three circular garbage cans, each of diameter 2 m. The three cans are touching as shown. Find, in metres, the perimeter of the rope encompassing the three cans.



(a) $2\pi+6$ (b) $3\pi+4$ (c) $4\pi+6$ (d) $6\pi+6$

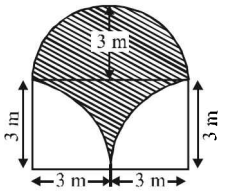
30. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC?



(a) 7.5 (b) 7
(c) 7.75 (d) None of these

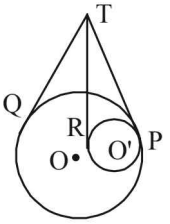
31. Two different sides of a parallelogram are 8 cm and 6 cm and the ratio of the diagonals is 3 : 4. Find the difference between the lengths of the diagonals?
(a) 5 cm (b) 7 cm (c) 6 cm (d) $\sqrt{8}$ cm

32. In the adjoining figure is a park in which shaded area is to be covered by grass. If the rate of covering with grass is ₹ 0.70 per sq. m.



Find the expenditure of covering its field with grass ($\pi=22/7$)
(a) ₹ 12.60 (b) ₹ 6.30 (c) ₹ 9.30 (d) ₹ 10.30

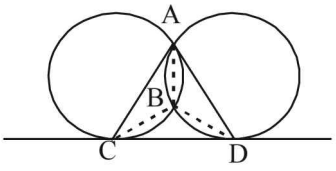
33. In the given figure, there are two circle with the centres O and O' touching each other internally at P. Tangents TQ and TP are drawn to the larger circle and tangents TP and TR are drawn to the smaller circle. Find TQ : TR.



(a) 8 : 7 (b) 7 : 8 (c) 5 : 4 (d) 1 : 1

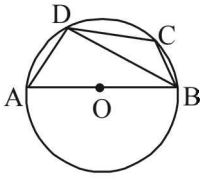
34. In the given figure, CD is a direct common tangent to two circles intersecting each other at A and B, then :

$\angle CAD + \angle CBD = ?$



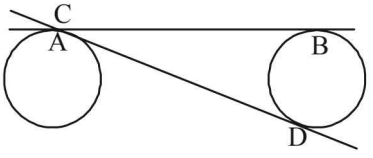
(a) 120° (b) 90° (c) 360° (d) 180°

35. In the adjoining figure AB is a diameter of the circle and $\angle BCD = 130°$. What is the value of $\angle ABD$?



(a) 30° (b) 50°
(c) 40° (d) None of these

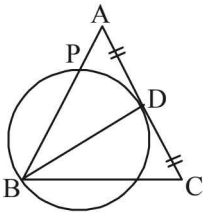
36. There are two circles each with radius 5 cm. Tangent AB is 26 cm. The length of tangent CD is :



(a) 15 cm (b) 21 cm
(c) 24 cm (d) can't be determined

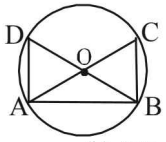
37. In the figure, ABC is a triangle in which AB = AC. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, find the value of AB :

GP_3386



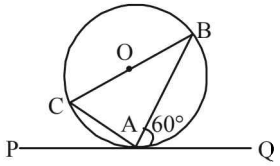
(a) 2 AP (b) 3 AP
(c) 4 AP (d) None of these

38. In the given diagram ABCD is a cyclic quadrilateral $\angle OCB = 50°$ and $\angle BOC = 110°$. Find the value of $\angle DAO$:



(a) 20° (b) 30°
(c) 50° (d) can't be determined

39. In the given figure, PAQ is the tangent. BC is the diameter of the circle. m $\angle BAQ = 60°$, find m $\angle ABC$:



(a) 25° (b) 30° (c) 45° (d) 60°

40. Two circles touch each other internally. Their radii are 2 cm and 3 cm. The biggest chord of the other circle which is outside the inner circle, is of length :

(a) $2\sqrt{2}$ cm (b) $3\sqrt{2}$ cm
(c) $2\sqrt{3}$ cm (d) $4\sqrt{2}$ cm

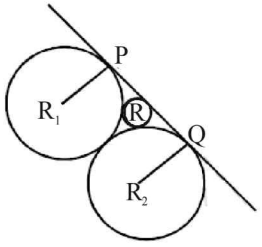
41. Three equal circles of unit radius touch each other. Then, the area of the circle circumscribing the three circles is :

(a) $6\pi(2+\sqrt{3})^2$ (b) $\frac{\pi}{6}(2+\sqrt{3})^2$
(c) $\frac{\pi}{3}(2+\sqrt{3})^2$ (d) $3\pi(2+\sqrt{3})^2$

42. Two circles with centres P and R touch each other externally at O. A line passing through O cuts the circles at T and S respectively. Then

(a) PT and RS are of equal length
(b) PT and RS are perpendicular to each other
(c) PT and RS are intersecting
(d) PT and RS are parallel

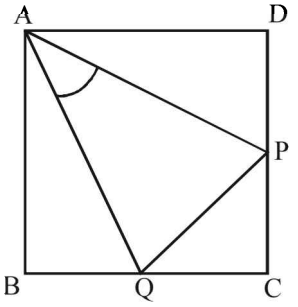
43. Three circles with radii R1, R2 and r touch each other externally as shown in the adjoining figure. If PQ is their common tangent and R1 > R2, then which of the following relations is correct?



(a) $R_1 - R_2 = r$ (b) $R_1 + R_2 = 2r$
(c) $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{r}$ (d) $\frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} = \frac{1}{\sqrt{r}}$

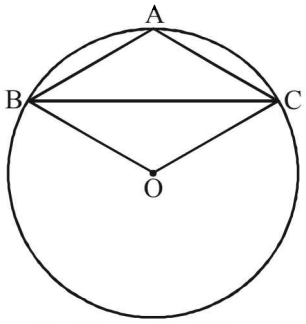
44. ABC is a triangle in which AB = 4 cm, BC = 5 cm and AC = 6 cm. A circle is drawn to touch side BC at P, side AB extended at Q and side AC extended at R. Then, AQ equals
(a) 7.0 cm (b) 7.5 cm (c) 6.5 cm (d) 15.0 cm

45. In the figure, ABCD is a square of side 1 dm and $\angle PAQ = 45°$. The perimeter (in dm) of the triangle PQC is



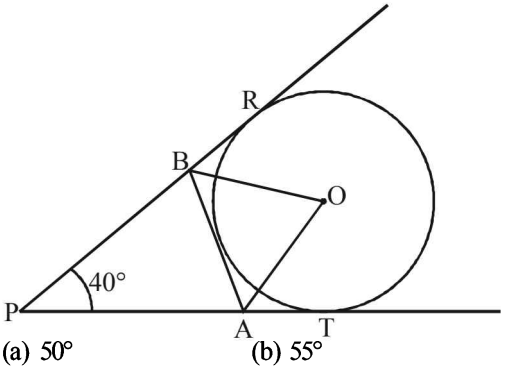
(a) 2 (b) $1+\sqrt{2}$
(c) $2\sqrt{2}-1$ (d) $1+\sqrt{3}$

46. In the figure, BC is a chord of the circle with centre O and A is a point on the minor arc BC. Then, $\angle BAC - \angle OBC$ is equal to



(a) 30° (b) 60° (c) 80° (d) 90°

47. In the figure, ΔAPB is formed by three tangents to the circle with centre O. If ∠APB = 40°, then the measure of ∠BOA is

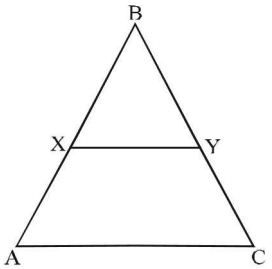


(a) 50° (b) 55°
(c) 60° (d) 70°

48. In ΔABC, D is a point on BC such that 3BD = BC. If each side of the triangle is 12 cm, then AD equals

(a) $4\sqrt{5}$ (b) $4\sqrt{6}$ (c) $4\sqrt{7}$ (d) $4\sqrt{11}$

49. In ΔABC, $\overline{XY}$ is paralled to $\overline{AC}$ and divides the triangle into the two parts of equal area. Then the $\frac{AX}{AB}$ equals



(a) $\frac{\sqrt{2}+1}{2}$ (b) $\frac{2-\sqrt{2}}{2}$

(c) $\frac{2+\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}-1}{2}$

50. In ΔABC, angle B is obtuse. The smallest circle which covers the triangle is the
(a) Circumcircle
(b) Circle with AB as diameter
(c) Circle with BC as diameter
(d) Circle with AC as diameter

51. Which of the following polygons are uniquely determined when all the sides are given?
(a) Quadrilateral (b) Triangle
(c) Pentagon (d) Hexagon

52. If an angle of a regular polygon is 165°, then the number of sides of the polygon is
(a) 30 (b) 24 (c) 18 (d) 15

53. If the angles A, B, C and D of a quadrilateral ABCD in the same order are in the ratio 3 : 7 : 6 : 4, then ABCD is a
(a) parallelogram (b) rhombus
(c) trapezium (d) kite

54. In a ΔABC, AB = 4 cm and AC = 8 cm. If M is the mid point of BC and AM = 3 cm, then length of BC, in cm, is

(a) $2\sqrt{26}$ (b) $2\sqrt{31}$ (c) $\sqrt{31}$ (d) $\sqrt{26}$

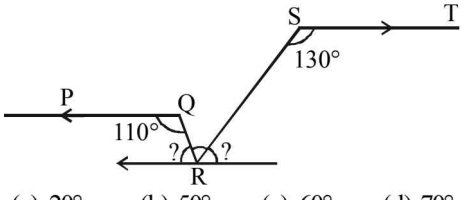
55. If x denotes an angle between any two lines of symmetry of a regular hexagon, then the minimum value of x is
(a) 30° (b) 45° (c) 60° (d) 90°

56. In a quadrilateral ABCD, if AB || CD, ∠D = 2, ∠B, AD = b and CD = a, then the side AB is of length

(a) $\frac{a}{2}+2b$ (b) a + 2b (c) 2a – b (d) a + b

57. ΔABC is a triangle with AB = AC and BC = 9 cm. If the height from A to BC is 6 cm, then the height from C to AB, in cm, is
(a) 6.0 (b) 7.2 (c) 7.5 (d) 8.0

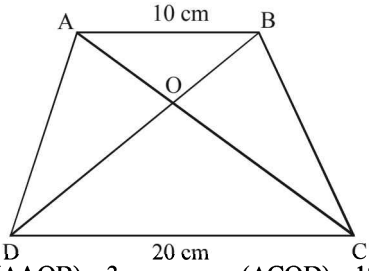
58. In a given figure PQ || ST, ∠PQR = 110°, ∠RST = 130°, then value of ∠QRS is-



(a) 20° (b) 50° (c) 60° (d) 70°

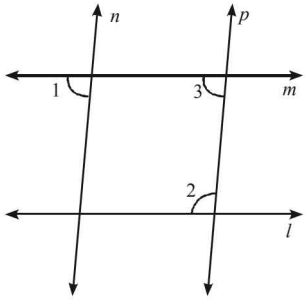
59. Let ABCD be a cyclic quadrilateral inscribed in a circle of radius 1 unit. If AB.BC.CD.DA ≥ 4, then the quadrilateral ABCD is a _____.
(a) Parallelogram (b) Square
(c) Rectangle (d) Rhombus

60. ABCD is a trapezium, AB || DC. Diagonals of trapezium intersect to each other at point O:



or (ΔAOB) = 3 sq. cm or (ΔCOD) = 12 sq. cm
or (ABCD) = ...
(a) 27 sq. cm (b) 45 sq. cm
(c) 36 sq. cm (d) 18 sq. cm

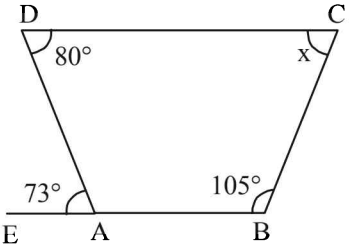
61. In figure if $l \parallel m$, $n \parallel p$, and $\angle 1 = 85°$, then $\angle 2$ is.



(a) 85° (b) 95° (c) 180° (d) 75°

62. The angles of a quadrilaterals are in the ratio 3 : 5 : 9 : 13. All the angles of the quadrilaterals are.
(a) 36°, 60°, 108°, 156°
(b) 30°, 50°, 90°, 130°
(c) 39°, 65°, 117°, 169°
(d) 30°, 65°, 100°, 150°

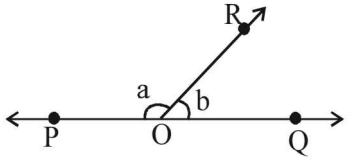
63. Use the information given in figure to choose the correct value of x.



(a) $x = 86°$ (b) $x = 60°$ (c) $x = 68°$ (d) $x = 107°$

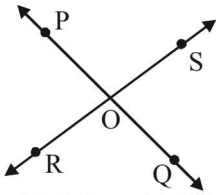
64. A park, in the shape of a quadrilateral $ABCD$, has $\angle C = 90°$, $AB = 9$ m, $BC = 12$ m, $CD = 5$ m, and $AD = 8$ m.
How much area does it occupy?
(a) $30\,m^2$ (b) $13\,m^2$ (c) $169\,m^2$ (d) $12\,m^2$

65. In figure, ∠POR and ∠QOR form a linear pair. If a – b = 80°, then the values of a and b are



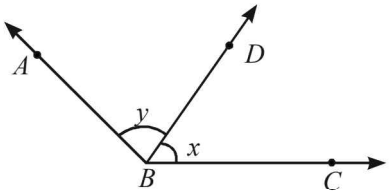
(a) a = 50°, b = 130° (b) a = 130°, b = 50°
(c) a = 260°, b = 60° (d) a = 60°, b = 260°

66. In figure, $\angle POR : \angle ROQ = 5 : 7$. The value of $\angle SOQ$ is



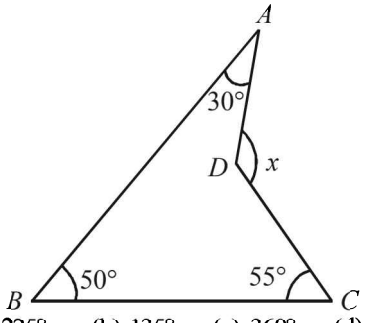
(a) 60° (b) 65° (c) 75° (d) 90°

67. For what value of $x + y$ in the given figure, will ABC be a line?



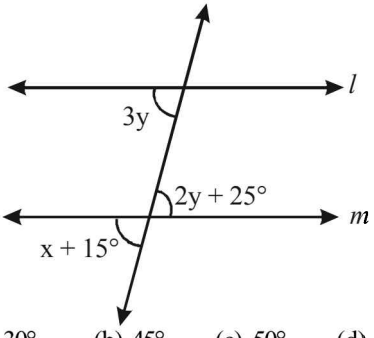
(a) 180° (b) 90° (c) 45° (d) 360°

68. Value of $\angle ADC$ from the given figure.



(a) 225° (b) 135° (c) 360° (d) 50°

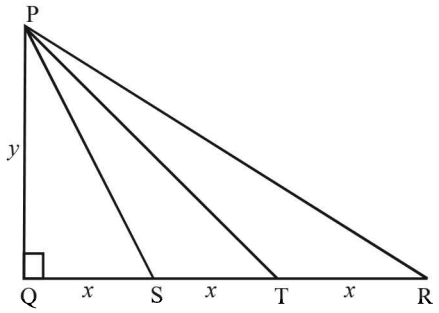
69. In figure, if $l \parallel m$, then the value of x is



(a) 30° (b) 45° (c) 50° (d) 60°

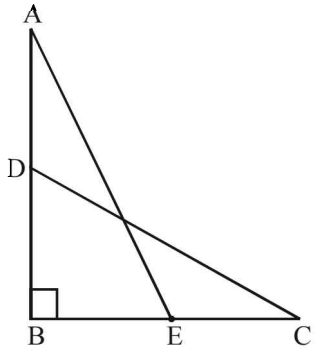
70. In ΔABC, if $\angle A = 50°$ and $\angle B = 60°$, then the shortest and largest sides of the triangle are
(a) AC and AB (b) BC and AB
(c) AC and BC (d) BC and AC

71. In the given figure, S and T trisect the side QR of a right triangle PQR. Then which of the following is correct?



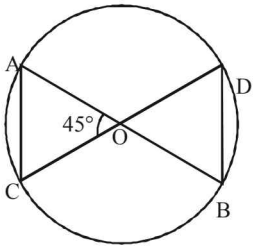
(a) $8PT^2 = 3PR^2 + 5PS^2$
(b) $8PR^2 = 8PT^2 + 8PS^2$
(c) $8PT^2 - 4PR^2 = 6PS^2$
(d) $8PT^2 = 7RP^2 - 6PS^2$

72. In the figure given below, $\angle B = 90°$, $AE = DC = 13$ cm, $BE = 5$ cm and $AD = EC = x$ cm. Find the value of x.



(a) 5 (b) 7 (c) 2 (d) 3

73. If in fig. O is the point of intersection of two chords AB and CD such that OB = OD, then triangles OAC and ODB are



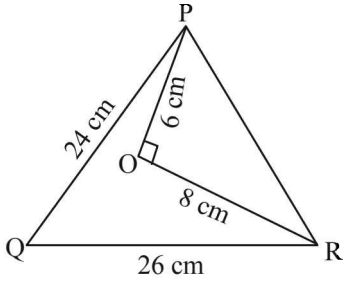
(a) equilateral but not similar
(b) isosceles but not similar
(c) equilateral and similar
(d) isosceles and similar

74. ABC be a rt. triangle with rt. angle at A. If AD ⊥ BC, then AD in terms of 'b' and 'c' provided AC = b and AB = c is

(a) $\frac{b}{\sqrt{b^2c}}$ (b) $\frac{bc}{\sqrt{b^2c+c}}$

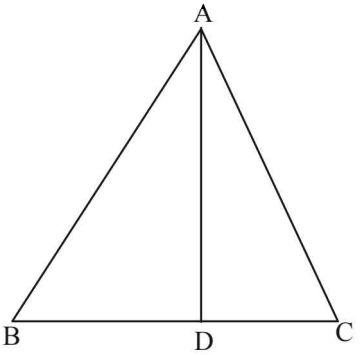
(c) $\frac{bc}{\sqrt{b^2+c^2}}$ (d) $\frac{c}{\sqrt{b^2+1}}$

75. In figure given below , O is a point inside ΔPQR such that $\angle POR = 90°$, OP = 6 cm and OR = 8 cm. If PQ = 24 cm, QR = 26 cm. Then



(a) $\angle QRP = 90°$
(b) $\angle PRQ = 90°$
(c) $\angle QPR = 90°$
(d) ΔPQR is an isosceles

76. In Δ ABC, $\frac{AB}{AC} = \frac{BD}{DC}$, $\angle B = 70°$ and $\angle C = 50°$. Then $\angle BAD = ?$

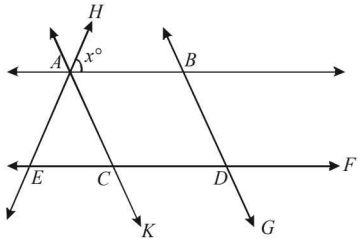


(a) 30° (b) 40° (c) 50° (d) 45°

GP_3386

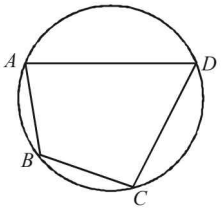
LEVEL - 2

1. In the given figure $AB \parallel CD$ and $AC \parallel BD$. If $\angle EAC = 40°$, $\angle FDG = 55°$, $\angle HAB = x°$, then find the value of x.



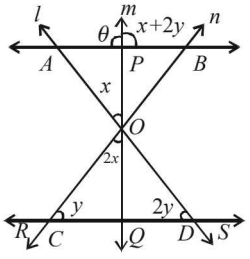
(a) 85° (b) 75° (c) 65° (d) 55°

2. In the figure given, $ABCD$ is a cyclic quadrilateral and $AB = 25$cm, $BC = 39$ cm, $CD = 52$ cm and $AD = 60$ cm. What is the diameter of the circle?



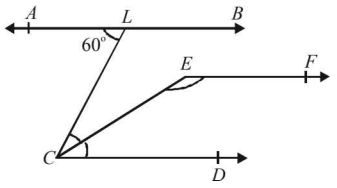
(a) 60 cm (b) 65 cm
(c) 72 cm (d) 78 cm

3. Give that segment AB and CD are parallel. If lines ℓ, m and n intersect at point O. Find the ratio of θ to $\angle ODS$



(a) 2 : 3 (b) 3 : 2
(c) 3 : 4 (d) Data insufficient

4. In the given figure, $AB \parallel CD$, $\angle ALC = 60°$, EC is the bisector of $\angle LCD$ and $EF \parallel AB$. Then, find the measure of $\angle CEF$.

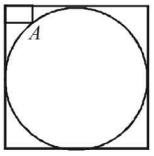


(a) 80° (b) 130° (c) 120° (d) 150°

5. $ABCD$ is a paralellogram in which $\angle B = 70°$. Find the number of points X in the plane of the parallelogram such that it is equidistant from its vertices.

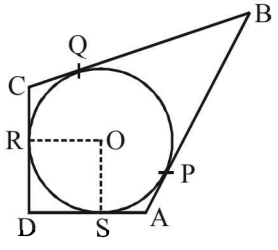
(a) zero (b) one
(c) two (d) three

6. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle . What is the radius of the circle in cm?



(a) 10 cm (b) 40 cm
(c) 50 cm (d) None of these

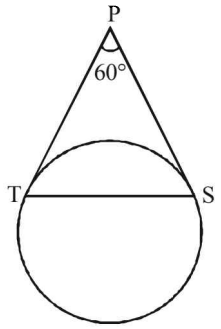
7. In the given figure, $\angle ADC = 90°$, BC = 38 cm, CD = 28 cm and BP = 25 cm, then find the radius of the circle.



(a) 15 cm (b) 10 cm
(c) 8 cm (d) 9 cm

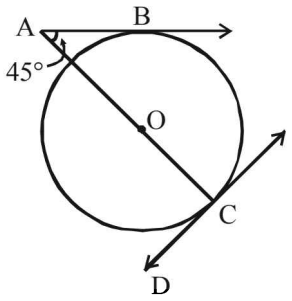
8. In the given figure, PT and PS are tangents to a circle from a point P such that PT = 5 cm and

$\angle$ TPS = 60°, then find the length of chord TS.



(a) 5 cm (b) 7 cm (c) 4 cm (d) 6 cm

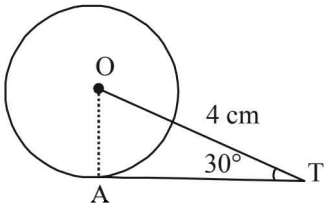
9. The diagram shows a circle with centre O. Line AB is tangent to the circle at point B and line DC is tangent to the circle at point C.
If the radius of the circle is 2 cm, what is the measure of AC ?



(a) $\left(2\sqrt{2}+2\right)$cm (b) 1.414 cm

(c) $3\sqrt{2}$cm (d) 2 cm

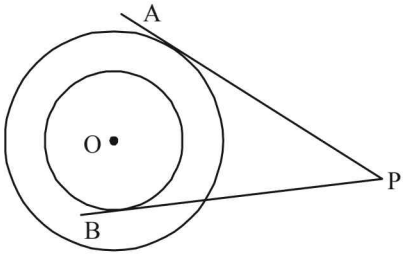
10. In the given figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30°$. Find the length of AT.



(a) 2 cm (b) $2\sqrt{3}$ cm

(c) $3\sqrt{3}$ cm (d) 4 cm

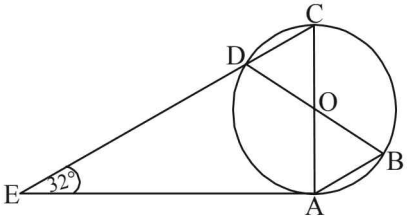
11. In the given figure, two concentric circles with centre O are of radii 5cm and 3cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, then find BP.



(a) $4\sqrt{10}$ cm (b) 4 cm

(c) 5 cm (d) 3 cm

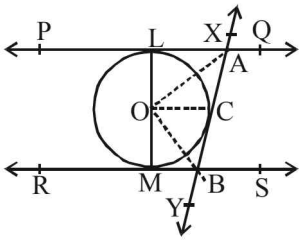
12. In the diagram, O is the centre of the circle ABCD. The straight lines AC and BD intersect at O. The tangent at A meets CD produced at E and $\angle$ AED = 32° $\angle$ABD and $\angle$AOD



(a) 60°, 120° (b) 58°, 116°

(c) 114°, 72° (d) 50°, 108°

13. PQ and RS are two parallel tangents to a circle with centre O and another tangent XY with point of contact C intersect PQ at A and RS at B. Find $\angle$AOB.



(a) 30° (b) 60°

(c) 90° (d) 120°

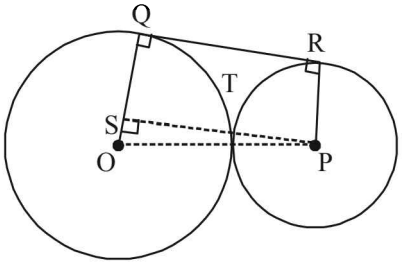
14. The radius of the incircle of a triangle is 4 cm and the secant into which the side is divided by the point of contact are 6 cm, 8 cm. Find the other two sides of the triangle.

(a) 15 cm, 13 cm (b) 20 cm, 10 cm
(c) 18 cm, 11 cm (d) 10 cm, 15 cm

15. The tangents drawn at the end points of two perpendicular diameters of a circle are parallel to each other, which form a square and whose length of side is 2 cm. The radius of the circle is

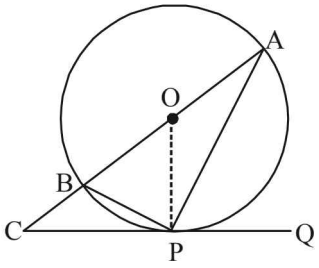
(a) 2 cm (b) 4 cm
(c) 0.35 cm (d) 1 cm

16. Two circles with centres O and P, and radii 8 cm and 4 cm touch each other externally. Find the length of their common tangent QR.



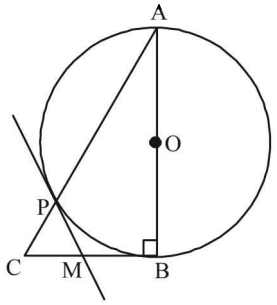
(a) 8 cm (b) 7 cm
(c) $8\sqrt{2}$ cm (d) $7\sqrt{3}$ cm

17. A tangent CQ touches a circle with centre O at P. Diameter AB is produced to meet the tangent at C. If $\angle ACP = a°$ and $\angle BPC = b°$, find the relation connecting a and b.



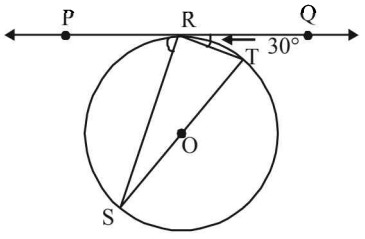
(a) $a° + b° = 180°$ (b) $a° + 2b° = 90°$
(c) $a° - b° = 60°$ (d) $2a° + b° = 100°$

18. In ΔABC, $\angle B = 90°$. If a circle drawn with AB as diameter intersects the hypotenuse AC at P, which of the following is true?



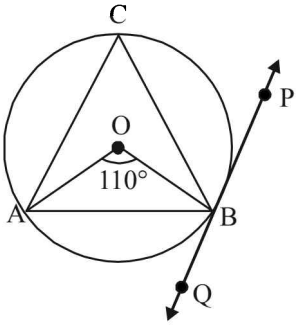
(a) The tangent drawn to the circle at P bisects the side BC.
(b) The tangent drawn to the circle at A bisects the side AB.
(c) The tangent drawn to the circle at B bisects the side AC.
(d) The tangent drawn to the circle at Cbisects the side BC.

19. In the figure, PQ is tangent at a point R of the circle with centre O. If $\angle TRQ = 30°$, find $\angle PRS$.



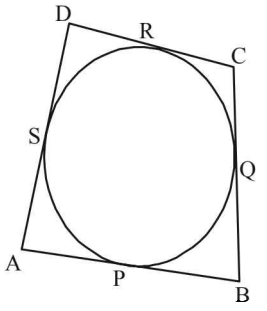
(a) 50° (b) 90° (c) 110° (d) 60°

20. In figure, AB is a chord of circle, and PQ is a tangent at point B of the circle. If $\angle AOB = 110°$, then $\angle ABQ$ is



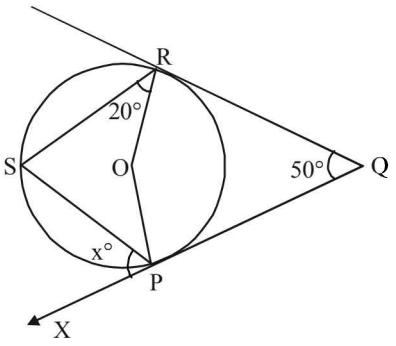
(a) 45° (b) 70° (c) 55° (d) 35°

21. In the given fig., quadrilateral ABCD is circumscribed touching the circle at P, Q, R and S. If AP = 6 cm, BP = 5 cm, CQ = 3 cm and DR = 4 cm, then perimeter of quadrilateral ABCD is



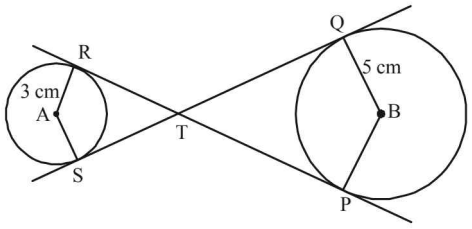
(a) 18 cm (b) 27 cm (c) 36 cm (d) 22 cm

22. In the diagram, PQ and QR are tangents to the circle centre O, at P and R respectively. Find the value of x.



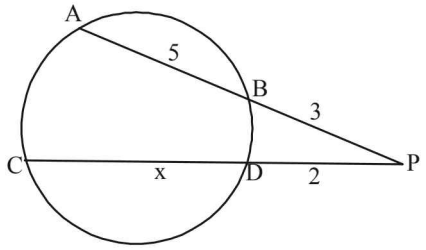
(a) 25 (b) 35 (c) 45 (d) 55

23. In the fig., RTP and STQ are common tangents to the two circles with centres A and B. The radii of the two circles are 3 cm and 5 cm respectively. If ST : TQ = 1 : 3 and RT = 4 cm. Find the length of QT and AB.



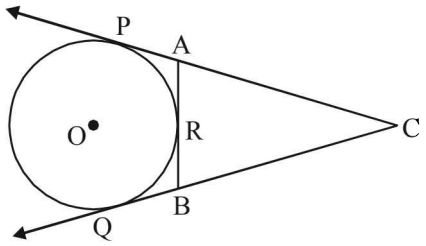
(a) QT = 10 cm, AB = 15 cm
(b) QT = 15 cm, AB = 12 cm
(c) QT = 18 cm, AB = 18 cm
(d) QT = 12 cm, AB = 18 cm

24. Two chords AB and CD of a circle intersect each other at P outside the circle. If AB = 5 cm, BP = 3 cm and PD = 2 cm, find CD.



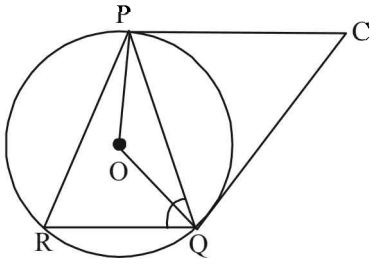
(a) 4 cm (b) 5 cm
(c) 8 cm (d) 10 cm

25. In the given figure, CP and CQ are tangents to a circle wih centre O. ARB is another tangent touching the cirlce at R. If CP = 11 cm and BC = 7 cm, then find the length of BR.



(a) 4 cm (b) 10 cm (c) 3.5 cm (d) 8 cm

26. In the given figure, PQ = QR, ∠RQP = 68°, PC and CQ are tangents to the circle with centre O. Calculate the values of ∠QOP and ∠QCP



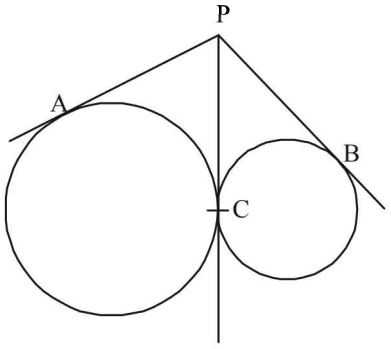
(a) ∠QOP = 112°, ∠QCP = 68°
(b) ∠QOP = 68°, ∠QCP = 112°
(c) ∠QOP = 100°, ∠QCP = 40°
(d) ∠QOP = 80°, ∠QCP = 60°

27. With the vertices ΔABC as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 9 cm. 7 cm. and 6 cm, find the radii of the circle.

(a) 3 cm, 6 cm, 9 cm (b) 4 cm, 5 cm, 2 cm

(c) 2 cm, 3 cm 11 cm (d) 9 cm, 7 cm, $\sqrt{3}$ cm

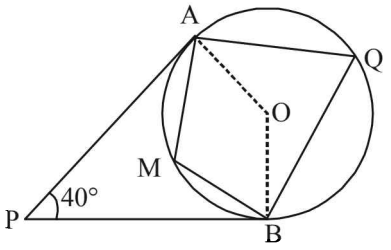
28. Two circles touch each other externally at a point C and P is a point on the common tangent at C. If PA and PB tangents to the two circles, then



(a) PA = 2 PB (b) PC = $\frac{1}{2}$PB

(c) PB = $\frac{1}{2}$PA (d) PA = PB

29. In the given figure, PA and PB are two tangents to the circle with centre O. If ∠APB = 40°, find ∠AQB and ∠AMB.



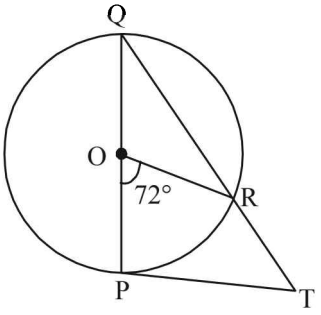
(a) ∠AQB = 70°, ∠AMB = 110°

(b) ∠AQB = 110°, ∠AMB = 70°

(c) ∠AQB = 100°, ∠AMB = 50°

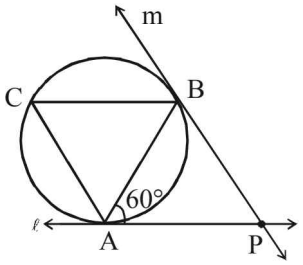
(d) ∠AQB = 60°∠AMB = 40°

30. In the given figure PQ is a diameter of a circle with cente O and PT is a tangent at P. QT meets the circle at R. If ∠POR = 72°, find ∠PTR.



(a) 50° (b) 54° (c) 27° (d) 108°

31. In the diagram below , if ℓ and m are two tangents and AB is a chord making an angle of 60° with the tangent ℓ, then the angle between ℓ and m is

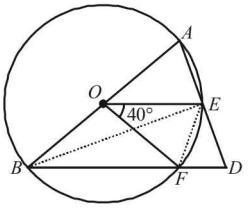


(a) 45° (b) 30° (c) 60° (d) 90°

32. 'O' is a centre of a circle, AC is a tangent to a circle at point A. If ΔOAC is an isosceles triangle, then find the measure of ∠OCA.

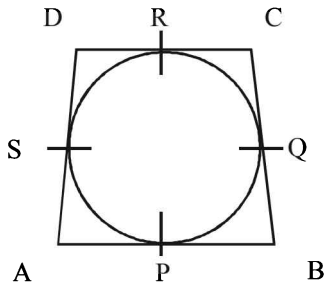
(a) 30° (b) 60° (c) 90° (d) 45°

33. In the given figure, *O* is the centre of a circle and *AB* is a diameter. If ∠*EOF* = 40°, then ∠*EDF* is



(a) 70° (b) 45° (c) 90° (d) 180°

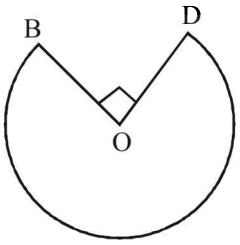
34. In figure, a circle touches all the four sides of a quadrilateral ABCD, whose sides AB = 6 cm, BC = 7 cm, and CD = 4 cm Length of AD is



(a) 10 cm (b) 7 cm (c) 3 cm (d) 4 cm

35. The shape of the top of a table in a restaurant is that of a segment of a circle with centre O and $\angle BOD = 90°$. If $BO = OD = 60$ cm. The perimeter of the table is

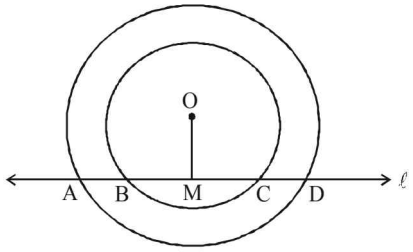
(Take $\pi = 3.14$)



(a) 402 cm (b) 402.6 cm

(c) 401 cm (d) 400 cm

36. Two concentric circles with centre O have A, B, C, D as the points of intersection with the line ℓ as shown in the figure. If AD = 12 cm and BC = 8 cm, the length of AB, CD, AC and BD are respectively.



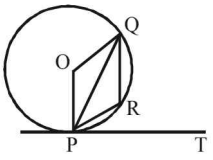
(a) 2 cm, 2 cm, 10 cm, 10 cm
(b) 10 cm, 10 cm, 2 cm, 2 cm
(c) 10 cm, 2 cm, 10 cm, 2 cm
(d) 2 cm, 10 cm, 2 cm, 10 cm

37. AD is a diameter of a circle. Two more circles pass through A and intersect AD in B and C respectively, such that AB and AC are diameters of these circles and AD > AC > AB. If the circumference of the middle circle is average of the circumference of the other two, then given AB = 4 units and CD = 2 units, what is the area, in sq. units of the largest circle?

(a) 128π (b) 64π

(c) 48π (d) 16π

38. In the figure, PQ is a chord of a circle with centre O and PT is the tangent at P such that $\angle QPT = 70°$. Then the measure of $\angle PRQ$ is equal to

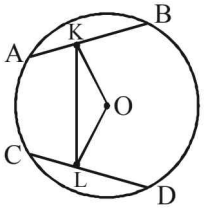


(a) 135° (b) 150° (c) 120° (d) 110°

39. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distance between them is 17 cm, the radius of the circle is

(a) 14 cm (b) 10 cm (c) 13 cm (d) 15 cm

40. In the given circle with centre 'O', the mid points of two equal chords AB & CD are K & L respectively. If $\angle OLK = 25°$, Then $\angle LKB = ?$



(a) 125° (b) 115° (c) 105° (d) 90°

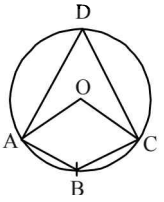
41. The length 'L' of a tangent, drawn from a point 'A' to a circle is $\frac{4}{3}$ of the radius 'r'. The shortest distance from A to the circle is :

(a) $\frac{1}{2}r$ (b) r (c) $\frac{1}{2}L$ (d) $\frac{2}{3}L$

GP_3386

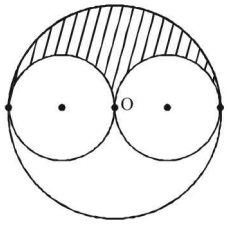
42. If two circles are such that one is not contained in the other and are non-intersecting, then number of common tangents are :
(a) One (b) Two (c) Three (d) Four

43. In the adjacent figure, if $\angle AOC = 110°$, then the value of $\angle D$ and $\angle B$ respectively



(a) 55°, 125° (b) 55°, 110°
(c) 110°, 25° (d) 125°, 55°

44. In the figure given below, radius of a greater circle is r cm. Find the area of non-shaded portion :

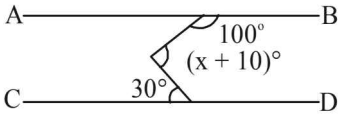


(a) $\frac{\pi r^2}{4}$ sq. cm (b) $\frac{3\pi r^2}{4}$ sq. cm

(c) $2\pi r^2$ sq. cm (d) $\frac{4\pi r^2}{3}$ sq. cm

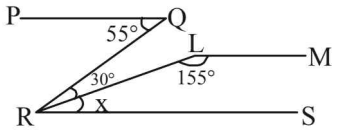
45. An equilateral triangle *BPC* is drawn inside a square *ABCD*. What is the value of the angle *APD* in degrees?
(a) 75° (b) 90° (c) 120° (d) 150°

46. AB || CD, shown in the figure. Find the value of x :



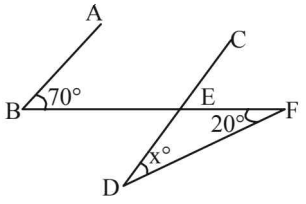
(a) 100° (b) 90° (c) 110° (d) 140°

47. In the figure PQ || LM || RS. Find the value of $\angle LRS$:



(a) 30° (b) 25° (c) 35° (d) 40°

48. In the figure AB || CD, find x° (i.e., $\angle CDF$) :

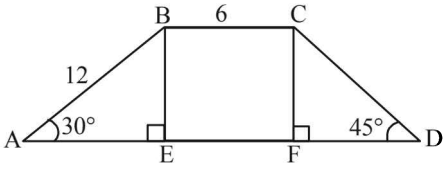


(a) 50° (b) 90° (c) 30° (d) 70°

49. ABCD is a parallelogram. m $\angle DAB = 30°$, BC = 20 cm and AB = 20 cm. Find the area of parallelogram :
(a) 150 cm^2 (b) 200 cm^2
(c) 400 cm^2 (d) 260 cm^2

50. If a rectangle and a parallelogram are equal in area and have the same base and are situated on the same side, then the ratio of perimeter of rectangle and that of parallelogram is k, then k is :
(a) $k > 1$ (b) $k < 1$
(c) $k = 1$ (d) Can't be determined

51. In the trapezium ABCD, $\angle BAE = 30°$, $\angle CDF = 45°$, BC = 6 cm and AB = 12 cm. Find the area of trapezuim :



(a) $18(3+\sqrt{3})\text{cm}^2$ (b) $36\sqrt{3}\text{cm}^2$

(c) $12(3+2\sqrt{3})\text{cm}^2$ (d) None of these

52. P is point in the interor of an equilateral triangle with side a units. If P_1, P_2 and P_3 are the distance of P form the three sides of the triangle, the $P_1 + P_2 - P_3$

(a) equals $\frac{2a}{3}$ units

(b) equals $\frac{a\sqrt{3}}{2}$ units

(c) is more than a unit

(d) cannot be determined unless the location of P is specified

53. If P is a point inside the scalene triangle ABC such that ΔAPB, ΔBPC and ΔCPA have the same area then P must be

(a) in centre of ΔABC

(b) circumcentre of ΔABC

(c) centroid of ΔABC

(d) orthocentre of ΔABC

54. If the line segment joining the midpoint of the consecutive side of quadrilateral ABCD form a recatangle then $\square ABCD$ must be

(a) rhombus (b) square

(c) kite (d) all of these

55. A has a pair of triangles with corresponding sides proportional, and B has a pair of pentagons with corresponding sides proportional.

$S_1 \equiv$ A's triangles must be similar

$S_2 \equiv$ B's pentagons must be similar

Which of the following statement is correct?

(a) S_1 is true, but S_2 is not true.

(b) S_2 is true, but S_1 is not true.

(c) Both S_1 and S_2 are true.

(d) Neither S_1 nor S_2 is true

56. ΔABC is an equilateral triangle of side $2\sqrt{3}$ cm. P is any point in the interior of ΔABC. If x, y, z are the distances of P from the sides of the triangle, then $x + y + z =$

(a) $2 + \sqrt{3}$ cm. (b) 5 cm.

(c) 3 cm. (d) 4 cm.

57. ABCD is a quadrilateral whose diagonals intersect each other at the point O such that OA = OB = OD. If $\angle OAB = 30°$, then the measures of $\angle ODA$ is

(a) 30° (b) 45° (c) 60° (d) 90°

58. In a triangle ABC, AB = AC. Points D and E are on the sides BC and AC respectively such that AD = AE. If $\angle BAD = 30°$, then the measure of $\angle EDC$ is

(a) 10° (b) 15° (c) 20° (d) 25°

59. E is the midpoint of diagonal BD of a parallelogram ABCD. If the point E is joined to a point F on DA such that $DF = \frac{1}{3}$ DA, then the ratio of the area of ΔDFE to the area of quadrilateral ABEF is

(a) 1 : 3 (b) 1 : 4 (c) 1 : 5 (d) 2 : 5

60. If the line segments joining the points (a, b) and (c, d) subtends a right angle at the origin, then which of the equation is correct?

(a) $ac - bd = 0$ (b) $ac + bd = 0$

(c) $ab + cd = 0$ (d) $ab - cd = 0$

61. If in an isosceles triangle 'a' is the length of the base and 'b' is the length of one of the equal side, then its area is equal to

(a) $a^2\sqrt{b^2 - 4b^2}$ (b) $\frac{a^2}{4}\left(\sqrt{4b} - a^2\right)$

(c) $\frac{a}{4}\sqrt{4b^2 - a^2}$ (d) $\frac{1}{4}\sqrt{a^2 + b^2}$

GP_3386

Hints and Solutions

Level 1

1. (a) $\angle EDC = \angle BAD = 45°$ (alternate angles)
$\therefore\ x = DEC = 180° - (50° + 45°) = 85°$.

2. (c) Let n be the number of sides of the polygon
Now, sum of interior angles = 8 × sum of exterior angles
i.e. $(2n-4)\times\frac{\pi}{2} = 8\times 2\pi$
or $(2n-4) = 32^2 \Rightarrow n = 18$

3. (b) $\angle BOE' = \angle ABO = 40°$ (alternate angles)
$\angle EOD = \angle CDO = 30°$ (alternate angles)
$\therefore\ \angle BOD = (40° + 30°) = 70°$. So, $x = 70$.

4. (b) At 11 : 15, the minute hand is at 3 and hour hand slightly advanced from 11.
Angle through which hour hand shifts in
15 minute $= \left(15\times\frac{1}{2}\right)^\circ = 7.5^\circ$
$\therefore$ Required angle $= (30\times 4)^\circ - 7.5^\circ = 112.5^\circ$

5. (b) Go through option for quicker answer
Exterior angle $= \frac{360}{15} = 24°$ (for $n = 15$)
$\therefore$ Interior angle $= 180° - 24° = 156°$
$\therefore$ Interior – Exterior $= 156 - 24 = 132°$

6. (b) The sum of the interior angles of a polygon of n sides is given by the expression $(2n-4)$
$\frac{\pi}{2} \Rightarrow (2n-4)\times\frac{\pi}{2} = 1620\times\frac{\pi}{180}$
$(2n-4) = \frac{1620\times 2}{180} = 18$
or $2n = 22 \Rightarrow n = 11$

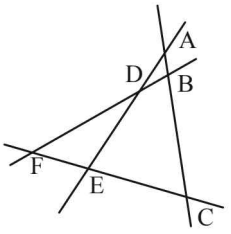
7. (b) Using the quality of similar triangles,
$\frac{CP}{PB} = \frac{CD}{AB} = \frac{1}{3}$;
In ΔBPQ and BCD,
$\frac{CD}{PQ} = \frac{BC}{BP} = \frac{4}{3} = 1:0.75$

8. (b) $9\times 180 - 2\times 360$
$= 180\times 5 = 900°$
$\because \left\{\begin{matrix} n\times 180 - 2\times 360 \\ = 180(n-4) \end{matrix}\right\}$

9. (b) $2x + 2y = 180° \Rightarrow x + y = 90°\ \therefore\ \angle COD = 90°$

10. (c) $\frac{AG}{BH} = \frac{CE}{DF} \Rightarrow \frac{120}{100} = \frac{30}{DF} \Rightarrow DF = 25$ cm

11. (b) ${}^4C_2 = \frac{4!}{2!2!} = \frac{4.3.2.1}{(2.1).(2.1)} = 6$

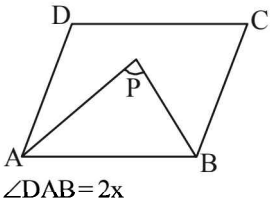


12. (b) $\angle AKH = \angle KHD = 25°$
$\angle EGH = \angle AKL = 180° - 60° = 120°$
$\therefore\ \angle HKL = 120° + 25° = 145°$

13. (b) $x + 2x + 3x + 4x = 360 \Rightarrow x = 36$
$\therefore$ The angles of quadrilateral (in order) are $36°, 72°, 108°, 144°$
Since two pairs of adjacent angles are supplementary, therefore AB || CD. Hence it is a trapezium.

14. (b) $3x + 4x + 5x + 6x = 360°$
$\Rightarrow x = 20\ (\therefore\ 6x = 120°)$
Hence, the largest angle of a quadrilateral is 120°.

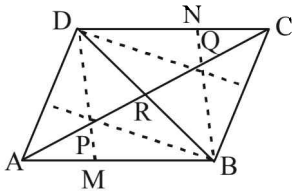
15. (a)



$\angle DAB = 2x$
$\angle ABC = 2y$
$2x + 2y = 180$ or $x + y = 90°$
i.e., $\angle PAB + \angle PBA = 90°$
$(\therefore\ \angle APB = 90°)$

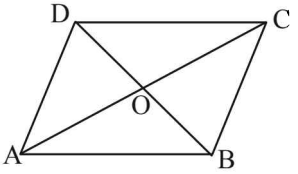
16. (a) Prove that opposite sides are parallel, and diagonals are equal. Also all the interior anlges are right angle.

17. (b) Let AC and BD bisects each other at R, then AP = 2PR and QC = 2RQ



(∵ AR and CR are the medians of ΔADB and ΔBDC)
and AP = QC and PR = RQ
(∵ ΔDAB ≅ ΔBCD)
Hence, AP = PQ

18. (c) Let BD = 6 cm and AC = 8 cm
∴ AO = 4 cm and BO = 3 cm



let AB = 5 cm (∴ ∠AOB = 90°)
⇒ ∠BOC = ∠AOD = ∠DOC = 90°
∴ BC is also 5 cm.
Hence, ABCD is a rhombus

∴ Area of rhombus ABCD $= \frac{AC \times BD}{2}$

$= \frac{6 \times 8}{2} = 24 \text{ cm}^2$

19. (d) Each interior angle = 140°
∴ Exterior angle = 180° – 140° = 40°
∴ Number of sides

$= \frac{360°}{\text{Exterior angle}} = \frac{360°}{40°} = 9$

20. (d) Sum of all the interior angles = (n – 2) 180°
1260° = (n – 2) × 180°
⇒ n = 9 (n → number of sides)

21. (c) $\frac{n(n-3)}{2} = 54 \Rightarrow n^2 - 3n = 108 \Rightarrow n = 12$

22. (c) Sum of all exterior angles = 360°
∴ Sum of interior angles = 4 × 360° = 1440°
∴ 1440° = (n – 2) 180°
⇒ n = 10 (no. of sides)

23. (c) Exterior angle $= \frac{360°}{9} = 40°$

Interior angle = 180° – 40° = 140°
∴ Interior angle : exterior angle = 140° : 40 = 7 : 2

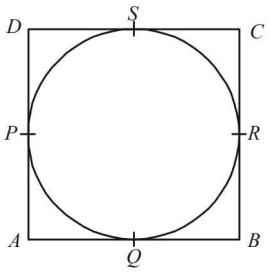
24. (d) Interior angle = 3 × exterior angle

$180° - \frac{360°}{n} = 3 \times \left(\frac{360}{n}\right) \Rightarrow n = 8$

25. (c) $b = \frac{1}{2}(48°)$

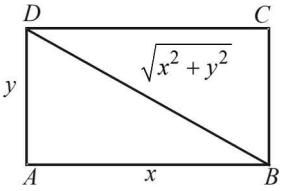
(∠at centre = 2 at circumference on same PQ) 24°
$\angle AQB = 90°$ (∠In semi-circle)
$\angle QXB = 180° - 90° - 24°$ (∠sum of Δ) = 66°

26. (b) Since $ABCD$ is a quadrilateral
Again AP, AQ are tangents to the circle from the point A.



∴ $AP = AQ$
Similarly $BR = BQ$
$CR = CS$
$DP = DS$
∴ $(AP + DP) + (BR + CR) = AQ + DS + BQ + CS$
$= (AQ + BQ) + (CS + DS)$
⇒ $AD + BC = AB + CD$

27. (d)



According to question,

$(x + y) - \sqrt{x^2 + y^2} = \frac{x}{2}$

$(x + y) - \frac{x}{2} = \sqrt{x^2 + y^2}$

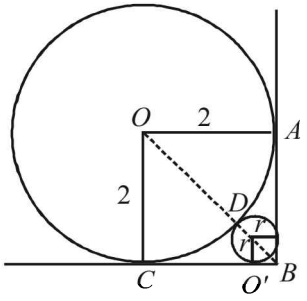
$\left(\frac{x}{2} + y\right)^2 = x^2 + y^2$

$$\frac{x^2}{4} + y^2 + xy = x^2 + y^2$$

$$x^2 + 4xy = 4x^2$$

$$4xy = 3x^2 \Rightarrow 4y = 3x \Rightarrow \frac{y}{x} = \frac{3}{4}$$

28. (d)



$OABC$ is square with side = 2

$$\therefore OB = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$OB = 2\sqrt{2} = OD + r + O'B = 2 + r + r\sqrt{2}$$

$$\Rightarrow r(\sqrt{2}+1) = 2(\sqrt{2}-1)$$

$$\Rightarrow r = \frac{2(\sqrt{2}-1)}{(\sqrt{2}+1)} = \frac{2(\sqrt{2}-1)^2}{2-1} = 2(2+1-2\sqrt{2})$$

$$= 6 - 4\sqrt{2}$$

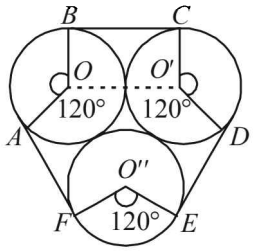
29. (a) $\angle AOB = \angle CO'D = \angle FO''E = 120°$

Distance between 2 centres = 2 m

$\therefore$ $BC = DE = FA = 2\,m$

Perimeter of the figure = $BC + DE + FA$ + circumference of sectors AOB, $CO'D$ and $FO''E$.

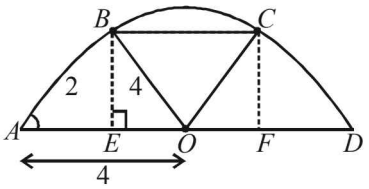
But three equal sectors of 120° = 1 full circle of same radius.



Therefore, perimeter of surface

$= 2\pi r + BC + DE + FA = (2\pi + 6)m$

30. (b)



BO = radius = 4 = AO

$AE = 2\cos A$

$$= 2 \times \left(\frac{2^2 + 4^2 - 4^2}{2 \times 2 \times 4} \right) = \frac{2}{4} = \frac{1}{2}$$

$$\therefore BC = AD - AE - FD = 8 - \frac{1}{2} - \frac{1}{2} = 7$$

$(\because AE = FD)$

31. (d) As for the diagonals X & Y of a parallelogram.

$X^2 + Y^2 = 2(A^2 + B^2)$; where A & B are the sides of the parallelogram.

Sides of a parallelogram are 8 cm and 6 cm.

And ratio of the diagonals is 3 : 4 such that $X = 4k$ and $Y = 3k$

$16k^2 + 9k^2 = 2(8^2 + 6^2)$, so, $K = \sqrt{8}$

Therefore, the required difference in $4k - 3k$

$= k = \sqrt{8}$ cm.

32. (a) From the fig. the shaded area

= (Area of the rectangle – 2 × quarter of circle) + area of semicircle

$$= \left[\left(3 \times 6 - 2 \times \frac{\pi}{4} \times 3^2 \right) + \frac{1}{2}\pi \times 3^2 \right] \text{sq. m}$$

$$= \left[18 - \frac{9\pi}{2} + \frac{9\pi}{2} \right] = 18 \text{ sq. m}$$

$$\therefore \text{Cost of covering with grass} = ₹ \frac{18 \times 70}{100}$$

$$= ₹ \frac{630 \times 2}{100} = ₹ 12.60$$

33. (d) TQ = TP and TP = TR

$\therefore$ TQ = TP = TR

34. (d) $\angle CAB = \angle BCD$

and $\angle DAB = \angle BDC$

$\therefore$ $\angle CAD = \angle CAB + \angle DAB = \angle BCD + \angle BDC$

$\Rightarrow$ $\angle CAD = \angle BCD + \angle BDC$

$\therefore$ $\angle CAD + \angle CBD = \angle BCD + \angle BDC + \angle CBD = 180°$

35. **(c)** $\angle BAD = 180° - 130° = 50°$

$(\because$ ▭ ABCD in cyclic$)$

and $\angle ADB = 90°$ (angle in semicircle)

$\therefore$ $\angle ABD = 180 - (90 + 50) = 40°$

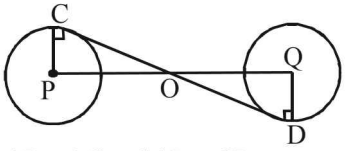
36. **(c)** $AB = PQ = 26$ CM

AND $PO = OQ = 13$CM

$CO = \sqrt{(PO)^2 - (PC)^2}$

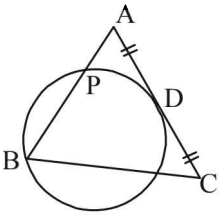
$CO = \sqrt{(13)^2 - (5)^2} = CO = 12$ cm

$\therefore CD = 2\,CO = 24$ cm



37. **(c)** $AB = AC$ and $AD = CD$

$\therefore$ $AB = 2AD$



Now, since AD is a tangent

$\therefore$ $AD^2 = AP \times AB$

$\left(\frac{AB}{2}\right)^2 = AP \times AB$

$\Rightarrow$ $AB = 4\,AP$

38. **(a)** $\angle DOA = 110°$

and $\angle ADB = 50°$

$\therefore$ $\angle DAO = 180° - (110° + 50°) = 20°$

39. **(b)** $\angle BAC = 90°$

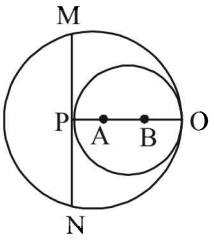
$\angle BCA = 60°$ $(\because \angle BCA = BAQ)$

$\therefore$ $\angle ABC = 180° - (90° + 60°)$

$\angle ABC = 30°$

40. **(d)** Let A be the centre of larger circle and B be the centre of the smaller circle,

$OA = 3$ cm and $BO = 2$ cm



and $AB = (OA - OB) = 1$ cm

also $BP = 2$ cm

$\therefore$ $AP = 1$ cm

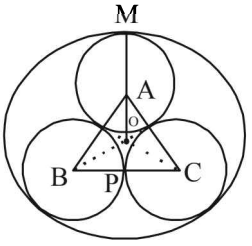
P is the point of contact of tangent MN

$\therefore$ $MP = \sqrt{AM^2 - AP^2} = \sqrt{3^2 - 1^2} = 2\sqrt{2}$ cm

$\therefore$ $MN = 2\,MP = 4\sqrt{2}$ cm

41. **(c)** $AB = BC = AC = 2$ cm

$(\because$ radius of each circle $= 1$ cm$)$



$\therefore$ $AP = \frac{\sqrt{3}}{2} \times 2$

$AP = \sqrt{3}$ cm

Let O be the centroid, then

$OA = \frac{2}{3} \times \sqrt{3} = \frac{2}{\sqrt{3}}$ cm

and $AM = 1$ cm

$\therefore$ $OM = \frac{2}{\sqrt{3}} + 1 = \frac{2+\sqrt{3}}{\sqrt{3}}$ cm

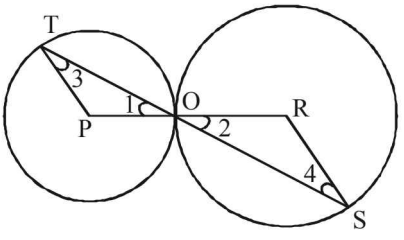
OM is the radius of the larger circle.

$\therefore$ Area of the circumscribing circle $= \pi R^2$

$= \pi \times \left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)^2$

$= \frac{\pi}{3}(2+\sqrt{3})^2$

42. **(d)**



$\angle 1 = \angle 2$ (Vertically Opposite Angles)

PO = PT (Radii of same circle)
So, $\angle 1 = \angle 3$
(Angles opposite to equal sides are equal)
Similarly, $\angle 2 = \angle 4$
$\therefore$ $\angle 3 = \angle 4$
As alternate interior angles are equal, we can say that
PT || RS

43. (d) Here, PQ is the common tangent to the three circles
So,

$$PR = \sqrt{(R_1 + r)^2 - (R_1 - r)^2} = \sqrt{4R_1 r} \quad ...(a)$$

$$RQ = \sqrt{4R_2 r} \quad .....(b)$$

$$PQ = \sqrt{4R_1 R_2} \quad .....(c)$$

We know that,
PQ = PR = RQ

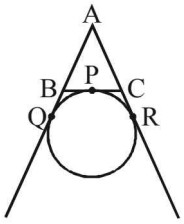
$$\Rightarrow \sqrt{4R_1R_2} = \sqrt{4R_1r} + \sqrt{4R_2r}$$

($\because$ From (a), (b) and (c))

$$\Rightarrow \sqrt{R_1R_2} = \sqrt{R_1r} + \sqrt{R_2r}$$

$$\therefore \frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R_2}} + \frac{1}{\sqrt{R_1}}$$

44. (b)



Perimeter of $\Delta ABC = AB + BC + CA$
$\Rightarrow$ $15 = (AQ - BQ) + (BP + PC) + (AR - CR)$
$\Rightarrow$ $15 = 2AQ$
(BQ = BP, PC = RC, AQ = AR as tangents from external point to a circle are equal)
$\therefore$ AQ = 7.5 cm

45. (a) Let $\angle DAQ = x°$
Now, in ΔADQ,

$$\tan x = \frac{DQ}{AD}$$

$$\Rightarrow \tan x = \frac{DQ}{1} \quad (\because \text{side of square} = 1)$$

$\tan x = DQ$
Now, QC = 1 – DQ
= 1 – tan x
Now, in ΔABP

$$\tan(45 - x) = \frac{BP}{BA} = \frac{BP}{1}$$

$$\frac{\tan 45° - \tan x}{1 + \tan 45° \tan x} = BP$$

$$\Rightarrow \frac{1 - \tan x}{1 + \tan x} = PB \quad (\because \tan 45° = 1)$$

Now, PC = 1 – PB

$$= 1 - \left[\frac{1 - \tan x}{1 + \tan x}\right] = \frac{1 + \tan x - 1 + \tan x}{1 + \tan x}$$

$$\Rightarrow PC = \frac{2 \tan x}{1 + \tan x}$$

Now $PQ^2 = QC^2 + PC^2$

$$= (1 - \tan x)^2 + \frac{(2 \tan x)^2}{(1 + \tan x)^2}$$

$$PQ^2 = \frac{(1 - \tan x^2)^2 + 4 \tan^2 x}{(1 + \tan x)^2} = \frac{(1 + \tan^2 x)^2}{(1 + \tan x)^2}$$

$$PQ = \frac{1 + \tan^2 x}{1 + \tan x}$$

Now required perimeter = PQ + QC + PC

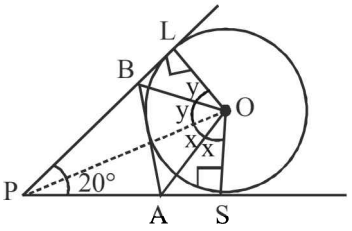
$$= \frac{1 + \tan^2 x}{1 + \tan x} + 1 - \tan x + \frac{2 \tan x}{1 + \tan x}$$

$$= \frac{1 + \tan^2 x + 1 - \tan^2 x + 2 \tan x}{1 + \tan x}$$

$$= \frac{2 + 2 \tan x}{1 + \tan x} = 2$$

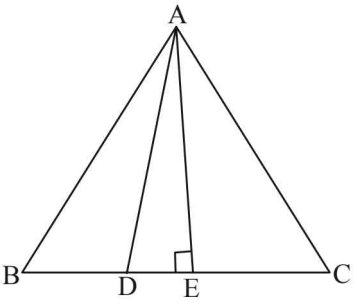
46. (d) Let $\angle BOC = 2x$
then $\angle BAC = 180° - x$
and $\angle OBC = 90° - x$
Now $\angle BAC - \angle OBC = 180° - x - 90° + x$
$= 90°$

47. (d) We redraw the figure, In ΔOPS, $\angle POS = 70°$ and In ΔPOL, $\angle POL = 70°$



From figure $2x + 2y = 140°$
$\angle BOA = x + y = 70°$

48. (c)



Here AB = AC = BC = 12 cm.
Let AE ⊥ BC, then BE = 6 cm,

$AE = \sqrt{(AB)^2 - (BE)^2}$

$= \sqrt{144 - 36}$

$= \sqrt{108}$

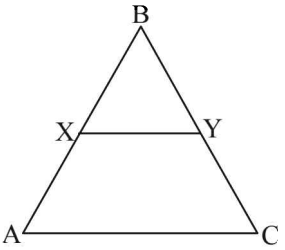
BD = 4 cm, DE = 6 – 4 = 2 cm.

$AD = \sqrt{(AE)^2 + (DE)^2}$

$= \sqrt{(\sqrt{108})^2 + (2)^2}$

$= \sqrt{108 + 4} = \sqrt{112} = 4\sqrt{7}$ cm

49. (b)



Here, $\Delta BXY \sim \Delta BAC$

$\Rightarrow \frac{\text{Area } (\Delta BXY)}{\text{Area } (\Delta BAC)} = \frac{BX^2}{AB^2}$

$\Rightarrow \frac{\text{Area } (\Delta BXY)}{2\text{Area } (\Delta BXY)} = \frac{BX^2}{AB^2}$

$\Rightarrow \frac{BX}{AB} = \frac{1}{\sqrt{2}}$

$AB = \sqrt{2}\, BX$

$AB = \sqrt{2}\, (AB - AX)$

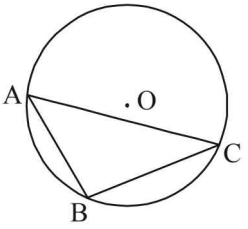
$AB = \sqrt{2}\, AB - \sqrt{2}\, AX$

$\sqrt{2}\, AX = (\sqrt{2} - 1)\, AB$

$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$

50. (a)



Here angle B is obtuse, so smaller circle which covers the triangle is the circum circle.

51. (b) Triangle (By SSS criteria).

52. (b) Each angle of a regular polygon = $\frac{(n-2)180°}{n}$

$165° = \frac{(n-2)180°}{n}$ (where n = no. of sides)

$165°\, n = 180°n - 360°$

$180°n - 165°n = 360°$

$15°n = 360°$

$\therefore\ n = 24$

53. (c) Let angles be 3x, 7x, 6x and 4x.
Total sum of angles of a quadilateral = 360°

$\Rightarrow 3x + 7x + 6x + 4x = 360°$

$\Rightarrow 20x = 360°$

$\Rightarrow x = 18°$

$\therefore$ Angles are $3 \times 18° = 54°$

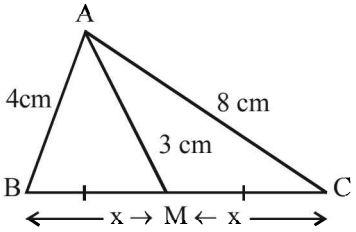
$7 \times 18° = 126°$

$6 \times 18° = 108°$

$4 \times 18° = 72°$

$\therefore$ Figure ABCD is trapezium. As all angles are different.

54. (b)



$AB^2 + AC^2 = 2(AM^2 + BM^2)$ [By Theorem]

$4^2 + 8^2 = 2\,(3^2 + x^2)$

$16 + 64 = 2\,(9 + x^2) \Rightarrow 80 = 18 + 2x^2$

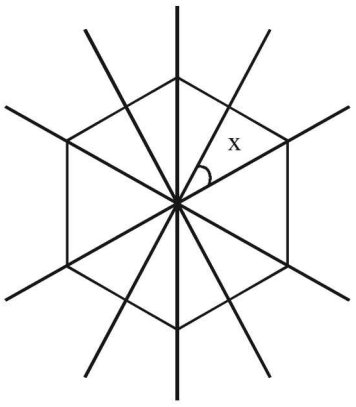
$\Rightarrow 62 = 2x^2$

$\Rightarrow 31 = x^2 \Rightarrow x = \sqrt{31}$

$BM = x$

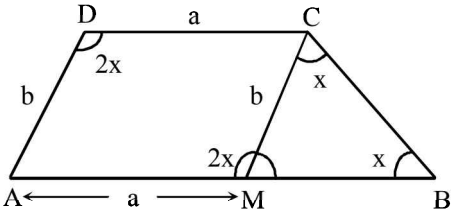
$BC = 2 \times x = 2\sqrt{31}$

55. (a)



$\frac{360°}{12°} = x° \Rightarrow x° = 30°$

56. (d)



$AD = b$

$\therefore CM = b$

$CD = a$

$\therefore Am = a$

Hence, AMCD is a parallelogram, AB||CD is also given

$\therefore \angle D = \angle AMC$

$\angle AMC = 2x$

In ΔMCB

$2x = \angle MCB + x$ ($\therefore$ Exterior angle is equal to sum of opposite interior angle)

$\therefore \angle MCB = x$

$\angle MCB = \angle B = x$

$\therefore MB = MC$ [As sides opposite to equal angles are equal]

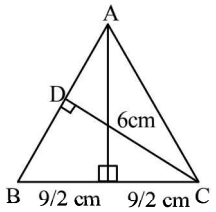
To calculate, $AB = AM + MB$

$AB = a + b$

57. (b) $\text{ar}(\Delta ABC) = \frac{1}{2} \times 9 \times 6 \text{ cm}^2 = 27 \text{ cm}^2$

$AB = \sqrt{6^2 + \left(\frac{9}{2}\right)^2} = \sqrt{36 + \frac{81}{4}} \text{cm} = \sqrt{\frac{144+81}{4}}$

$= \frac{15}{2} \text{cm}$

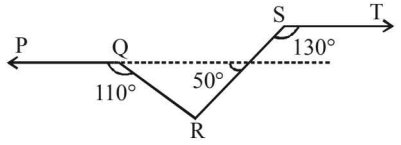


$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times CD$

$27 = \frac{1}{2} \times \frac{15}{2} \times CD$

$\therefore CD = 7.2$

58. (c)



$\angle QRS = 110° - (50°) = 60°$

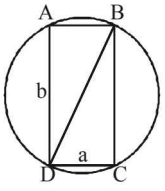
59. (c) Centre of a cyclic quadrilateral coincides with that of circle and diameter is the diagonal so, angles are all right angle.

So, it is either rectangle or square

$a^2 + b^2 = 2^2 = 4$

$AB \cdot BC \cdot CD \cdot AD = a.b.a.b = a^2b^2$

If $a \neq b$, let $b = 2a$



So, $a^2 + (2a)^2 = 4 \Rightarrow a^2 = \frac{4}{5}$

$AB \cdot BC \cdot CD \cdot AD = a^2b^2 = a^2 (2a)^2 = 4a^4$

$4(a^2)^2 \Rightarrow 4\left(\frac{4}{5}\right)^2 = \frac{64}{25} > 4$

If $a = b$, then $2a^2 = 4 \Rightarrow a = \sqrt{2}$ and then a^4

$= \left(\sqrt{2}\right)^4 = 4$

So, $AB \cdot BC \cdot CD \cdot DA \geq 4$

It is rectangle

60. (a) Area of $\Delta AOB = \frac{1}{2} \times 10 \times h_1$

$3 = \frac{1}{2} \times 10 \times h_1 \Rightarrow h_1 = \frac{6}{10} = \frac{3}{5}$

Area of $\Delta COD = \frac{1}{2} \times 20 \times h_2$

$12 = \frac{1}{2} \times 20 \times h_2 \Rightarrow h_2 = \frac{12}{10} = \frac{6}{5}$

Now, height of trapezium ABCD $= h_1 + h_2$

$= \frac{3}{5} + \frac{6}{5} = \frac{9}{5}$

$\therefore$ Area of trapezium

$= \frac{1}{2} \times [(h_1 + h_2) \times (10 + 20)]$

$= \frac{1}{2} \times \frac{9}{5} \times 30 = \frac{54}{2} = 27$ sq. cm.

61. (b) $\because n \parallel p$ and m is transversal

$\therefore \angle 1 = \angle 3 = 85°$ (Corresponding angles)

Also, $m \parallel l$ & p is transversal

$\therefore \angle 2 + \angle 3 = 180°$

($\because$ Consecutive interior angles)

$\Rightarrow \quad \angle 2 + 85° = 180°$

$\Rightarrow \quad \angle 2 = 180° - 85°$

$\Rightarrow \quad \angle 2 = 95°$

62. (a) Given ratio of angles are 3 : 5 : 9 : 13

Let four angles of the quadrilateral are $3k$, $5k$, 9k and $13k$.

We know that sum of four angles of a quadrilateral is 360°

Therefore, $3k + 5k + 9k + 13k = 360°$

$\Rightarrow 30k = 360° \Rightarrow k = 12$

Therefore four angles are

$(3 \times 12)°, (5 \times 12)°, (9 \times 12)°$ and $(13 \times 12)°$

or 36°, 60°, 108°, 156°.

63. (c) Since, EAB is a straight line

$\therefore \angle DAE + \angle DAB = 180°$

$\Rightarrow 73° + \angle DAB = 180°$

i.e., $\angle DAB = 180° - 73° = 107°$

Since, the sum of the angles of quadrilateral $ABCD$ is 360°

$\therefore 107° + 105° + x + 80° = 360°$

$\Rightarrow 292° + x = 360°$

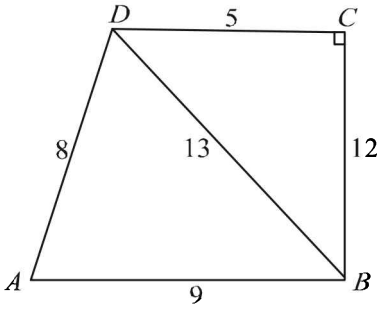
and $x = 360° - 292° = 68°$

64. (a) Given, a quadrilateral $ABCD$ in which AB=9m, BC = 12 m, CD = 5m, and AD = 8m. We divide the quadrilateral in two triangular region ABD and BCD.

Now, In ΔBCD, right angle at C.

We have $BD^2 = BC^2 + CD^2$

[By pythagorous theorem]



$= 52 + 122 = 25 + 144 = 169\ m^2$

$BD = \sqrt{169} = 13$m

$\therefore$ Area of $\Delta BCD = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 5 \times 12 = 30\ m^2$

65. (b) $\because \angle POR$ and $\angle QOR$ form a linear pair

$\therefore \angle POR + \angle QOR = 180°$

(Linear pair axiom)

or $a + b = 180°$ ..... (a)

But $a - b = 80°$ ..... (b) [Given]

Adding eqs (a) and (b), we get

$2a = 260° \therefore a = \frac{260}{2} = 130°$

Substituting the value of a in (a), we get

$130° + b = 180°$

$b = 180° - 130° = 50°$

66. (c) $\angle POR + \angle ROQ = 180°$ ...(1) [Linear Pair]

But $\frac{\angle POR}{\angle ROQ} = \frac{5}{7}$ [Given]

$\Rightarrow \quad \angle ROQ = \frac{7}{5} \angle POR$ ...(2)

From (1) and (2), we have

$\angle POR + \frac{7}{5} \angle POR = 180°$

$\Rightarrow \frac{12}{5} \angle POR = 180°$

$\Rightarrow \angle POR = \frac{180° \times 5}{12}$

GP_3386

$\Rightarrow \angle POR = 75°$

Now, $\angle SOQ = \angle POR = 75°$

(Vertically opposite $\angle s$)

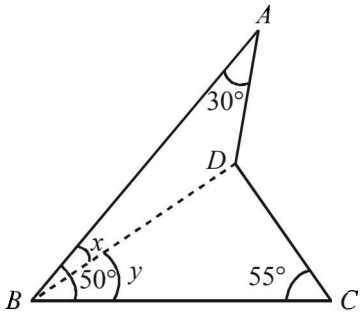
67. (a) For A, B, C must be lie in a line $x + y = 180°$.

68. (b) Construction: Join BD

Now in, ΔABD

$\angle x + \angle ADB + 30° = 180°$ ...(1)

In ΔBDC, $\angle y + \angle BDC + 55° = 180°$...(2)



By adding (1) and (2)

$\angle x + \angle ADB + 30° + \angle y + \angle BDC + 55° = 360°$

Also, $\angle x + \angle y = 50°$

Then $\angle ADB + \angle BDC + 135° = 360°$

$\Rightarrow \angle ADB + \angle BDC = 225°$

i.e., $x = \angle ADC = 135°$

69. (d) Since $l \parallel m$, then

$3y = 2y + 25°$ [Alternate $\angle s$]

$\Rightarrow 3y - 2y = 25°$

$\Rightarrow y = 25°$ ... (1) [Vertically opposite $\angle s$]

Also $x + 15° = 2y + 25°$

$\Rightarrow x + 15° = 2(25°) + 25°$ [using (a)]

$\Rightarrow x + 15° = 50° + 25°$

$\Rightarrow x = 75° - 15°$

$\Rightarrow x = 60°$

70. (b) We have

$\angle A = 50°$ and $\angle B = 60°$

$\therefore \angle A + \angle B + \angle C = 180°$

[Angle sum property of a Δ]

$\Rightarrow 50° + 60° + \angle C = 180°$

$\Rightarrow 110° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 110°$

$\Rightarrow \angle C = 70°$

Since $\angle$A and $\angle$C are the smallest and largest angles, therefore sides BC and AB are the smallest and largest sides of the triangle.

71. (a) S and T trisect the side QR.

Let $QS = ST = TR = x$ units

Let $PQ = y$ units

In right ΔPQS, $PS^2 = PQ^2 + QS^2$

(By Pythagoras Theorem)

$= y^2 + x^2$ ...(i)

In right ΔPQT, $PT^2 = PQ^2 + QT^2$

(By Pythagoras Theorem)

$= y^2 + (2x)^2 = y^2 + 4x^2$ ...(ii)

In right ΔPQT, $PR^2 = PQ^2 + QR^2$

(By Pythagoras Theorem)

$= y^2 + (3x)^2 = y^2 + 9x^2$ ...(iii)

R.H.S. $= 3PR^2 + 5PS^2$

$= 3(y^2 + 9x^2) + 5(y^2 + x^2)$ [From (i) and (iii)]

$= 3y^2 + 27x^2 + 5y^2 + 5x^2 = 8y^2 + 32x^2$

$= 8(y^2 + 4x^2) = 8PT^2 =$ L.H.S. [From (ii)]

Thus $8PT^2 = 3PR^2 + 5PS^2$

72. (b) In ΔABE, $\angle B = 90°$

So, $(13)^2 = AB^2 + 5^2 \Rightarrow AB = 12$ cm

From figure, $BD = AB - AD = (12 - x)$ cm

and $BC = BE + EC = (5 + x)$ cm.

In ΔBCD, $\angle B = 90°$, $(13)^2 = (12 - x)^2 + (5 + x)^2$

$\Rightarrow 2x^2 - 14x = 0 \Rightarrow x = 7$

73. (d) Isosceles and similar

74. (c) In right angled ΔCAB, BC^2

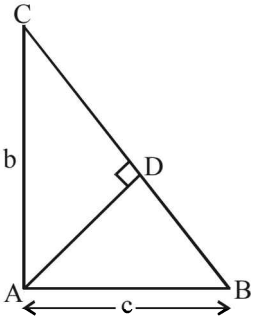
$= AB^2 + AC^2 = c^2 + b^2$

or $BC = \sqrt{b^2 + c^2}$

Area of ΔABC = 1/2 base × height = 1/2 bc

Also area of ΔABC = 1/2 (BC) (AD)

$= \frac{1}{2}\sqrt{b^2 + c^2} \times AD$



$\therefore \frac{1}{2}\left(\sqrt{b^2 + c^2}\right) AD = \frac{1}{2} bc$.

Hence $AD = \frac{bc}{\sqrt{b^2 + c^2}}$

75. (c) In right angled triangle POR

$PR^2 = PO^2 + OR^2 = (6)^2 + (8)^2 = 36 + 64 = 100$

$\therefore PR = 10$ cm

Again in right angled triangle

PQR, $QR^2=(26)^2=676$
$PQ^2+PR^2=(24)^2+(10)^2=576+100=676$
$\therefore QR^2=PQ^2+PR^2$
$\therefore \Delta PQR$ is a right angled triangle with right angle at P.
i.e., $\angle QPR=90°$

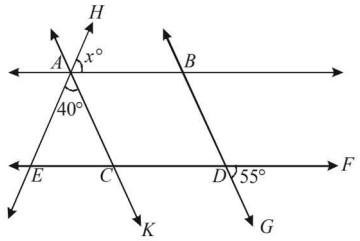
76. (a) $\angle A=180°-(70°+50°)=60°$.

$\frac{BD}{DC}=\frac{AB}{AC}$ means AD is the bisector of $\angle A$.

$\therefore \angle BAD=\frac{1}{2}\times 60°=30°$

Level 1

1. (a) $\angle DCK=\angle FDG=55°$ (corr. $\angle s$)



$\therefore \angle AEC=180°-(\angle EAC+\angle ACE)$
$\therefore \angle HAB=\angle AEC=85°$ (corr. $\angle s$)
Hence, $x=85°$

2. (b) *BC*, *CD* and *BD* constitute a right = angle triangle (13 × 3, 13 × 4) and 13 × 5). If *BD* = 65, then it satisfies the other set of values of *AB* and *AD* also.

3. (c) Let the line *m* cut *AB* and *CD* at point *P* and *Q* respectively
$\angle DQO=y+2x$ (exterior angle)
And, $x+2y$ (corresponding angle)
$\therefore y=x$ ...(1)
Also. $\angle DOQ=x$ (vertically opposite angles)
In ΔOCD, sum of the angles = 180°
$\therefore y+2y+2x+x=180°$
$3x+3y=180°$
$x+y=60$ ...(2)
From (1) and (2) $x=y=30 \Rightarrow 2y=60$
$\therefore \angle ODS=180-60=120°$
$\therefore \theta=180-3x=180-3(30)=180-90=90°$.
$\therefore$ The required ratio = 90 : 120 = 3 : 4.

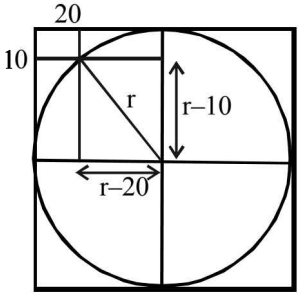
4. (d) $\angle LCD=\angle ALC=60°$ (alternate angles)

$\angle DCE=\frac{1}{2}\angle LCD=30°$. (*EC* is the angle bisector)

$\therefore \angle FEC=(180°-30°)=150°$.

5. (a) No such point is possible.

6. (c)



In the right angled Δ,
$r^2=(r-20)^2+(r-10)^2$
$=2r^2-60r+500$
Solving, we get, $r=50$.

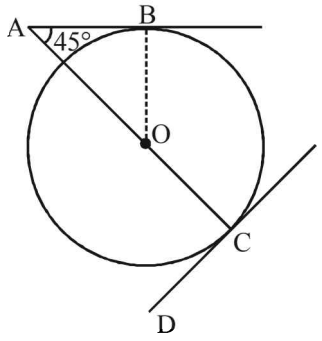
7. (a) Join OR and OS and let *r* be the radius of circle. Given, BC = 38 cm, CD = 28 cm
BP = 25 cm = BQ
($\because$ tangent drawn from external point B to the circle)
Then, CQ = BC – BQ
= 38 – 25 = 13 cm
$\therefore$ CR = CQ = 13 cm
Then, RD = CD – CR
= 28 – 13 = 15 cm
Clearly, quadrilateral DROS is a square.
($\because$ Three angles are 90°)
$\therefore$ r = OR = RD = 15 cm

8. (a) We know that, length of tangents drawn from an external point contact to the circle are equal.
$\therefore$ PS = PT = 5 cm
Then, $\angle PTS=\angle PST$
($\because$ angles opposite to equal sides are equal)
In ΔPTS, we have
$\angle PTS+\angle PST+\angle TPS=180°$
($\because$ sum of three angles of a triangle is 180°)
$\Rightarrow \angle PTS+\angle PTS+60°=180°$
$\Rightarrow 2\angle PTS=120°$
$\Rightarrow \angle PTS=60°$
Thus, $\angle PTS=\angle PST=\angle TPS=60°$
$\therefore$ ΔPTS is an equilateral triangle.
Hence, TS = 5 cm

9. (a) Join OB.
$\angle OBA=90°$ [Since radius is perpendicular to tangent]
By Pythagoras theorem $AB^2+OB^2=AO^2$.
In ΔABO, $\angle ABO=90°$ and $\angle BAO=45°$
$\Rightarrow \angle BOA=45°$.

So, OBA is an isosceles right angled triangle

$\therefore$ OB = AB = 2 cm



$\Rightarrow AO^2 = 2^2 + 2^2 \Rightarrow AO = 2\sqrt{2}$ cm and AC $= AO + OC = 2\sqrt{2} + 2$.

10. (b) $\frac{AT}{OT} = \cos 30° \Rightarrow \frac{\sqrt{3}}{2} OT = AT$

$\Rightarrow \quad AT = \frac{\sqrt{3}}{2}(d) = 2\sqrt{3}$ cm

11. (a) Join OA, OB and OP.

$OA \perp AP$, $OB \perp BP$.

In ΔOAP, $\angle OAP = 90°$, $OP^2 = OA^2 + AP^2 = 5^2 + 12^2 = 169$

$\Rightarrow OP = 13$ cm.

In ΔOBP, $\angle OBP = 90°$, $BP^2 = OP^2 - OB^2 = 13^2 - 3^2 = 160$

$\Rightarrow BP = 4\sqrt{10}$ cm

12. (b) Since A, B, C, D are four points on circle having centre O and AC and BD intersect at O.

So, OA = OB = OC = OD. Since EA is tangent at A, $OA \perp EA$.

In Δ AOB, $OA = OB \Rightarrow \angle BAO = \angle ABO$.

So, it is isosceles.

Similarly, ΔDOC is also isosceles

$\angle ABD = \angle CAB = \angle BDC = \angle ACD$.

From right angle ΔEAC

$\angle ACE = 90 - 32 = 58°$

$\angle ACE$ is $\angle ACD = \angle ABD$.

So, $\angle ABD = 58°$

$\angle AOB = \angle DOC$ and $\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow \angle AOB + 2 \times 58 = 180$

$\Rightarrow \angle AOB = 180 - 116$

$\Rightarrow \angle AOB = 64° = \angle DOC$

$\angle AOD = 180 - \angle DOC = 180 - 64$

So, $\angle AOD = 116°$.

13. (c) In ΔLAO and ΔAOC, AL = AC, (tangent from A are equal)

AO = AO .....(common)

OL = OC (each = radius)

$\Delta LAO \cong \Delta AOC$ (SSS)

$\therefore \quad \angle LAO = \angle CAO$

$\Rightarrow \quad \angle LAC = 2\angle CAO$ ....(i)

Similarly, $\angle MBO = \angle CBO$

$\Rightarrow \quad \angle CBM = 2\angle CBO$ ....(ii)

Now, $\angle LAC + \angle CBM = 180°$

($\because$ PQ || RS and ACB is a transverse line)

$\Rightarrow 2\angle CAO + 2\angle CBO = 180°$

$\Rightarrow \angle CAO + \angle CBO = 90°$

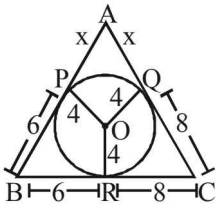
$\Rightarrow \angle AOB = 180° - 90° = 90°$

(sum of the angle of a triangle = 180°)

$\angle AOB = 90°$

14. (a) Let AQ = x

$\therefore$ AP = x (Tangent drawn from external point)



BC = 6 + 8 = 14 cm., AC = 8 + x, AB = 6 + x

$\therefore$ 2s = AB + BC + CA

= 6 + x + 8 + x + 14 = 28 + 2x

$\Rightarrow$ s = 14 + x

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where a = BC, b = AC, c = AB

$= \sqrt{(14+x)(x)(6)(8)} = \sqrt{48x(14+x)}$

Now, area of ΔABC = ar (ΔOBC) + ar (ΔOCA) + ar(ΔOAB)

$= \frac{14 \times 4}{2} + \frac{(8+x) \times 4}{2} + \frac{(6+x) \times 4}{2} = 56 + 4x$

$\therefore \sqrt{48x(14+x)} = 56 + 4x$ ....(i)

Squaring both sides of eqn. (i), we get

$48x(14+x) = 16(14+x)^2 \Rightarrow 3x(14+x) = (14+x)^2$

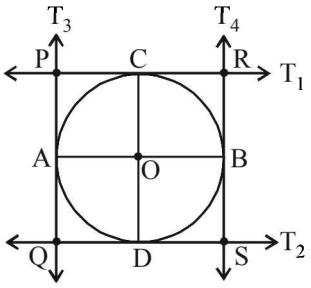
$\Rightarrow 3x = 14 + x \Rightarrow 2x = 14 \Rightarrow x = 7$

$\therefore$ AC = 8 + 7 = 15cm, AB = 6 + 7 = 13 cm.

15. (d) Given *AB* and *CD* are two perpendicular diameters.

T_1, T_2 are tangents at *C* and *D* respectively.

T_3, T_4 are tangents at *A* and *B* respectively.



From figure, $PQSR$ is a square.
Since, $PR \parallel QS$, therefore $AB \parallel RP$ and $AB \parallel QS$
Since, $PR = 2$ cm, therefore $AB = 2$ cm

So, radius $= \frac{AB}{2}$

$= \frac{2}{2} = 1$ cm

16. (c) Join O to P and Q. Join P to R. Draw SP $\perp$ OQ.
Now SP = QR, as they are opposite sides of rectangle PRQS.
OP = 8 cm + 4 cm = 12 cm
OS = 8 cm – 4 cm = 4 cm

$\therefore$ SP $= \sqrt{OP^2 - OS^2}$

$= \sqrt{12^2 - 4^2}$ cm $= 8\sqrt{2}$ cm

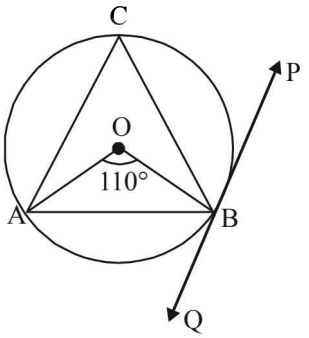
$\therefore$ QR $= 8\sqrt{2}$ cm

17. (b) Given $\angle BPC = b°$ and $\angle ACP = a°$.
Also, $\angle OPA = \angle OAP = b°$ (Angles in an isosceles triangle OAP, angle in deternate segment.)
$\angle CPO = 90°$
$\therefore \angle CPA = 90° + b°$
In ΔACP, $\angle ACP = 180° - [(b° + 90°) + b°]$
$\Rightarrow a° + 2b° = 90°$

18. (a) Join P to B Now, $\angle BPC = 90°$
$\Rightarrow \angle BPM + \angle MPC = 90°$
But $\angle BPM = A$
$\Rightarrow \angle A + \angle MPC = 90°$
$\therefore \angle A = 90° - \angle MPC$
(from ΔABC)
$\therefore \angle A = 90° - \angle C$
$\therefore \angle MPC = C$
$\therefore$ MP = MC but, MP = MB
$\Rightarrow$ MC = MB

19. (d) Since ST is a diameter of the circle with centre O,
So $\angle SRT = 90°$. [Angle in a semicircle]
Now, $\angle PRS + \angle SRT + \angle TRQ = 180°$ [Linear pair]
$\angle PRS + 90° + 30° = 180°$
$\angle PRS = 180° - 120° = 60°$

20. (c) In ΔOAB, OA = OB (radii of the circle)
$\therefore \angle OAB = \angle OBA$



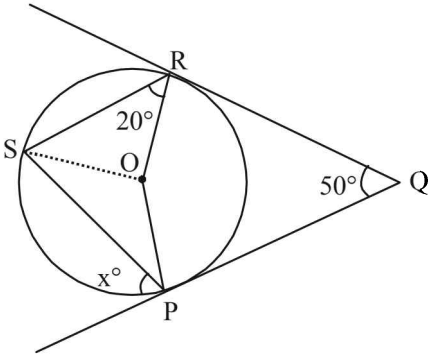
In ΔOAB, $\angle AOB + \angle OAB + \angle OBA = 180°$
$2\angle OBA = 180° - 110°$
$\Rightarrow \angle OBA = 35°$
Since BQ is a tangent at B
$\therefore \angle OBQ = 90°$
$\angle OBA + \angle ABQ = 90°$, $35° + \angle ABQ = 90°$
$\angle ABQ = 90° - 35° = 55°$

21. (c) Perimeter of quadrilateral is
AB + BC + CD + AD
we know that four sides of a quadrilateral ABCD are tangential to a circle.
$\therefore$ AB + CD = BC + AD
$\therefore$ perimeter of quadrilateral = 2(AB + CD)
Now AB = AP + PB = 6 + 5 = 11 cm and CD = DR + CR = 4 + 3 [$\because$ CR = CQ]
= 7 cm
So, perimeter
= 2 (11 + 7) = 2 × 18 = 36 cm

22. (c) $\angle POR + \angle PQR = 180°$
$\therefore \angle POR = 180° - 50° = 130°$

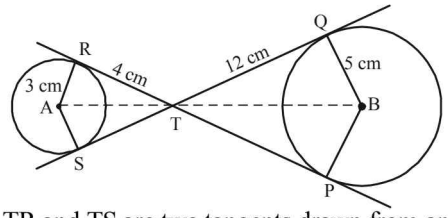
$\angle PSR = \frac{1}{2} \angle POR$

$\therefore \angle PSR = \frac{1}{2} \times 130° = 65°$



$\Rightarrow \angle PSR = \angle OSP + \angle OSR$
$\Rightarrow \angle PSR = \angle OSP + 20°$ [$\because \angle OSP = \angle OSR$]
$\Rightarrow 65° = \angle OSP = 20°$
$\Rightarrow \angle OSP = 45°$
$\Rightarrow \angle OPX = 90°$ [$\because$ PX is a tangent]
$\Rightarrow \angle SPX + \angle OPS = 90°$
$x° + \angle OSP = 90°$ [$\because \angle OPS = \angle OSP$]
$x° = 90° - 45° = 45°$

23. (d) We have AR = 3 cm
BQ = 5 cm, Rt = 4 cm
and ST : TQ = 1 : 3



TR and TS are two tangents drawn from an external point T to the the circle with centre A

$\therefore$ TR = TS and $\frac{ST}{TQ} = \frac{1}{3} \Rightarrow \frac{TR}{TQ} = \frac{1}{3}$

$\Rightarrow \frac{4}{TQ} = \frac{1}{3}$ [$\because$ TR = 4 cm]

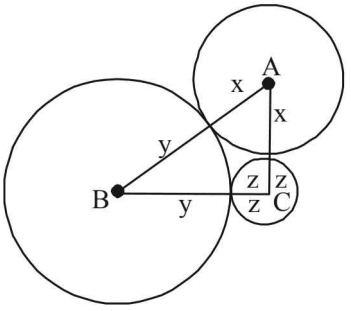
$\Rightarrow$ TQ = 12 cm
Now, In ΔART, $\angle R = 90°$
$AT^2 = AR^2 + RT^2$
$\Rightarrow AT^2 = (3)^2 + (4)^2 = 9 + 16 = 25$
$\Rightarrow$ AT = 5 cm
and In ΔBQT $\angle Q = 90°$
$BT^2 = BQ^2 + TQ^2$
$\Rightarrow BT^2 = (5)^2 + (12)^2 = 25 + 144 = 169$
$\Rightarrow$ BT = 13 cm
Now $\therefore$ AB = AT + BT
Hence QT = 12 cm and AB = 18 cm

24. (d) Since two chords AB and CD of the circle are intersecting at P, when produced.
$\therefore$ PA. PB = PC.PD
[Each = (length of the tangent from P)^2]
$\Rightarrow$ (AB + PB). (PB) = (PD + DC) PD
$\Rightarrow (5 + 3)(c) = (2 + x)2$
$\Rightarrow 24 = (2 + x)2$
$\Rightarrow 12 = 2 + x \quad \Rightarrow x = 10$cm
$\Rightarrow$ CD = 10 cm

25. (a) CQ = CP = 11
[$\because$ length of tangents from C to the circle are equal]
$\therefore$ CQ = 11cm
But BC = 7 cm $\therefore$ 7 + BQ = 11
[$\because$ CQ = BC + BQ]
$\therefore$ BQ = 11 – 7 = 4
Since BR = BQ = 4
[$\because$ length of tangents from B to the circle are equal]
Thus BR = 4 cm.

26. (a) In ΔPQR,
PQ = QR
$\Rightarrow \angle PRQ = \angle QPR$
[$\angle$s opp. to equal sides of a Δ are equal]
Also, $\angle QPR + \angle RQP + \angle PRQ = 180°$
[Sum of $\angle$s of a Δ is 180°]
$\Rightarrow 68° + 2\angle PRQ = 180°$
$\Rightarrow 2\angle PRQ = (180° - 68°) = 112°$
$\Rightarrow \angle PRQ = 56°$
$\therefore \angle QOP = 2\angle PRQ = (2 \times 56°) = 112°$
[Angle at the centre is double the angle on the circle]
Since the radius through the point of contact is perpendicular to the tangent, we have
$\angle OQC = 90°$ and $\angle OPC = 90°$
Now, $\angle OQC + \angle QOP + \angle OPC + \angle QCP = 360°$ [Sum of the $\angle$s is equal 360°]
$\Rightarrow 90° + 112° + 90° + \angle QCP = 360°$
$\Rightarrow \angle QCP = (360° - 292°) = 68°$

27. (b) Let AB = 9 cm, BC = 7 cm and CA = 6 cm
Let, x, y, z be the radii of circles with centre A, B, C respectively.
Then, x + y = 9, y + z = 7 and z + x = 6.
Adding, we get 2(x + y z) = 22
$\Rightarrow$ x + y + z = 11



$\therefore x = [(x+y+z) - (y+z)] = (11 - 7)$ cm = 4 cm.
Similarly, y = (11 – 6) cm = 5 cm and z = (11 – 9) cm = 2 cm.
Hence, the radii of circle with centres A, B, C are 4 cm, 5 cm and 2 cm respectively.

28. (d) Since tangents from same external points are equal,

$\therefore$ PA = PC ....(i)
and PC = PB ....(ii)
From (i) and (ii) we get
PA = PB

29. (a) Since OA $\perp$ PA and OB $\perp$ PB,
$\therefore$ In quadrilateral AOBP,
$\Rightarrow 40° + 90° + 90° + \angle AOB = 360°$
$\Rightarrow \angle AOB = 140°$

also, $\angle AQB = \frac{1}{2}$ of $\angle AOB = 70°$

and $\angle AMB = \frac{1}{2}$ of reflex $\angle AOB$

$= \frac{1}{2} \times (360° - 140°) = \frac{1}{2} \times 220 = 110°$

[$\because$ The angle subtended by an arc at the centre is double the angle subtended by the arc at any point on the remaining part of the cirlce.]

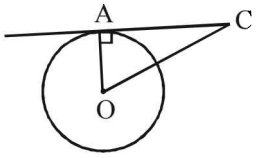
30. (b) Since PT is a tangent to the circle.
$\therefore$ OP $\perp$ PT or $\angle QPT = 90°$ ....(i)
In ΔOQR, $\angle POR = \angle OQR + \angle ORQ$
(exterior angle property)
also, $\angle OQR = \angle ORQ$ (angles opp. to equal sides are equal)
$\therefore \angle POR = 2\angle OQR$

$\frac{72°}{2} = \angle OQR$

$\Rightarrow \angle OQR = 36°$
again, in ΔQPT,
$\angle PQT + \angle QPT + \angle QTP = 180°$
(angle sum property)
$\angle QTP = 180° - 90° - 36°$
$\angle QTP = 54°$
or $\angle PTR = 54°$

31. (c) Here, PA = PB
$\therefore \angle PAB = \angle ABP = 60°$
Now, $\angle APB + \angle PBA + \angle BAP = 180°$
$\Rightarrow \angle APB = 180° - 60° - 60° = 60°$

32. (d) Let AC be the tangent to circle of centre O.
OA $\perp$ AC
($\because$ Radius always $\perp$ to tangent)
$\therefore$ $\angle OAC = 90°$ and ΔOAC is isosceles triangle
$\therefore \angle O = \angle C = x$

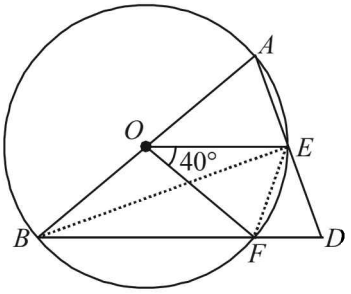


Now, by angle sum property of Δ We have
$90° + x + x = 180° \Rightarrow x = 45°$
Hence, $\angle OCA = 45°$

33. (c) Join *BE* and *EF*.
Since *AB* is the diameter of the circle
$\angle AEB = 90°$ [Angle in the semicircles is 90°]

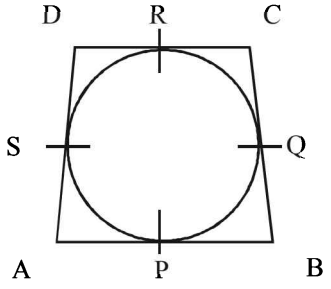
$\angle EBF = \frac{1}{2}\angle EOF = \frac{40°}{2} = 20°$

[Angle made by a chord at any point on the circumference is half the angle made by the chord at the centre of the circle.]



Also $\angle AEB = 90°$
$\therefore \angle BED = 180° - 90° = 90°$
In ΔBFD,
$\angle BED + \angle EBD + \angle EDB = 180°$
$90° + \angle EBF + \angle EDB = 180°$
[$\because \angle EBD = \angle EBF$]
$90° + 20° + \angle EDB = 180°$
$\angle EDB = 180° - 110° = 70°$
i.e. $\angle EDF = 90°$

34. (c) **Given :** A circle inscribed in a quadrilateral ABCD, AB = 6 cm, BC = 7 cm, CD = 4 cm,



To find: AD
Proof : Let the circle touch the sides AB, BC, CD, DA, at P, Q, R and S, respectively.

AP = AS
BP = BQ
DR = DS
CR = CQ
} {Lengths of two tangents drawn from an external point of circle, are equal}

Adding all these, we get
(AP + BP) + (CR + RD) = (BQ + QC) + (DS + SA)
AB + CD = BC + DA $\Rightarrow$ 6 + 4 = 7 + AD
$\Rightarrow$ AD = 10 – 7 = 3 cm.

35. (b) Given that $\angle BOD = 90°$, so the top of the table is $\frac{3}{4}$ of a circle of radius 60 cm.

Perimeter of the table

$$= \left(\frac{3}{4} \times 2\pi r + 2r\right) \text{cm}$$

$$= \left(\frac{3}{4} \times 2 \times 3.14 \times 60 + 2 \times 60\right) \text{cm}$$

= (90 × 3.14 + 120) cm = 402.6 cm.

36. (a) Since OM ⊥ BC, a chord of the circle.
∴ it bisects BC.

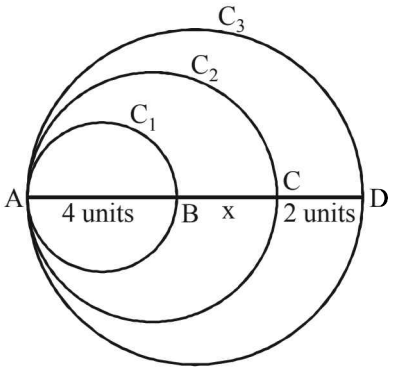
$$\therefore \quad BM = CM = \frac{1}{2}(BC) = \frac{1}{2}(8) = 4 \text{ cm.}$$

Since OM ⊥ AD, a chord of the circle.
∴ it bisects AD.

$$\therefore \quad AM = MD = \frac{1}{2}AD = \frac{1}{2}(12) = 6 \text{ cm.}$$

Now, AB = AM – BM = 6 – 4 = 2 cm.
CD = MD – MC = 6 – 4 = 2 cm
AC = AM + MC = 6 + 4 = 10 cm
BD = BM + MD = 4 + 6 = 10 cm.

37. (d)



$C_1 = 2\pi \times 2 = 4\pi$ ($\therefore$ r = 2 units)
AC = AB + BC $\Rightarrow$ AC = 4 + x

Radius of $C_2 = \frac{4+x}{2}$

$$C_2 = 2\pi\frac{(4+x)}{2} = \pi(4+x)$$

$$C_3 = 2\pi\frac{(6+x)}{2} = (6+x)\pi$$

[$\therefore$ AD = AB + BC + CD $\Rightarrow$ AD = 4 + x + 2
$\Rightarrow$ AD = 6 + x; radius = $\frac{6+x}{2}$]

According to question

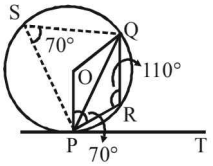
$$C_2 = \frac{C_1 + C_3}{2} \Rightarrow \pi(4+x) = \frac{4\pi + \pi(6+x)}{2}$$

$\Rightarrow$ 2 (4 + x) = 4 + 6 + x
$\Rightarrow$ 8 + 2x = 10 + x $\Rightarrow$ x = 2

Radius of $C_3 = \frac{6+x}{2} \Rightarrow \frac{6+2}{2} = 4$

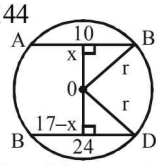
Area of $C_3 = \pi r^2 = \pi(4)^2 \Rightarrow 16\pi$

38. (d) $\angle PSQ = \angle QPT = 70°$
(Angles in alternate segment of circle are equal)



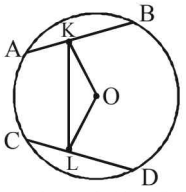
$\therefore \angle PRQ = 180° - \angle PSQ = 180° - 70° = 110°$

39. (c) $r^2 = x^2 + 25$
$r^2 = (17 - x)^2 + 144$
Now $x^2 + 25 = (17 - x)^2 + 144$
$\Rightarrow$ x = 12
$r^2 = 144 + 25$
r = 13 cm



40. (b) O is the centre of circle. K and L are mid points of Chords AB and CD respectively.

$\therefore$ OK ⊥ AB and OL ⊥ CD
As AB = CD

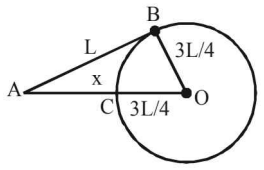


$\therefore$ OK = OL. (equal chords are

equidistant from centre)
So, ΔOKL is an isosceles.
$\therefore$ $\angle OKL = \angle OLK = 25°$ (given)
Therefore $\angle LKB = \angle OKL + \angle OKB$
$= 25° + 90° = 115°$

41. (b) Given the length of tangent $L = \frac{4}{3}r$, where r is the radius.

or $r = \frac{3L}{4}$.



From the figure $L^2 + \left(\frac{3L}{4}\right)^2 = \left(x + \frac{3L}{4}\right)^2$

$$L^2 + \left(\frac{3L}{4}\right)^2 = x^2 + \left(\frac{3L}{4}\right)^2 + 2x\left(\frac{3L}{4}\right)$$

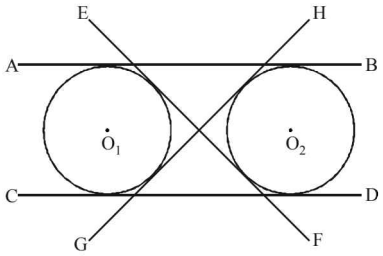
$$x^2 + 2\left(\frac{3L}{4}\right)x - L^2 = 0$$

$2x^2 + 3Lx - 2L^2 = 0$ or $(x + 2L)(2x - L) = 0$

$\Rightarrow$ $x = -2L$ or $\frac{L}{2}$

We reject $x = -2L$. Hence $x = \frac{L}{2}$.

42. (d)



There are four common tangents, AB, CD, EF and GH.

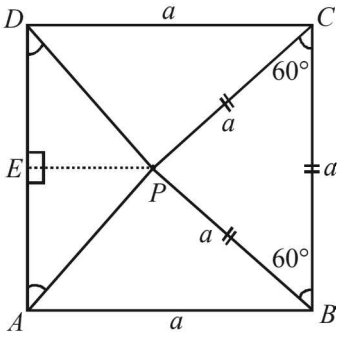
43. (a) Given $\angle$ AOC By property of circles, we have
$\angle AOC = 2\angle ADC \Rightarrow \angle ADC = 110° \div 2 = 55°$
Again we have 110°
$\angle AOC = 2\angle ADC$
$\angle ADC = 55°$
$\angle B + \angle D = 180°$
$\angle B = 125°$

44. (b) Required area
= Area of 2 small semi-circles
+ Area of big semi circle

$$\Rightarrow \frac{\pi\left(\frac{r}{2}\right)^2}{2} + \frac{\pi\left(\frac{r}{2}\right)^2}{2} + \frac{\pi r^2}{2} \Rightarrow \pi\left(\frac{r}{2}\right)^2 + \frac{\pi r^2}{2}$$

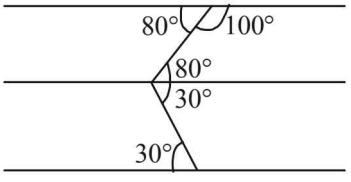
$$\Rightarrow \frac{\pi r^2}{4} + \frac{\pi r^2}{2} = \frac{3\pi r^2}{4} \text{ sq.cm.}$$

45. (d)



$\angle PBA = \angle ABC - \angle PBC = 90° - 60° = 30°$
Further in ΔABP
$PB = AB = a \Rightarrow \angle BPA = \angle BAP$
Further $2(\angle BPA) + \angle PBA = 180°$
$\Rightarrow 2\angle BPA = 180° - 30° = 150°$
$\Rightarrow \angle BPA = 75° = \angle BAP$
Similarly
$\angle PAD = 90° - \angle PAB = 90° - 75° = 15°$
Again in right angled ΔAPE,
$\angle EPA = 90° - \angle PAE = 90° - 15° = 75°$
Similarly we can calculate that $\angle DPE = 75°$
$\therefore$ $\angle DPA = 75° + 75° = 150°$

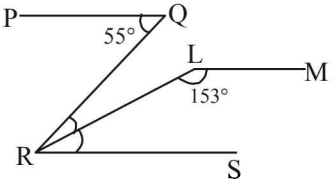
46. (a) $100° + (x + 10°) - 30° = 180°$
$\Rightarrow$ $x = 100°$
Alternatively :



$\therefore$ $x + 10° = 80° + 30° \Rightarrow x = 100°$

GP_3386

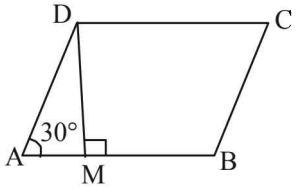
47. (b)



$\angle PQR = \angle QRS$
$55° = \angle QRS$
$55° = \angle QRL + \angle LRS$
$55° = 30° + \angle LRS$
$\Rightarrow \angle LRS = 25°$

48. (a) $\angle CEF = \angle ABF = 70°$
$\therefore \angle FED = 180° - 70° = 110°$
$\angle EDF + \angle DFE + \angle FED = 180°$
$\therefore \angle FDE = 180° - (10° + 20°) = 50°$

49. (b) $AD = BC = 20$ cm
$AB = 20$

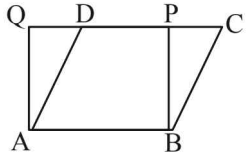


In ΔADM, $\frac{DM}{AD} = \sin 30° = \frac{1}{2}$

$\therefore \quad DM = \frac{1}{2} AD = 10\text{cm}$

$\therefore$ Area of parallelogram = base × height
$= 20 \times 10 = 200\ \text{cm}^2$

50. (b) ABCD is a parallelogram and ABPQ is a rectangle having same base and equal perpendicular height. Therefore the area of both is same.



Now since $AD > AQ$
and $BC > BP$
$\therefore 2(AB + BC) > 2(AB + BP)$
$\therefore$ Perimeter of parallelogram > perimeter of rectangle
$\Rightarrow k < 1$

51. (a) $\frac{BE}{AB} = \sin 30° = \frac{1}{2}$

$\therefore \quad BE = \frac{1}{2} \times AB = 6\text{ cm} = CF$

and $\frac{CF}{DF} = \tan 45° = 1$

$\therefore \quad DF = CF = 6\text{ cm}$

and $\frac{AE}{AB} = \cos 30° = \frac{\sqrt{3}}{2}$

$\therefore \quad AE = 6\sqrt{3}\text{cm}$

$\therefore \quad AD = AE + EF + FD = 6\sqrt{3} + 6 + 6$

$= 6(2 + \sqrt{3})\text{ cm}$

Area of trapezium ADCB

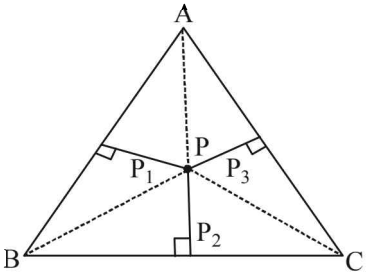
$= \frac{1}{2} \times (AD + BC) \times BE$

$= \frac{1}{2} \times [6(2 + \sqrt{3}) + 6] \times 6$

$= \frac{6}{2}(2 + \sqrt{3} + 1) \times 6 = 18 \times (3 + \sqrt{3})$

$= 18(3 + \sqrt{3})\text{ cm}^2$

52. (b)



Here equilateral triangle
$AB = BC = AC = a$
Area of equilateral triangle = Sum of area of triangle (APB) + BPC + APC

$\Rightarrow \frac{\sqrt{3}}{4}(a)^2 = \frac{1}{2} \times a(P_1 + P_2 + P_3)$

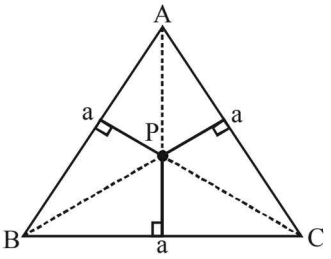
$\Rightarrow P_1 + P_2 + P_3 = \frac{\sqrt{3}}{2} a$

53. (c) We know that median divides triangle in equal area and meeting point of median are called centroid.

54. (d) When we join the mid point of a quadilateral ABCD then it form a rectangle that ABCD must be rhombus, square and kite.

55. (a) For similarty of triangles we have SSS criteria. So S_1 in true.
But for polygon : two polygon to be similar if the corresponding sides are in same ratio then corresponding angle must be same. So S_2 is not correct.

56. (c)

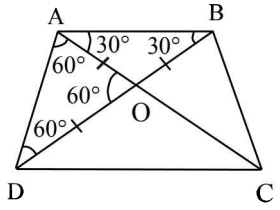


$a = 2\sqrt{3}$ cm

$$\frac{1}{2}a(x+y+z) = \frac{\sqrt{3}}{4}a^2$$

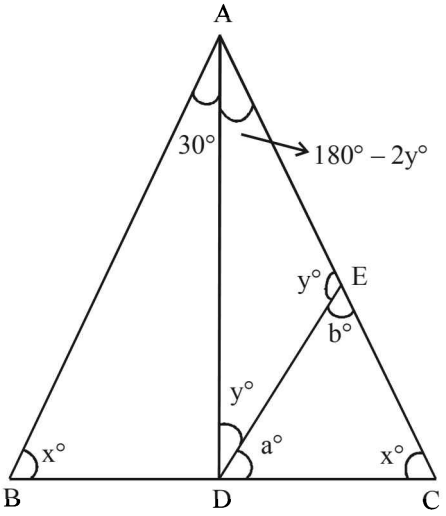
$$x+y+z = \frac{\sqrt{3}}{4}a = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3 \text{ cm}$$

57. (c) OA = OB (Given)
$\therefore$ $\angle OAB = \angle OBA$ (opp. angle of equal side is equal)
$\angle OAB = \angle OBA = 30°$
In ΔOAB
$\angle OAB + \angle AOB + \angle OBA = 180°$
$30° + \angle AOB + 30° = 180°$
$\angle AOB = 120°$
As DOB is a straight line



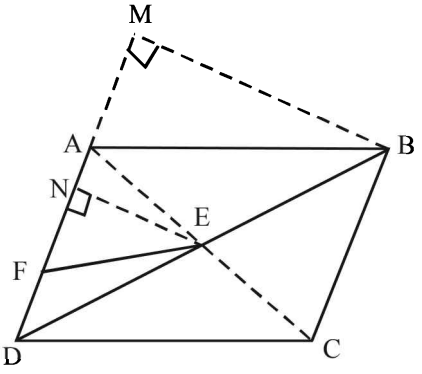
$\therefore$ $\angle DOB = 180°$
$\angle DOA + \angle AOB = 180°$
$\angle DOA = 60°$
Now, in ΔAOD
$\angle ODA + \angle DOA + \angle DAO = 180°$
$2\angle ODA + 60° = 180°$ [$\because \angle ODA = \angle DOA$ as OA = OD]
$2\angle ODA = 120°$
$\angle ODA = 60°$

58. (b)



AB = AC
$\Rightarrow \angle B = \angle C = x°$
AD = AE
$\Rightarrow \angle D = \angle E = y°$
In ΔDEC < y is external angle
$\Rightarrow y° = a + x° \Rightarrow a° = y - x°$
In ΔABC
$2x° + 30° + 180° - 2y° = 180°$
$2(y - x) = 30°$
$y - x = 15° = a°$
$\therefore$ $\angle EDC = 15°$

59. (c)



GP_3386

As, $\Delta DEN \sim \Delta DBM$

$\therefore$ $BM = 2EN$

$$ar(\Delta DFE) = \frac{1}{2} DF \times EN$$

$$= \frac{1}{2}\left(\frac{1}{3} DA\right) \times \frac{1}{2} BM$$

$$= \frac{1}{12}(DA \times BM)$$

$$ar(\Delta DFE) : ar(\Delta ABD)$$

$$= \frac{1}{12}(DA \times BM) : \frac{1}{2}(DA \times BM) = 1 : 6$$

$ar(\Delta ABD) = 6(\Delta DEF)$

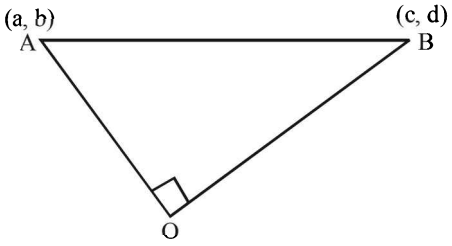
As, $ar(\Delta ABD) = ar(\Delta DFE) + ar(ABEF)$

$6(\Delta DEF) = ar(\Delta DFE) + ar(ABEF)$

$5[ar(\Delta DFE)] = ar(ABEF)$

$$\therefore \quad \frac{ar(\Delta DFE)}{ar(ABEF)} = \frac{1}{5}$$

60. (a)



Let A = (a, b)

B = (c, d)

In right ΔAOB,

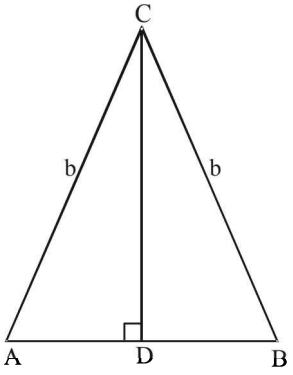
$OA^2 + OB^2 = AB^2$

$a^2 + b^2 + c^2 + a^2 = (a - c)^2 + (b - d)^2$

$\Rightarrow a^2 + b^2 + c^2 + d^2 = a^2 + c^2 - 2ca + b^2 + d^2 - 2bd$

$\Rightarrow 2(ac - bd) = 0 \Rightarrow ac - bd = 0$

61. (c) Let ABC be an isosceles triangle, where base AB = a and equal sides $AC = BC = b$. Let CD be the perpendicular on AB.



So, $AD = DB = \frac{1}{2} AB = \frac{a}{2}$

Altitude, CD = height of the ΔABC is given by

$$h = \sqrt{AC^2 - AD^2}$$

$$\Rightarrow h = \frac{1}{2}\sqrt{4b^2 - a^2}$$

Area of the $\Delta ABC = \frac{1}{2}$ base $\times$ altitude

$$= \frac{1}{2} \times a \times \frac{1}{2}\sqrt{4b^2 - a^2} = \frac{a}{4}\sqrt{4b^2 - a^2}.$$

CHAPTER

20

Geometry-II: Triangles

(Including Similarity and Congruence)

TRIANGLE

A triangle is a convex polygon having three sides.
A triangle is represented by the symbol Δ.
Triangles can be classified on the basis of their sides or angles.

On the basis of sides, triangles are of the following types:

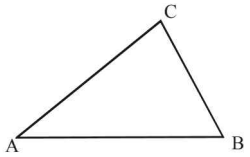
(a) **Equilateral triangle:** All the three sides are equal
(b) **Isosceles triangle:** Two sides are equal
(c) **Scalene triangle:** All the three sides are unequal.

On the basis of angles, triangles are of the following types:

(a) **Acute angled triangle:** Each interior angle is less than 90°.
(b) **Right angled triangle:** One of the interior angle is equal 90°.
(c) **Obtuse angled triangle:** One of the interior angle is more than 90°.

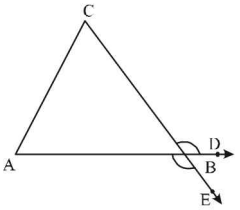
Basic Properties and Some Important Theorems of Triangles

1. Sum of measures of the interior angles of a triangle is 180°.



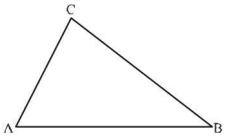
In ΔABC, $\angle CAB + \angle ABC + \angle ACB = 180°$
or $\angle A + \angle B + \angle C = 180°$

2. The exterior angle of a triangle is equal to the sum of the opposite (not adjacent) interior angles



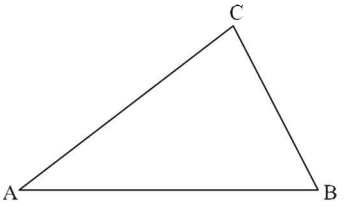
In ΔABC, $\angle CBD = \angle A + \angle C = \angle ABE$

3. Sum of the lengths of any two sides of a triangle is greater than the length of the third side.



(i) $AB + AC > BC$
(ii) $AC + BC > AB$
(iii) $AB + BC > AC$

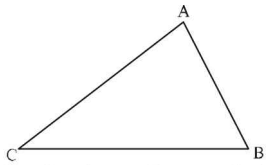
4. Difference between the lengths of any two sides of a triangle is smaller than the length of the third side.



(i) $|AB - BC| < AC$
(ii) $|AC - AB| < BC$
(iii) $|AC - BC| < AB$

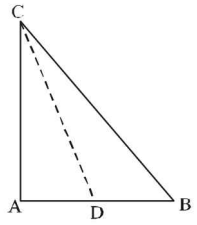
5. In any triangle, side opposite to greatest angle is largest and side opposite to smallest angle is smallest.

GP_3386



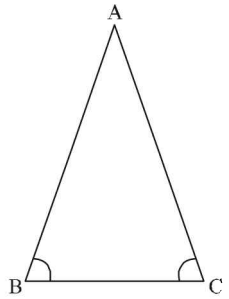
In ΔABC, if ∠A > ∠B > ∠C, then *BC* is the largest side and *AB* is the smallest side.

6. In any triangle, line joining any vertex to the mid point of its opposite side is called a **median** of the opposite side of the triangle.



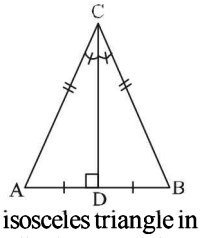
In ΔABC, D is the mid point of AB
Hence, CD is a median of ΔABC.
A triangle can have 3 medians.
Any median of a triangle divides the triangle into two triangles of equal areas.

7. Sides opposite to equal angles in a triangle are equal.



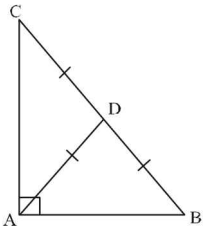
In ΔABC, $\angle B = \angle C \therefore AB = AC$
Converse of this property is also true.

8. In an isosceles triangle, if a perpendicular is drawn to unequal side from its opposite vertex, then
(a) The perpendicular is the median
(b) The perpendicular bisects the vertex angle.



ΔABC is an isosceles triangle in which AC = BC.
CD is perpendicular to AB, hence CD is a median and ∠ACD = ∠BCD

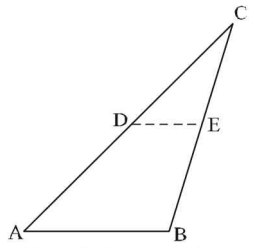
9. In a right angled triangle, the line joining the vertex of the right angle to the mid point of the hypotenuse is half the length of the hypotenuse.



In ΔABC, ∠BAC = 90° and D is the mid point of BC, then

$$AD = \frac{1}{2}BC = BD = CD$$

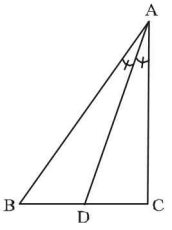
10. **Mid-point theorem:** In any triangle, line segment joining the mid points of any two sides is parallel to the third side and equal to half of the length of third side.



In ΔABC, D and E are mid points of sides *AC* and *BC*, then *DE* is parallel to *AB* i.e. $DE \parallel AB$ and $DE = \frac{1}{2}AB$

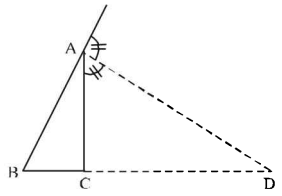
11. **Angle Bisector Theorem:** Bisector of an angle (internal or external) of a triangle divides the opposite side (internally or externally) in the ratio of the sides containing the angle.

For example:



In figure AD is the bisector of exterior ∠BAC

$$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$$

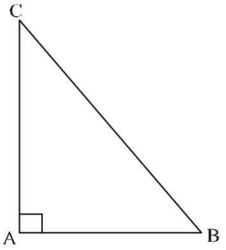


In figure AD is the bisector of exterior ∠BAC.

$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$

Converse of the angle bisector theorem is also true.

12. **Pythagoras Theorem:** In a right angled triangle, Square of longest or hypotenuse = Sum of square of other two sides.

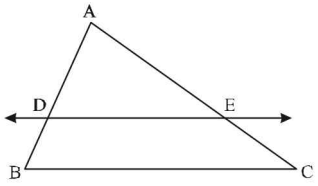


In figure ΔABC is a triangle right angled at A.

$\therefore \ (BC)^2 = (AB)^2 + (AC)^2$

Converse of this theorem is also true.

13. **Basic Proportionality Theorem (BPT):** If a line is drawn parallel to one side of a triangle which intersects the other two sides in distinct points, the other two sides are divided in the same ratio.



In ΔABC, DE || BC,

Then, $\frac{AD}{DB} = \frac{AE}{EC}$

This theorem is also known as Thales theorem.

Converse of this theorem is also true.

Illustration – 1

In a triangle ABC, ∠A = x, ∠B = y, and ∠C = y + 20. If 4x – y = 10, then the triangle is :

(a) Right-angled (b) Obtuse-angled
(c) Equilateral (d) None of these

Solution : (a) We have, x + y + (y + 20) = 180

or x + 2y = 160 ...(1)

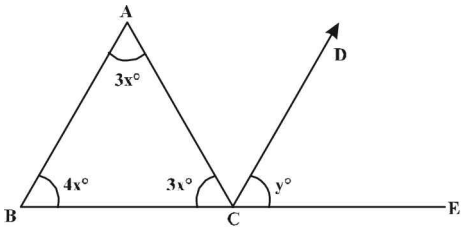
and 4x – y = 10 ...(2)

From (1) and (2), y = 70, x = 20

Angles of the triangles are 20°, 70°, 90°. Hence the triangle is a right angled.

Illustration – 2

In the given figure, CD || AB. Find *y*.



(a) 79° (b) 72° (c) 74° (d) 77°

Solution : (b) In ΔABC,

∠ABC + ∠BCA + ∠CAB = 180°

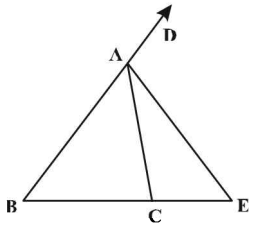
⇒ $4x + 3x + 3x = 180° \Rightarrow 10x = 180° \Rightarrow x = 18°$

Now, ∠ABC = ∠DCE (corresponding angles are equal)

⇒ $\angle DCE = 4x° \Rightarrow y = 4 \times 18° = 72°$

Illustration – 3

In the adjoining figure, AE is the bisector of exterior ∠*CAD* meeting *BC* produced in *E*. If *AB* = 10 cm, *AC* = 6 cm and *BC* = 12 cm, then *CE* is equal to



(a) 6 cm (b) 12 cm (c) 18 cm (d) 20 cm

Solution : (c) $\frac{BE}{CE} = \frac{AB}{AC}$ as AE is an exterior angle bisector.

Let CE = *x*, BE = BC + EC = 12 + *x*

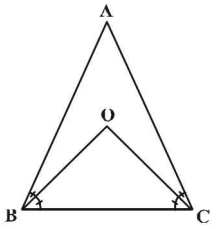
⇒ $\frac{12 + x}{x} = \frac{10}{6} \Rightarrow (12 + x)\,6 = 10x$

⇒ $72 + 6x = 10x \Rightarrow 4x = 72 \Rightarrow x = 18$ cm

GP_3386

Illustration – 4

OB and OC are respectively the bisectors of ∠ABC and ∠ACB. Then, ∠BOC is equal to

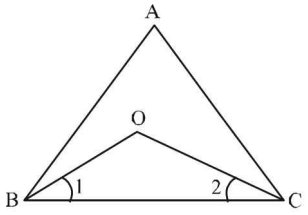


(a) $90° - \frac{1}{2}\angle A$ **(b)** $90° + \angle A$

(c) $90° + \frac{1}{2}\angle A$ **(d)** $180° - \frac{1}{2}\angle A$

Solution : (c) In ΔBOC,

$\angle 1 + \angle 2 + \angle BOC = 180° \Rightarrow \angle A + \angle B + \angle C = 180°$.... (1)



$$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90°$$

$$\Rightarrow \frac{1}{2}(\angle A) + \angle 1 + \angle 2 = 90°$$

$$\Rightarrow \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$$

Put $\angle 1 + \angle 2$ in Eq. (1), we get

$$\angle BOC = 180° - \left(90° - \frac{1}{2}\angle A\right) = 90° + \frac{1}{2}\angle A$$

Basic Pythagorean Triplets

A Pythagorean triplet is a set of three natural numbers a, b and c, which are length of the sides of a right angled triangle.

Hence, if $a^2 + b^2 = c^2$, $b^2 + c^2 = a^2$ or $c^2 + a^2 = b^2$, then the set of natural numbers a, b and c is a Pythagorean triplet.

Since $3^2 + 4^2 = 5^2$, hence 3, 4, 5 form a Pythagorean triplet.

General Rule To Find Pythagorean Triplet: If r and s are two natural numbers such that $r > s$, $r - s$ is odd and GCD of r and s is 1, then the Pythagorean triplet a, b, c are defined by

$a = r^2 - s^2$, $b = 2rs$ and $c = r^2 + s^2$.

NOTE: If each term of any Pythagorean triplet is multiplied or divided by such a positive number that the products or quotients obtained respectively are natural numbers then the new products or quotients are also form Pythagorean triplets.
Since 3, 4, 5 form a Pythagorean triplet, therefore 9, 12 and 15 also form a Pythagorean triplet.

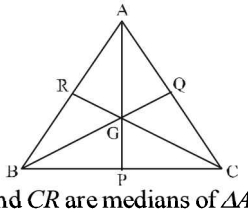
Determination of Nature of Triangle

Let length of three sides of a triangle are a, b and c.

- If c be the length of longest side and $c^2 = a^2 + b^2$, then the triangle is right-angled triangle.
- If c be the length of longest side and $c^2 > a^2 + b^2$, then the triangle is an obtuse-angled triangle.
- If c be the length of longest side and $c^2 < a^2 + b^2$, then the triangle is an acute-angled triangle.

Important Terms Related to a Triangle

1. **Medians and Centroid:** We know that a line segment joining the mid point of a side of a triangle to its opposite vertex is called a median.



AP, *BQ* and *CR* are medians of ΔABC where *P*, *Q* and *R* are mid points of sides BC, CA and AB respectively.

(i) Three medians of a triangle on concurrent. The point of concurrent of three medians is called Centroid of the triangle denoted by G.

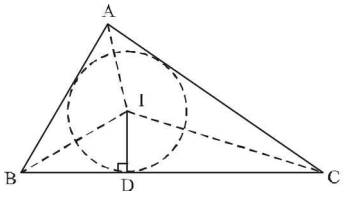
(ii) Centroid of the triangle divides each median in the ratio 2 : 1.
i.e. $AG : GP = BG : GQ = CG : GR = 2 : 1$, where G is the centroid of ΔABC.

2. **Angle Bisectors and Incentre:** Lines bisecting the interior angles of a triangle are called angle bisectors of triangle.

(i) Angle bisectors of a triangle are concurrent and the point of concurrency is called Incentre of the triangle, denoted by I.

(ii) With I as centre and radius equal to length of the perpendicular drawn from I to any side, a circle can be drawn touching the three sides of the triangle. So this is called incircle of the triangle. Incentre is equidistant from all the sides of the triangle.

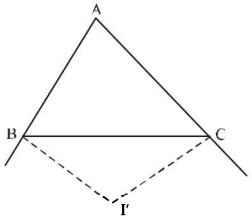
(iii) Angle formed by any side at the incentre is always 90° more than half the vertex angle opposite to the side.



In figure AI, BI, CI are angle bisectors of ΔABC. Hence I is the incentre of the ΔABC and

$$\angle BIC = 90° + \frac{1}{2}\angle A, \angle AIC = 90° + \frac{1}{2}\angle B$$

and $\angle AIB = 90° + \frac{1}{2}\angle C$

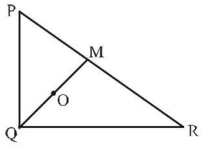


If *BI'* and CI' be the angle bisectors of exterior angles at B and C, then

$$\angle BI'C = 90° - \frac{1}{2}\angle A$$

Illustration – 5

If in the given figure ∠PQR = 90°, O is the centroid of ΔPQR, PQ = 5 cm and QR = 12 cm, then OQ is equal to



(a) $3\frac{1}{2}$cm **(b)** $4\frac{1}{3}$cm

(c) $4\frac{1}{2}$cm **(d)** $5\frac{1}{3}$cm

Solution : (b) By Pythagoras theorem,

$PR = \sqrt{PQ^2 + QR^2} = \sqrt{5^2 + 12^2} = 13$ cm

∴ O is centroid ⇒ QM is median and M is mid-point of PR.

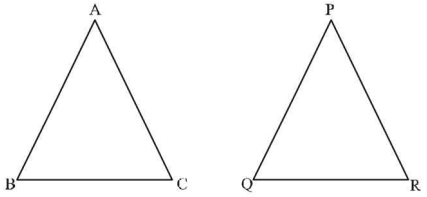
$$QM = PM = \frac{13}{2}$$

∴ Centroid divides median in ratio 2 : 1.

$$\therefore OQ = \frac{2}{3}QM = \frac{2}{3}\times\frac{13}{2} = \frac{13}{3} \qquad \therefore OQ = 4\frac{1}{3}\text{ cm}$$

CONGRUENCY OF TWO TRIANGLES

Two triangles are congruent if they are of the same shape and size i.e .if any one of them can be made to superpose on the other it will cover exactly.



If two triangles *ABC* and *PQR* are congruent then 6 elements (i.e. three sides and three angles) of one triangle are equal to corresponding 6 elements of other triangle.

(i) ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R

(ii) AB = PQ, BC = QR, AC = PR

This is symbolically written as ΔABC ≅ ΔPQR

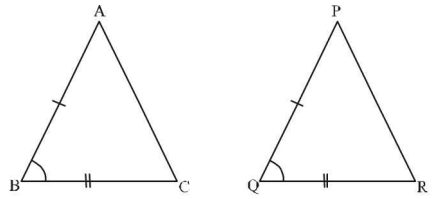
NOTE: In two congruent triangles, sides opposite to equal angles are corresponding sides and angles opposite to equal sides are corresponding angles.

Conditions of Congruency

There are 4 conditions of congruency of two triangles.

1. **SAS (Side-Angle-Side) Congruency:** If two sides and the included angle between these two sides of one triangle is equal to corresponding

two sides and included angle between these two sides of another triangle, then the two triangles are congruent.



In ΔABC and ΔPQR

$AB = PQ$,

$BC = QR$

and $\angle ABC = \angle PQR$

$\therefore$ $\Delta ABC \cong \Delta PQR$ [by SAS congruency]

Here $\cong$ is the sign of congruency.

2. **ASA (Angle-Side-Angle) Congruency:** If two angles and included side between these two angles of one triangle are equal to corresponding angles and included side between these two angles of another triangle, then two triangles are congruent.

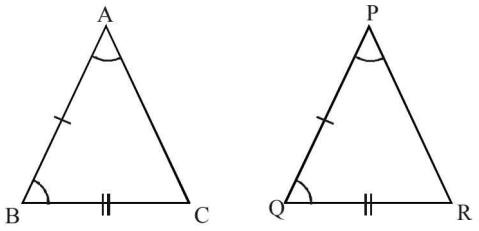
In ΔABC and ΔPQR

$\angle A = \angle P$

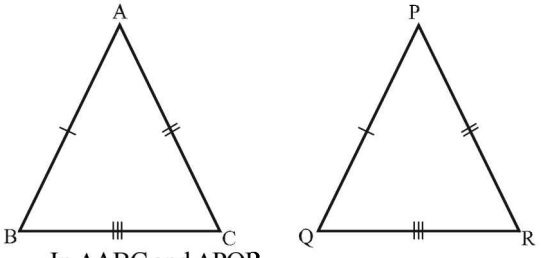
$\angle B = \angle Q$

$AB = PQ$

$\therefore \Delta ABC \cong \angle PQR$ [by ASA congruency]



3. **SSS (Side-Side-Side) Congruency:** If three sides of one triangle are equal to corresponding three sides of another triangle, the two triangles are congruent.



In ΔABC and ΔPQR

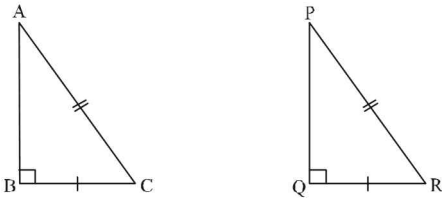
AB = PQ

BC = QR

CA = RP

$\therefore$ $\Delta ABC \cong \Delta PQR$ [by SSS congruency]

4. **RHS (Right angle-Hypotenuse-Side) Congruency:** Two right angled triangles are congruent to each other if hypotenuse and one side of one triangle are equal to hypotenuse and corresponding side of another triangle.



In ΔABC and ΔPQR

$\angle ABC = \angle PQR = 90°$

AC = PR

BC = QR

$\therefore$ $\Delta ABC \cong \Delta PQR$ [by RHS congruency]

SIMILARITY OF TWO TRIANGLES

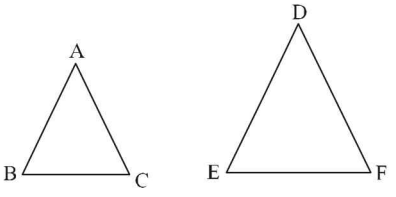
Two triangles are said to be similar, if their shapes are the same but their size may or may not be equal.

When two triangles are similar, then

(i) all the corresponding angles are equal and

(ii) all the corresponding sides are in the same ratio (or proportion)

NOTE: In two similar triangles, sides opposite to equal angles are called corresponding sides. And angles opposite to side proportional to each other are called corresponding angles.



If ΔABC and ΔDEF are similar, then

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

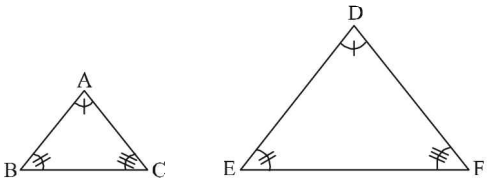
ΔABC ~ ΔDEF, read as triangle ABC is similar to triangle DEF. Here ~ is the sign of similarity.

Conditions of Similarity

There are 4 conditions of similarity.

1. **AAA (Angle–Angle–Angle) Similarity:** Two triangles are said to be similar, if their all corresponding angles are equal.

For example:



In ΔABC and ΔDEF, if

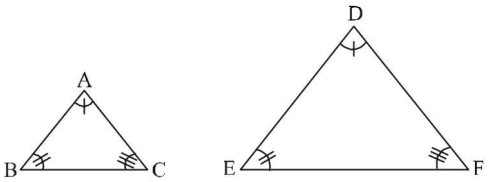
$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

Then ΔABC ~ ΔDEF [By AAA Similarity]

Corollary AA (Angle-Angle) Similarity: If two angles of one triangle are respectively equal to two angles of another triangles, then two triangles are similar.



In ΔABC and ΔDEF, if

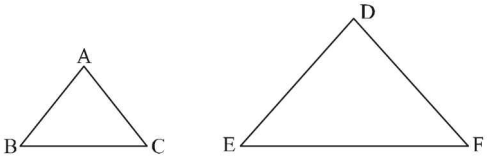
$\angle A = \angle D$

$\angle B = \angle E$

then ΔABC ~ ΔDEF [By AA Similarity]

2. **SSS (Side–Side–Side) Similarity:** Two triangles are said to be similar, if sides of one triangle are proportional (or in the same ratio of) to the sides of the other triangle:

For example:



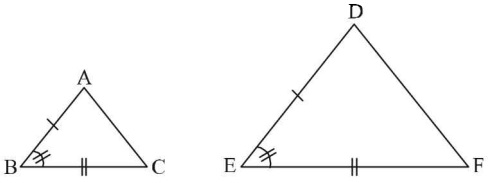
In ΔABC and ΔDEF, if

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$

Then ΔABC ~ ΔDEF [By SSS Similarity]

3. **SAS (Side–Angle–Side) Similarity:** Two triangles are said to be similar if two sides of a triangle are proportional to the two sides of the other triangle and the angles included between these sides of two triangles are equal.

For example:



In ΔABC and ΔDEF, if

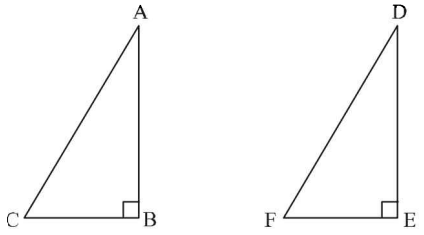
$$\frac{AB}{DE} = \frac{BC}{EF}$$

and $\angle B = \angle E$

Then, ΔABC ~ ΔDEF [By SAS Similarity]

4. **RHS (Right angle-Hypotenuse-Side) Similarity:** Two triangles are said to be similar if one angle of both triangle is right angle and hypotenuse of both triangles are proportional to any one other side of both triangles respectively.

For example:



In ΔABC and ΔDEF, if

$\angle B = \angle E [= 90°]$

$$\frac{AC}{DF} = \frac{AB}{DE}$$

Then DABC ~ DDEF [By RHS similarity]

NOTE: In similar triangles,
Ratio of medians = Ratio of corresponding heights
= Ratio of circumeradii
= Ratio of in radii

Theorem

If two triangles are similar, then ratio of areas of two similar triangle is equal to the ratio of square of corresponding sides.

GP_3386

Illustration – 6

D and E are the points on the sides AB and AC respectively of a ΔABC and AD = 8 cm, DB = 12 cm, AE = 6 cm and EC = 9 cm, then BC is equal to

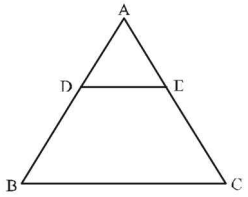
(a) $\frac{2}{5}DE$ **(b)** $\frac{5}{2}DE$

(c) $\frac{3}{2}DE$ **(d)** $\frac{2}{3}DE$

Solution : (b) As in ΔADE and ΔABC

$\frac{AD}{AB}=\frac{8}{20}=\frac{2}{5}, \frac{AE}{AC}=\frac{6}{15}=\frac{2}{5}$

So, $\frac{AD}{AB}=\frac{AE}{EC}$



and $\angle A = \angle A$ (common)

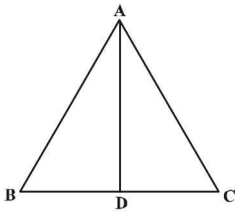
$\Delta ADE \sim \Delta ABC$

$\therefore \quad \frac{DE}{BC}=\frac{AD}{AB} \Rightarrow \frac{DE}{BC}=\frac{2}{5}$

$\Rightarrow BC=\frac{5}{2}DE$

Illustration – 7

In a right angled ΔABC in which ∠A = 90°. If AD ⊥ BC, then the correct statement is



(a) $AB^2 = BD \times DC$ **(b)** $AB^2 = BD \times AD$

(c) $AB^2 = BC \times DC$ **(d)** $AB^2 = BC \times BD$

Solution : (d) Clearly, $\Delta ABD \sim \Delta CBA$

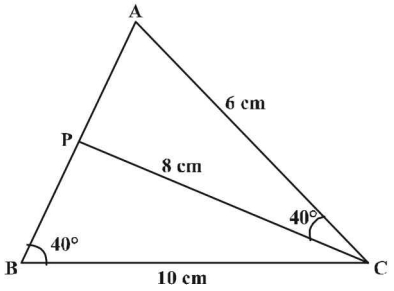
$\Rightarrow \quad \frac{AB}{BD}=\frac{CB}{BA}$

$\Rightarrow \quad AB^2 = BC \times BD$

Illustration – 8

From the adjoining diagram, calculate

(i) AB **(ii) AP**



Solution : In ΔAPC and ΔABC,

$\angle ACP = \angle ABC$ & $\angle A = \angle A$

$\Rightarrow \quad \Delta ACP \sim \Delta ABC \Rightarrow \frac{AP}{AC}=\frac{PC}{BC}=\frac{AC}{AB}$

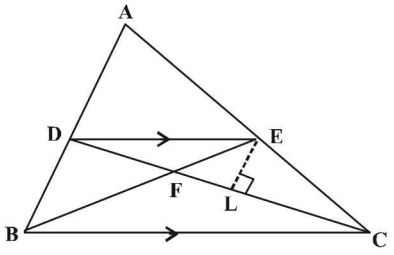
$\therefore \quad \frac{AP}{6}=\frac{8}{10}=\frac{6}{AB}$

$\Rightarrow \quad AP = 6\times\frac{8}{10}=4.8$ and $AB=\frac{60}{8}=7.5$

$\Rightarrow$ AP = 4.8 cm and AB = 7.5 cm

Illustration – 9

In the adjoining figure, DE || BC and AD : DB = 4 : 3



Find $\frac{AD}{AB}$ and then $\frac{DE}{BC}$

Solution : Since the sides of similar triangles are proportional, we have $\frac{AD}{AB}=\frac{DE}{BC}$

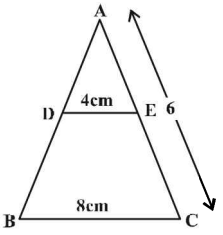
But, $\frac{AD}{DB}=\frac{4}{3} \Rightarrow \frac{AD}{AD+DB}=\frac{4}{4+3} \Rightarrow \frac{AD}{AB}=\frac{4}{7}$

$\therefore \quad \frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{7}$

Illustration – 10

In the given figure, DE parallel to BC. If AD = 2 cm, DB = 3 cm and AC = 6 cm, then AE is

(a) 2.4 cm **(b) 1.2 cm**
(c) 3.4 cm **(d) 4.8 cm**



Solution : (a)

The triangles *ADE* and *ABC* are similar.

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{2}{5} = \frac{AE}{6}$

$\Rightarrow AE = \frac{12}{5} = 2.4\text{cm}$

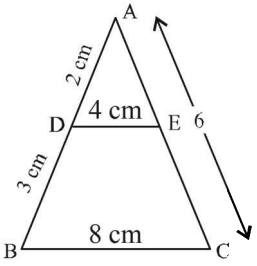
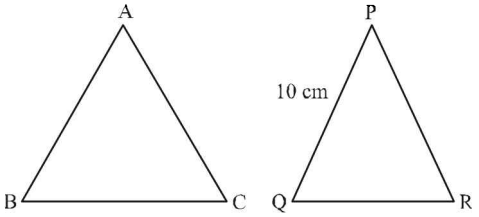


Illustration – 11

The perimeters of two similar triangles ABC and PQR are 36 cm, and 24 cm, respectively. If PQ = 10 cm, then the length of AB is :

(a) 16 cm **(b) 12 cm**
(c) 14 cm **(d) 15 cm**

Solution : (d)



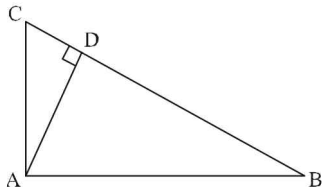
ΔABC and ΔPQR are similar.

$\frac{AB}{PQ} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} \Rightarrow \frac{AB}{PQ} = \frac{36}{24}$

$\Rightarrow AB = 15$

IMPORTANT POINTS

1. In ΔABC right angled at A, if AD is perpendicular to BC.



$\Delta ABC \sim \Delta DBA \sim \Delta DAC$

Now $\Delta ABC \sim \Delta DBA$

$\Rightarrow \frac{AB}{BC} = \frac{DB}{BA} \Rightarrow AB^2 = DB \times BC$

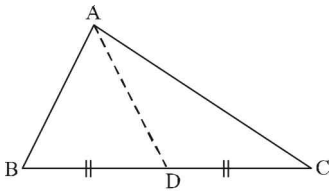
And $\Delta ABC \sim \Delta DAC$

$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC} \Rightarrow AC^2 = DC \times BC$

And $\Delta DBA \sim \Delta DAC$

$\Rightarrow \frac{DA}{DB} = \frac{DC}{DA} \Rightarrow DA^2 = DB \times DC$

2. In a triangle, the sum of the square of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side.

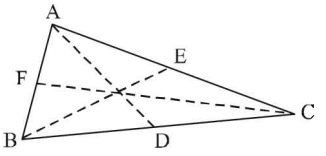


In the figure, AD is the median.

$\therefore \quad AB^2 + AC^2 = 2\left[AD^2 + \left(\frac{BC}{2}\right)^2\right]$

3. In a triangle,

$3 \times \begin{pmatrix}\text{Sum of square of}\\ \text{three sides of}\\ \text{a triangle}\end{pmatrix} = 4 \times \begin{pmatrix}\text{Sum of the square of}\\ \text{three medians of}\\ \text{the triangle}\end{pmatrix}$

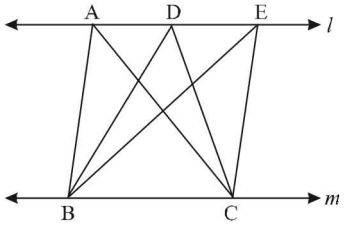


GP_3386

In figure AD, BE and CF are medians of ΔABC.

$\therefore$ $3 \times (AB^2 + BC^2 + CA^2) = 4 \times (AD^2 + BE^2 + CF^2)$

4. Area of triangles on the same base and lie between the same pair of parallel lines are equal.



In the figure, ΔABC, ΔDBC and ΔEBC are on the same base BC and lie between the same pair of parallel lines l and m.

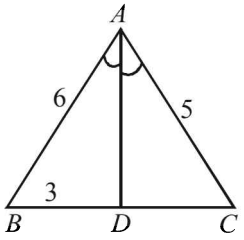
$\therefore$ area of ΔABC = area of ΔDBC = area of ΔEBC.

5. (a) In an equilateral triangle centroid, incentre, circumcentre, orthocentre coincide at the same point.

(b) Circumradius = 2 × in radius

LEVEL - 1

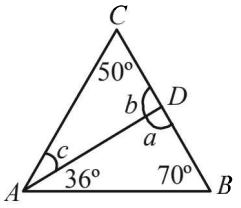
1. In the given figrue, AD is the bisector of $\angle BAC$, $AB = 6$ cm, $AC = 5$ cm and $BD = 3$ cm. Find DC.



(a) 11.3 cm (b) 2.5 cm
(c) 3.5 cm (d) 4 cm

2. A point P is 26 cm away form the centre O of a circle and the length PT of the tangent draw from P to the circle is 10cm. Find radius of the circle
(a) 24 cm (b) 32 cm
(c) 22 cm (d) 42 cm

3. Given the adjoining figure. Find a, b, c

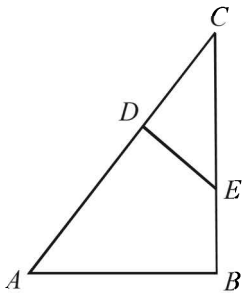


(a) 74°, 106°, 24° (b) 90°, 20°, 24°
(c) 60°, 30°, 24° (d) 106°, 24°, 74°

4. The centroid, circumcenter, orthocenter in a triangle–
(a) are always coincident.
(b) are always collinear.
(c) are always the inside the triangular area.
(d) always coincide in a equilateral triangle and otherwise collinear.

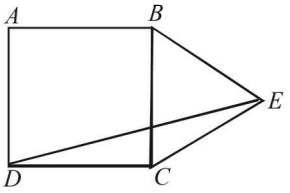
5. Which one of the following cannot be the ratio of angles in a right angled triangle?
(a) 1 : 2 : 3 (b) 1 : 1 : 2
(c) 1 : 3 : 6 (d) None of these

6. ABC and CDE are right angled triangle. $\angle ABC = \angle CDE = 90°$. D lies on AC and E lies on BC. $AB = 24$ cm, $BC = 60$ cm. If $DE = 10$ cm, then CD is:



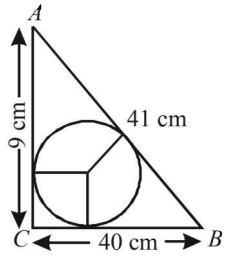
(a) 28 cm (b) 35 cm
(c) 25 cm (d) can't be determined

7. If $ABCD$ is a square and BCE is an equilateral triangle, what is the measure of the angle DEC?



(a) 15° (b) 30°
(c) 20° (d) 45°

8. What is the inradius of the incircle shown in the figure?



(a) 9 cm
(b) 4 cm
(c) can't be determined
(d) None of these

9. The sides of a triangle are in the ratio of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is

(a) 9 cm (b) 10 cm
(c) 11 cm (d) 12 cm

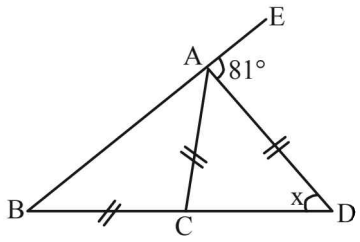
10. ΔABC has sides AB, AC measuring 2001 and 1002 units respectively. How many such triangles are possible with all integral sides?
(a) 2001 (b) 1002
(c) 2003 (d) 1004

11. a, b and c are sides of a triangle . If $a^2 + b^2 + c^2 = ab + bc + ac$ then the triangle will be
(a) equilateral (b) isosceles (c) right angled (d) obtuse angle

12. Two equal circles of radius 4 cm intersect each other such that each passes through the centre of the other. The length of common chord is.
(a) $2\sqrt{3}$ cm (b) $4\sqrt{3}$ cm
(c) $2\sqrt{2}$ cm (d) 8 cm

13. The radius of the incircle of triangle when sides are 18, 24 and 3 cms is
(a) 2 cm. (b) 4 cm.
(c) 6 cm. (d) 9 cm.

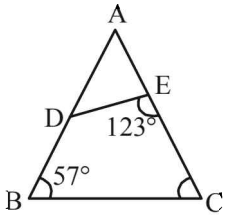
14. In the given figure, BC = AC = AD and ∠EAD = 81°. Find the value of x :



(a) 45° (b) 54°
(c) 63° (d) 36°

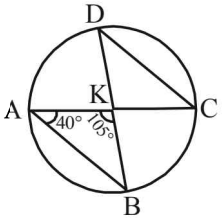
15. If the angles of triangle are 60°, 90° and 30°, then what is the ratio of the sides opposite to these angles?
(a) $\sqrt{3} : \sqrt{2} : 1$ (b) $1 : \sqrt{2} : 2$
(c) $2 : \sqrt{3} : 1$ (d) $\sqrt{3} : 2 : 1$

16. In the figure AD = 12 cm, AB = 20 cm and AE = 10 cm. Find EC :



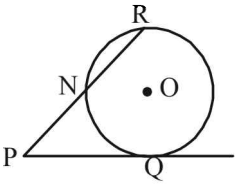
(a) 14 cm (b) 10 cm
(c) 8 cm (d) 15 cm

17. In the given figure, ∠CAB = 40° and ∠AKB = 105°. Find ∠KCD :



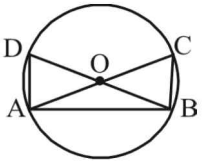
(a) 65° (b) 35°
(c) 40° (d) 72°

18. In the given figure, PQ is the tangent of the circle. Line segment PR intersects the circle at N and R. PQ = 15 cm, PR = 25 cm, find PN :



(a) 15 cm (b) 10 cm
(c) 9 cm (d) 6 cm

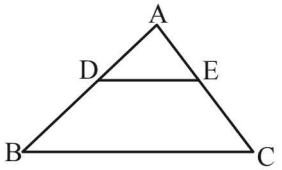
19. In the given figure O is the centre of the circle and ∠BAC = 25°, then the value of ∠ADB is :



(a) 40° (b) 55°
(c) 50° (d) 65°

20. The areas of the similar triangles are in the ratio of 25 : 36. What is the ratio of their respective heights?
(a) 5 : 6 (b) 6 : 5
(c) 1 : 11 (d) 2 : 3

21. In ΔABC, AC = 5 cm. Calculate the length of AE where DE ∥ BC. Given that AD = 3 cm and BD = 7 cm :



(a) 2 cm (b) 1 cm
(c) 1.5 cm (d) 2.5 cm

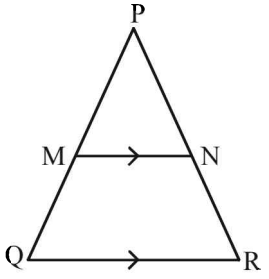
22. In a triangle PQR, PQ = 20 cm and PR = 6 cm, the side QR is :
(a) equal to 14 cm
(b) less than 14 cm
(c) greater than 14 cm
(d) None of these

23. The incentre of a triangle is determined by the :
(a) medians
(b) angle bisector
(c) perpendicular bisectors
(d) altitudes

24. The circumcentre of a triangle is determined by the:
(a) altitudes
(b) median
(c) perpendicular bisector
(d) angle bisector

25. E and F are points on the sides PQ and PR respectively of a ΔPQR. In which of the following options is EF || QR?
(a) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm
(b) PE = 4 cm, EQ = 4.5 cm, PF = 8 cm, FR = 9 cm
(c) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.8 cm, PF = 0.52 cm
(d) Both (b) and (c)

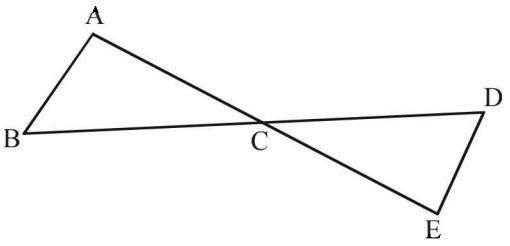
26. In the triangle PQR, MN || QR and MN divides the triangle into two parts of equal areas, then $\frac{QM}{PQ}$



(a) $\frac{1}{\sqrt{2}+1}$ (b) $\frac{2-\sqrt{2}}{2}$

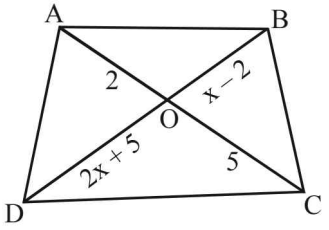
(c) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}-1}$

27. In the figure, triangle ABC is similar to triangle EDC:



If we have AB = 4 cm, ED = 3 cm, CE = 4-2 cm and CD = 4-8 cm, then the values of CA and CB respectively are
(a) 6 cm, 6.6 cm (b) 4.8 cm, 6.6 cm
(c) 5.4 cm, 6.4 cm (d) 5.6 cm, 6.4 cm

28. In a given figure in trapazium ABCD if AB || CD, then value of x is:



(a) $\frac{29}{8}$ (b) $\frac{8}{29}$

(c) 20 (d) $\frac{1}{20}$

29. $\Delta ABC \sim \Delta PQR$ and $\frac{\text{area } \Delta ABC}{\text{area } \Delta PQR} = \frac{16}{9}$ If PQ = 18 cm and BC = 12 cm. then AB and QR are respectively
(a) 9 cm, 24 cm (b) 24 cm, 9 cm
(c) 32 cm, 6.75 cm (d) 13.5 cm, 16 cm

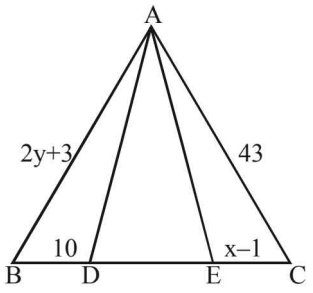
30. E and F are respectively, the mid points of the sides AB and AC of Δ ABC and the area of the quadrilateral BEFC is k times the area of Δ ABC. The value of k is

(a) $\frac{1}{2}$ (b) 3

(c) $\frac{3}{4}$ (d) 4

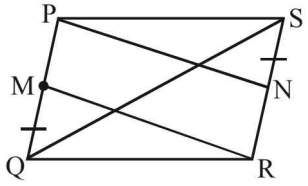
GP_3386

31. In the given figure, AD = AE and ∠BAD = ∠EAC, then



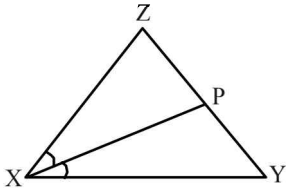
(a) x = 11 (b) x = 13
(c) y = 21 (d) y = 11

32. PQRS is a parallelogram and M, N are the mid-points of PQ and RS respectively. Which of the following is not true ?



(a) RM trisects QS (b) PN trisects QS
(c) ΔPSN ~ Δ QMR (d) MS is not parallel to QN

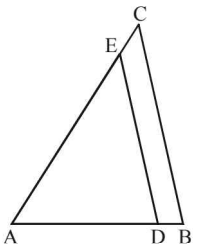
33. In a ΔXYZ, if the internal bisector of ∠X meets YZ at 'P', then ....



(a) $\frac{XY+XZ}{XZ}=\frac{YZ}{PZ}$ (b) $\frac{XY}{PZ}=\frac{XZ}{YP}$

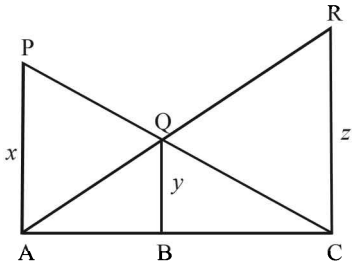
(c) $\frac{XY}{XZ}=\frac{PZ}{YP}$ (d) $\frac{XZ}{XY}=\frac{YP}{YZ}$

34. In Fig. $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, find the value of x.



(a) 4 (b) 7 (c) 5 (d) 2

35. In the given figure PA, QB and RC, each are perpendicular to AC.



Which of the following is correct ?

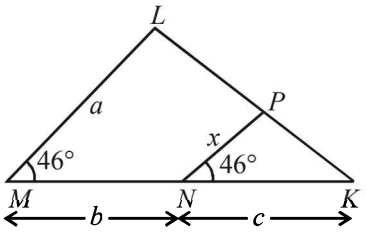
(a) $y + z = x$ (b) $\frac{1}{x+z}=\frac{1}{y}$

(c) $\frac{1}{y}=\frac{1}{x}+\frac{1}{z}$ (d) None of these

36. Given $\Delta ABC \sim \Delta DEF$. If $AB = 2DE$ and area of ΔABC is 56 cm^2 find the area of ΔDEF.

(a) 14 sq.cm (b) 5 sq.cm
(c) 18 sq.cm (d) 56 sq.cm

37. In the given figure, express x in terms of a, b and c.



(a) $x=\frac{ab}{a+b}$ (b) $x=\frac{ac}{b+c}$

(c) $x=\frac{bc}{b+c}$ (d) $x=\frac{ac}{a+c}$

38. In a Δ ABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, find the value of x.

(a) 6 (b) 5 (c) 4 (d) 1

39. ABC is an isosceles triangle in which AB = AC = 10 cm. BC = 12 cm. PQRS is a rectangle inside the isosceles triangle. Given PQ = SR = y cm. and PS = QR = 2x cm. then x =

(a) $6-\frac{3y}{4}$ (b) $6+6y$

(c) $6+\frac{4y}{3}$ (d) $\frac{7x+8y}{4}$

40. ΔABC is an isosceles triangle right angled at B. Similar triangles ACD and ABE are constructed on sides AC and AB. Ratio between the areas of ΔABE and ΔACD is

(a) 1 : 4 (b) 2 : 1 (c) 1 : 2 (d) 4 : 3

41. The internal bisectors of ∠B and ∠C of ΔABC meet at O. If ∠A = 80° then ∠BOC is :

(a) 50° (b) 160°

(c) 100° (d) 130°

42. Incentre of a triangle lies in the interior of :

(a) an isosceles triangle only

(b) a right angled triangle only

(c) any equilateral triangle only

(d) any triangle

43. What is the ratio of side and height of an equilateral triangle ?

(a) 2 : 1 (b) 1 : 1

(c) $2:\sqrt{3}$ (d) $\sqrt{3}:2$

44. If the medians of a triangle are equal, then the triangle is :

(a) right angled (b) isosceles

(c) equilateral (d) scalene

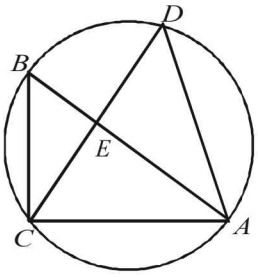
45. The number of common tangents that can be drawn to two given circles is at the most :

(a) 1 (b) 2

(c) 3 (d) 4

LEVEL - 2

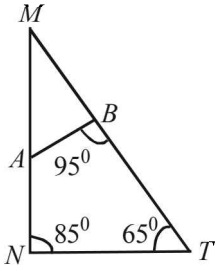
1. In the adjoining the figure, points *A*, *B*, *C* and *D* lie on the circle. *AD* = 24 and *BC* = 12. What is the ratio of the area of the triangle *CBE* to that of the triangle *ADE*



(a) 1 : 4 (b) 1:2

(c) 1 : 3 (d) Insufficient data

2. In the figure, if $\frac{NT}{AB}=\frac{9}{5}$ and if *MB* = 10, find *MN*.



(a) 5 (b) 4

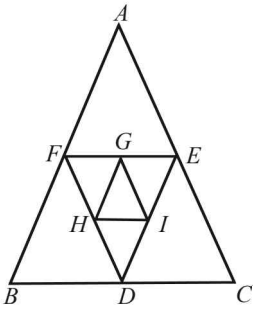
(c) 28 (d) 18

3. Two medians AD and BE of ΔABC intersect at G at right angles. If AD = 9 cm and BE = 6 cm, then the length of BD (in cm) is.

(a) 10 (b) 6

(c) 5 (d) 3

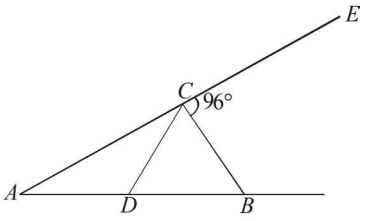
4. *D*, *E*, *F* are midpoints of *BC*, *CA* and *AB* respectively. *G*, *H*, *I* are midpoints of *FE*, *FD*, *DE* respectively. Areas of Δ*DHI* and Δ*AFE* are in the ratio



(a) 1 : 3 (b) 1 : 4

(c) 1 : 9 (d) 1 : 16

5. In the figure (not drawn to scale) given below, if $AD = CD = BC$, and $\angle BCE = 96°$, how much is $\angle DBC$?



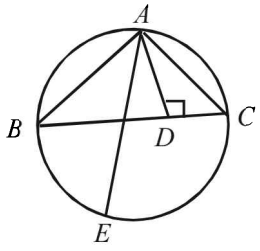
(a) 32°
(b) 84°
(c) 64°
(d) Cannot be determined

6. In a triangle ABC, the internal bisector of the angle A meets BC at D. If $AB = 4$, $AC = 3$ and $\angle A = 60°$, then the length of AD is

(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$

(c) $15\sqrt{\frac{3}{8}}$ (d) $6\sqrt{\frac{3}{7}}$

7. In ΔABC, $AB = 8$, $AC = 6$, Altitude $AD = 4.8$ AE is the diameter of the circumcircle. Find the circumradius.

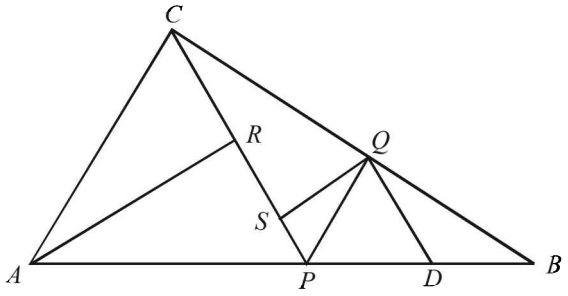


(a) 5
(b) 10
(c) 15
(d) Cannot be determined

8. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?

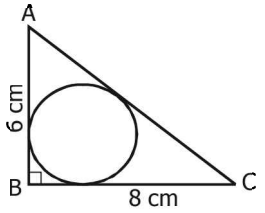
(a) 5 (b) 21
(c) 10 (d) 15

9. In the figure (not drawn to scale) given below, P is a point on AB such that $AP : PB = 4 : 3$. PQ is parallel to AC and QD is parallel to CP. In ΔARC, $\angle ARC = 90°$, and in ΔPQS, $\angle PSQ = 90°$. The length of QS is 6 cms. What is ratio $AP : PD$?



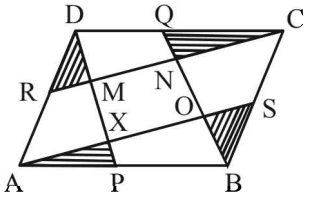
(a) 10 : 3 (b) 2 : 1
(c) 7 : 3 (d) 8 : 3

10. The radius of the incircle in the given diagram will be



(a) 1.8 cm (b) 2 cm
(c) 2.5 cm (d) 3.6 cm

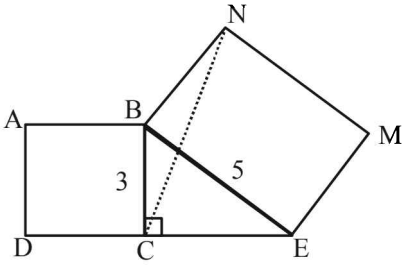
11. In the || gm ABCD, P, Q, R and S are mid-points of sides AB, CD, DA and BC respectively. AS, BQ, CR and DP are joined. Find the ratio of the area of the shaded region to the area of the || gm ABCD.



(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{4}{15}$ (d) $\frac{1}{6}$

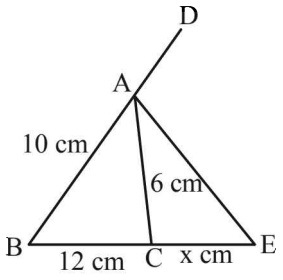
12. In the given figure, there is a square of 3 cm. If an another square of 5 cm with side BE is formed. In triangle BCE, C is right angle. Find the length of CN?



(a) $\sqrt{56}$ cm (b) $\sqrt{57}$ cm

(c) $\sqrt{58}$ cm (d) $\sqrt{59}$ cm

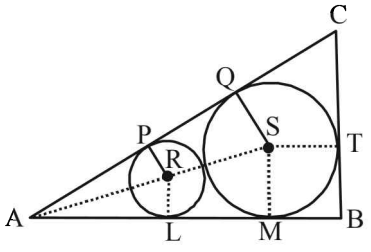
13. In the given figure, AE is the bisector of the exterior ∠CAD meeting BC produced in E. If AB = 10 cm, AC = 6 cm and BC = 12 cm, find CE.



(a) 10 cm (b) 12 cm

(c) 16 cm (d) 18 cm

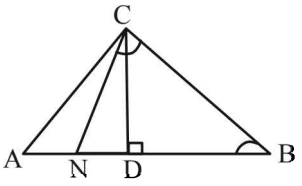
14. Two circles inscribed in a right angled triangle have radii is 12 cm & 3 cm. Find length of AP.



(a) 3 cm (b) 4 cm

(c) 5 cm (d) 7 cm

15. In the adjoining figure CD is perpendicular on AB. CN is the angle bisector of ∠ACB. ∠ACN = 50°, then the angle NCD is :

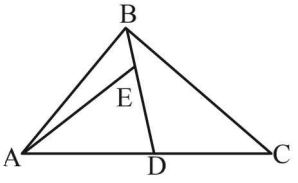


(a) $\frac{1}{3}\angle ACB$ (b) $\angle CAB - \angle CBA$

(c) $\frac{1}{2}(\angle CBA \sim \angle BAC)$

(d) can't be determined

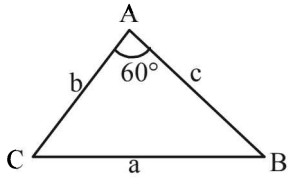
16. In the given figure ABC is a triangle in which 3AD = CD and E lies on BD, DE = 2BE. What is the ratio of area of ΔABE and area of ΔABC?



(a) 1/12 (b) 1/3

(c) 1/2 (d) None of these

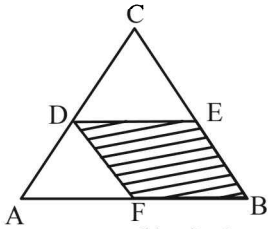
17. In the adjoining figure ∠BAC = 60° and BC = a, AC = b and AB = c, then :



(a) $a^2 = b^2 + c^2$ (b) $a^2 = b^2 + c^2 - bc$

(c) $a^2 = b^2 + c^2 + bc$ (d) $a^2 = b^2 + 2bc$

18. ABC is a triangle in which D, E and F are the mid-points of the sides AC, BC and AB respectively. What is the ratio of the area of the shaded to the unshaded region in the triangle?



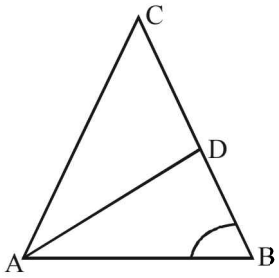
(a) 1 : 1 (b) 3 : 4

(c) 4 : 5 (d) None of these

19. If in a triangle ABC, D is the mid-point of side BC ∠ADB = 45° and ∠ACD = 30°. then ∠BAD and ∠ABC are respectively equal to

(a) 15°, 105° (b) 30°, 105°
(c) 30°, 100° (d) 60°, 100°

20. In the figure, ABC is a triangle in which AD bisects ∠A, AC = BC, ∠B = 72° and CD = 1 cm. Length of BD (in cm) is



(a) 1 (b) $\frac{1}{2}$

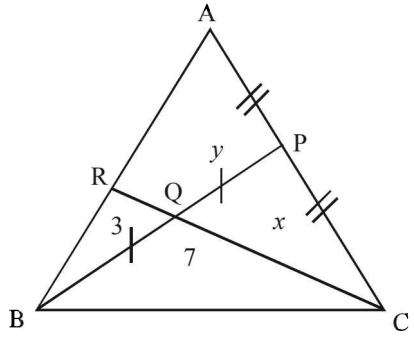
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{3}+1}{2}$

21. Three circles each of radius r units are drawn inside an equilateral triangle of side a units, such that each circle touches the other two and two sides of the triangle as shown in the figure, (P, Q and R are the centres of the three circles). Then relation between r and a is

(a) $a = 2(\sqrt{3}+1)r$ (b) $a = (\sqrt{3}+1)r$

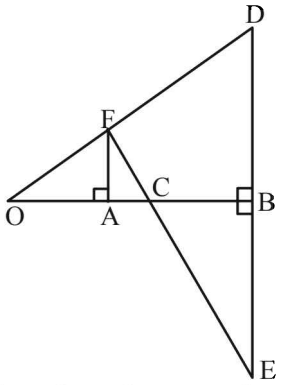
(c) $a = (\sqrt{3}+2)r$ (d) $a = 2(\sqrt{3}+2)r$

22. In the figure given below, points P and Q are mid points on the sides AC and BP respectively. Area of each part is shown in the figure, then find the value of $x + y$.



(a) 11 (b) 4
(c) 7 (d) 18

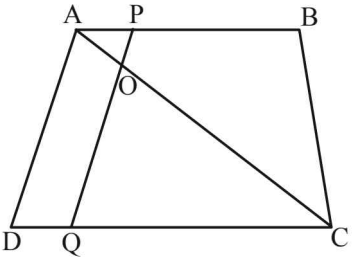
23. In the given fig., OB is the perpendicular bisector of the line segment DE, FA ⊥ OB and FE intersects OB at the point C.



(a) $\frac{1}{OA}+\frac{1}{OC}=\frac{1}{OB}$ (b) $\frac{1}{OA}+\frac{1}{OB}=OC^2$

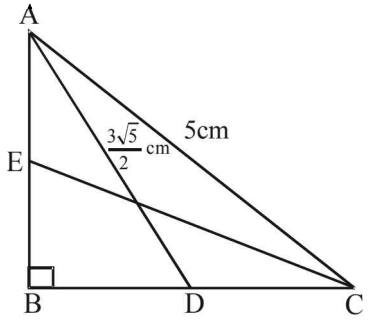
(c) $\frac{1}{OA}=\frac{1}{OC^2}+\frac{1}{OB^2}$ (d) $\frac{1}{OA}+\frac{1}{OB}=\frac{2}{OC}$

24. In the given fig., if AB || DC and AC and PQ intersect each other at the point O. Then



(a) $(OA)^2 = OC.AP$ (b) $OA.CQ = OC.OP$
(c) $OA + CQ = AP$ (d) $OA.CQ = OC.AP$

25. In the given figure, ABC is a right triangle, right angled at B. AD and CE are the two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



(a) $5\sqrt{2}$ cm (b) $2\sqrt{5}$ cm
(c) 6 cm (d) $\sqrt{7}$ cm

Hints and Solutions

Level 1

1. (b) In the given figure, ΔABD is similar to ΔACD

Then $\frac{AB}{BD}=\frac{AC}{DC} \Rightarrow \frac{6}{3}=\frac{5}{DC} \Rightarrow DC$

$=2.5$ cm

2. (a) $26^2-10^2=r^2, r=24$

3. (a) $a+36°+70°=180°$ (sum of angles of triangle)

$\Rightarrow a=180°-36°-70°=74°$

$b=36°+70°$(Ext. angle of triangle) $=106°$

$c=a-50°$ (Ext. angle of triangle) $=74°-50°$ $=24°$.

4. (d) The centroid, circumcenter, orthocenter always coincide in a equilateral triangle and otherwise collinear.

5. (c) Clearly option (a) shows the angles would be 30, 60 and 90. It can be the ratio of angle in a right angled triangle.

Option (b) shows the angles would be 45, 45 and 90, then it can be the ratio of angle in a right angled triangle.

But option (c) cannot form the ratio of angles of right angled triangle.

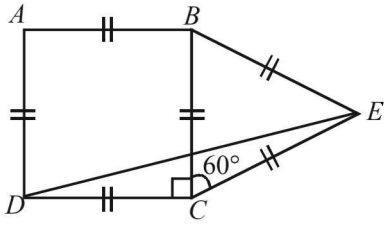
6. (c) ΔABC is similar to ΔEDC

$\therefore \quad \frac{AB}{ED}=\frac{BC}{DC}=\frac{AC}{EC}$

$\therefore \quad \frac{AB}{DE}=\frac{BC}{DC} \Rightarrow \frac{24}{10}=\frac{60}{DC}$

$\Rightarrow \quad DC=25$ cm

7. (a)



In $\Delta DEC, \angle DCE=90+60=150°$

$\angle CDE=\angle DEC=\frac{180-150}{2}=15°$

8. (b) Inradius of right angled triangle

$=\frac{AB+BC-AB}{2}$

$=\frac{9+40-41}{2}=4$ cm

9. (d) $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$

$6x+4x+3x=52$, or $13x=52$, or $x=4$

Required length $=12$ cm.

10. (c) Value of BC will lie in between 999 and 3003.

Hence $999<BC<3003$.

So, the total values possible for $BC=2003$.

11. (a) $a^2+b^2+c^2=ab+bc+ac$

Put $a=b=c=k$ we get $3k^2=3k^2$, which satisfies the above equation. Thus the triangle is equilateral.

12. (b) According to question

AB is a common chord.

O and O' is the centre of the circle

In ΔODA

$AO^2=AD^2+OD^2$

$(4)^2=(AD)^2+(2)^2$

$AD^2=12$

$AD=2\sqrt{3}$

$\therefore AB=2\times 2\sqrt{3}=4\sqrt{3}$

13. (c) $r=\frac{\text{Area of triangle}}{s}=\frac{\Delta}{s}$

$s=\frac{a+b+c}{2}=\frac{18+24+30}{2}=36$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

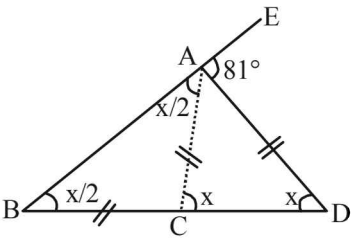
$\Rightarrow \Delta=\sqrt{36(36-18)(36-24)(36-30)}$

$\Delta=\sqrt{36\times 18\times 12\times 6}=216$

So, radius of incircle $=\frac{216}{36}=6$ cm.

GP_3386

14. (b)

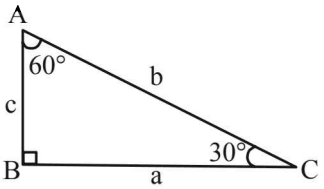


$x + \frac{x}{2} = 81°$ [Sum of two interior angles is equal to opposite exterior angle]

$\Rightarrow$ $2x + x = 162°$

$x = \frac{162}{3} = 54°$

15. (d)



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = x$

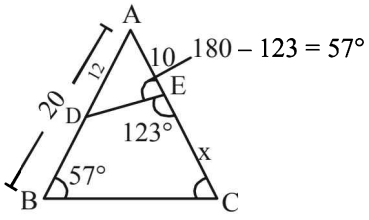
$a = \sin 60°.x = \frac{\sqrt{3}x}{2}$ $a : b : c$ $\frac{\sqrt{3}}{2}x : x : \frac{x}{2}$

$b = \sin 90°.x = x$

$c = \sin 30°.x = \frac{x}{2}$

Hence, $a : b : c = \sqrt{3} : 2 : 1$

16. (a)



$\Delta ABC \sim \Delta AED$

$\frac{AB}{AE} = \frac{AC}{AD}$

$\Rightarrow$ $\frac{20}{10} = \frac{10 + x}{12}$

$\Rightarrow$ $x = 14$ cm.

17. (b) $\angle ABK = 180° - (105° + 40°) = 35°$

$\therefore \angle ACD = \angle ABD = 35°$

18. (c) $PN \times PR = PQ^2$

$PN \times 25 = (15)^2$

$\therefore$ $PN = 9$ cm

19. (d) $\angle ACB = 180 - (90 + 25) = 65°$

$(\because \angle ABC = 90°)$

But since $\angle ADB = \angle ACB$

$\therefore \angle ADB = 65°$

20. (a)

21. (c) By mid-point theorem.

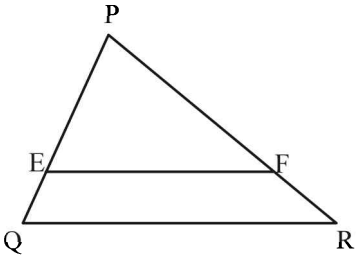
$AE = \frac{3 \times 5}{10} = 1.5$

22. (c) Since sum of two sides of a triangle is greater than third side.

23. (b) Since angle bisectors meet at incentre.

24. (c) Since perpendicular bisectors of sides meet at circumcentre.

25. (b)



Here, $EF \parallel QR$

By the converse of basic proportional by

$\frac{PE}{EQ} = \frac{PF}{FR}$

Out of all the given options, the following option fulfils the above condition.

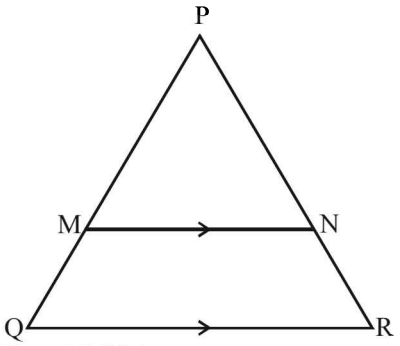
(b) $PE = 4$ cm, $QE = 4.5$cm, $PF = 8$ cm, $RF = 9$cm

$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$

$\frac{PF}{FR} = \frac{8}{9}$

So, $\frac{PE}{EQ} = \frac{PF}{FR}$

26. (c)



Since MN||QR

$\therefore \angle PMN = \angle PQR$ [Corr. Angles]

Hence $\Delta PMN \sim \Delta PQR$ [By AA]

$$\therefore \frac{PM^2}{PQ^2} = \frac{\text{Area of } \Delta PMN}{\text{Area of } \Delta PQR} = \frac{1}{2}$$

$$\Rightarrow \frac{PM}{PQ} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1 - \frac{PM}{PQ} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{PQ - PM}{PQ} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\Rightarrow \frac{QM}{PQ} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

27. (d) $\frac{AB}{DE} = \frac{BC}{DC} = \frac{AC}{CE}$

$$\frac{4}{3} = \frac{BC}{4.8} = \frac{AC}{4.2}$$

BC = 6.4 AC = 5.6

28. (c) $\Delta AOB \sim \Delta COD$

$$\frac{AO}{OC} = \frac{OB}{OD} = \frac{AB}{CD}$$

$$\frac{2}{5} = \frac{x-2}{2x+5}, x = 20$$

29. (b) $\frac{16}{9} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2$

$$\Rightarrow \frac{16}{9} = \left(\frac{AB}{18}\right)^2 \text{ and } \frac{16}{9} = \left(\frac{12}{QR}\right)^2$$

$$\Rightarrow \frac{4}{3} = \frac{AB}{18} \text{ and } \frac{4}{3} = \frac{12}{QR}$$

$\Rightarrow AB = 24$ cm, $QR = 9$ cm

30. (c) $\Delta ABC \sim \Delta AEF$

$$\frac{\text{ar.}(\Delta AEF)}{\text{ar.}(\Delta ABC)} = \left(\frac{AE}{AB}\right)^2 = \frac{1}{4}$$

$\Rightarrow$ Area of quadrilatral BEFC

= ar. (ΔABC) – Ar. (ΔAEF)

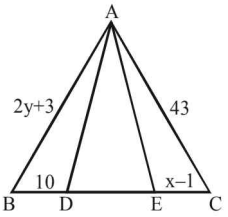
$$= \text{ar. } (\Delta ABC) = \frac{1}{4} \text{ ar. } (\Delta ABC) = \frac{3}{4} \text{ ar. } (\Delta ABC)$$

31. (a) ΔADE is isosceles (as AD = AE given)

So $\angle ADE = \angle AED$

$180° - \angle ADE = 180° - \angle AED$

$\angle ADB = \angle AEC$



Now in ΔADB and ΔAEC

$\angle BAD = \angle EAC$ (given)

AD = AE (given)

$\angle ADB = \angle AEC$ (proved)

$\therefore \Delta ADB \cong \Delta AEC$ (ASA congruence)

So AB = AC and BD = CE (cpct)

or 2y + 3 = 43 and x – 1 = 10

so y = 20, x = 11.

32. (c) Since in ΔPSN and ΔQMR

Corresponding angles are not equal

$\therefore \Delta PSN \sim \Delta QMR$.

33. (a) The internal bisector of $\angle X$ meets YZ at P

$$\frac{XY}{XZ} = \frac{YP}{PZ}$$

Add 1 on both the sides

$$\Rightarrow \frac{XY}{XZ} + 1 = \frac{YP}{PZ} + 1$$

$$\Rightarrow \frac{XY + XZ}{XZ} = \frac{YP + PZ}{PZ} = \frac{YZ}{PZ}$$

GP_3386

34. (a) In ΔABC, we have $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$ [By Thale's Theorem]

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4 \Rightarrow x = 4$

35. (c) In ΔPAC and ΔQBC, we have

$\angle PAC = \angle QBC$ [Each = 90°]

$\angle PCA = \angle QCB$ [Common]

$\therefore \quad \Delta PAC \sim \Delta QBC$

$\therefore \quad \frac{x}{y} = \frac{AC}{BC}$ *i.e.* $\frac{y}{x} = \frac{BC}{AC}$ ...(i)

Similarly $\frac{z}{y} = \frac{AC}{AB}$ *i.e.* $\frac{y}{z} = \frac{AB}{AC}$ ...(ii)

Adding (i) and (ii), we get

$$\frac{BC + AB}{AC} = \frac{y}{x} + \frac{y}{z} = y\left(\frac{1}{x} + \frac{1}{z}\right)$$

$$\frac{AC}{AC} = y\left(\frac{1}{x} + \frac{1}{z}\right) \Rightarrow 1 = y\left(\frac{1}{x} + \frac{1}{z}\right)$$

$$\Rightarrow \frac{1}{y} = \frac{1}{x} + \frac{1}{z}$$

36. (a) Given, $AB = 2DE$ and $\Delta ABC \sim \Delta DEF$

Hence, $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DEF)} = \frac{AB^2}{DE^2}$

or $\frac{56}{\text{area}(\Delta DEF)} = \frac{4DE^2}{DE^2} = 4$

$[\because AB = 2DE]$

area $(\Delta DEF) = \frac{56}{4} = 14$ sq.cm.

37. (b) In ΔKPN and ΔKLM, we have

$\angle KNP = \angle KML = 46°$

$\angle K = \angle K$ (Common)

$\therefore \quad \Delta KNP \sim \Delta KML$

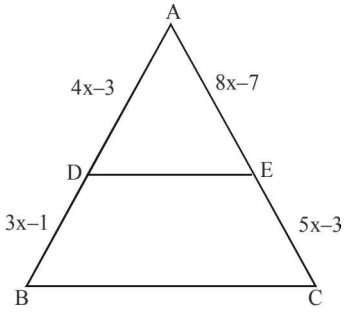
(By A-A criterion of similarity)

$\Rightarrow \frac{KN}{KM} = \frac{NP}{ML} \Rightarrow \frac{c}{b+c} = \frac{x}{a}$

38. (d) In ΔABC, we have $DE \parallel BC$

$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$

(By basic proportionality theorem)



$\Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

$\Rightarrow 20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

$\Rightarrow 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

$\Rightarrow 2x^2 - x - 1 = 0 \Rightarrow (2x+1)(x-1) = 0$

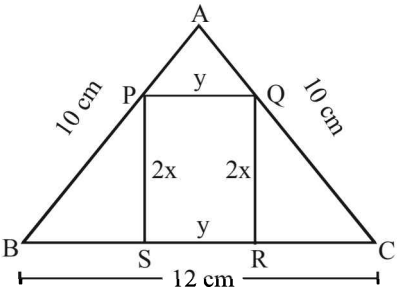
$\Rightarrow x = 1$ or $x = -1/2$

So, the required value of x is 1. [$x = -1/2$ is neglected as length cannot be negative]

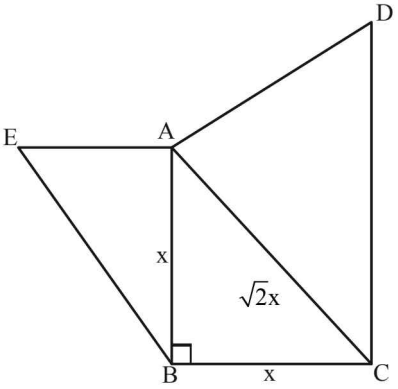
39. (a) Using Pythagoras theorem in ΔABL we have AL = 8cm,

Also, $\Delta BPQ \sim \Delta BAL$

$\therefore \frac{BQ}{PQ} = \frac{BL}{AL} \Rightarrow \frac{6-x}{y} = \frac{6}{8}$ or $x = 6 - \frac{3}{4}y$



40. (c)



Let AB = BC = x.

Since Δ ABC is right-angled with

$\angle B = 90°$

$\therefore AC^2 = AB^2 + BC^2 = x^2 + x^2 = 2x^2$

$\Rightarrow AC = \sqrt{2}x$

Since $\Delta ABE \sim \Delta ACD$

$$\therefore \frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{AB^2}{AC^2} = \frac{x^2}{2x^2} = \frac{1}{2}.$$

$$\text{Thus } \frac{\text{Area}(\Delta ABE)}{\text{Area}(\Delta ACD)} = \frac{1}{2}$$

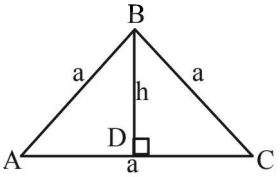
Thus reqd. ratio is 1 : 2.

41. (d) $\angle BOC = 90° + \frac{1}{2}\angle A$

$= 90° + 40° = 130°$

42. (d) Since angle bisectors of interior angles always meet inside a triangle.

43. (c)



$$a^2 = h^2 + \left(\frac{a}{2}\right)^2 \text{ [By Pythagorus theorem]}$$

$$\Rightarrow h^2 = \frac{3}{4}a^2 \Rightarrow h = \frac{\sqrt{3}}{2}a$$

$$\Rightarrow \frac{a}{h} = \frac{2}{\sqrt{3}}$$

44. (c) Only in equilateral triangle, length of medians are equal.

45. (b) Maximum two tangents can be drawn to two circles.

Level 2

1. (a) $AD = 24,\ BC = 12$

In ΔBCE & ΔADE

since $\angle CBA = \angle CDA$ (Angles by same arc)

$\angle BCE = \angle DAE$ (Angles by same arc)

$\angle BEC = \angle DEA$ (Opp. angles)

$\therefore \angle BCE$ & $\angle DAE$ are similar Δs with sides in the ratio 1 : 2

Ratio of area = 1:4 (i.e square of sides)

2. (d) $\angle MBA = 180° - 95° = 85°$

$\angle AMB = \angle TMN$ ...(Same angles with different names)

$\therefore \Delta MBA \sim \Delta MNT$ .....(*AA* test for similarity)

$$\frac{MB}{MN} = \frac{AB}{NT} \text{ .....(proportional sides)}$$

$$\frac{10}{MN} = \frac{5}{9} \qquad \therefore MN = \frac{90}{5} = 18.$$

3. (c) According to question. G is the centroid which divides the median in 2 : 1

$\therefore$ AD = 3 units = 9 cm

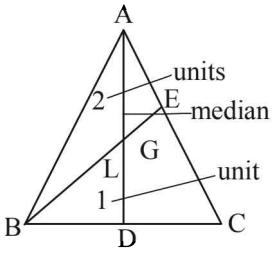
3 units = 9 cm $\therefore$ 1 unit = 3 cm

$\therefore$ GD = 3 cm

BE = 3 units = 6 cm

3 units = 6 cm

$$1 \text{ unit} = \frac{6}{3}$$



$$2 \text{ units} = \frac{6}{3} \times 2 = 4 \text{ cm}$$

$\therefore$ BG = 4 cm

ΔBGD is a right angle triangle

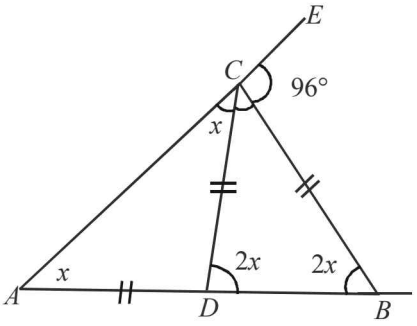
$BD^2 = BG^2 + GD^2$

GP_3386

$BD^2 = (4)^2 + (3)^2$
$BD^2 = 16 + 9 = 25$
$\therefore$ $BD = 5$ cm

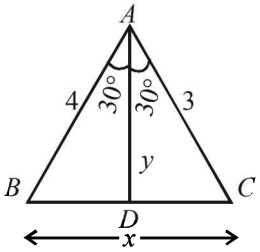
4. (b) We have area of triangle $AFE = A/4$. (If A = Area of triangle ABC) and area of triangle $DHI = (A/4)/4 = A/16$. Hence, ratio = 1 : 4.

5. (c)



Let $\angle CAD = \angle ACD = x$
At point C,
$x + (180° - 4x) + 96° = 180°$
$\Rightarrow 180° - 3x + 96° = 180°$
$\therefore$ $x = 32°$
Hence, $\angle DBC = 2 \times 32 = 64°$

6. (b)



Let $BC = x$ and $AD = y$
Using the theorem of angle of bisector,

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3}$$

$$\Rightarrow BD = \frac{4}{7}x \text{ and } DC = \frac{3}{7}x$$

In ΔABD, by sine rule, $\frac{\sin 30}{4/7x} = \frac{\sin B}{y}$

...(a)

In ΔABC, by sine rule; $\frac{\sin 60}{x} = \frac{\sin B}{3}$

$$\Rightarrow \frac{\sqrt{3}}{2x} = \frac{\sin 30.y}{4/7x \times 3}$$

[Putting value of sin B from (a)]

$$\Rightarrow y = \frac{\sqrt{3}}{2x} \times \frac{4}{7}x \times 3 \times \frac{2}{1} = \frac{12\sqrt{3}}{7}$$

7. (a) In ΔABD and ΔAEC
$m\angle ADB = m\angle ACE$ (both 90°)
$m\angle ABC = m\angle ABC$ (intercept the same area AC)
$\therefore$ $\Delta ABD - \Delta AEC$ (AA similarity)

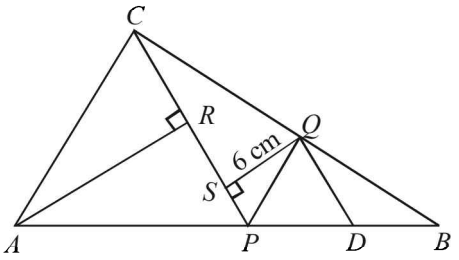
$$\frac{AB}{AE} = \frac{AD}{AC}$$ (Proportional sides)

$$\therefore \quad AE = \frac{AB}{AD} \times AC = \frac{8}{4.8} \times 6 = 10$$

$\therefore$ Circumradius = 5,

8. (c) If 'a', 'b' and 'c' are the length of sides of a obtuse triangle and 'a' be the length of longest side.
Then $a^2 > b^2 + c^2$
Case (i) : If length of one longest side be 15 cm, then
$225 > 64 + x^2$
$\Rightarrow$ $x^2 < 161$
$\therefore$ $x = 8, 9, 10, 11, 12$
[Since, the value of x is less than 8, because sum of length of any two sides of a triangle is greater than the longest side.
Case (ii) : If length of longest side be x cm, then
$x^2 > 225 + 64$
$\Rightarrow$ $x^2 > 289$
$\therefore$ $x = 18, 19, 20, 21$ and 22
[Since the value of x is less than 23]
Therefore, total number of values of x is 10 and hence total number of triangles is 10.

9. (c) From figure



Given that,
$PQ \| AC$,

$$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$$

Again, $QD \| CP$,

$$\therefore \frac{PD}{BD} = \frac{CQ}{QB} = \frac{4}{3}$$

As $\frac{PD}{DB} = \frac{4}{3}$

$$\Rightarrow \frac{PD}{DB+PD} = \frac{4}{3+4} \Rightarrow \frac{PD}{PB} = \frac{4}{7}$$

$$\Rightarrow PD = \frac{4}{7}PB$$

$$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4} \times \frac{AP}{PB} = \frac{7}{4} \times \frac{4}{3} = 7:3$$

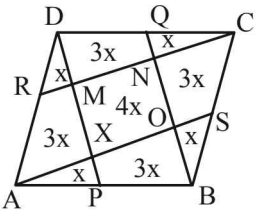
10. (b) $AC^2 = AB^2 + BC^2 \Rightarrow AC = 10$

We have $r = (A/s)$; $A = \frac{1}{2} \times (6 \times 8) = 24$

$s = (6 + 8 + 10)/2 = 12$

$r = A/s = 24/12 = 2$.

11. (a)



$\Delta QCN \sim \Delta DCM$

$$\frac{\text{area}(QCN)}{\text{area}(DCM)} = \frac{QC^2}{DC^2}$$

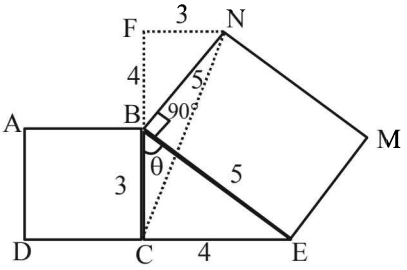
$$\Rightarrow \frac{\text{area (QCN)}}{\text{area}(DCM)} = \frac{1}{4}$$

If area QCN = x

Then area DQNM = 4x – x = 3x

$$\text{Required ratio} = \frac{4x}{20x} = \frac{1}{5}$$

12. (c)



$CN = \sqrt{CF^2 + FN^2}$

$= \sqrt{7^2 + 3^2} = \sqrt{58}$ cm

Alternate Method : –

$$\cos(90 + \theta) = \frac{BN^2 + BC^2 - CN^2}{2 \times BN \times BC}$$

$$\Rightarrow -\sin\theta = \frac{5^2 + 3^2 - CN^2}{2 \times 5 \times 3}$$

$$\Rightarrow -\frac{4}{5} = \frac{34 - CN^2}{30}$$

$\Rightarrow -24 - 34 = -CN^2$

$CN^2 = 58$

$CN = \sqrt{58}$ cm

13. (d) Given : AB = 10 cm, AC = 6 cm and BC = 12 cm,

By exterior angle bisector theorem

CE = x (Let)

$$\frac{BE}{CE} = \frac{AB}{AC}$$

$$\frac{12 + x}{x} = \frac{10}{6}$$

$6(12 + x) = 10x$ [by cross multiplication]

$72 + 6x = 10x$

$72 = 10x - 6x$

$72 = 4x$

$x = 18$

CE = 18 cm

14. (b) $PQ = \sqrt{(RS)^2 - (QS - PR)^2}$

$PQ = \sqrt{(15)^2 - (12 - 3)^2}$

PQ = 12 cm = LM

$\because$ STBM is a square

MB = BT = ST = SM = 12 cm.
In ΔAPR & ΔASQ,

$$\frac{AP}{AQ} = \frac{PR}{SQ}$$

$$\frac{AP}{AP+PQ} = \frac{3}{12}$$

$$\frac{AP}{AP+12} = \frac{1}{4}$$

$4AP = AP + 12$
$3AP = 12$
$\therefore$ AP = 4 cm.

15. (c) Go back to the basics

16. (a) Area of $\Delta BAE = \frac{1}{4}AC \times \frac{1}{3}BD$

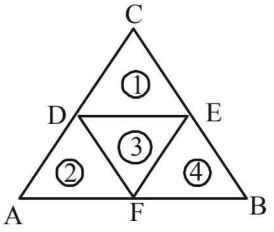
$$= \frac{1}{12}\text{Area of } \Delta ABC$$

17. (b) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (cosine rules)

$$\frac{1}{2} = \frac{b^2 + c^2 - a^2}{2bc} \quad (\because \cos 60° = 1/2)$$

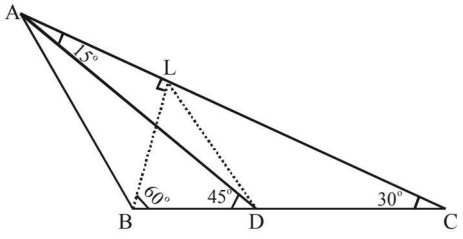
$\therefore$ $a^2 = b^2 + c^2 - bc$

18. (a) The 4 triangles (smaller) are congruent.



So, out of these 4 triangles, 2 triangles are taken thus the ratio of the shaded to the unshaded region is 1 : 1 (since two triangle are shaded and 2 are unshaded).

19. (b)



Draw BL $\perp$ AC and join L to D.
In right ΔBLC,
$\angle CBL = 60°$ ($\because \angle BCL = 30°$)

Also, $\sin 30° = \frac{BL}{BC} \Rightarrow BL = \frac{BC}{2}$

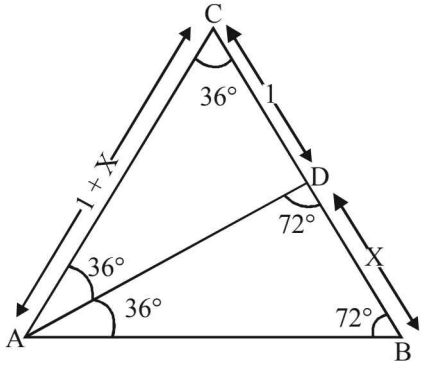
But, $BD = \frac{BC}{2}$ ($\because$ D is mid-point of BC)

So, BL = BD
$\therefore$ $\angle BLD = \angle BDL$ (Angles opposite to equal sides are equal)
In Δ BDL,
$\angle DBL + \angle BLD + \angle BDL = 180°$
$\Rightarrow$ $60° + 2\angle BLD = 180°$ ($\because \angle BLD = \angle BDL$)
$\therefore$ $\angle BLD = \angle BDL = 60°$
So, $\angle ADL = \angle BDL - \angle BDA = 60° - 45° = 15°$
Thus, we have LD = LA = LB
$\Rightarrow$ L is the circumcentre of ΔBDA

$\therefore$ $\angle BAD = \frac{1}{2}\angle BLD = \frac{1}{2} \times 60° = 30°$

Now, in ΔABD,
$30° + 45° + \angle ABC = 180°$
Hence, $\angle ABC = 105°$

20. (c) Let BD = x cm
Since AC = BC, therefore ΔABC is an isoscele triangle.
$\Rightarrow$ $\angle B = \angle CAB = 72°$
Since AD bisects $\angle A$
$\therefore$ $\angle DAB = 36°$ so, In ΔADB, $\angle ADB = 72°$
$\Rightarrow$ ΔADB is an isoscele triangle
$\therefore$ AB = AD = 1cm
$\Rightarrow$ AB = 1 cm
Similarly, ΔADC is also an isoscele triangle.
$\therefore$ AD = CD $\Rightarrow$ AD = 1 cm



Now $\frac{AC}{AB}=\frac{CD}{BD}$

$\Rightarrow \quad \frac{1+x}{1}=\frac{1}{x}$

$\Rightarrow \quad x+x^2-1=0$

$\Rightarrow$

$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(-1)}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

$$BD=\frac{\sqrt{5}-1}{2}$$

21. (a) $\angle A=\angle B=\angle C=60°$

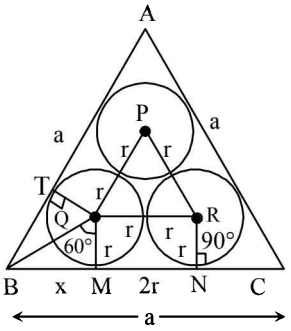
$MN=QR=2r$ in quadrilateral BTQM

$\angle B+\angle BMQ+\angle MQT+\angle QTB=360°$

$60°+90°+90°+\angle QTB=360°$

$\Rightarrow \angle QTB=120°$

$\Delta BTQ\cong\Delta BQM$



$\Rightarrow \angle TQB=\angle MQB=60°$

In ΔBQM

$\tan 60°=\frac{BM}{QM}=\frac{x}{r}$

$\sqrt{3}=\frac{x}{r}\Rightarrow x=r\sqrt{3}$

Similarly $CN=r\sqrt{3}$

$BC=M+MN+CN=a$

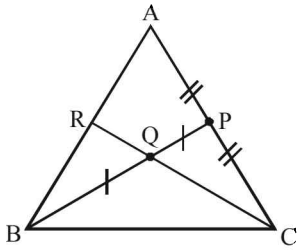
$2x+2r=a$

$\Rightarrow 2(x+r)=a$

$\Rightarrow 2(r\sqrt{3}+r)=a\Rightarrow a=2r(\sqrt{3}+1)$

22. (d) We have a theorem which says

If ABC is a triangle in which D is the mid-point of BC and E is the mid-point of AD then area



$(\Delta BED)=\frac{1}{4}$ area (ΔABC)

So, by applying this theorem we get

$ar(\Delta PQC)=\frac{1}{4}ar(\Delta BAC)$

$\Rightarrow x=\frac{1}{4}(10+x+y)\Rightarrow 3x=10+y$ ..(a)

Also, $ar(\Delta BPC)=ar(\Delta BAP)$

($\because$ BP is median)

$\Rightarrow 7+x=3+y\Rightarrow 4=y-x$ ...(b)

from (a) and (b)

$3x=10+(4+x)$

$\Rightarrow 3x=14+x\Rightarrow x=7$ and $y=11$

Thus, $x+y=7+11=18$

23. (d) In ΔAOF and ΔBOD

$\angle O=\angle O$ (same angle)

and $\angle A=\angle B$ (each 90°)

Therefore, $\Delta AOF\sim\Delta BOD$ (AA similarity)

So, $\frac{OA}{OB}=\frac{FA}{DB}$ ....(i)

Also, in ΔFAC and $\angle EBC$, $\angle A=\angle B$ (each = 90°) and $\angle FCA=\angle ECB$ (Vertically opposite angles).

Therefore, $\Delta FAC\sim\Delta EBC$ (AA similarity)

$\Rightarrow \frac{FA}{EB}=\frac{AC}{BC}$ ....(ii)

But EB = DB (B is mid-points of DE)

$\Rightarrow \frac{FA}{DB}=\frac{AC}{BC}$

Therefore, from (i) and (ii) we have :

$\frac{AC}{BC}=\frac{OA}{OB}$

i.e, $\frac{OC-OA}{OB-OC}=\frac{OA}{OB}$ or.

$OB.OC-OA.OB=OA.OB-OA.OC$

or $OB.OC+OA.OC=2OA.OB$ or

$(OB+OA).OC=2OA.OB$

GP_3386

or $\frac{1}{OA}+\frac{1}{OB}=\frac{2}{OC}$ [Dividing both the sides by OA. OB. OC]

24. (d) Since AB || DC and AC is transversal
$\therefore \angle OAB = \angle OCQ$ (alternate angles)
Also PQ is a transversal
$\therefore \angle APO = \angle OQC$ (alternate angles)
In Δs OAP and OQC, $\angle OAP = \angle OCQ$, $\angle APO = \angle OQC$, $\angle AOP = \angle QOC$ (vertically opposite angles)
$\therefore \Delta OAP$ and ΔOQC are equiangular and
$\therefore$ they are similar

$\therefore \frac{OA}{OC}=\frac{OP}{OQ}=\frac{AP}{CQ}$

$\Rightarrow OA.CQ = OC.AP$

25. (b) In right ΔABD, $AD^2 = AB^2 + BD^2$

$\Rightarrow AD^2 = AB^2 + \left(\frac{1}{2}BC\right)^2$

$\Rightarrow AD^2 = AB^2 + \frac{1}{4}BC^2$ ... (i)

Similarly, in right

ΔEBC, $EC^2 = BC^2 + \frac{1}{4}AB^2$ ... (ii)

Adding (i) and (ii), we get

$AD^2 + EC^2 = \frac{5}{4}AB^2 + \frac{5}{4}BC^2$

$\Rightarrow \left(\frac{3\sqrt{5}}{2}\right)^2 + EC^2 = \frac{5}{4}(5)^2$

$\Rightarrow EC^2 = \frac{125}{4} - \frac{45}{4}$

$\Rightarrow EC^2 = \frac{80}{4} = 20$ cm $\Rightarrow CE = 2\sqrt{5}$ cm

CHAPTER

21

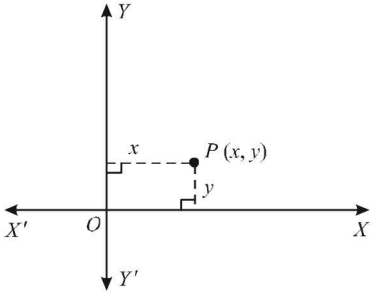
Coordinate Geometry

RECTANGULAR COORDINATE AXES

Let XOX' be a horizontal straight line and YOY' be a vertical straight line drawn through a point O.
Then
The line XOX' is called ***x-axis***
The line YOY' is called ***y-axis***
Plane of paper is called ***coordinate plane***.
x-axis and y-axis together are called ***co-ordinate axes***.
The point O is called the ***origin***.

Coordinates

Position of any point in a coordinate plane can be described by their coordinates. The ordered pair of perpendicular distances first from y-axis and second from x-axis of a point P is called *coordinates* of P.



If the coordinates of point P are (x, y), then x is called abscissa or ***x-coordinate*** of P and y is called the ***ordinate*** or ***y-coordinate*** of point P.

Coordinate of a Point on *x* or *y*-axis

(i) Coordinate of origin is (0, 0)

(ii) Coordinate of any point on the x-axis but right side of origin is of the form $(x, 0)$, where $x > 0$.

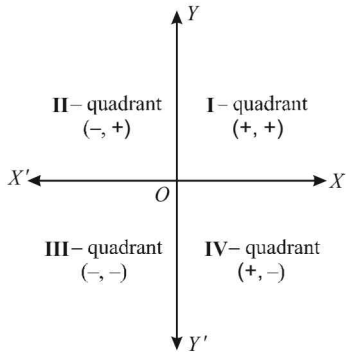
(iii) Coordinate of any point on the x-axis but left side of origin is of the form $(-x, 0)$, where $x > 0$.

(iv) Coordinate of any point on the y-axis but above the origin is of the form $(0, y)$, where $y > 0$.

(v) Coordinate of any point on the y-axis but below the origin is of the form $(0, -y)$, where $y > 0$.

Quadrants of Coordinate Plane and Sign of x and y-Coordinate of a Point in Different Quadrants

x and y-axis divide the coordinate plane in four parts. Each part is called a **quadrant**.
The four quadrants are written as I-quadrant (XOY), II-quadrant (YOX'), III-quadrant (X'OY') and IV-quadrant (Y'OX).



(i) Any of the four quadrants does not include any part of x or y-*axis*.

(ii) In the first quadrant both x and y-coordinates of any point are $+ve$.

(iii) In second quadrant x-coordinate of any point is $-ve$ but y-coordinate of any point is $+ve$.

(iv) In third quadrant, both x and y-coordinates of any point are $-ve$.

(v) In fourth quadrant, x-coordinate of any point is $+ve$ but y-coordinate of any point is $-ve$ as shown in the above diagram.

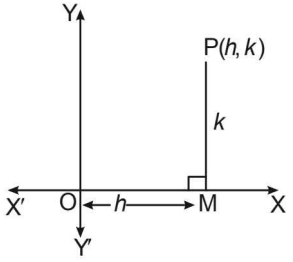
GP_3386

Plotting Point whose Coordinates are Known

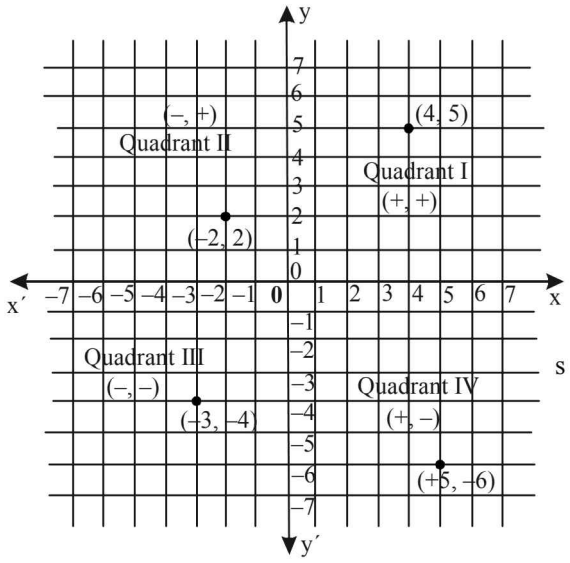
The point can be plotted by measuring its proper distances from both the axes. Thus, any point P whose coordinates are (h, k) can be plotted as follows:

(i) Measure OM equal to h (i.e. x-coordinate of point P) along the x-axis.

(ii) Now perpendicular to OM equal to k.
Mark point P above M such that PM is parallel to y-axis and $PM = k$ (i.e. y-coordinate of point P)



Coordinate geometry requires us to work with the rectangular coordinate system. In a plane, two perpendicular number lines are drawn, as shown in the figure below. The horizontal line is called the x - axis and the vertical one is called the y-axis. The point of intersection of the two axis is called the origin and is denoted by O.



Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance of point $P(x, y)$ from the origin $= \sqrt{x^2 + y^2}$

Illustration – 1

If distance between the point (x, 2) and (3, 4) is 2, then find the value of x.

Solution : $2 = \sqrt{(x-3)^2 + (2-4)^2}$

$\Rightarrow 2 = \sqrt{(x-3)^2 + 4}$

Squaring both sides

$4 = (x-3)^2 + 4 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

Illustration – 2

Find the distance between each of the following points : A(– 6, –1) and B(– 6, 11)

Solution : Here the points are A(–6, –1) and B(–6, 11)

By using distance formula, we have

$AB = \sqrt{\{-6-(-6)\}^2 + \{11-(-1)\}^2} = \sqrt{0^2 + 12^2} = 12$

Hence, AB = 12 units.

Applications of Distance Formula

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. After finding the distances AB, BC and CA; we shall find that the points are

(a) collinear, if the sum of any two distances is equal to the third.

(b) vertices of an equilateral triangle if $AB = BC = CA$

(c) vertices of an isosceles triangle of $AB = BC$, $BC = CA$ or $CA = AB$

(d) vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ or $BC^2 + CA^2 = AB^2$ or $CA^2 + AB^2 = BC^2$

(ii) For given four points A, B, C, D;

(a) $AB = BC = CD = DA$; $AC = BD$ $\Rightarrow$ ABCD is a square

(b) $AB = BC = CD = DA \Rightarrow$ ABCD is a rhombus

(c) $AB = CD$, $BC = DA$; $AC = BD$ $\Rightarrow$ ABCD is a rectangle

(d) $AB = CD$, $BC = DA \Rightarrow$ ABCD is a parallelogram

Illustration – 3

What kind of triangle is formed by A(1, 2), B(4, 3) and C(5, 6)?

Solution : $AB^2 = (4-1)^2 + (3-2)^2 = 9 + 1 = 10$

$BC^2 = (5-4)^2 + (6-3)^2 = 1 + 9 = 10$

$CA^2 = (5-1)^2 + (6-2)^2 = 16 + 16 = 32$

$AB^2 = BC^2 \Rightarrow$ it is isosceles.

$CA^2 > AB^2 + BC^2$, since $32 > 10 + 10$
$\Rightarrow \angle B$ is obtuse
Hence, ABC is an obtuse isosceles Δ.

Section Formula

Co-ordinates of a point which divides the line segment joining two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ are :

(i) $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

If $m_1 = m_2$, then the point will be the **mid point** of PQ, whose co-ordinates

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(ii) When we need to find the ratio in which a point on a line segment divides it, we suppose the required ratio as k : 1 or m/n : 1.

Illustration – 4

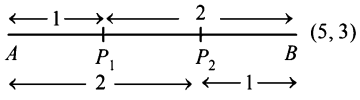
Find the ratio in which the line $3x + 4y = 7$ divides the line segment joining the points (1, 2) and (– 2, 1).

Solution : Ratio $= -\frac{3(1) + 4(2) - 7}{3(-2) + 4(1) - 7} = -\frac{4}{-9} = \frac{4}{9} = 4 : 9$

Illustration – 5

Find the points of trisection of line joining the points A (2, 1) and B (5, 3).

Solution :



$P_1(x, y) = \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times 3 + 2 \times 1}{1 + 2}\right) = \left(3, \frac{5}{3}\right)$

$P_2(x, y) = \left(\frac{2 \times 5 + 1 \times 2}{2 + 1}, \frac{2 \times 3 + 1 \times 1}{2 + 1}\right) = \left(4, \frac{7}{3}\right)$

Illustration – 6

Prove that points A (1, 1), B (– 2, 7) and C (3, – 3) are collinear.

Solution : $AB = \left|\sqrt{(1+2)^2 + (1-7)^2}\right| = \left|\sqrt{9 + 36}\right| = 3\sqrt{5}$

$BC = \left|\sqrt{(-2-3)^2 + (7+3)^2}\right| = \left|\sqrt{25 + 100}\right| = 5\sqrt{5}$

$CA = \left|\sqrt{(3-1)^2 + (-3-1)^2}\right| = \left|\sqrt{4 + 16}\right| = 2\sqrt{5}$

Clearly, BC = AB + AC. Hence A, B, C are collinear.

Illustration – 7

Find the ratio in which the join of (– 4, 3) and (5, –2) is divided by (i) x-axis (ii) y-axis.

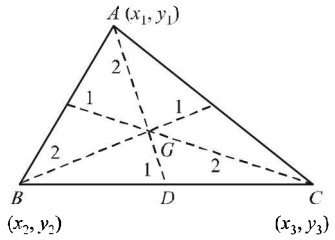
Solution : (i) x-axis divides the join of (x_1, y_1) and (x_2, y_2) in the ratio of $-y_1 : y_2 = -3 : -2 = 3 : 2$.
(ii) y-axis divides, in the ratio of $-x_1 : x_2 \Rightarrow 4 : 5$.

Coordinates of Some Particular Points

Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of any triangle ABC, then

Centroid

Centroid is the point of intersection of the medians of a triangle. Centroid divides each median in the ratio of 2 : 1. A median is a line segment joining the mid point of a side to its opposite vertex of a triangle.

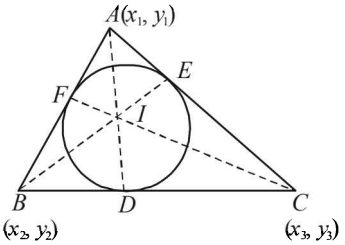


Co-ordinates of centroid, G

$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Incentre

Incentre is the point of intersection of internal bisectors of the angles of a triangle. Also incentre is the centre of the circle touching all the sides of a triangle.



Co-ordinates of incentre,

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right),$$

where a, b, c are length of the sides opposite to vertices A, B, C respectively of triangle ABC.

(i) Angle bisector divides the opposite sides in the ratio of the sides included in the angle.

GP_3386

For example: $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$.

(ii) Incentre divides the angle bisectors AD, BE and CF in the ratio (b + c) : a, (c + a) : b and (a + b) : c respectively.

Illustration – 8

Find incentre (I) of triangle whose vertices are A (– 36, 7), B (20, 7), C (0, – 8).

Solution : Using distance formula

$$a = BC = \left|\sqrt{20^2 + (7+8)^2}\right| = 25$$

$$b = CA = \left|\sqrt{36^2 + (7+8)^2}\right| = 39$$

$$c = AB = \left|\sqrt{(36+20)^2 + (7-7)^2}\right| = 56$$

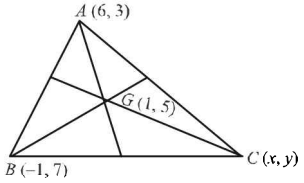
$$I = \left(\frac{25(-36) + 39(20) + 56(0)}{25+39+56}, \frac{25(7) + 39(7) + 56(-8)}{25+39+56}\right)$$

$I = (-1, 0)$.

Illustration – 9

The two vertices of a triangle are (6, 3) and (–1, 7) and its centroid is (1, 5). Find the third vertex.

Solution : Let ABC be a triangle whose vertices are A = (6, 3), B = (–1, 7), C = (x, y) and centroid G = (1, 5)



Then using the formula, for coordinates of centroid

$$1 = \frac{6 + (-1) + x}{3} \text{ and } 5 = \frac{3+7+y}{3} \Rightarrow x = -2 \text{ and } y = 5$$

Hence, the third vertex is C = (–2, 5)

Area of a Triangle

Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of a triangle, then area of the triangle ABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

When one vertex is origin i.e. if the vertices are (0, 0), (x_1, y_1) and (x_2, y_2) then area of the triangle

$$= \frac{1}{2}\left|x_1 y_2 - x_2 y_2\right|.$$

Illustration – 10

The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on y = x + 3. Find the third vertex.

Solution : Let the third vertex be (x_3, y_3), area of triangle $= \frac{1}{2}|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$

As, $x_1 = 2, y_1 = 1, x_2 = 3, y_2 = 2$, Area of $\Delta = 5$

$$\Rightarrow \quad 5 = \frac{1}{2}|2(-2 - y_1) + 3(y_3 - 1) + x_3(1+2)|$$

$$\Rightarrow \quad 10 = |3x_3 + y_3 - 7| \Rightarrow 3x_3 + y_3 = \pm 19$$

Taking positive sign, $3x_3 + y_3 - 7 = 10$

$$\Rightarrow \quad 3x_3 + y_3 = 17 \qquad \ldots(1)$$

Taking negative sign, $3x_3 + y_3 - 7 = -10$

$$\Rightarrow \quad 3x_3 + y_3 = -3 \qquad \ldots(2)$$

Given that (x_3, y_3) lies on $y = x + 3$

So, $\quad -x_3 + y_3 = 3 \qquad \ldots(3)$

Solving eqs. (1) and (3), $x_3 = \frac{7}{2}, y_3 = \frac{13}{2}$

Solving eqs. (2) and (3), $x_3 = \frac{-3}{2}, y_3 = \frac{3}{2}$.

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

Image of a Point

Let (x, y) be any point, then its image with respect to

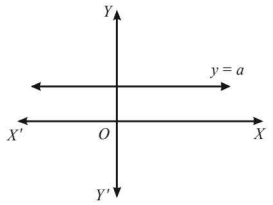
(i) x-axis is $(x, -y)$ **(ii)** y-axis is $(-x, y)$

(iii) origin is $(-x, -y)$ **(iv)** line $y = x$ is (y, x)

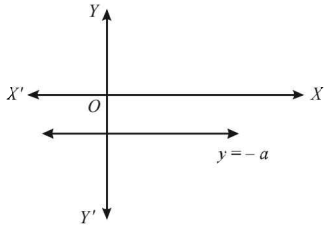
Equation of Straight Line Parallel to an Axis

(i) Equation of x-axis is $y = 0$

(a) Equation of a line parallel to x-axis (or perpendicular to y-axis) at a distance 'd' above the y-axis is $y = a$.

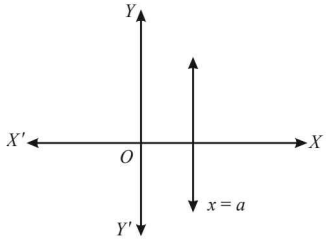


(b) Equation of a line parallel to x-axis (or perpendicular to y-axis) at a distance 'a' below the y-axis is $y = -a$.

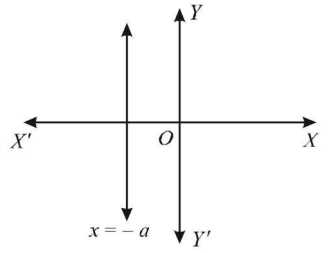


(ii) Equation of y-axis is $x = 0$

(a) Equation of a line parallel to y-axis (or perpendicular to x-axis) at a distance 'a' towards right side of y-axis is $x = a$.



(b) Equation of a line parallel to y-axis (or perpendicular to x-axis) at a distance 'a' towards left side of y-axis is $x = -a$.



For example: Equation of a line which is parallel to x-axis and at a distance of 4 units below the y-axis is $y = -4$.

Slope of a Straight Line

Slope is the inclination of a line with reference to the x-axis. It is denoted by m.

(i) Slope (m) of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(ii) Slope m of line $ax + by + c = 0$ is $-\frac{a}{b}$ i.e., $m = -\frac{a}{b}$.

(iii) If a line is slanting up from left to right, then its slope is positive.

(iv) If a line is slanting down from left to right, then its slope is negative.

(v) If a line is parallel to x-axis, then its slope is zero.

(vi) If a line is parallel to y-axis, then its slope is undefined.

(vii) Two parallel lines have the same slope.

(viii) The slopes of two perpendicular lines are negative reciprocals of each other. So, if the slope of line A is 3, then the slope of line B, which is perpendicular to line A, will be $-\frac{1}{3}$.

Condition of Collinearity of Three Points

Three points A (x_1, y_1) B (x_2, y_2) and C (x_3, y_3) will be collinear if one of the following three conditions is satisfied.

(i) If sum of any two distances AB, BC and CA is equal to the third distance.

(ii) $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$

(iii) Slope of AB = Slope of BC

(iv) Area of $\Delta ABC = 0$

Equation of Straight Lines

A linear relation between x and y which is satisfied by coordinates of every point lying on a straight line is called the equation of that Straight Line. Every linear equation in two variables x and y always represents a straight line.

For example: $4x + 9y = 40, -10x + 6y = 8, x + 3y = 0$, etc.

General form of the equation of a straight line is given by $ax + by + c = 0$.

Different Forms of the Equation of a Straight Line

General Form

$ax + by + c = 0$,

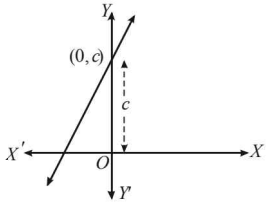
where a, b, c are any real numbers but a and b cannot be zero simultaneously.

Slope Intercept Form

$y = mx + c$,

where m is the slope of the line and c is the y-intercept of the line.

y-intercept of a line is the y-coordinate of the point where the line intersect the y-axis.



Slope Point Form

The equation of a line with slope m and passing through a point (x_1, y_1) is $y - y_1 = m(x - x_1)$.

GP_3386

Two Point Form

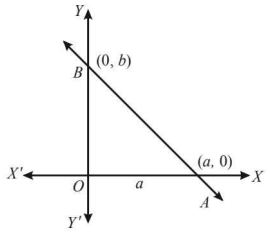
The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Intercept Form

$$\frac{x}{a} + \frac{y}{b} = 1$$

where a and b are x and y-intercept respectively of the line.



x-intercept of a line is the x-coordinate of the point where the line intersects the x-axis. y-intercept of a line is the y-coordinate of the point where the line intersects the y-axis.

Illustration – 11

Find the equation of a line which is passes through (3, – 4) and makes an angle of 45° with x-axis.

Solution : $y-(-4) = \tan 45°\,(x-3)$

$\Rightarrow y+4 = x-3 \quad \Rightarrow \quad x-y-7=0$ [$\tan 45° = 1$]

Illustration – 12

Find the equation of a line which makes intercepts 3 and 4 on x-axis and y-axis respectively.

Solution : $\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12.$

Illustration – 13

Find the equation of the line passing through the points (–1, 1) and (2, –4).

Solution : The line passes through the points A(–1, 1) B(2, –4)

Equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

We have $x_1 = -1, y_1 = 1, x_2 = 2, y_2 = -4$

$\therefore$ Equation of AB is $y - 1 = \frac{-4-1}{2+1}(x+1)$

$\Rightarrow y - 1 = \frac{-5}{3}(x+1) \quad \Rightarrow 3(y-1) = -5(x+1)$

$3y - 3 = -5x - 5 \Rightarrow 5x + 3y - 3 + 5 = 0 \Rightarrow 5x + 3y + 2 = 0$

LEVEL - 1

1. If distance between the point $(x, 2)$ and $(3, 4)$ is 2, then the value of x =
(a) 0 (b) 2 (c) 3 (d) 4

2. The mid-point of the line segment joining the points $(-2, 4)$ and $(6, 10)$ is
(a) $(2,5)$ (b) $(2,7)$ (c) $(3,7)$ (d) $(3,8)$

3. For which value of k given below the points $A(-1, 4)$, $B(2, 5)$ and $C(3, k)$ are collinear ?
(a) 16/3 (b) 16 (c) 5 (d) –1

4. The line $x + y = 4$ divides the line joining the points $(-1, 1)$ and $(5, 7)$ in the ratio
(a) 2 : 1 (b) 1 : 2
(c) 1 : 2 externally (d) None of these

5. The centroid of a triangle, whose vertices are $(2, 1)$, $(5, 2)$ and $(3, 4)$ is
(a) $\left(\frac{8}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{10}{3}, \frac{7}{3}\right)$
(c) $\left(-\frac{10}{3}, \frac{7}{3}\right)$ (d) $\left(\frac{10}{3}, -\frac{7}{3}\right)$

6. If the coordinates of the points A, B, C be $(4, 4)$, $(3, -2)$ and $(3, -16)$ respectively, then the area of the triangle ABC is:
(a) 27 (b) 15 (c) 18 (d) 7

7. The equation of the line passing through $(2, -4)$ and parallel to $X - 2Y - 5 = 0$, is
(a) $2X + Y + 3 = 0$ (b) $X - 2Y - 10 = 0$
(c) $X - 2Y + 8 = 0$ (d) $X - 2Y + 13 = 0$

8. The equation of the line passing through the points $(5, 3)$ and $(3, 5)$ is
(a) $X - Y + 8 = 0$ (b) $X + Y + 8 = 0$
(c) $X - Y - 8 = 0$ (d) $X + Y - 8 = 0$

9. The point which divides the line segment joining the points $(7, -6)$ and $(3, 4)$ in ratio 1 : 2 internally lies in the
(a) I quadrant (b) II quadrant
(c) III quadrant (d) IV quadrant

10. Which of the following two lines are perpendicular?
1. $x + 2y = 5$ 2. $2x - 4y = 6$
3. $2x + 3y = 4$ 4. $2x - y = 4$
(a) 1 and 2 (b) 2 and 4
(c) 2 and 3 (d) 1 and 4

11. Ratio in which the line $3x + 4y = 7$ divides the line segment joining the points $(1, 2)$ and $(-2, 1)$ is
(a) 3 : 5 (b) 4 : 6
(c) 4 : 9 (d) None of these

12. For what value of k, are the lines $x + 2y - 9 = 0$ and $kx + 4y + 5 = 0$ parallel?
(a) 2 (b) – 1 (c) 1 (d) 0

13. The area of the triangle formed by the line $5x - 3y + 15 = 0$ with coordinate axes is
(a) 15 sq. units (b) 5 sq. units
(c) 8 sq. units (d) $\frac{15}{2}$ sq. units

14. For what value of k, the equations $3x - y = 8$ and $9x - ky = 24$ will have infinitely many solutions ?
(a) 6 (b) 5 (c) 3 (d) 1

15. Which of the following points is the nearest to the origin?
(a) $(0, -6)$ (b) $(-8, 0)$ (c) $(-3, -4)$ (d) $(7, 0)$

16. The line $mx + ny = 1$ passes through the points $(1, 2)$ and $(2, 1)$. What is the value of m?
(a) 1 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

17. The vertices of triangle ABC are $A(4, 4)$, $B(6, 3)$, $C(2, -1)$; then angle ABC is equal to
(a) 45° (b) 90°
(c) 60° (d) None of these

18. What is the inclination of the line
$\sqrt{3}x - y - 1 = 0$?
(a) 30° (b) 60° (c) 135° (d) 150°

19. The perimeter of the triangle whose vertices are $(-1, 4)$, $(-4, -2)$, $(3, -4)$, will be
(a) 38 (b) 16
(c) 42 (d) None of these

20. Find the value of 'A' if the distance between the points (8, A) and (4, 3) is 5.
(a) 6 (b) 0
(c) Both (a) and (b) (d) Not

21. Find the value of C if the distance between the point (C, 4) and the origin is 5 units
(a) 3 (b) –3 (c) Both a & b (d) 4

22. If two opposite vertices of a rectangle are (4, 2) and (10, 6) and the equation of other diagonal is x – 3y + k = 0. Find the value of k?
(a) $\frac{1}{2}$ (b) 3 (c) 5 (d) 4

GP_3386

23. For what value of k, the lines $(k+1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular to each other?

(a) $\frac{1}{3}$ (b) $\frac{-5}{3}$ (c) $-\frac{1}{3}$ (d) 5

24. A (–2, –1), B(1, 0), C (4, 3) and D (1, 2) are the four points of a quadrilateral. The quadrilateral is a :

(a) Square (b) Rhombus
(c) Parallelogram (d) None of these

25. Find the co-ordinates of the centroid of a triangle whose vertices are (0, 6) (8, 12) and (8, 0).

(a) $\left(\frac{16}{3}, 6\right)$ (b) $\left(6, \frac{16}{3}\right)$ (c) (6, 5) (d) (6, 3)

26. The angle which the line joining the points $(\sqrt{3}, 1)$ and $(\sqrt{15}, \sqrt{5})$ makes with x–axis is :

(a) 30° (b) 45° (c) 60° (d) 90°

27. The equation of a line parallel to x–axis and passing through (–6, –5) is :

(a) y=–5 (b) x = –6
(c) y = – 5x (d) x= – 6x–5

28. The length of perpendicular from the origin to the line $12x + 5y + 7 = 0$ is

(a) 2 units (b) 1 unit
(c) $\frac{7}{13}$ units (d) $\frac{7}{11}$ units

29. The value of k for which the system of equations $kx + y = 7$, $9x + 3y = 17$ has a unique solution :

(a) k=3 (b) k=0
(c) $k \neq 3$ (d) $k \neq 0$

30. Find the equation of the line which cuts off intercepts 2 and 3 on the axis.

(a) 9x – 7y = 6 (b) 3x – 2y = 5
(c) 4x – 3y = 7 (d) 3x + 2y = 6

31. The two vertices of a triangle are (3, –5) and (–7, 4). If its centroid is (2, – 12), find the third vertex.

(a) (10, – 35) (b) (–2, 10)
(c) (10, 35) (d) (–3, 10)

32. The lines whose equations are $2x - 5y + 7 = 0$ and $8x - 20y + 28 = 0$ are :

(a) parallel (b) perpendicular
(c) coincident (d) intersecting

33. The ratio in which the line segment joining A (3, – 5) and B (5, 4) is divided by x–axis is :

(a) 4 : 5 (b) 5 : 4 (c) 5 : 7 (d) 6 : 5

34. The ratio in which the point $P\left(1, \frac{10}{3}\right)$ divides the line segment joining the points A (–3, 2) and B (3, 4) is

(a) 2 : 3 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

35. Let the vertices of a triangle ABC be (4, 4), (3, 5) and (–1, –1), then the triangle is :

(a) scalene (b) equilateral
(c) right angled (d) None of these

36. Find the mid-point of the line-segment joining two points (3, 4) and (5, 12).

(a) (–4, 8) (b) (0, 8)
(c) (4, 8) (d) (4, 0)

37. The points A (– 4, – 1), B (–2, – 4), C (4, 0) and D (2, 3) are the vertices of a

(a) Parallelogram (b) Rectangle
(c) Rhombus (d) Square

38. The centroid of the triangle whose vertices are (3, 10), (7, 7), (–2, 1) is

(a) (8/3, 6) (b) (6, 8/3)
(c) (– 4, – 7/3) (d) None of these

39. The coordinates of the centroid G of a triangle with vertices at (3, 7), (5, 5) and (–3, 2) is

(a) (10/3, 14/3) (b) (10/3, 10/3)
(c) (5/3 14/3) (d) (11/3, 10/3)

40. Distance between P (x, 4) and Q (3, –6) is 10 units and x is positive integer, then x =

(a) 3 (b) 9
(c) 7 (d) 11

41. In what ratio is the line segment made by the points (7, 3) and (– 4, 5) divided by the y-axis?

(a) 2 : 3 (b) 4 : 7
(c) 3 : 5 (d) 7 : 4

42. Find the third vertex of the triangle whose two vertices are (–3, 1) and (0, –2) and the centroid is the origin.

(a) (2, 3) (b) $\left(\frac{-4}{3}, \frac{14}{3}\right)$
(c) (3, 1) (d) (6, 4)

43. If the origin gets shifted to (2, 2), then what will be the new coordinates of the point (4, –2)?

(a) (– 2, 4) (b) (2, 4)
(c) (4, 2) (d) (2, – 4)

44. The points (3,0), (– 3, 0), (0, $-3\sqrt{3}$) are the vertices of

(a) equilateral triangle
(b) isosceles triangle
(c) right triangle
(d) scalene triangle

LEVEL - 2

1. The vertices of a parallelogram in order are $A(1, 2), B(4, y), C(x, 6), D(3, 6)$, then $(x, y) =$
(a) (6,3) (b) (3,6) (c) (5,6) (d) (1,4)

2. How many squares are possible if two of the vertices of a quadrilateral are (1, 0) and (2, 0)?
(a) 1 (b) 2 (c) 3 (d) 4

3. The straight line joining (1, 2) and (2, –2) is perpendicular to the line joining (8, 2) and $(4, p)$. What will be the value of p?
(a) –1 (b) 1
(c) 3 (d) None of these

4. What is the slope of the line perpendicular to the line $\frac{x}{4}+\frac{y}{3}=1$?
(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $-\frac{4}{3}$ (d) $\frac{4}{3}$

5. If the points $A(1, 2)$, $B(2, 4)$ and $C(3, a)$ are collinear, what is the length BC?
(a) $\sqrt{2}$ unit (b) $\sqrt{3}$ unit
(c) $\sqrt{5}$ unit (d) 5 unit

6. If area of Δ formed by joining mid-points of the sides of ΔABC is 2sq. units, then area of $\Delta ABC =$
(a) 8 (b) 4 (c) 2 (d) 1

7. Find the equation of a straight line passing through (2, –3) and having a slope of 1 unit.
(a) $y-x+5=0$ (b) $x+y+5=0$
(c) $-x-y=-5$ (d) $x-y-5=0$

8. The middle point of $A(1, 2)$ and $B(x, y)$ is $C(2, 4)$. If BD is perpendicular to AB such that $CD = 3$ unit, then what is the length BD?
(a) $2\sqrt{2}$ unit (b) 2 unit
(c) 3 unit (d) $3\sqrt{2}$ unit

9. Which one of the following points on the line $2x - 3y = 5$ is equidistant from (1, 2) and (3, 4)?
(a) (7,3) (b) (4, 1) (c) (1, – 1) (d) (–2, –3)

10. Find the area of quadrilateral intercepted by straight lines $4x + 2y = 12$, $8x + 4y = 48$ between the axis.
(a) 36 sq. units (b) 27 sq. units
(c) 18 sq. units (d) 9 sq. units

11. The area of the triangle formed by the lines $5x + 7y = 35$, $4x + 3y = 12$ and x-axis is :
(a) $\frac{160}{13}$ sq.units (b) $\frac{150}{13}$ sq.units
(c) $\frac{140}{13}$ sq.units (d) 10 sq. units

12. The value of p for which the lines $3x + 8y + 9 = 0$ and $24x + py + 19 = 0$ are perpendicular is :
(a) – 12 (b) – 9 (c) – 11 (d) 9

13. The angle between the lines represented by the equations $2y - \sqrt{12}x - 9 = 0$ and $\sqrt{3}y - x + 7 = 0$, is :
(a) 30° (b) 45° (c) 60° (d) $22\frac{1^\circ}{2}$

14. If $A(3, 5)$, $B(–3, – 4)$, $C(7, 10)$ are the vertices of a parallelogram taken in the order, then the co-ordinates of the fourth vertex D are
(a) (10,19) (b) (15,10)
(c) (19,10) (d) (13,19)

15. The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is
(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$
(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

16. The coordinates of a point which divides the join of (5, –5) and (2, – 3) in the ratio 4 : 3, externally, are:
(a) (3,4) (b) (–7,3)
(c) (– 7, 9) (d) (8,3)

17. If the coordinates of the mid-point of the line segment joining the points (2, 1) and (1, – 3) is (x, y), then the relation between x and y can be best described by
(a) $3x + 2y = 5$ (b) $6x + y = 8$
(c) $5x - 2y = 4$ (d) $2x - 5y = 4$

18. If the point $R(–1, 2)$ divides externally the line segment joining $P(2, 5)$ and Q in the ratio 3 : 4, what will be the coordinates of Q?
(a) (– 3, 6) (b) (2, – 4)
(c) (3,6) (d) (1,2)

19. If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(– 6, 5)$ and $R(– 2, 3)$, then the value of a is
(a) – 4 (b) – 12 (c) 12 (d) – 6

GP_3386

Hints and Solutions

Level 1

1. (c) $2=\sqrt{(x-3)^2+(2-4)^2}$

$\Rightarrow 2=\sqrt{(x-3)^2+4}$

Squaring both sides

$4=(x-3)^2+4 \Rightarrow x-3=0 \Rightarrow x=3$

2. (b) $\left(\frac{-2+6}{2}, \frac{4+10}{2}\right)=(2,7)$

3. (a) $\frac{5-4}{2+1}=\frac{k-5}{3-2} \Rightarrow k=\frac{16}{3}$

4. (b) Ratio $=-\left(\frac{-1+1-4}{5+7-4}\right)=\frac{1}{2}$

5. (b) $x=\frac{2+5+3}{3}=\frac{10}{3}$; $y=\frac{1+2+4}{3}=\frac{7}{3}$

6. (d) $\frac{1}{2}[4(-2+16)+3(-16-4)+3(4+2)]=7$

7. (b) Line parallel to $X-2Y-5=0$ will be $X-2Y+K=0$.

Put the point in this equation we have

$2+8+K=0$

or $K=-10$ or line is $X-2Y-10=0$

8. (d) The equation will be

$Y-3=\frac{(5-3)}{(3-5)}\times(X-5)$

$\Rightarrow Y-3=5-X$ or $X+Y-8=0$

9. (d) The point is $\left(\frac{3+14}{3}, \frac{4-12}{3}\right)$ i.e., $\left(\frac{17}{3}, \frac{-8}{3}\right)$

10. (d) $(1\times 2)+(2\times(-1))=2-2=0$

11. (c) $-\frac{3(1)+4(2)-7}{3(-2)+4(1)-7}=-\frac{4}{-9}=\frac{4}{9}$

12. (a) Given equation of lines are

$x+2y-9=0 \Rightarrow 2y=-x+9$

$\Rightarrow y=-\frac{1}{2}x+\frac{9}{2}$ ...(1)

and $kx+4y+5=0 \Rightarrow 4y=-kx-5$

$\Rightarrow y=\frac{-k}{4}x-\frac{5}{4}$ ...(2)

Since line (1) and line (2) are parallel therefore their slopes are equal.

$\frac{-1}{2}=\frac{-k}{4} \Rightarrow k=2$

13. (d) $\frac{5x}{-15}+\frac{3y}{15}=1 \Rightarrow \frac{x}{-3}+\frac{y}{5}=1$

$\therefore$ area of $\Delta=\frac{1}{2}\times 3\times 5=\frac{15}{2}$

14. (c) For infinite solution $\frac{3}{9}=\frac{-1}{-k}=\frac{8}{24}$

$\Rightarrow \frac{1}{3}=\frac{1}{k} \Rightarrow k=3$

15. (c) Distance between origin and

$(-3,-4)=\sqrt{(-3)^2+(-4)^2}=5$ units

Similarly, distances of points $(0,-6)$, $(-8,0)$, $(7,0)$ from origin are 6 units, 8 units and 7 units respectively

16. (d) Since line $mx+ny=1$ passes through $(1,2)$ and $(2, 1)$, therefore they satisfy the equation.

$\Rightarrow \quad m+2n=1$ ...(i)

and $\quad 2m+n=1$ ...(ii)

From eqs. (i) and (ii), we get $m=n=\frac{1}{3}$

17. (d) Angle ABC is the angle between the lines AB and BC.

Now slope of line $AB=m_1=\frac{3-4}{6-4}=-\frac{1}{2}$

and slope of line $BC=m_2$

$=\frac{-1-3}{2-6}=\frac{-4}{-4}=1$

Now, $\tan\theta=\frac{m_1-m_2}{1+m_1m_2}=-1 \Rightarrow \theta=135°$

18. (b) Given equation can be written as

$y=\sqrt{3}x-1$ on comparing with $y=mx+c$

We get $\tan\theta=\sqrt{3} \Rightarrow \theta=60°$

19. (d) The three length AB, BC, AC will be

$$AB = \sqrt{[(-1+4)^2 + (4+2)^2]} = \sqrt{45}$$

$$BC = \sqrt{[(-4-3)^2 + (-2+4)^2]} = \sqrt{7^2 + 2^2} = \sqrt{53}$$

$$AC = \sqrt{4^2 + 8^2} = \sqrt{80}$$

Perimeter $= AB + BC + AC = 7\sqrt{5} + \sqrt{53}$

20. (c) Distance between the points

$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

So, $5 = \sqrt{(8-4)^2 + (A-3)^2}$

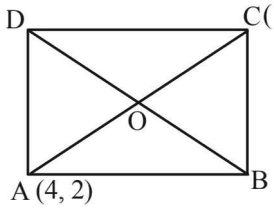
$25 = (4)^2 + (A-3)^2 \Rightarrow A^2 - 6A = 0$

$\Rightarrow A(A-6) = 0 \Rightarrow A = 6, 0$

21. (c) $(5)^2 = (C-0)^2 + (4-0)^2$

$25 = C^2 + 16 \Rightarrow C^2 = 9 \Rightarrow C = \pm 3$

22. (c)



O is the mid-point of diagonal AC and BD

co-ordinate of $O = \left(\frac{4+10}{2}, \frac{2+6}{2}\right) = (7,4)$

Point O is also situated on line

$x - 3y + k = 0$

$\Rightarrow 7 - 3 \times 4 + k = 0 \Rightarrow 7 - 12 + k = 0 \Rightarrow k = 5$

23. (b) $(k + 1)x + ky = 3$ and $5x - 2y = 7$ are perpendicular

For perpendicular : $a_1a_2 + b_1b_2 = 0$

$\Rightarrow (k+1) \times 5 + k \times (-2) = 0$

$$\Rightarrow 5k + 5 - 2k = 0 \Rightarrow k = \frac{-5}{3}$$

24. (c) Let A, B, C and D be the vertices of the quadrilateral whose coordinates are (–2, –1), (1, 0), (4, 3) and (1, 2) respectively.

Now,

$AB = \sqrt{10}, BC = \sqrt{18}, DC = \sqrt{10}, AD = \sqrt{18}$

$\therefore$ AB = DC and BC = AD

i.e., the opposite sides are equal

Hence ABCD is a parallelogram.

25. (a) Centroid of a triangle

$$\equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\equiv \left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right) \equiv \left(\frac{16}{3}, 6\right)$$

26. (a) The slope of the line is

$$\frac{\sqrt{5}-1}{\sqrt{15}-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}} \text{ or } \theta = 30°$$

27. (a) The equation of a line parallel to x-axis is y = b.

Since, it passes through (–6, –5), so b = – 5

$\therefore$ The required equation is, y = – 5.

28. (c) Length of perpendicular

$$= \frac{12 \times 0 + 5 \times 0 + 7}{\sqrt{12^2 + 5^2}} = \frac{7}{13} \text{ units}$$

29. (c) $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution if

$$\therefore \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{k}{9} \neq \frac{1}{3} \Rightarrow k \neq 3$$

30. (d) a = 2, b = 3.

$$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{2} + \frac{y}{3} = 1 \Rightarrow 3x + 2y = 6$$

31. (a) Vertices of triangle are (3, – 5) (–7, 4) and (h, k) whereas centroid is (2, – 12)

$$(2, -12) \equiv \left(\frac{3-7+h}{3}, \frac{-5+4+k}{3}\right)$$

On solving we get (h, k) = (10, – 35)

32. (c) Here, $\frac{a_1}{a_2} = \frac{2}{8} = \frac{1}{4}, \frac{b_1}{b_2} = \frac{-5}{-20} = \frac{1}{4}$

and $\frac{c_1}{c_2} = \frac{7}{28} = \frac{1}{4}$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

So the given lines are coincident.

33. (b) Let the ratio be k : 1

The ordinate of a point lying on x–axis must be zero

$$\therefore \frac{4k - 5 \times 1}{k+1} = 0 \Rightarrow 4k = 5 \Rightarrow k = \frac{5}{4}$$

$\therefore$ Required ratio is $\frac{5}{4} : 1 = 5 : 4$

34. (c) Let the ratio be k : 1

$$\therefore \quad \frac{3k - 3 \times 1}{k+1} = 1 \Rightarrow 3k - 3 = k + 1$$

$\Rightarrow 2k = 4 \Rightarrow k = 2$

$\therefore$ Required ratio is 2 : 1

GP_3386

35. (c) Let $A=(4,4), B=(3,5), C=(-1,-1)$

Then AB $=\sqrt{(3-4)^2+(5-4)^2}=\sqrt{2}$

BC $=\sqrt{(-1-3)^2+(-1-5)^2}=\sqrt{52}$

AC $=\sqrt{(-1-4)^2+(-1-4)^2}=\sqrt{50}$

$\therefore$ $AB^2 + AC^2 = BC^2$

Hence by the Pythagoras theorem ABC is a right-angled triangle.

36. (c) Let $A(3,4)$ and $B(5,12)$ be the given points. Let $C(x, y)$ be the mid-point of AB. Using mid-point formula, we have,

$x=\frac{3+5}{2}=4$ and $y=\frac{4+12}{2}=8$

$\therefore$ $C(4, 8)$ are the co-ordinates of the mid-point of the line segment joining two points (3, 4) and (5, 12).

37. (b) Let $A(-4,-1), B(-2,-4), C(4,0), D(2,3)$ be the given points. then

$AB=\sqrt{(-2-(-4))^2+(-4-(-1))^2}=\sqrt{13}$

$BC=\sqrt{(4-(-2))^2+(0-(-4))^2}=2\sqrt{13}$

$CD=\sqrt{(2-4)^2+(3-0)^2}=\sqrt{13}$

$AD=\sqrt{(2-(-4))^2+(3-(-1))^2}=2\sqrt{13}$

$\therefore$ $AB=CD$ and $BC=AD$

Now, $AC=\sqrt{(4-(-4))^2+(0-(-1))^2}=\sqrt{65}$

$AB^2+BC^2=CD^2+AD^2=65=AC^2$

Hence, ABCD is a rectangle.

38. (a) Centroid $=\left(\frac{3+7-2}{3},\frac{10+7+1}{3}\right)=\left(\frac{8}{3},6\right)$

39. (c) Let G be (X, Y), then $X=\{3+5+(-3)\}/3=5/3$
$Y=(7+5+2)/3=14/3$ $\Rightarrow$ G is (5/3, 14/3)

40. (a) The distance between two points is

$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

$10=\sqrt{(-6-4)^2+(3-x)^2}\Rightarrow x=3$

41. (d) Coordinates of the point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2)g in the ratio m : n is given by.

$\Rightarrow\left[\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right]$

Let y-axis divide the line segment joining P (7, 3) and Q (–4, 5) in the ratio (K : 1), so,

$=\left[\frac{K(-4)+1(7)}{K+1},\frac{K(5)+1(3)}{K+1}\right]$

Since x coordinate of a point
On the y-axis is 0

So, $\frac{K(-4)+7}{K+1}=0\Rightarrow K=7/4$

42. (c) As given that, the centroid is the origin.

So, $\frac{x_1+x_2+x_3}{3}=0$, $\frac{y_1+y_2+y_3}{3}=0$

$\frac{-3+0+x_3}{3}=0$, $\frac{1+(-2)+y_3}{3}=0$

$\Rightarrow x_3=3, y_3=1$

Hence, the third vertex by the triangle is (3, 1)

43. (d) Let the new coordinates be (X, Y)

$\therefore$ $X=4-2=2$
$Y=-2-2=-4$

Thus the new coordinates are $(2, -4)$

44. (a) Find the three lengths separately

$AB=6$, $BC=\sqrt{3^2+(3\sqrt{3})^2}=6$,

$AC=\sqrt{3^2+(3\sqrt{3})^2}=6$

Hence, the point are the vertices of an equilateral triangle.

Level 1

1. (a) Mid-point of AC is $\left(\frac{1+x}{2},\frac{2+6}{2}\right)$ *i.e.*, $\left(\frac{1+x}{2},4\right)$;

Mid-point of BD is $\left(\frac{4+3}{2},\frac{y+5}{2}\right)$

In a || gm, diagonals bisect each other

$\therefore$ $\frac{1+x}{2}=\frac{7}{2}$ and $\frac{y+5}{2}=4\Rightarrow x=6, y=3$

2. (c) When we consider these two points to be the vertices of a side, then two squares can be possible. Again, if we consider the vertices of diagonal, the one more square can be constructed.

3. (b) Equation of line passing through $(1,2)$ and $(2,-2)$ is

$(y-2)=\left(\frac{-2-2}{2-1}\right)(x-1)\Rightarrow 4x+y-6=0$

Equation of line passing through $(8,2)$ and $(4,p)$ is $(y-2)=\left(\frac{p-2}{4-8}\right)(x-8)$

$\Rightarrow (p-2)x+4y-8p+8=0$

Now, $4(p-2)+(1\times 4)=0 \Rightarrow p=1$

4. (d) Given equation of the line $\frac{x}{4}+\frac{y}{3}=1$ can be written as

$3x+4y=12 \Rightarrow y=\frac{-3}{4}x+\frac{12}{4}$

The slope of the line $\frac{x}{y}+\frac{y}{3}=1$ is $\frac{-3}{4}$

$\therefore$ Slope of the line perpendicular to this line $=-\left(\frac{-1}{3/4}\right)=\frac{4}{3}$

5. (c) Since the points are collinear, therefore

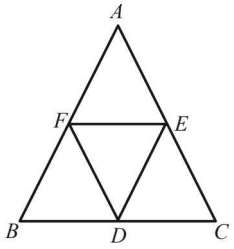
$\Rightarrow \quad (4-a)-2(2-3)+1(2a-12)=0$

$\Rightarrow \quad 4-a+2+2a-12=0 \Rightarrow a=6$

Thus, coordinates of C are (3, 6).

$\therefore BC=\sqrt{(3-2)^2+(6-4)^2}=\sqrt{5}$ unit

6. (a) Area of $\Delta ABC = 4$ area of ΔDEF



where D, E, F are the mid-points of BC, CA, AB respectively.

Reqd. area = 4 (2) = 8 sq. units

7. (a) Here slope = 1

And point given is (2, –3).

So, we will use point-slope formula for finding the equation of straight line. This formula is given by:

$(y-y_1)=m(x-x_1)$

So, equation of the line will be $y-(-3)=1(x-2)$

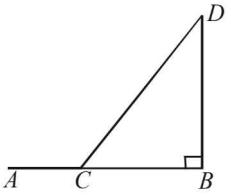
$\Rightarrow y+3=x-2 \Rightarrow y-x+5=0$

8. (b) Given that mid point of $A(1,2)$ and $B(x,y)$ is $C(2,4)$,

$\frac{1+x}{2}=2$ and $\frac{2+y}{2}=4 \Rightarrow x=3$ and $y=6$

This given coordinates of B are (3, 6).

Given that $BD \perp AB$ and $CD=3$ unit



$BC=\sqrt{(2-3)^2+(4-6)^2}=\sqrt{1+4}=\sqrt{5}$

In right angled ΔBCD, $CD^2=BC^2+BD^2$

$\Rightarrow 9=5+BD^2 \Rightarrow BD^2=4 \Rightarrow BD=2$ unit

9. (b) Let point $P(x_1,y_1)$ be equidistant from point $A(1,2)$ and $B(3,4)$.

$\therefore PA=PB \Rightarrow PA^2=PB^2$

$\Rightarrow (1-x_1)^2+(2-y_1)^2=(3-x_1)^2+(4-y_1)^2$

$\Rightarrow 1+x_1^2-2x_1+4+y_1^2-4y_1$

$=9+x_1^2-6x_1+16+y_1^2-8y_1$

$\Rightarrow x_1+y_1=5$ ....(1)

As $P(x_1,y_1)$ lies on $2x-3y=5$

$\therefore 2x_1-3y_1=5$ ....(2)

On solving Eqs. (1) and (2), we get

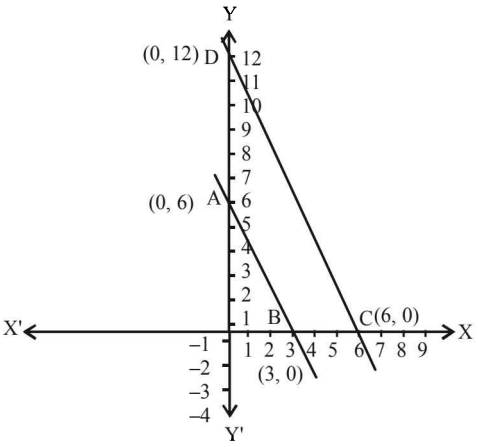
$x_1=4$ and $y_1=1$

$\therefore$ Coordinates of P are (4, 1).

10. (b) $4x+2y=12$

$8x+4y=48$

$\Rightarrow \frac{x}{3}+\frac{y}{6}=1 \Rightarrow \frac{x}{6}+\frac{y}{12}=1$



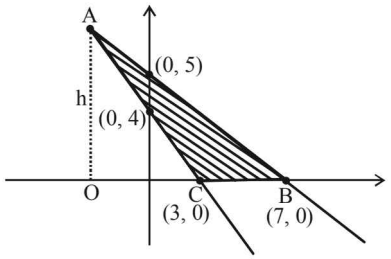
Area of quadrilateral

= Area of ΔOCD – Area of ΔOAB

$=\frac{1}{2}\times 6\times 12-\frac{1}{2}\times 3\times 6=27$ sq. units

11. (a) $5x+7y=35$ or $\frac{x}{7}+\frac{y}{5}=1$ ....(i)

and $4x + 3y = 12$ or $\frac{x}{3} + \frac{y}{4} = 1$ ....(ii)



$\therefore$ Base of $\Delta ABC = BC = 7 - 3 = 4$ unit

let height $(OA) = h$

From (i) and (ii) we get $y = \frac{80}{13}$

$\therefore$ ar $(\Delta ABC) = \frac{1}{2} \times 4 \times \frac{80}{13} = \frac{160}{13}$ sq.unit

12. (b) Condition of perpendicularism,

$a_1a_2 + b_1b_2 = 0 \therefore 3 \times 24 + 8 \times p = 0$

$\Rightarrow p = -9$

Alternatively

$3x + 8y + 9 = 0$

$\Rightarrow y = \left(-\frac{3}{8}\right)x - \frac{9}{8} \Rightarrow m_1 = -\frac{3}{8}$

$24x + py + 19 = 0$

$\Rightarrow y = \left(-\frac{24}{p}\right)x - \frac{19}{p} \Rightarrow m_2 = -\frac{24}{p}$

For perpendicularism, $m_1.m_2 = -1$

$\therefore \quad \left(-\frac{3}{8}\right)\left(-\frac{24}{p}\right) = -1 \Rightarrow p = -9$

13. (a) $2y - \sqrt{12}\,x - 9 = 0$

$\Rightarrow y = \frac{\sqrt{12}}{2}x + \frac{9}{2} \Rightarrow m_1 = \frac{\sqrt{12}}{2} = \sqrt{3}$

$\sqrt{3}y - x + 7 = 0 \Rightarrow m_2 = \frac{1}{\sqrt{3}}$

$\therefore \quad \tan\theta = \frac{m_1 - m_2}{1 + m_1.m_2} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}.\frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$

So, $\theta = 30°$

14. (d) Mid-point of $A(3, 5)$ and $C(7, 10)$

$= M\left(5, \frac{15}{2}\right)$

$\therefore$ Mid-point of $BD = M\left(5, \frac{15}{2}\right)$

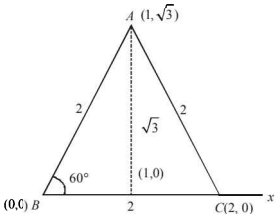
$B(-3, -4)$ and $D(x, y)$

$\therefore \quad \frac{-3 + x}{2} = 5, \quad x = 10 + 3 = 13$

$\frac{-4 + y}{2} = \frac{15}{2}, \; y = 15 + 4 = 19$

Co-ordinates of fourth vertex $D = (13, 19)$

15. (d) Clearly, the triangle is equilateral.



So, the incentre is the same as the centroid.

$\therefore$ Incentre

$= \left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$

16. (b) Let the ratio be 4 : 3 or 4/3 : 1.

Now $X = \frac{\frac{4}{3} \times 2 - 5}{\frac{4}{3} - 1} = \frac{\frac{8}{3} - 5}{\frac{1}{3}} = \frac{-\frac{7}{3}}{\frac{1}{3}} = -7$

$Y = \frac{\frac{4}{3} \times (-3) + 5}{\frac{4}{3} - 1} = \frac{1}{\frac{1}{3}} = 3.$

Hence $(-7, 3)$ is required point

17. (b) The mid-point joining the points given will be

$\{(2 + 1)/2, (1 - 3)/2\} = \left\{\frac{3}{2}, -1\right\}$

Only (b) option satisfies it.

18. (c) Using section formula, we get.

x- coordinate of point $R = \left(\frac{mx_2 - nx_1}{m - n}\right)$

$-1 = \frac{3x - 4 \times 2}{3 - 4} \Rightarrow 1 = 3x - 8 \Rightarrow x = 3$

y-coordinate of point R

$= \frac{(my_2 - n\,y_1)}{(m - n)} \Rightarrow 2 = \frac{3 \times y - 4 \times 5}{3 - 4} \Rightarrow y = 6$

19. (b) As given that $P\left(\frac{a}{3}, 4\right)$ is the mid-point of line segment joining the point Q (–6, 5) and R (–2, 3)

Since, P is the mid-point of QR, then

$\frac{a}{3} = \left(\frac{-6 + (-2)}{2}\right) \Rightarrow a = -12$

CHAPTER

22

Mensuration-Plane Figures

BASIC CONVERSION OF UNITS

(i) Length:

1 m = 10 dm
1 dm = 10 cm
1 cm = 10 mm
1 feet (ft) = 12 inches
1 inch = 2.54 cm
1 yard (y) = 3 feet (ft)

(ii) Surface Area:

Surface areas are measured in square units.
1 square metre = 1m × 1m = 100 cm × 100 cm = 10000 cm^2
1 square yard = 1y × 1y = 3ft × 3ft = $9ft^2$

(iii) Mass:

1 kg = 1000 grams (g)
1 quintal = 100 kg
1 tonne = 10 quintal = 1000 kg

(iv) Volume:

Volumes are measured in cubic units.
1 m^3 = 10000 litres
1 litre = 1000 cm^3

Note that $\sqrt{2} = 1.414$; $\sqrt{3} = 1.732$,

$\pi = \frac{22}{7}$ or 3.14

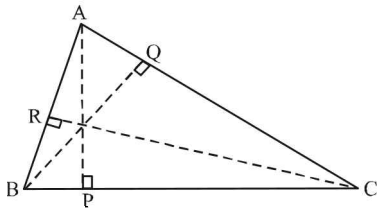
PLANE FIGURES

Perimeter: The perimeter of a plane geometrical figure is the total length of sides (or boundary) enclosing the figure. Units of measuring perimeter can be cm, m, km, etc.

Area: The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

Area of a Triangle

1. If in a triangle, we draw a perpendicular AP from vertex A on opposite side BC then AP is called **altitude** (or **height**) of the triangle ABC corresponding to base BC.



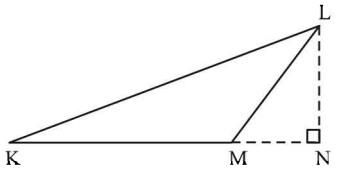
Similarly, BQ and CR are the altitudes of ΔABC corresponding to, bases AC and AB respectively.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{corresponding altitude}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AP = \frac{1}{2} \times AC \times BQ$$

$$= \frac{1}{2} \times AB \times CR$$

Note that in ΔKLM, LN is the perpendicular on KM produced.

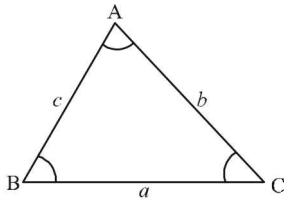


Here, LN is the altitude corresponding to the base KM of ΔKLM.

$$\therefore \text{Area of } \Delta KLM = \frac{1}{2} \times KM \times LN$$

GP_3386

2. Let in ΔABC, $BC = a$, $AC = b$ and $AB = c$; then perimeter of $\Delta ABC = a + b + c$



Semi-perimeter of ΔABC, $s = \frac{a+b+c}{2}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

(Heron's formula)

3. Area of $\Delta ABC = \frac{1}{2} \times$ (Product of two sides) $\times$ (Sine of the included angle)

$= \frac{1}{2} ac \sin B$ or $\frac{1}{2} ab \sin C$ or $\frac{1}{2} bc \sin A$

Note that $\sin 30° = \frac{1}{2}$, $\sin 45° = \frac{1}{\sqrt{2}}$,

$\sin 60° = \frac{\sqrt{3}}{2}$, $\sin 90° = 1$

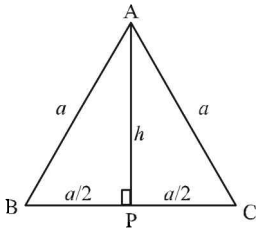
Area of an Equilateral Triangle

Since, ΔABC is an equilateral triangle.

$\therefore AB = BC = CA = a$ (say)

From ΔAPC,

$$AP^2 = AC^2 - PC^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$



$$AP = \frac{\sqrt{3}}{2}a \Rightarrow h = \frac{\sqrt{3}}{2}a$$

$\therefore$ Area of an equilateral $\Delta = \frac{1}{2} \times a \times \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$.

where a is the length of its one side

Note that

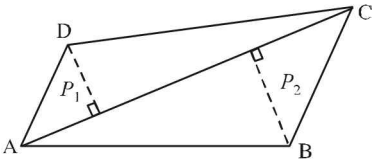
(i) among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.

(ii) for a given area of triangle, the perimeter of equilateral triangle is minimum.

Area of a Quadrilateral

1. Area of quadrilateral $ABCD$

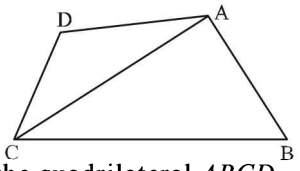
$= \frac{1}{2} \times$ (Length of the longest diagonal) $\times$ (Sum of length of perpendiculars to the longest diagonal from its opposite vertices)



$= \frac{1}{2} \times d \times (p_1 + p_2)$, where d

$= AC$ (i.e. longest diagonal)

2. If length of four sides and one of its diagonals of quadrilateral $ABCD$ are given, then



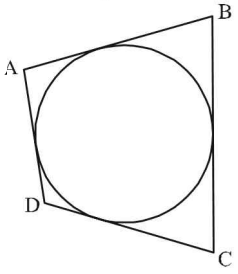
Area of the quadrilateral $ABCD$

= Area of ΔABC + Area of ΔADC

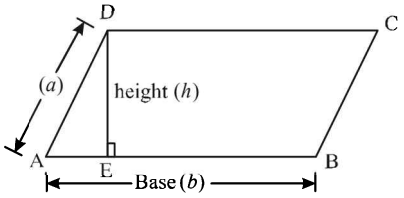
3. Area of circumscribed quadrilateral

$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

where $s = \frac{a+b+c+d}{2}$ and a, b, c, d are length of sides of quadrilateral $ABCD$.

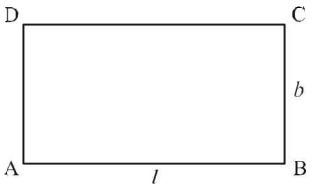


Area of a Parallelogram



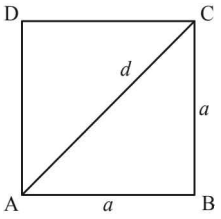
Area of parallelogram = Base × Corresponding height
$A = b \times h$
Perimeter of a parallelogram = $2(a + b)$,
where a and b are length of adjacent sides.

Area of a Rectangle



Area of a rectangle = Length × Breadth = $l \times b$
Perimeter of a rectangle = $2(l + b)$

Area of a Square



Area of square = side × side = $a \times a = a^2$

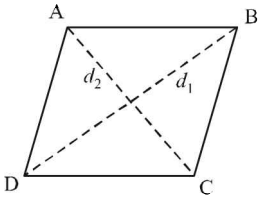
Length of diagonal (d) = $a\sqrt{2}$
(by Pythagoras theorem)

$$\Rightarrow a = \frac{d}{\sqrt{2}}$$

Hence, area of the square = $\left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$

Perimeter of square = 4 × side = $4 \times a$
For a given perimeter of a rectangle, a square has maximum area.

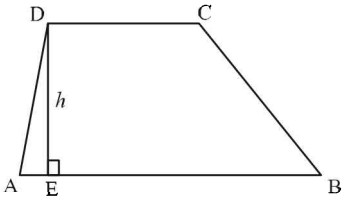
Area of a Triangle



Area of a rhombus = $\frac{1}{2}$ × product of diagonals

$$= \frac{1}{2} \times d_1 \times d_2$$

Area of a Trapezium



Shorttest distance between parallel sides of a trapezium is called **height** of trapezium.
Fig. $ABCD$ is a trapezium, whose sides AB and CD are parallel,
Here $DE = h$ = Height of the trapezium
= Distance between || sides.

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of } \| \text{ sides}) \times \text{height}$$

$$= \frac{1}{2} \times (AB + CD) \times DE$$

Illustration – 1

A rectangular parking space is marked out by painting three of its sides. If the length of the unpainted side is 9 feet, and the sum of the lengths of the painted sides is 37 feet, then what is the area of the parking space in square feet?
(a) 46 (b) 81 (c) 126 (d) 252

Solution : (c) Clearly, we have : $l = 9$ and $l + 2b = 37$ or $b = 14$.
$\therefore$ Area = $(l \times b) = (9 \times 14)$ sq. ft. = 126 sq. ft.

Illustration – 2

A square carpet with an area 169 m² must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangular room. What is the area of rectangular room?
(a) 180 m² (b) 164 m² (c) 152 m² (d) 143 m²

Solution : (d) Side of square carpet
$= \sqrt{\text{Area}} = \sqrt{169} = 13m$
After cutting one side,
Measure of one side = 13 – 2 = 11 m
and other side = 13 m (remain same)
$\therefore$ Area of rectangular room = 13 × 11 = 143 m²

Illustration – 3

The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12 km/hr completes one round in 8 minutes, then the area of the park (in sq. m) is:

(a) 15360 **(b) 153600**
(c) 30720 **(d) 307200**

Solution : (b) Perimeter = Distance covered in 8 min.

$= \left(\frac{12000}{60} \times 8\right)$ m = 1600m.

Let length = $3x$ metres and breadth = $2x$ metres.
Then, $2(3x + 2x) = 1600 \Rightarrow x = 160$.
$\therefore$ Length = 480 m and Breadth = 320 m.
$\therefore$ Area = (480×320) m^2 = 153600 m^2.

Illustration – 4

The length and breadth of a playground are 36m and 21m respectively. Poles are required to be fixed all along the boundary at a distance 3m apart. The number of poles required will be

(a) 39 **(b) 38** **(c) 37** **(d) 40**

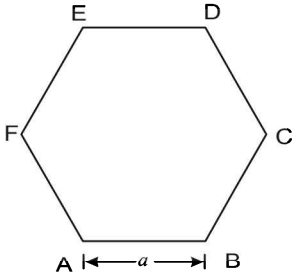
Solution : (b) Given, playground is rectangular.
Length = 36 m, Breadth = 21 m
Perimeter of playground = 2(21 + 36) = 114m
Poles are fixed along the boundary at a distance 3 m.

$\therefore$ Required no. of poles = $\frac{114}{3} = 38$.

Area of a Regular Hexagon

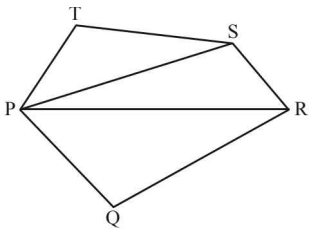
Area = $\frac{3\sqrt{3}}{2}a^2$,

where 'a' is the length of each side of the regular hexagon.



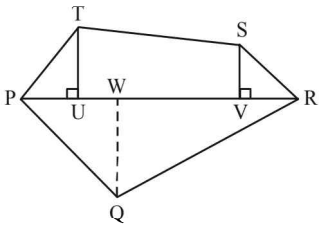
Area of Irregular Plane Figures

1. By drawing all the diagonals from anyone vertex, the polygon is divided into several triangles.
   Hence area of the polygon
   = Sum of areas of all the triangles



$\therefore$ Area of pentagon $PQRST$
= Area of ΔPQR + Area of ΔPRS + Area of ΔPST

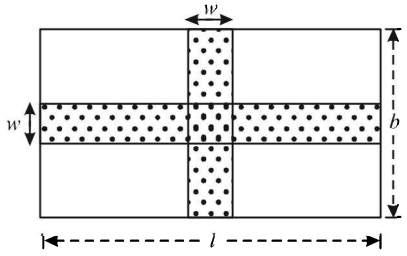
2. By drawing longest diagonal and perpendicular from all vertices on two sides of the longest diagonal to the longest diagonals, the polygon is divided into several right triangles and trapeziums. By finding the sum of all the triangles and trapeziums, so formed we get the area of the polygon.



Here, area of pentagon $PQRST$ = Area of ΔPTU + Area of trapezium ($TUVS$) + Area of ΔSVR + Area of ΔRQW + Area of ΔQWP.

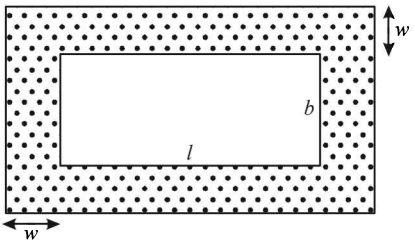
Paths

1. Pathways Running Across the Middle of a Rectangle



Area of the path = $l.w + b.w - w.w$
$= (l + b - w).w$
Perimeter of the path = $2l + 2b - 4w$
$= 2(l + b - 2w)$
Here w is the width of the path.

2. Pathways Outside a Rectangle

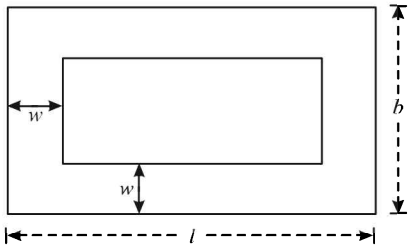


Area of path = $2(lw) + 2(b.w) + 4(w.w)$
$= (l + b + 2w)2w$

Perimeter of path
= (Internal perimeter) + (External perimeter)
$= 2(l + b) + 2(l + b + 4w) = 4(l + b + 2w)$

Here w is the width of the path.

3. Pathway Inside a Rectangle



Area of path = $2(l.w) + 2(b.w) - 4(w.w)$
$= (l + b - 2w).2w$

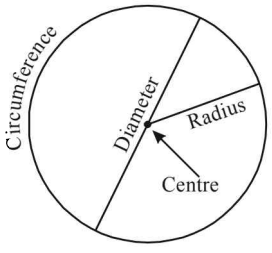
Perimeter of path
= Length of outer path + Length of inner path
$= 2(l + b) + 2(l + b - 4w) = 4(l + b - 2w)$

AREA RELATED TO A CIRCLE

Circle

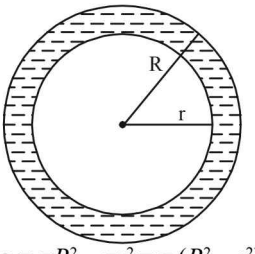
Set of all points in a plane which are at a fixed distance from a fixed point in the same plane is called a **circle**. The fixed point is called **centre** of the circle and the fixed distance is called **radius** of the circle.

Circumference or ***perimeter*** of a circle of radius r is
$c = 2\pi r = \pi d$ $(2r = d =$ diameter$)$
Area of the circle $= \pi r^2$



Circular Ring

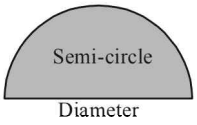
Region enclosed between two concentric circles of different radii in a plane is called a **ring**.



Area of the ring $= \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$
Circumference of the ring = (External circumference) + (Internal circumference) $= 2\pi R + 2\pi r = 2\pi(R + r)$

Semi-circle

A semi-circle is a figure enclosed by a diameter and one half of the circumference of the circle.

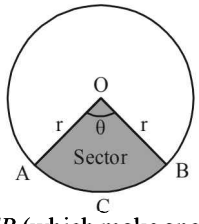


Area of the semi-circle $= \dfrac{\pi r^2}{2}$

Circumference of the semi-circle $= \pi r + 2r = r(\pi + 2)$

Sector of a Circle

Sector of a circle is the portion of a circle enclosed by two radii and an arc of the circle. *OACB* is a sector of the circle.



Length of arc *ACB* (which make angle θ at the centre)

$= (2\pi r) \times \dfrac{\theta}{360} = \dfrac{\pi r \theta}{180}$

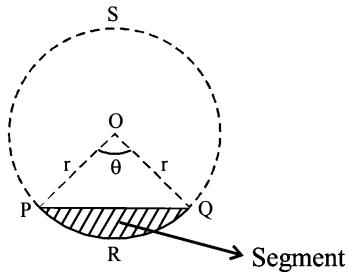
Perimeter of the sector $OACB = 2r + \dfrac{\pi r \theta}{180}$

Area of sector $OACB = (\pi r^2) \times \dfrac{\theta}{360}$

Segment of a Circle

A *segment of a circle* is a region enclosed by a chord and an arc of the circle.

GP_3386



Any chord of a circle which is not a diameter divides the circle into two segments, one of which is the major segment and other is ***minor segment***.

Perimeter of the segment $PRQP$
= Length of the arc PRQ + Length of PQ

$$= \frac{\pi r\theta}{180} + 2r\sin\frac{\theta}{2}$$

Area of (minor) segment PQR = (Area of sector $OPRQO$ – Area of ΔOPQ)

Area of (major) segment PSQ
= (Area of circle – Area of minor segment PQR)

Illustration – 5

A circular grass lawn of 35 metres in radius has a path 7 metres wide running around it on the outside. Find the area of path.

(a) 1694 m^2 **(b) 1700 m^2**
(c) 1598 m^2 **(d) None of these**

Solution : (a) Radius of a circular grass lawn (without path) = 35 m

$\therefore$ Area = $\pi r^2 = \pi(35)^2$

Radius of a circular grass lawn (with path) = 35 + 7 = 42 m

$\therefore$ Area of lawn with path = $\pi r^2 = \pi(42)^2$

$\therefore$ Area of path = $\pi(42)^2 - \pi(35)^2 = \pi\left(42^2 - 35^2\right)$

$= \pi(42+35)(42-35)$

$= \frac{22}{7} \times 77 \times 7 = 1694\ m^2$

Illustration – 6

There are two concentric circular tracks of radii 100 m and 102 m, respectively. A runs on the inner track and goes once round on the inner track in 1 min 30 sec, while B runs on the outer track in 1 min 32 sec. Who runs faster?

(a) Both A and B are equal
(b) A
(c) B
(d) None of these

Solution : (b) Radius of the inner track = 100 m and time taken by A = 1 min 30 sec ≡ 90 sec.

Also, Radius of the outer track = 102 m and time taken by B = 1 min 32 sec ≡ 92 sec.

Now, speed of A who runs on the inner track

$$= \frac{2\pi(100)}{90} = \frac{20\pi}{9} = 6.98$$

And speed of B who runs on the outer track

$$= \frac{2\pi(102)}{90} = \frac{51\pi}{23} = 6.96$$

Since, speed of A > speed of B

$\therefore$ A runs faster than B.

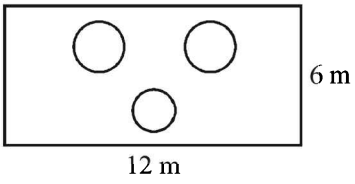
Illustration – 7

A rectangular plate is of 6 m breadth and 12 m length. Two apertures of 2 m diameter each and one apertures of 1 m diameter have been made with the help of a gas cutter. What is the area of the remaining portion of the plate?

(a) 68.5 sq. m. **(b) 62.5 sq m**
(c) 64.5 sq. m **(d) None of these**

Solution : (c) Given, Length = 12 m and Breadth = 6 m

$\therefore$ Area of rectangular plate = $12 \times 6 = 72\ m^2$



Since, two apertures of 3 m diameter each have been made from this plate.

Area of these two apertures = $\pi(1)^2 + \pi(1)^2 = 2\pi$

Area of 1 aperture of 1m diameter = $\pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

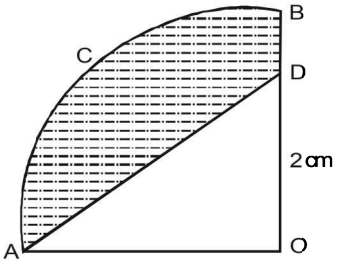
$\therefore$ Total area of aperture = $2\pi + \frac{\pi}{4} = \frac{9\pi}{4} = \frac{99}{14}$

$\therefore$ Area of the remaining portion of the plate

$$= 72 - \frac{99}{14}\ \text{sq. m} = \frac{909}{14}\ \text{sq. m} \approx 64.5\ \text{sq.m}$$

Illustration – 8

In the adjoining figure, *AOBCA* represents a quadrant of a circle of radius 3.5 cm with centre *O*. Calculate the area of the shaded portion.



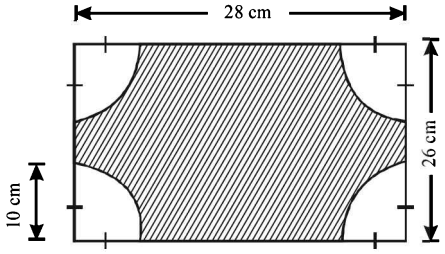
(a) 35 cm² **(b) 7.875 cm²**
(c) 9.625 cm² **(d) 6.125 cm²**

Solution : (d) Area of shaded portion
= Area of quadrant – Area of triangle

$$\Rightarrow \frac{\pi r^2}{4} - \frac{1}{2} \times 3.5 \times 2 = \frac{3.14 \times (3.5)^2}{4} - 3.5 = 6.125 \text{cm}^2$$

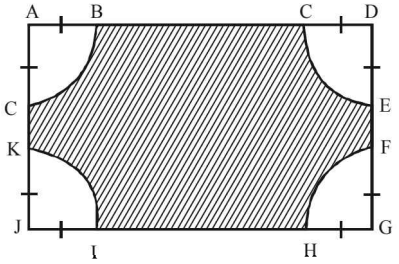
Illustration – 9

Find the perimeter and area of the shaded portion of the adjoining diagram:



(a) 90.8 cm, 414 cm²
(b) 181.6 cm, 423.7 cm²
(c) 90.8 cm, 827.4 cm²
(d) 181.6 cm, 827.4 cm²

Solution : (a)



KJ = radius of semicircles = 10 cm
4 quadrants of equal radius = 1 circle of that radius
Area of shaded portion = Area of rectangle – Area of circle
$(28 \times 26) - (3.14 \times 102) \Rightarrow 414 \text{ cm}^2$
$BC = 28 - (10 + 10) = 8$ and $EF = 26 - (10 + 10) = 6$
Perimeter of shaded portion = 28 cm + $2\pi r$
$\therefore$ 414 cm² = Area and Perimeter = 90.8 cm.

GP_3386

LEVEL - 1

1. The side and the height of a rhombus are 20 and 13 cms respectively. Find the area.
(a) 260 cm^2 (b) 275 cm^2
(c) 290 cm^2 (d) None of these

2. The area of a triangle is 615 m^2. If one of its sides is 123 metre, find the length of the perpendicular dropped on that side from opposite vertex.
(a) 15 metres (b) 12 metres
(c) 10 metres (d) None of these

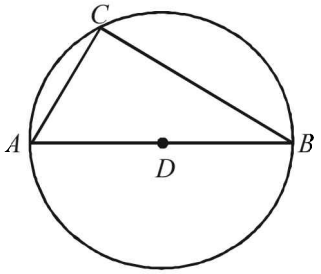
3. A wire can be bent in the form of a circle of radius 56 cm. If it is bent in the form of a square, then its area will be:
(a) 3520 cm^2 (b) 6400 cm^2
(c) 7744 cm^2 (d) 8800 cm^2

4. From a circular sheet of paper with a radius of 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the areas of uncut to the cut portion?
(a) 1 : 3 (b) 4 : 1 (c) 3 : 1 (d) 4 : 3

5. One diagonal of a rhombus is 24 cm whose side is 13 cm. Find the area of the rhombus.
(a) 25 sq. cm (b) 312 sq. cm.
(c) 125 sq. cm. (d) 120 sq. cm.

6. The area of a right angled isosceles triangle whose hypotenuse is equal to 270 m is
(a) 19000 m^2 (b) 18225 m^2
(c) 17256 m^2 (d) 18325 m^2

7. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm. long, find the area of ΔABC



(a) 60 sq. cm (b) 30 sq. cm
(c) 40 sq. cm (d) 52 sq. cm.

8. Find the area of an isosceles triangle whose equal sides are 8 cm each and the third side is 10 cm ?
(a) 10 cm^2 (b) 48 cm^2
(c) $5\sqrt{39}$ cm^2 (d) $10\sqrt{10}$ cm^2

9. What is the area of a regular hexagon inscribed in a circle of radius r ?
(a) $2\sqrt{3}\,r^2$ sq. units (b) $\frac{3\sqrt{3}}{2}r^2$ sq. units
(c) $\frac{2}{\sqrt{3}}r^2$ sq. units (d) $\frac{\sqrt{3}}{2}r^2$ sq. units

10. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. The area of the triangle is
(a) 72 cm^2 (b) 60 cm^2
(c) 66 cm^2 (d) None of these

11. The area of a square field is 576 km^2. How long will it take for a horse to run around at the speed of 12 km/h ?
(a) 12 h (b) 10 h (c) 8 h (d) 6 h

12. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is
(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

13. What is the greatest area (in sq. units) of a rectangle the sum of whose 3 sides is equal = 100.
(a) 625 (b) 1250 (c) 883.33 (d) 666.66

14. If the perimeter of a square and a rectangle are the same, then the areas A and B enclosed by them would satisfy the inequality :
(a) $A > B$ (b) $A \geq B$ (c) $A < B$ (d) $A \leq B$

15. A rectangular field costs ₹ 110 for levelling at 50 paise per square metre. If the ratio of length : breadth is 11 : 5. Find the length of the fields:
(a) 16m (b) 21m
(c) 2m (d) None of these

16. A drawing room is 7.5 m long, 6.5 m broad and 6 m high. Find the length of paper 2.5 dm wide to cover its walls allowing 8 sq. m for doors:
(a) 368m (b) 640m (c) 625m (d) 888m

17. The length and breadth of a rectangular field are 120 m and 80 m respectively. Inside the field, a park of 12 m width is made around the field. The area of the park is :
(a) 2358 m^2 (b) 7344 m^2
(c) 4224 m^2 (d) 3224 m^2

18. The ratio of the area of a square to that of the square drawn on its diagonal is:
(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

19. The length and breadth of a square are increased by 60% and 40% respectively. The area of the resulting rectangle exceeds the area of the square by :
(a) 224% (b) 24% (c) 124% (d) 100%

20. If every side of a triangle is doubled, then increase in area of the triangle is :
(a) 200% (b) 300%
(c) 400% (d) None of these

21. If the altitude of an equilateral triangle is $2\sqrt{3}$, then its area is :
(a) $4\sqrt{3}$ cm^2 (b) $12\sqrt{3}$ cm^2
(c) $\frac{8}{\sqrt{3}}$ cm^2 (d) None of these

22. The two adjacent sides of a parallelogram are 25 cm and 40 cm respectively. The altitude drawn on the longer side is 18 cm, then the area of the parallelogram is :
(a) 50 cm^2 (b) 720 cm^2
(c) 500 cm (d) None of these

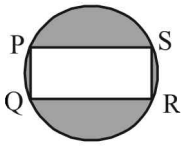
23. Area of a rhombus is 256 cm^2. One of the diagonal is half of the other diagonal. The sum of the diagonals is :
(a) 38 cm (b) 48 cm (c) 28 cm (d) 56 cm

24. The area of a hexagon whose one side is 4 m is :
(a) $6\sqrt{3}\ m^2$ (b) $24\sqrt{3}\ m^2$
(c) $42\sqrt{3}\ m^2$ (d) $24\ m^2$

25. If the driving wheel of a bicycle makes 560 revolutions in travelling 1.1 km. Find the diameter of the wheel :
(a) 31.5 cm (b) 30.5 cm
(c) 62.5 cm (d) None of these

26. A wire is in the form of a circle of radius 42 m is cut and again bent in the form of a square. What is the diagonal of the square?
(a) 66 m (b) $66\sqrt{3}$ m
(c) $66\sqrt{2}$ m (d) None of these

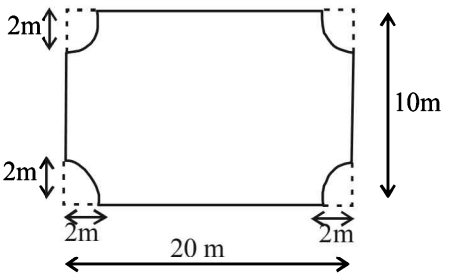
27. In the adjoining figure PQRS is a rectangle 8 cm × 6 cm, inscribed in the circle. The area of the shaded portion will be:



(a) 48 cm^2 (b) 42.50 cm^2
(c) 32.50 cm^2 (d) 30.57 cm^2

28. If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its perimeter is :
(a) 15 cm (b) 42 cm (c) 60 cm (d) 70 cm

29. The perimeter of the figure given below correct to one decimal place is :



(a) 56 m (b) 56.6 m (c) 57.2 m (d) 57.9 m

30. The area of the largest triangle that can be inscribed in a semicircle of radius r is :
(a) r^{2}cm^2 (b) $\left(\frac{r}{3}\right)^2$ cm^2
(c) $r\sqrt{2}$ cm^2 (d) $3\sqrt{3r}$ cm^2

31. If a regular hexagon is inscribed in a circle of radius r, then its perimeter is :
(a) $6\sqrt{3}$r (b) 6r (c) 3r (d) 12r

32. Area of a rhombus is 144 cm^2 and the ratio of length of two diagonals is 1 : 2. The sum of lengths of its diagonals are :
(a) 72 cm (b) 40 cm
(c) 36 cm (d) $18\sqrt{2}$cm

33. There are two concentric circles whose areas are in the ratio of 9 : 16 and the difference between their diameters is 4 cm. What is the area of the outer circle?
(a) 32cm^2 (b) 64π cm^2
(c) 36 cm^2 (d) 48 cm^2

34. How many squares are there in a 5 inch by 5 inch square grid, if the grid is made up one inch by one inch squares?
(a) 50 (b) 150 (c) 55 (d) 25

35. A circle and a rectangle have the same perimeter. The sides of the rectangle are 18 cm and 26 cm. What is the area of the circle ?
(a) 88 cm^2 (b) 154 cm^2
(c) 1250 cm^2 (d) 616 cm^2

GP_3386

36. In a parallelogram, the length of one diagonal and the perpendicular dropped on that diagonal are 30 and 20 metres respectively. Find its area.
(a) 600 m^2 (b) 540 m^2
(c) 680 m^2 (d) 574 m^2

37. A square carpet with an area 169 m^2 must have 2 metres cut-off one of its edges in order to be a perfect fit for a rectangualar room. What is the area of rectangular room?
(a) 180 m^2 (b) 164 m^2
(c) 152 m^2 (d) 143 m^2

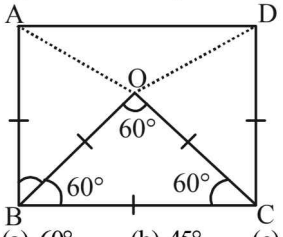
38. The altitude drawn to the base of an isosceles triangle is 8 cm and the perimeter is 32 cm. The area of the triangle is
(a) 72 cm^2 (b) 48 cm^2
(c) 66 cm^2 (d) None of these

39. From a circular sheet of paper with a radius of 20 cm, four circles of radius 5 cm each are cut out. What is the ratio of the areas of uncut to the cut portion?
(a) 1 : 3 (b) 4 : 1 (c) 3 : 1 (d) 4 : 3

40. Area of rectangular field is 3584 m^2 and the length and the breadth are in the ratio 7 : 2 respectively. What is the perimeter of the rectangle ?
(a) 246m (b) 292m (c) 286m (d) 288m

41. The length of the two sides forming the right angle of a right-angled triangle are 6 cm and 8 cm. The length of its circumradius is :
(a) 5 cm (b) 7 cm (c) 6 cm (d) 10 cm

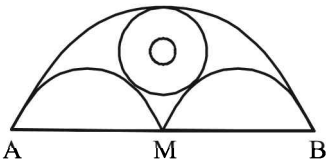
42. ABCD is a square in which ΔOBC is an equilateral triangle then find $\angle DOA$.



(a) 60° (b) 45° (c) 150° (d) 90°

43. In a || gm, the lengths of adjacent sides are 11 cm and 13 cm. If the length of one diagonal is 16 cm, find the length of other diagonal.
(a) 9 cm (b) 13 cm (c) 16 cm (d) 18 cm

44. In the given figure, AB is a line of length 2a, with M as mid-point. Semi–circles are drawn on one side with AM, and AB as diameters. A circle with centre O and radius r is drawn such that this circle touches all the three semi-circles. What is the value of r?



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

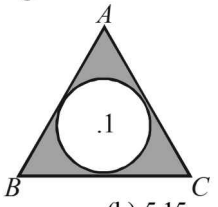
45. The difference between the length and breadth of a rectangle is 23 m. If its perimeter is 206 m, then its area is:
(a) 1520 m^2 (b) 2420 m^2 (c) 2480 m^2 (d) 2520 m^2

46. A rectangular park 60 × 40 m^2 has two cross-roads running in the middle of the park and the rest park has been lawn. If the area of the lawn is 2109 m^2. What is width of the road?
(a) 2m (b) 3m (c) 4m (d) 5m

47. What is the radius of circle drawn outside the triangle whose length of sides are 15, 20 and 25 cm?
(a) 10 cm (b) 12.5 cm (c) 15 cm (d) 20 cm

48. What is the radius of the incircle of a triangle with sides 10, 24 and 30 cm?
(a) 2 cm (b) 3 cm (c) 1.5 cm (d) 4.5 cm

49. Find the area of the shaded region if ΔABC is an equilateral triangle of side 6 cm.



(a) 6.15 cm. (b) 5.15 cm.
(c) 4 cm. (d) 3.12 cm.

50. If the ratio of areas of two squares is 9 : 1, the ratio of their perimeter is:
(a) 9 : 1 (b) 3 : 4 (c) 3 : 1 (d) 1 : 3

LEVEL - 2

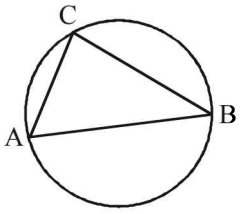
1. In a triangle ABC, points P, Q and R are the mid-points of the sides AB, BC and CA respectively. If the area of the triangle ABC is 20 sq. units, find the area of the triangle PQR

(a) 10 sq. units (b) 5.3 sq. units
(c) 5 sq. units (d) None of these

2. What is the side of the largest possible regular octagon that can be cut out of a square of side 1 cm?

(a) $\sqrt{2}-1$ (b) $\frac{1}{\sqrt{2}+2}$ cm
(c) $\frac{1}{\sqrt{2}-1}$ cm (d) $\frac{\sqrt{2}}{\sqrt{2}+1}$ cm

3. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of triangle ABC



(a) 60 sq. cm. (b) 30 sq. cm.
(c) 40 sq. cm. (d) 52 sq. cm.

4. The sides of a triangle are 5, 12 and 13 units respectively. A rectangle is constructed which is equal in area to the triangle and has a width of 10 units. Then the perimeter of the rectangle is

(a) 30 (b) 26
(c) 13 (d) None of these

5. A garden is 24 m long and 14 m wide. There is a path 1 m wide outside the garden along its sides. If the path is to be constructed with square marble tiles 20 cm × 20 cm, the number of tiles required to cover the path is

(a) 1800 (b) 200 (c) 2000 (d) 2150

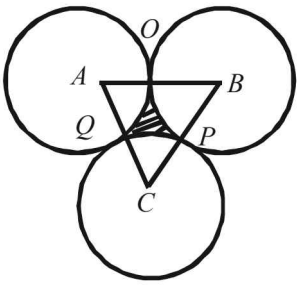
6. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12km / hr completes one round in 8 minutes, then the area of the park (in sq. m) is:

(a) 15360 (b) 153600 (c) 30720 (d) 307200

7. The length and breadth of a playground are 36m and 21 m respectively. Poles are required to be fixed all along the boundary at a distance 3m apart. The number of poles required will be

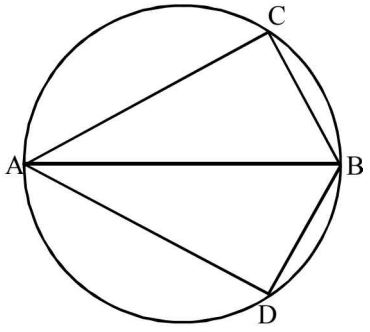
(a) 39 (b) 38 (c) 37 (d) 40

8. Three circles with centres A, B and C and with unit radii touch each other at O, P and Q. Find the area of the shaded region .



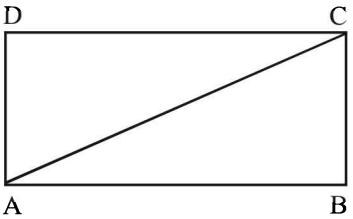
(a) 0.16 sq. units (b) 1.21 sq. units
(c) 0.03 sq. units (d) 0.32 units

9. AB is the diameter of the given circle, while points C and D lie on the circumference as shown. If AB is 15 cm. AC is 12 cm and BD is 9 cm, find the area of the quadrilateral $ABCD$.



(a) 54 sq. cm (b) 216 sq. cm
(c) 162 sq. cm (d) None of these

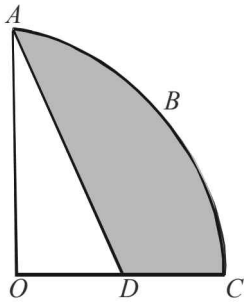
10. In the given figure, $AC + AB = 5\,AD$ and $AC - AD = 8$. Then the area of the rectangle $ABCD$ is



(a) 36 (b) 50
(c) 60 (d) Cannot be answered

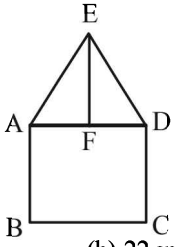
GP_3386

11. In the figure given below, $ABCO$ represents a quadrant of a circle of radius 10.5 cm with centre O. Calculate the area of shaded portion, if $OD = DC$.



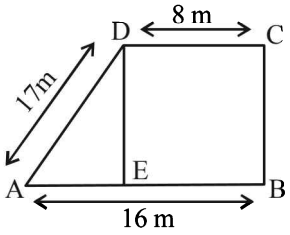
(a) 59 cm^2 (b) 69 cm^2
(c) 79 cm^2 (d) 49 cm^2

12. ABCD is a square, AC = BD = $4\sqrt{2}$ cm, AE = DE = 2.5 cm. Find the area of the adjoining figure ABCDE :



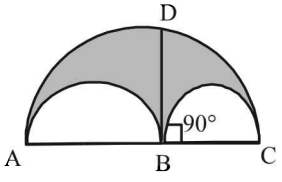
(a) 19 cm^2 (b) 22 cm^2
(c) 17 cm^2 (d) None of these

13. In the given figure, ABCD is a trapezium in which the parallel sides AB, CD are both perpendicular to BC. Find the area of the trapezium :



(a) 140 m^2 (b) 168 m^2 (c) 180 m^2 (d) 156.4 m^2

14. In the adjoining figure there are three semicircles in which BC = 6 cm and BD = $6\sqrt{3}$ cm. What is the area of the shaded region (in cm) :

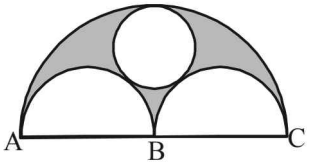


(a) 12π (b) 9π (c) 27π (d) 28π

15. A copper wire when bent in the form of a square, encloses an area of 121 m^2. If the same wire is bent to form a circle, the area enclosed by it would be :

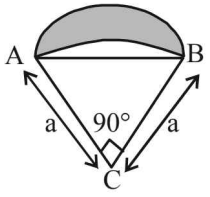
(a) 122 m^2 (b) 112 m^2
(c) 154 m^2 (d) 308 m^2

16. In the following figure AB = BC and AC = 84 cm. The radius of the inscribed circle is 14 cm . B is the centre of the largest semi circle. What is the area of the shaded region?



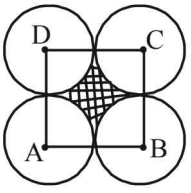
(a) 335 cm^2 (b) 770 cm^2
(c) 840 cm^2 (d) 650 cm^2

17. In the adjoining figure ACB is a quadrant with radius 'a'. A semicircle is drawn outside the quadrant taking AB as a diameter. Find the area of shaded region :



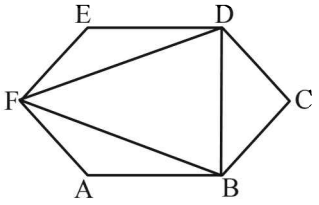
(a) $\frac{1}{4}(\pi - 2a^2)$ (b) $\left(\frac{1}{4}\right)(\pi a^2 - a^2)$
(c) $\frac{a^2}{2}$ (d) Can't be determined

18. ABCD is a square, 4 equal circles are just touching each other whose centres are the vertices A, B, C, D of the square. What is the ratio of the shaded to the unshaded area within square?



(a) $\frac{8}{11}$ (b) $\frac{3}{11}$ (c) $\frac{5}{11}$ (d) $\frac{6}{11}$

19. ABCDEF is a regular hexagon of side 6 cm. What is the area of triangle BDF?



(a) $32\sqrt{3}$ cm^2 (b) $27\sqrt{3}$ cm^2
(c) 24 cm^2 (d) None of these

20. Four equal circles are described about the four corners of a square so that each touches two of the others. If a side of the square is 14 cm, then the area enclosed between the circumferences of the circles is :
(a) 24 cm^2 (b) 42 cm^2
(c) 154 cm^2 (d) 196 cm^2

21. The ratio between the length and the breadth of a rectangular park is 3 : 2. If a man cycling along the boundary of the park at the speed of 12km / hr completes one round in 8 minutes, then the area of the park (in sq. m) is:
(a) 15360 (b) 153600 (c) 30720 (d) 307200

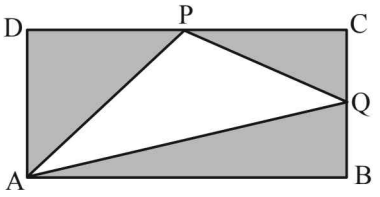
22. The length and breadth of a playground are 36m and 21 m respectively. Poles are required to be fixed all along the boundary at a distance 3m apart. The number of poles required will be
(a) 39 (b) 38 (c) 37 (d) 40

23. The length of a rectangular field is double its width. Inside the field there is a square-shaped pond 8 m long. If the area of the pond is 1/8 of the area of the field, what is the length of the field?
(a) 32m (b) 16m (c) 64m (d) 20m

24. A horse is tethered to one corner of a rectangular grassy field 40 m by 24 m with a rope 14 m long. Over how much area of the field can it graze?
(a) 154 cm^2 (b) 308 m^2
(c) 150 m^2 (d) None of these

25. The sides of a triangle are 5, 12 and 13 units respectively. A rectangle is constructed which is equal in area to the triangle and has a width of 10 units. Then the perimeter of the rectangle is
(a) 30 (b) 26
(c) 13 (d) None of these

26. In the given figure, ABCD is a rectangle. P and Q are the mid-points of sides CD and BC respectively. Then the ratio of area of shaded portion to the area of unshaded portion is :



(a) 5 : 4 (b) 3 : 5 (c) 5 : 3 (d) 5 : 8

27. Q is a point in the interior of a rectangle ABCD, if QA = 3 cm, QB = 4 cm and QC = 5 cm, then the length of QD (in cm) is
(a) $3\sqrt{2}$ (b) $5\sqrt{2}$ (c) $\sqrt{34}$ (d) $\sqrt{41}$

28. With P, Q & R centres, three circles are drawn such that they touch each other externally. If the sides of the ΔPQR are 4 cm, 6 cm & 8 cm, then what is the sum of radii of the circles?
(a) 9 cm (b) 18 cm (c) 10 cm (d) 15 cm

29. Find the perimeter of a square which is symmetrically inscribed in semi-circle of radius 10 cm.
(a) $\sqrt{80}$ cm (b) 80 cm
(c) $8\sqrt{24}$ cm (d) $16\sqrt{5}$ cm

30. A circle is inscribed in an equilateral triangle of side a. What is the area of any square inscribed in this circle?
(a) $\frac{a^2}{3}$ (b) $\frac{a^2}{4}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{8}$

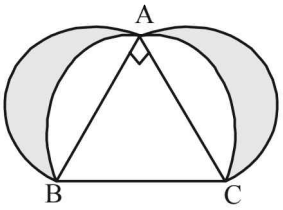
31. In a ΔABC, BD & CE are the two medians which intersect each other at right angle. AB = 22, AC = 19, find BC = ?
(a) 6.5 (b) 13 (c) 10 (d) 15

32. Chords AB and CD of a circle intersect at E and are perpendicular to each other. Segments AE, EB and ED are of lengths 2 cm, 6 cm and 3 cm respectively. Then the length of the diameter of the circle (in cm) is
(a) $\sqrt{65}$ (b) $\frac{1}{2}\sqrt{65}$ (c) 65 (d) $\frac{65}{2}$

33. A right angled isosceles triangle is inscribed in a semicircle of radius 7 cm. The area enclosed by the semi - circle but exterior to the triangle is:
(a) 14 cm^2 (b) 28 cm^2 (c) 32 cm^2 (d) 34 cm^2

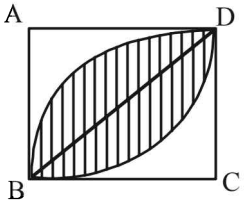
GP_3386

34. In the given figure, ΔABC is a right angled triangle, right angled at A. Semi-circles are drawn on the sides AB, BC and AC. Then the area of shaded portion is equal to which one of the following ?



(a) Area of ΔABC
(b) 2 times the area of ΔABC
(c) Area of semi-circle ABC
(d) None of the above

35. In the given figure, the side of square ABCD is 7 cm. What is the area of the shaded portion, formed by the arcs BD or the circles with centre at C and A?



(a) $7\,cm^2$ (b) $28\,cm^2$ (c) $14\,cm^2$ (d) $21\,cm^2$

36. Three circular laminas of the same radius are cut out from a larger circular lamina. When the radius of each lamina cut out is the largest possible, then what is the ratio (approximate) of the area of the residual piece of the original lamina to its original total area?

(a) 0.30 (b) 0.35 (c) 0.40 (d) 0.45

37. Find the area of the largest (or maximum sized) square that can be made inside a right angle triangle having sides 6 cm, 8 cm & 10 cm when one of vertices of the square coincide with the vertex of right angle of the triangle?

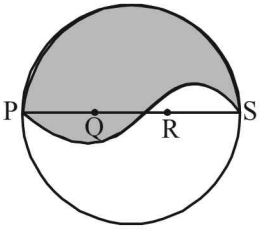
(a) $\frac{576}{49}cm^2$ (b) $24\,cm^2$

(c) $\frac{24}{7}cm^2$ (d) None of thes

38. The length of a room floor exceeds its breadth by 20 m. The area of the floor remains unaltered when the length is decreased by 10 m but the breadth is increased by 5 m. The area of the floor (in square metres) is :

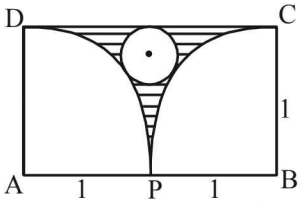
(a) 280 (b) 325 (c) 300 (d) 420

39. PQRS is a diameter of a circle of radius 6 cm as shown in the figure. The lengths PQ, QR and RS are equal. Semi - circles are drawn on PQ and QS as diameters. What is the perimeter of the shaded region?



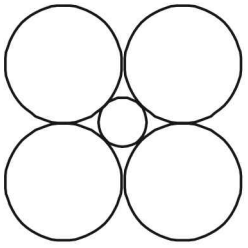
(a) 12π cm (b) 14π cm (c) 16π cm (d) 18π cm

40. In the following figure ABCD is a rectangle with AD and DC equal to 1 and 2 units respectively. Two quarter circles are drawn with centres at B and A respectively. Now a circle is drawn touching both the quarter circles and one of the sides of the rectangle. Find the area of the shaded region :



(a) 0.23 Sq. units (b) 0.41 Sq. units
(c) 0.35 Sq. units (d) 0.52 Sq. units

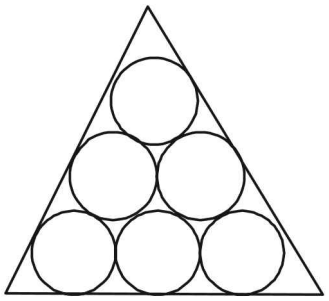
41. In the given figure, when all the outer circles have radii 'R' then the radius of the inner circle will be :



(a) $\left(\sqrt{2}+1\right)R$ (b) $\left(\sqrt{2}-1\right)R$

(c) $\left(\sqrt{3}+1\right)R$ (d) $\left(\sqrt{3}-1\right)R$

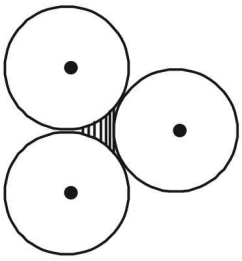
42. An equilateral triangle circumscribes all the circles, each with radius 10 cm. What is the perimeter of the equilateral triangle?



(a) $20\left(2+\sqrt{3}\right)$cm (b) $40\left(2+\sqrt{3}\right)$ cm

(c) $60\left(2+\sqrt{3}\right)$ cm (d) $40\left(2-\sqrt{3}\right)$ cm

43. Three circles of equal radius 'a' cm touch each other. The area of the shaded region is :



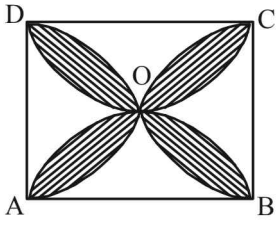
(a) $\left(\frac{\sqrt{3}+\pi}{2}\right)a^2$sq.cm

(b) $\left(\frac{6\sqrt{3}-\pi}{2}\right)a^2$sq.cm

(c) $\left(\sqrt{3}-\pi\right)a^2$sq.cm

(d) $\left(\frac{2\sqrt{3}-\pi}{2}\right)a^2$sq.cm

44. In the given figure ABCD is a square. Four equal semicircles are drawn in such a way that they meet each other at 'O'. Sides AB, BC, CD and DA are the respective diameters of the four semicircles. Each of the side of the square is 8 cm. Find the area of the shaded region.



(a) $16(\pi-2)$ cm^2 (b) $16(\pi+2)$ cm^2

(c) $32(\pi-2)$ cm^2 (d) $32(\pi+2)$ cm^2

45. Three circles of radii $\sqrt{2}+1, \sqrt{2}+1$ and 1 unit, touch each other externally, then find the perimeter of the surrounded part by three circles.

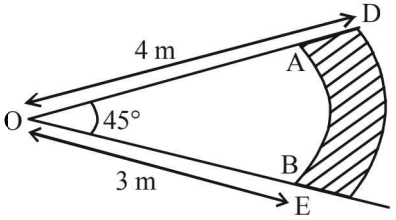
(a) $\frac{\pi}{2}\left(1+\sqrt{2}\right)$unit

(b) $\frac{\pi}{2}\left(\sqrt{2}-1\right)$ unit

(c) $\frac{\pi}{2}\left(2-\sqrt{2}\right)$ unit

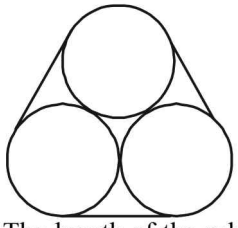
(d) $\frac{\pi}{2}\left(2+\sqrt{2}\right)$ unit

46. In the figure, OED and OBA are sectors of a circle with centre O. The area of the shaded portion.



(a) $\frac{11}{16}$m^2 (b) $\frac{11}{8}$m^2 (c) $\frac{11}{2}$m^2 (d) $\frac{11}{4}$m^2

47. Three circles of diameter 10 cm each are bound together by a rubber band as shown in the figure.



The length of the rubber band (in cm) if it is stretched is

(a) 30 (b) $30+10\pi$

(c) 10π (d) $60+20\pi$

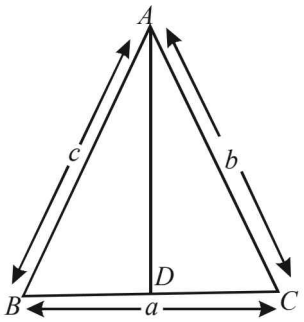
48. The median AD and BE of ΔABC are perpendicular. Find the length of AB if BC = 3 cm and AC = 4 cm.

(a) $\sqrt{5}$ cm (b) 5 cm

(c) $2\sqrt{5}$ cm (d) None of these

GP_3386

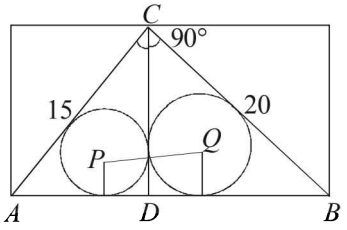
49. In a triangle ABC, AD is the angle bisector of $\angle BAC$ and $\angle BAD = 60°$. What is the length of AD?



(a) $\frac{b+c}{bc}$ (b) $\frac{bc}{b+c}$

(c) $\sqrt{b^2+c^2}$ (d) $\frac{(b+c)^2}{bc}$

50. In the figure given below, ACB is a right angled triangle. CD is the altitude. Circles are inscribed within the triangles ACD, BCD. P and Q are the centres of the circles. The distance PQ is



(a) 5 (b) $\sqrt{50}$ (c) 7 (d) 8

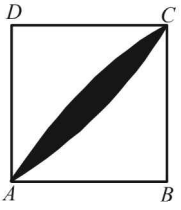
51. Two circles with same radii intersect each other at the points A and B. A straight line perpendicular to AB intersects the circles at C, D, E and F. If CD = 4.5 cm, then the measure of EF is

(a) 1.50 cm (b) 2.25 cm (c) 4.50 cm (d) 9.00 cm

52. In the given diagram a rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum if the bucket is raised by 11 m.

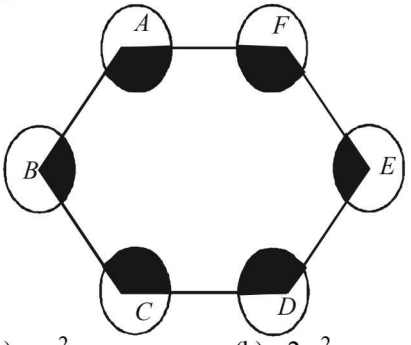
(a) 10 (b) 2.5 (c) 5 (d) 5.5

53. In the figure given below, $ABCD$ is a square of side 4 cm. Two quadrants of a circle with B and D as centres are drawn. The radius of each of the quadrants is 4cm. What is the area of the shaded portion?



(a) 4.56 sq. cm (b) 9.12 sq. cm
(c) 13.68 sq. cm (d) 7.76 sq.cm

54. Find the sum of the areas of the shaded sectors given that $ABCDEF$ is any hexagon and all the circles are of same radius r with different vertices of the hexagon as their centres as shown in the figure.



(a) πr^2 (b) $2\pi r^2$
(c) $5\pi r^2/4$ (d) $3\pi r^2/2$

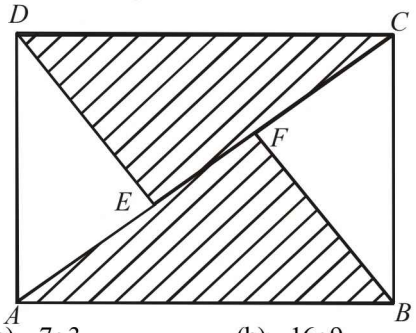
55. $ABCD$ is a quadrilateral. The diagonals of $ABCD$ intersect at the point P. The area of the triangles APD and BPC are 27 and 12, respectively. If the areas of the triangles APB and CPD are equal, then the area of triangle APB is

(a) 12 (b) 18 (c) 15 (d) 16

56. Two circles of radius 1 cm touch at point P. A third circle is drawn through the points A, B and C such that PA is the diameter of the first circle and BC perpendicular to AP is the diameter of the second circle. The radius of the third circle (in cm) is

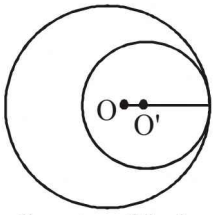
(a) $\frac{9}{5}$ (b) $\frac{7}{4}$ (c) $\frac{\sqrt{10}}{2}$ (d) $\frac{5}{3}$

57. *ABCD* is a rectangle of dimensions 6 cm × 8 cm. *DE* and *BF* are the perpendiculars drawn on the diagonal of the rectangle. What is the ratio of the shaded region to that of unshaded region?



(a) 7 : 3 (b) 16 : 9
(c) $4:3\sqrt{2}$ (d) Data insufficient

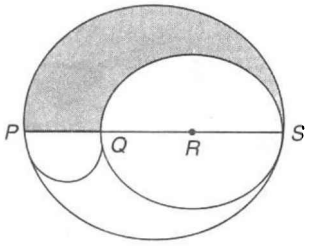
58. Two circles touch internally and their centres are *O* and *O'* as shown. The sum of their areas is 180π sq. cm. and the distance between their centres is 6 cm.



What is the diameter of the larger circle ?

(a) 16 cm (b) 12 cm
(c) 18 cm (d) 24 cm

59. PQRS is the diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semi-circles are drawn with PQ and QS as diameters as shown in the figure alongside. Find the ratio of the area of the shaded region to that of the unshaded region.



(a) 1:2 (b) 25 : 121
(c) 5 : 18 (d) 5 : 13

60. Two parallel chords of a circle of diameter 20 cm are 12 cm and 16 cm long. If the chords are in the same side of the centre, then the distance between them is

(a) 28 cm (b) 2 cm
(c) 4 cm (d) 8 cm

GP_3386

Hints and Solutions

Level 1

1. (a) Area of rhombus = side × height
$= 13 \times 20 = 260\ \text{cm}^2$

2. (c) In a triangle,

$\text{Area} = \frac{1}{2} \times \text{length of perpendicular} \times \text{base}$

or $615 = \frac{1}{2} \times \text{length of perpendicular} \times 123$

$\therefore$ Length of perpendicular $= \frac{615 \times 2}{123} = 10\ \text{m}.$

3. (c) Length of wire $= 2\pi \times R$

$= \left(2 \times \frac{22}{7} \times 56\right)\text{cm} = 352\ \text{cm}.$

Side of the square $= \frac{352}{4}\ \text{cm} = 88\ \text{cm}.$

Area of the square $= (88 \times 88)\ \text{cm}^2 = 7744\ \text{cm}^2.$

4. (c) $\frac{\text{Area of uncut portion}}{\text{Area of cut portion}} = \frac{(\pi \times 20 \times 20) - (100\pi)}{(4 \times \pi \times 5 \times 5)}$

$= \frac{300\pi}{100\pi} = \frac{3}{1}$

5. (d) $(\text{Side})^2 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2$

$+ \left(\frac{1}{2} \times \text{other diagonal}\right)^2$

$\Rightarrow 13^2 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2 + \left(\frac{1}{2} \times 24\right)^2$

$\Rightarrow 25 = \left(\frac{1}{2} \times \text{diagonal}\right)^2 \Rightarrow \text{diagonal} = 10$

$\therefore$ Area $= \frac{1}{2} \times 10 \times 24 = 120\ \text{sq. cm}.$

6. (b) Hypotenuse = 270 m
$\Rightarrow$ Hypotenuse^2 = Side^2 + Side^2 = 2 Side^2
$\Rightarrow$ Side$^2 = (270)^2/2 = 36450 \Rightarrow$ side = 190.91m
$\Rightarrow$ Required area = 1/2 × 190.91 × 190.91
= 36446.6/2 = 18225 m^2 (approx).

7. (b) $AD = 6.5$
$\therefore$ $AB = 13$ (diameter)

Now $\angle ACB = 90°$ (since the diameter of a circle subtends 90° at the circumference)
So by Pythagorus theorem, $CB = 12$ cm.

$\therefore$ area of $\Delta ACB = \frac{1}{2} \times 5 \times 12 = 30$ sq. cm

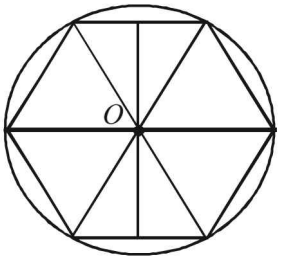
8. (c) Area of isosceles triangle $= \frac{b}{4}\left(\sqrt{4a^2 - b^2}\right)$
where b is the base and a is any of the equal sides.
Area of the required triangle

$= \frac{10}{4}\left(\sqrt{4(8)^2 - (10)^2}\right) = \frac{10}{4}\sqrt{156}$

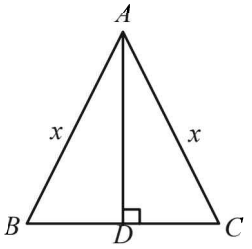
$= 5\sqrt{39}\ \text{cm}^2$

9. (b) We can divide the regular hexagon into 6 equilateral triangles. Since the hexagon is in a circle the radius r is the side of the equilateral triangle.
$\therefore$ Area of the hexagon

$= 6 \times \frac{\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{2} r^2$ sq. units.



10. (b) Let ABC be the isosceles triangle and AD be the altitude.
Let $AB = AC = x$. Then, $BC = (32 - 2x)$.



Since, in an isosceles triangle, the altitude bisects the base. So, $BD = DC = (16 - x)$.
In ΔADC, $AC^2 = AD^2 + DC^2$
$\Rightarrow x^2 = (8)^2 + (16 - x)^2 \Rightarrow 32x = 320 \Rightarrow x = 10.$

$\therefore$ $BC = (32 - 2x) = (32 - 20)\text{ cm} = 12\text{ cm}$.

Hence, required area $= \left(\frac{1}{2} \times BC \times AD\right)$

$= \left(\frac{1}{2} \times 12 \times 10\right) \text{cm}^2 = 60\text{ cm}^2$.

11. (c) Area of field = 576 km^2. Then,

each side of field = $\sqrt{576} = 24$ km

Distance covered by the horse

= Perimeter of square field = $24 \times 4 = 96$ km

$\therefore$ Time taken by horse = $\frac{\text{distance}}{\text{speed}} = \frac{96}{12} = 8$ h

12. (c) $2\pi r = \pi(36 + 20) \Rightarrow r = 28$

13. (b) Given is $2a + b = 100$, and area = ab. Assuming $2a = x$, we get $x + b = 100$, and area = $1/2xb$. Now area will be maximum when $x = b$, or $a = b/2$. Thus $a = 25$, & $b = 50$, maximum area = 1250.

14. (a) Consider some appropriate values.

4a = 16 cm 2 (l + b) = 16 cm

but $a^2 = 16\text{ cm}^2$ and $l \times b = 12\text{ cm}^2$

$\Rightarrow$ $a^2 > l \times b$

Remember, when a + b is constant, then the maximum value of a × b is at when a = b. So, area of square (i.e. l × b = a × a) is always greater than area of rectangle with same perimeter.

15. (c) Area $= \frac{110}{0.5} = 220$ sq.m

and $11x \times 5x = 220 \Rightarrow x = 2$

16. (b) Net area = Total area of 4 walls – $8m^2$

and Area of 4 walls = 2 (l + b) × h = $168m^2$

Length of paper $= \frac{160}{0.25} = 640$m

17. (c) $(120 + 80 - 24) \times 24 = 176 \times 24 = 4224$

18. (b) $a^2 : \left(a\sqrt{2}\right)^2 = 1 : 2$

19. (c) $1.6 \times 1.4 = 2.24$

$\Rightarrow$ increase = $1.24 \Rightarrow 124\%$

20. (b) If sides of triangle are a, b and c then

Area of triangle (A)

$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = a + b + c$

Now sides are $2a, 2b, 2c$

Now $s^1 = 2a + 2b + 2c = 2s$

Now area of triangle

$A^1 = \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)} = 4A$

% Increase in area

$\frac{4A - A}{A} \times 100 = 300\%$

21. (a) Height of an equilateral triangle

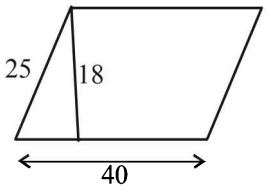
$= \frac{\sqrt{3}}{2} \times \text{side}$

$\therefore$ $2\sqrt{3} = \frac{\sqrt{3}}{2} \times \text{side} \Rightarrow \text{Side} = 4\text{ cm}$

$\therefore$ Area of an equilateral triangle

$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3}\text{cm}^2$

22. (b) Area = $40 \times 18 = 720\text{ cm}^2$



23. (b) $\frac{1}{2} \times x \times 2x = 256 \Rightarrow x^2 = 256$

$\Rightarrow x = 16$ $\therefore$ $2x = 32$

Hence $x + 2x = 16 + 32 = 48$

24. (b) $6 \times \frac{\sqrt{3}}{4} \times (\text{Side})^2 = \frac{3\sqrt{3}}{2}(\text{Side})^2$

$= \frac{3\sqrt{3}}{2} \times 4 \times 4 = 24\sqrt{3}\text{ m}^2$

25. (c) Circumference $= \frac{1100}{560} = \frac{110}{56} = 2\pi r$

$\therefore$ $2r = \frac{110}{56} \times \frac{7}{22} = \frac{5}{8}\text{m} = 62.5\text{ cm}$

26. (c) r = 42

$\therefore$ $2\pi r = 264 = 4a \Rightarrow a = 66$

$\therefore$ $d = a\sqrt{2} = 66\sqrt{2}$

27. (d) Diameter $= \sqrt{8^2 + 6^2} = 10cm$

Area of shaded portion

$$= \frac{22}{7} \times 5 \times 5 - 8 \times 6 = 30.57 \text{ cm}^2$$

28. (c) Side $= \sqrt{9^2 + 12^2} = 15 \text{ cm}$

29. (b) Required perimeter

$$= 4 \times \left(\frac{2 \times 22 \times 2}{4 \times 7} \right) + 2(16 + 6) = 56.57 \text{ m}$$

30. (a) The area of triangle is largest when height of the triangle is equal to the radius.

$$\therefore \text{ Area } = \frac{1}{2} \times r \times 2r = r^2 \text{ cm}^2$$

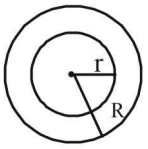
31. (b) In a regular hexagon inscribed in a circle, its side equals the radius.

32. (c) $\frac{1}{2} \times x \times 2x = 144 \Rightarrow x = 12$

$\therefore$ sum of lengths of diagonals
$= 12 + 2 \times 12 = 36 \text{ cm}$

33. (b) $\frac{r}{R} = \frac{3x}{4x}$

$\because$ $4x - 3x = 2$



$\Rightarrow$ $x = 2$

$\therefore$ Outer radius $= 8 \text{ cm}$

$\therefore$ Area of outer circle $= \pi \times (8)^2 = 64\pi \text{ cm}^2$

34. (d) Required no. of squares $= \frac{5^2}{1^2} = 25$

35. (d) Perimeter of the circle $= 2\pi r = 2(18 + 26)$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = 14$$

$\therefore$ Area of the circle

$$= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2.$$

36. (a) In a parallelogram.
Area = Diagonal × length of perpendicular on it.
$= 30 \times 20 = 600 \text{ m}^2$

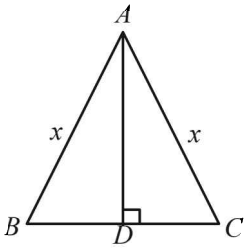
37. (d) Side of square carpet

$= \sqrt{\text{Area}} = \sqrt{169} = 13 \text{ m}$

After cutting of one side,
Measure of one side = 13 – 2 = 11 m and other side = 13 m (remain same)

$\therefore$ Area of rectangular room $= 13 \times 11 = 143 \text{ m}^2$

38. (b) Let ABC be the isosceles triangle and AD be the altitude.
Let $AB = AC = x$. Then, $BC = (32 - 2x)$.



Since, in an isosceles triangle, the altitude bisects the base. So, $BD = DC = (16 - x)$.
In ΔADC, $AC^2 = AD^2 + DC^2$

$\Rightarrow$ $x^2 = (8)^2 + (16 - x)^2 \Rightarrow 32x = 320 \Rightarrow x = 10.$

$\therefore$ $BC = (32 - 2x) = (32 - 20) \text{ cm} = 12 \text{ cm}.$

Hence, required area $= \left(\frac{1}{2} \times BC \times AD \right)$

$$= \left(\frac{1}{2} \times 12 \times 8 \right) \text{cm}^2 = 48 \text{ cm}^2.$$

39. (c) $\frac{\text{Area of uncut portion}}{\text{Area of cut portion}} = \frac{(\pi \times 20 \times 20) - (100\pi)}{(4 \times \pi \times 5 \times 5)}$

$$= \frac{300\pi}{100\pi} = \frac{3}{1}$$

40. (d) Area of field $= 3584 \text{ m}^2$

Let the length and breadth be $7x$ and $2x$

Then $7x \times 2x = 3584 \text{m}^2$

$14x^2 = 3584 \text{m}^2$

$x^2 = 256$

$x = 16 \text{m}$

Length $= 7x = 112 \text{m}$,

Breadth $= 2x = 16 \times 2 = 32 \text{m}$

Perimeter $= 2(l + b) = 2(112 + 32) = 288 \text{m}$

41. (a) In a right angled Δ, the length of circumradius is half the length of hypotenuse.

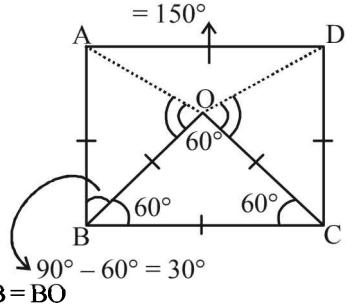
$\therefore$ $H^2 = 6^2 + 8^2$

$H^2 = 36 + 64 \Rightarrow 100$

$H = 10 \text{ cm}$

Circumradius $= 5 \text{ cm}$

42. **(c)** $360 - (75 + 75 + 60)$
$= 150°$



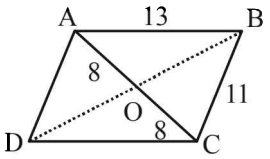
$AB = BO$

$$\angle BAO = \angle BOA = \frac{180 - 30}{2} = 75°$$

$\angle AOB = \angle DOC = 75°$

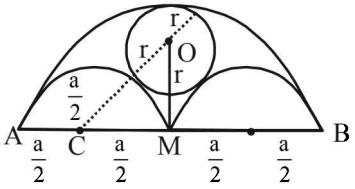
$\angle AOD = 360° - [\angle AOB + \angle DOC + \angle BOC]$
$= 360° - 210° = 150°.$

43. **(d)**



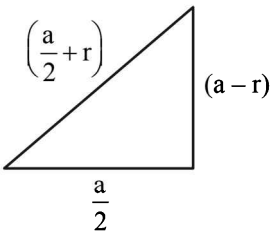
$AB^2 + BC^2 = 2(AO^2 + BO^2)$
$\Rightarrow 13^2 + 11^2 = 2(8^2 + BO^2)$
$\Rightarrow 169 + 121 = 2(64 + BO^2)$
$\Rightarrow BO^2 = 81 \Rightarrow BO = 9$
Diagonal $= 2 \times BO = 2 \times 9 = 18$ cm.

44. **(c)**



$$\left(\frac{a}{2} + r\right)^2 = \left(\frac{a}{2}\right)^2 + (a - r)^2$$

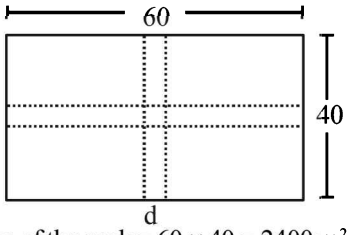
$$\Rightarrow \quad \frac{a^2}{4} + r^2 + ar = \frac{a^2}{4} + a^2 + r^2 - 2ar$$



$$\Rightarrow \quad 3ar = a^2 \Rightarrow r = \frac{a^2}{3a} \Rightarrow r = \frac{a}{3}$$

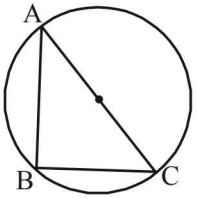
45. **(d)** $a - b = 23$ ....(i)
$2(a + b) = 206 \Rightarrow a + b = 103$ .... (ii)
From (i) and (ii), $a = 63$, $b = 40$
Area $= ab = 63 \times 40 = 2520$ m^2.

46. **(b)**



Area of the park $= 60 \times 40 = 2400$ m^2.
Area of the lawn $= 2109$ m^2.
Area of the cross-roads
$= 2400 - 2109 = 291$
Area of the cross-roads
$= 60.x + 40.x - x.x = 291$
$\Rightarrow 100x - x^2 = 291$
$\Rightarrow x^2 - 100x + 291 = 0$
$\Rightarrow x^2 - 97x - 3x + 291 = 0 \Rightarrow (x - 3)(x - 97) = 0$
$\Rightarrow x = 3, x = 97$
but $x = 97$ is not possible. Hence, width of the road $= 3$ m.

47. **(b)**

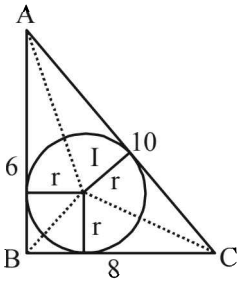


$$\text{Area of a triangle} = \frac{abc}{4R}$$

$$\Rightarrow \quad R = \frac{15 \times 20 \times 25}{4 \times 150} = 12.5 \text{ cm}$$

Shortcut : – $\dfrac{\text{hypotenuse}}{2} = \dfrac{25}{2} = 12.5$ cm

48. **(a)**



$$\text{Area } ABC = \frac{1}{2} \times AB \times AC$$

= Area AIB + Area BIC + Area AIC

$\Rightarrow \frac{1}{2}\times 6\times 8=\frac{1}{2}\times 8\times r+\frac{1}{2}\times 6\times r+\frac{1}{2}\times 10\times r$

$\Rightarrow \frac{1}{2}\times 6\times 8=\frac{1}{2}r(8+6+10)\Rightarrow r=\frac{48}{24}=2\text{cm}$

Alternate :-

Area ABC = Inradius × Semiperimeter

$\Rightarrow \frac{1}{2}\times 6\times 8=r\times\left(\frac{6+8+10}{2}\right)\Rightarrow r=2\text{ cm}$

Shortcut Approach :–

$\text{Inradius}=\frac{L+B-H}{2}=\frac{6+8-10}{2}=2\text{ cm}$

49. (a) $\text{Height}=AD=\frac{\sqrt{3}}{2}\times\text{side}=\frac{\sqrt{3}}{2}\times 6=3\sqrt{3}$ cm

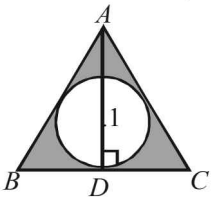
$\therefore \text{Inradius}=\frac{1}{3}\times\text{height}=\frac{1}{3}\times 3\sqrt{3}$

$=\sqrt{3}$ cm.

$A(\text{shaded region})=A(\Delta ABC)-A(\text{incircle})$

$=\frac{\sqrt{3}}{4}\times(\text{side})^2=\pi r^2$

$=\frac{\sqrt{3}}{4}\times 6\times 6-3.14\times\left(\sqrt{3}\right)^2$



$=15.57-9.42=6.15$ cm.

50. (c) Let the area of two squares be $9x$ and x respectively.

So, sides of both squares will be

$\sqrt{9x}$ and $\sqrt{x}$ respectively. [since, side $=\sqrt{\text{area}}$]

Now, perimeters of both squares will be

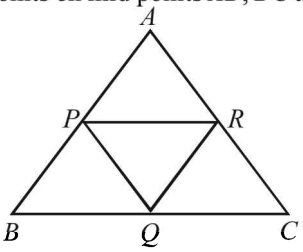
$4\times\sqrt{9x}$ and $4\sqrt{x}$ respectively.

[since , perimeter = 4 × side]

Thus, ratio of their perimeters $=\frac{4\sqrt{9x}}{4\sqrt{x}}=3:1$

Level 1

1. (c) Consider for an equilateral triangle. Hence ΔABC consists of 4 such triangles with end points on mid points AB, BC and CA



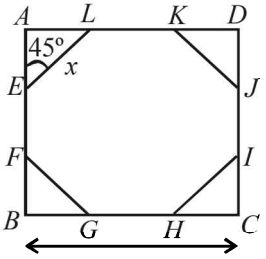
$\Rightarrow \frac{1}{4}ar(\Delta ABC)=ar(\Delta PQR)$

$\Rightarrow ar(\Delta PQR)=5$ sq. units

2. (a) Since the interior angle of an octagon is 135°, the angles in ΔAEL are each 45°.

If x is the side of the octagon $LE=x$.

Hence, in ΔALE.



$AL=AE=\frac{x}{\sqrt{2}}=KD$

$\therefore \quad AL+LK+KD$

$=\frac{x}{\sqrt{2}}+x+\frac{x}{\sqrt{2}}$

But this is the side of the square.

Hence, $\frac{2x}{\sqrt{2}}+x=1\Rightarrow\left(1+\sqrt{2}\right)x=1$

$\therefore \quad x=\frac{1}{1+\sqrt{2}}=\sqrt{2}-1$

3. (b) In the figure $\angle ACB$ is 90°

(angle subtended by diameter = 90°)

$AC=5, AB=13$

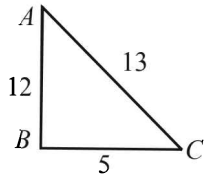
Using pythagoras theorem,

$AB^2=AC^2+CB^2$

$\Rightarrow CB=\sqrt{13^2-5^2}=12$

Area of $\Delta ABC=\frac{1}{2}\times 5\times 12=30$

4. (b)



ABC forms a right angled triangle

$\therefore$ Area $= \frac{1}{2} \times 12 \times 5 = 30$

Area of rectangle $= 30 = \ell \times 10$ or $\ell = 3$ units

$\therefore$ Perimeter $= 2(10 + 3) = 26$

5. (c) Given, length of garden = 24 m and breadth of garden = 14 m

$\therefore$ Area of the garden $= 24 \times 14\ m^2 = 336\ m^2$.

Since, there is 1 m wide path outside the garden

$\therefore$ Area of Garden (including path)
$= (24 + 2) \times (14 + 2) = 26 \times 16\ m^2 = 416\ m^2$.

Now, Area of Path = Area of garden (inculding path) – Area of Garden
$= 416 - 336 = 80\ m^2$.

Now, Area of Marbles $= 20 \times 20 = 400\ cm^2$

$\therefore$ Marbles required $= \frac{\text{Area of Path}}{\text{Area of Marbles}}$

$= \frac{80,0000}{400} = 2000$

6. (b) Perimeter = Distance covered in 8 min.

$= \left(\frac{12000}{60} \times 8\right) m = 1600$ m.

Let length $= 3x$ metres and breadth $= 2x$ metres.

Then, $2(3x + 2x) = 1600$ or $x = 160$.

$\therefore$ Length = 480 m and Breadth = 320 m.

$\therefore$ Area $= (480 \times 320)\ m^2 = 153600\ m^2$.

7. (b) Given, playground is rectangular.

Length = 36 m, Breadth = 21 m

Now, perimeter of playground
$= 2(21 + 36) = 114$

Now, poles are fixed along the boundary at a distance 3m.

$\therefore$ Required no. of poles $= \frac{114}{3} = 38$.

8. (a) Area of shaded region = Area of equilateral $\Delta ABC - 3$ (Area of sector AQO)

$= \frac{\sqrt{3}}{4} \times (2)^2 - 3 \times \frac{60}{360} \times \frac{22}{7} \times (1)^2$

$= \sqrt{3} - \frac{11}{7} = 1.73 - 1.57 = 0.16$ sq. units.

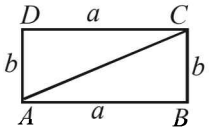
9. (d) Since AB is the diameter of the circle, $\angle ACB$ would be right angle. In this triangle, we know $AB = 15$ and $AC = 12$. So, we can find BC. Since 3–4–5 forms a triplet, $3 \times (3–4–5)$ also forms a triplet.

So, 9 – 12 – 15 forms a triplet. Hence, $BC = 9$. Since $BC = BD$, $ADB – ACB$ (similar triangles).

Hence, area of ΔABC = Area of ΔABD
$= 1/2\ AC \times CB = 1/2 \times 12 \times 9 = 54$

So, area of quadrilateral $ABCD = 2 \times 54$
$= 108$ sq. cm.

10. (c)



$AC + AB = 5AD$ or $AC + a = 5b$ ....(1)

$AC - AD = 8$ or $AC = b + 8$ ....(2)

Using (1) and (2),

$a + b + 8 = 5b$ or $a + 8 = 4b$ ...(3)

Using Pythagorous theorem,

$a^2 + b^2 = (b + 8)^2 = b^2 + 64 + 16b$

or $a^2 = 16b + 64 = (4b - 8)^2 = 16b^2 + 64 - 64b$ [From (3)]

$\Rightarrow 16b^2 - 80b = 0$ or $b = 0$ or 5

Putting $b = 5$ in (3), $a = 4b - 8 = 20 - 8 = 12$

Area of rectangle $= 12 \times 5 = 60$

11. (a) Since $ABCDOA$ is a quadrant of circle of radius 10.5 cm $OA = OC = r = 10.5$ cm and

$OD = DC = \frac{10.5}{2} = 5.25$ cm

Area of shaded portion = (Area of the quadrant) – (Area of ΔAOD)

$\text{Area} = \frac{\theta}{360} \pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{90}{360} \times \frac{22}{7} \times (10.5)^2 - \frac{1}{2} \times 5.25 \times 10.25$

$= 86.625 - 27.5625 = 59.06\ cm^2. \cong 59\ cm^2$

12. (a) AB = BC = CD = AD = 4 cm

$\left(\because \text{Side} = \frac{\text{Diagonal}}{\sqrt{2}}\right)$

and EF = 1.5 cm (by Pythagorus theorem)

$\therefore$ Area of ABCDE = Area of ABCD + Area of AED

$= (4)^2 + \frac{1}{2} \times 4 \times 1.5 = 19\ cm^2$

GP_3386

13. (c) $DE = \sqrt{17^2 - 8^2} = \sqrt{289-64} = 15m$

$$\text{Area} = \frac{1}{2}\times 8\times 15 + 8\times 15 = 180m^2$$

14. (c) Join AD and CD

Let $\angle BCD = \theta$

$$\therefore \tan\theta = \frac{BD}{BC} = \frac{6\sqrt{3}}{6} = \sqrt{3}$$

$$\theta = 60^\circ \Rightarrow \angle BAD = 30^\circ$$

$$\Rightarrow \tan 30^\circ = \frac{6\sqrt{3}}{AB} \Rightarrow AB = 18\,\text{cm}$$

$\therefore$ AC = 24 cm

Area of shaded portion

$$= \frac{\pi}{2}(12)^2 - \frac{\pi}{2}(3)^2 - \frac{\pi}{2}(9)^2 = 27\pi$$

15. (c) Side of square $= \sqrt{121} = 11$ m

Now, $2\pi r = 4\times 11 \Rightarrow r = 7$

$$\therefore \text{Area} = \frac{22}{7}\times 7\times 7 = 154\ \text{m}^2$$

16. (b) Area of shaded region

$$= \frac{1}{2}\times\frac{22}{7}\times 42\times 42 - \left[\frac{22}{7}\times 21\times 21 + \frac{22}{7}\times 14\times 14\right]$$

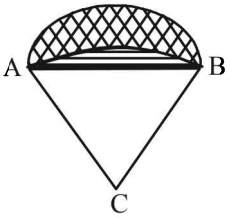
$$= 2772 - (1386+616) = 770\ \text{cm}^2$$

17. (c) Area of quadrant $= \frac{1}{4}\pi a^2$

Area of triangle ACB $= \frac{a^2}{2}$

▩ → Area of required region

▤ → Area of segment



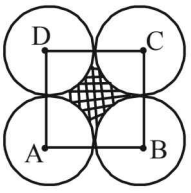
$\therefore$ Area of segment $= \frac{\pi a^2}{4} - \frac{a^2}{2} = \frac{a^2}{4}[\pi - 2]$

$\therefore$ Area of semi-circle $= \frac{1}{2}\pi\left(\frac{a\sqrt{2}}{2}\right)^2 = \frac{\pi a^2}{4}$

$\therefore$ Area of required region

$$= \frac{\pi a^2}{4} - \frac{a^2}{4}[\pi - 2] = \frac{a^2}{2}\ \text{sq. unit}$$

18. (b) Let the each side of the square be 2 cm, then



area of square = 4 cm^2 and area of 4 quadrants of the four circles (i.e., unshaded part inside the square)

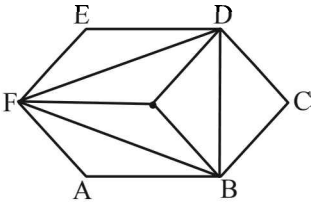
$$= 4\times\frac{1}{4}\pi\times(1)^2 = \pi\ \text{cm}^2$$

$\therefore$ Area of shaded region = $(4-\pi)$ cm^2

Therefore, the required ratio

$$= \frac{4-\pi}{\pi} = \frac{6/7}{22/7} = \frac{3}{11}$$

19. (b)



$$\frac{\text{Area of } \Delta BDF}{\text{Area of hexagon}} = \frac{1}{2}$$

As there is a perfect symmetricity.

$\therefore$ Area of hexagon

$$= \frac{3\sqrt{3}}{2}\times(6)^2 = 54\sqrt{3}\ \text{cm}^2$$

$\therefore$ Area of $\Delta BDF = 27\sqrt{3}\text{cm}^2$

Alternatively:

Subtract the areas of three triangles DEF, BAF, BCD from the area of hexagon.

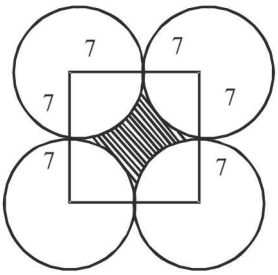
$$\text{Area of } \Delta DEF = \frac{1}{2}\times 6\times 6\sin 120^\circ = 9\sqrt{3}\ \text{cm}^2$$

$\therefore$ Area of all the three triangles $= 27\sqrt{3}$ cm

$\therefore$ Required area of $\Delta BDF = 54\sqrt{3} - 27\sqrt{3}$

$= 27\sqrt{3}\ \text{cm}^2$

20. (b)



The shaded area gives the required region.
Area of the shaded region
= Area of the square – area of four quadrants of the circles $= (14)^2 - 4\times\frac{1}{4}\pi(7)^2$

$= 196 - \frac{22}{7}\times 49 = 196 - 154 = 42\,cm^2$

21. (b) Perimeter = Distance covered in 8 min.

$= \left(\frac{12000}{60}\times 8\right)m = 1600$ m.

Let length = $3x$ metres and breadth = $2x$ metres.
Then, $2(3x + 2x) = 1600$ or $x = 160$.
$\therefore$ Length = 480 m and Breadth = 320 m.
$\therefore$ Area = $(480 \times 320)\,m^2 = 153600\,m^2$.

22. (b) Given, playground is rectangular.
Length = 36 m, Breadth = 21 m
Now, perimeter of playground
$= 2(21 + 36) = 114$
Now, poles are fixed along the boundary at a distance 3m.

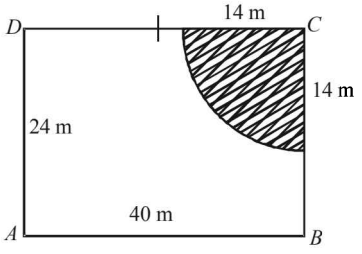
$\therefore$ Required no. of poles $= \frac{114}{3} = 38$.

23. (a) Let width of the field = b m
$\therefore$ length = $2b$ m
Now, area of rectangular field = $2b \times b = 2b^2$
Area of square shaped pond = $8 \times 8 = 64$
According to the question,

$64 = \frac{1}{8}(2b^2) \Rightarrow b^2 = 64\times 4 \Rightarrow b = 16$ m

$\therefore$ length of the field = $16 \times 2 = 32$ m

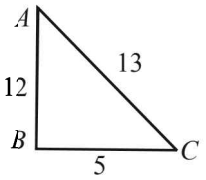
24. (a)



Area of the shaded portion

$= \frac{1}{4}\times\pi(14)^2 = 154\,m^2$

25. (b)

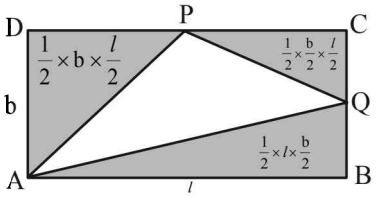


ABC forms a right angled triangle

$\therefore$ Area $= \frac{1}{2}\times 12\times 5 = 30$

Area of rectangle $= 30 = \ell \times 10$ or $\ell = 3$ units
$\therefore$ Perimeter $= 2(10 + 3) = 26$ $(\because AE = FD)$

26. (c)

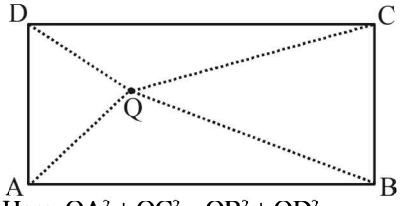


Shaded area $= \frac{lb}{4} + \frac{lb}{4} + \frac{lb}{8} = \frac{5lb}{8}$

Unshaded portion $= lb - \frac{5lb}{8} = \frac{3lb}{8}$

Required ratio $= \frac{5lb}{8} : \frac{3lb}{8} = 5 : 3$

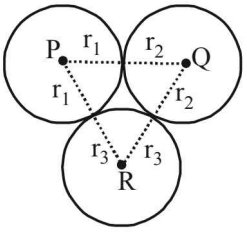
27. (a) If 'P' is a point inside a rectangle then, $AP^2 + PC^2 = PB^2 + PD^2$.



Here, $QA^2 + QC^2 = QB^2 + QD^2$
$\Rightarrow 3^2 + 5^2 = 4^2 + QD^2 \Rightarrow \quad QD^2 = 18$

$\Rightarrow QD = 3\sqrt{2}$ cm

28. (a)



PQ = 4; QR = 6 ; PR = 8

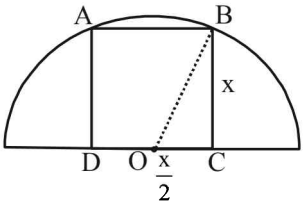
PQ + QR + PR = 18

$\Rightarrow r_1 + r_2 + r_2 + r_3 + r_3 + r_1 = 18$

$\Rightarrow 2(r_1 + r_2 + r_3) = 18$

$\Rightarrow r_1 + r_2 + r_3 = 9$cm.

29. (d)



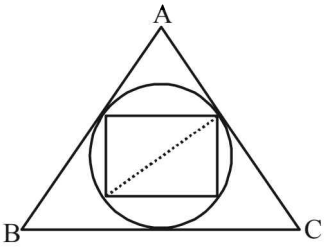
In ΔOBC, $OC^2 + BC^2 = BO^2$

$\Rightarrow (x)^2 + \left(\frac{x}{2}\right)^2 = (10)^2 \Rightarrow 4x^2 + x^2 = 400$

$\Rightarrow x^2 = 80 \Rightarrow x = \sqrt{80} = 4\sqrt{5}$ cm

Perimeter ABCD $= 4 \times 4\sqrt{5}$ cm $= 16\sqrt{5}$ cm.

30. (c)



Altitude of equilateral triangle $= \frac{\sqrt{3}}{2}a$

Radius of incircle $= \frac{\sqrt{3}}{2}a.\frac{1}{3} = \frac{a}{2\sqrt{3}}$

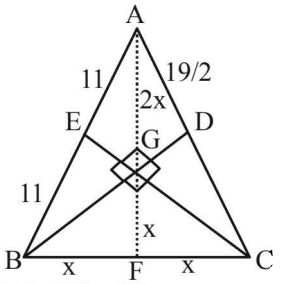
Diameter of incircle = Diagonal of square

Diameter $= \frac{a}{2\sqrt{3}} \times 2 = \frac{a}{\sqrt{3}}$

Diagonal of square $= \frac{a}{\sqrt{3}}$

$$\text{Area} = \frac{(\text{Diagonal})^2}{2} = \frac{\left(\frac{a}{\sqrt{3}}\right)^2}{2} = \frac{a^2}{6}.$$

31. (b)



BF = FC = GF

[Mid-point of hypotenuse is equidistant from the vertices]

$2(AF^2 + BF^2) = AB^2 + AC^2$

$\Rightarrow 2[(3x)^2 + x^2] = (22)^2 + (19)^2$

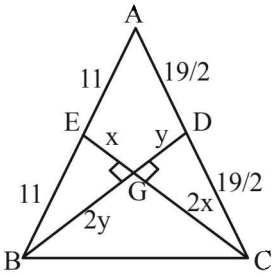
$\Rightarrow 2.10x^2 = 845 \Rightarrow x = 6.5$

$BC = 2x = 2 \times 6.5 = 13$.

Note : If two mediaus of a triangle is perpendicular to each other then the relation between the sides is given by $a^2 + b^2 = 5c^2$

Alternate :–

In ΔEGB, $(x)^2 + (2y)^2 = 11^2$ ...(i)



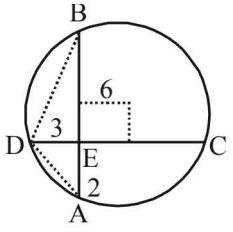
In ΔDGC ; $y^2 + (2x)^2 = \left(\frac{19}{2}\right)^2$ ...(ii)

Adding (i) and (ii),

$5y^2 + 5x^2 = \frac{845}{4} \Rightarrow x^2 + y^2 = \frac{169}{4}$

$BC = \sqrt{4x^2 + 4y^2} = \sqrt{169} = 13$

32. (a)



$$DB = \sqrt{(BE)^2 + (DE)^2} = \sqrt{6^2 + 3^2} = \sqrt{45}$$

$$DA = \sqrt{(DE)^2 + (AE)^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$$

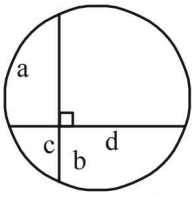
$$\text{Area ABD} = \frac{1}{2} \times DE \times AB = \frac{1}{2} \times 3 \times 8$$

$$\text{Also, Area ABD} = \frac{DB \times DA \times AB}{4R}$$

$$\Rightarrow \frac{1}{2} \times 3 \times 8 = \frac{\sqrt{45} \times \sqrt{13} \times 8}{4R} \Rightarrow R = \frac{\sqrt{65}}{2}$$

$$\text{Diameter} = \frac{\sqrt{65}}{2} \times 2 = \sqrt{65} \text{ cm}$$

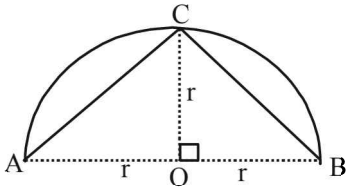
Shortcut 2nd method



$a^2 + b^2 + c^2 + d^2 = (2r)^2$

Here, $6^2 + 2^2 + 3^2 + 4^2 = (2r)^2 \Rightarrow 2r = \sqrt{65}$

33. **(b)** Area enclosed by semi-circle but exterior of triangle

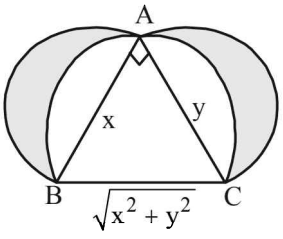


= Area of semi-circle – Area of triangle ACB.

$$= \frac{\pi(7)^2}{2} - \frac{1}{2} \times 7 \times 14$$

$$= \frac{49\pi}{2} - 49 = 77 - 49 = 28 \text{ cm}^2$$

34. **(a)**



Shaded area = Area of the semi-circle with diameter AB + Area of the semicircle, with diameter AC + Area of the triangle ABC – Area of the semicircle with diameter BC.

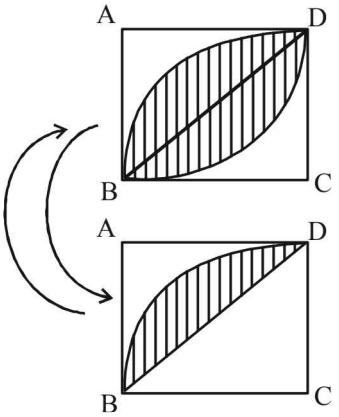
$$= \frac{\pi\left(\frac{x}{2}\right)^2}{2} + \frac{\pi\left(\frac{y}{2}\right)^2}{2} + \frac{1}{2} \times xy - \frac{\pi\left(\frac{\sqrt{x^2 + y^2}}{2}\right)^2}{2}$$

$$= \frac{\pi x^2}{8} + \frac{\pi y^2}{8} + \frac{1}{2}xy - \frac{\pi(x^2 + y^2)}{8}$$

$$= \frac{\pi}{8}(x^2 + y^2) - \pi\frac{(x^2 + y^2)}{8} + \frac{1}{2}xy$$

$$= \frac{1}{2}xy = \text{Area of } \Delta ABC.$$

35. **(b)**



$$\text{Shaded area} = 2 \times \left[\frac{49\pi}{4} - \frac{49}{2}\right] = 28 \text{ cm}^2$$

36. **(b)** Let r = Radius of 3 smaller laminas

In ΔADC, $(2r)^2 = r^2 + DC^2 \Rightarrow DC = \sqrt{3}r$

$$\because OC = \frac{2}{3}DC = \frac{2}{3} \times \sqrt{3}r = \frac{2r}{\sqrt{3}}$$

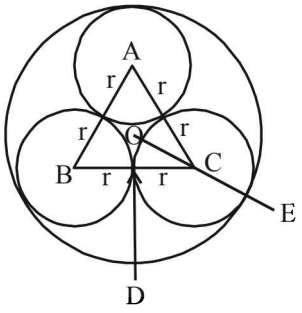
Radius of larger circular lamina = OE

$$OC + CE = \frac{2r}{\sqrt{3}} + r = \frac{(2 + \sqrt{3})r}{\sqrt{3}}$$

Area of 3 laminas = $3\pi r^2$

$$\text{Area of larger lamina} = \pi\left[\frac{(2 + \sqrt{3})}{\sqrt{3}}r\right]^2$$

$$= \pi \frac{\left(4+3+4\sqrt{3}\right)}{3} r^2 = \frac{\left(7+4\sqrt{3}\right)\pi r^2}{3}$$



Residual area

$$= \left(\frac{7+4\sqrt{3}}{3} - 3\right)\pi r^2 = \left(\frac{4\sqrt{3}-2}{3}\right)\pi r^2$$

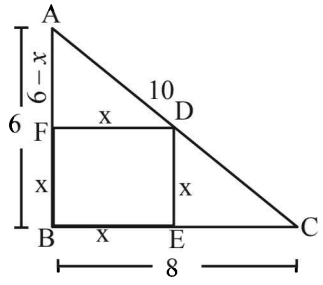
$$\therefore \quad \text{Required ratio} = \frac{\left(\frac{4\sqrt{3}-2}{3}\right)\pi r^2}{\frac{7+4\sqrt{3}}{3}\pi r^2}$$

$$= \frac{4\sqrt{3}-2}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

$$= \frac{28\sqrt{3}-48-14+8\sqrt{3}}{49-48}$$

$$= 36\sqrt{3} - 62 = 36 \times 1.732 - 62 = 0.35$$

37. (a)



$$\Delta ADF \sim \Delta ACB \Rightarrow \frac{AF}{AB} = \frac{FD}{BC}$$

$$\Rightarrow \frac{6-x}{6} = \frac{x}{8} \Rightarrow 48 - 8x = 6x \Rightarrow \quad 14x = 48$$

$$\Rightarrow x = \frac{48}{14} = \frac{24}{7}$$

Hence, area BEDF $= x^2 = \frac{576}{49}\text{cm}^2$

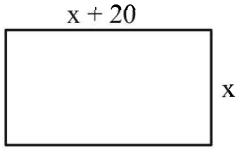
Alternate : –

Side of the largest square

$$= \frac{AB \times BC}{AB + BC} = \frac{8 \times 6}{8+6} = \frac{24}{7}$$

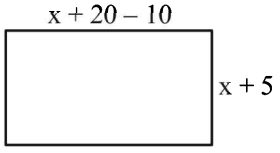
$$\text{Area} = \frac{576}{49}\text{cm}^2.$$

38. (c) **Case I :**



Area of the rectangle = (x + 20)x

Case II



Area of the rectangle = (x + 10) × (x + 5)

According to question,

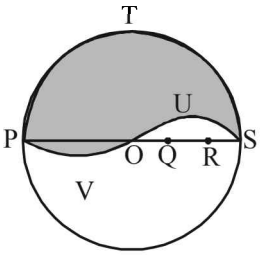
$(x+20)x = (x+10)(x+5)$

$\Rightarrow x^2 + 20x = x^2 + 5x + 10x + 50$

$\Rightarrow 5x = 50 \Rightarrow x = 10\text{ m}$

$\therefore$ Breadth = 10 m; Length = 30 m and Area = $10 \times 30 = 300\text{ m}^2$.

39. (a) Given, OS = 6 cm



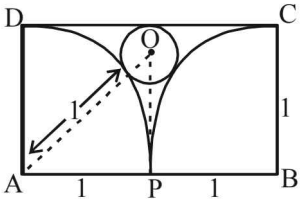
$\therefore$ PQ = QR = RS = 4 cm

$\therefore$ Perimeter of shaded region
= Perimeter of semi-circle PTS
+ Perimeter of semi-circle QUS
+ Perimeter of semi-circle PVQ
$= \pi(6) + \pi(4) + \pi(2) = 12\pi$ cm.

40. **(a)** Let radius of the circle is 'r' units

$OP = (1-r)$, $OA = (1+r)$ and $AP = 1$

In ΔAOP; $OA^2 = AP^2 + OP^2$



$\Rightarrow \quad (1+r)^2 = 1^2 + (1-r)^2 \Rightarrow \quad r = \frac{1}{4}$ units

$\therefore$ Area of smaller circle $= \pi\left(\frac{1}{4}\right)^2$

$= \frac{\pi}{16}$ sq units

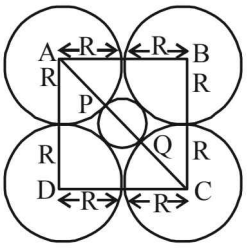
Sum of the area of the quarter circles

$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$ square units

Area of shaded region

$= 2 - \left(\frac{\pi}{16} + \frac{\pi}{2}\right) = 2 - \frac{9}{16}\pi = 0.23$ sq. units

41. **(b)**



Let radius of inner circle = r

A, B, C, D are the centres of the four outer circles

$\therefore$ ABCD is a square of side 2R

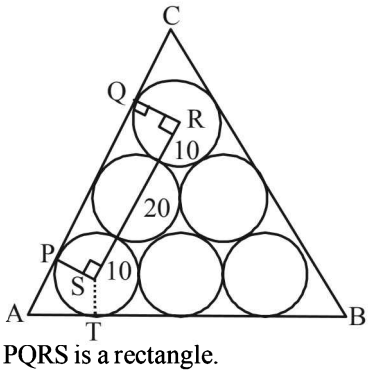
$\therefore \quad AC = \sqrt{2}\,(\text{side}) = \sqrt{2}\,(2R) = 2\sqrt{2}R$

$\therefore \quad PQ = AC - AP - QC$

$= 2\sqrt{2}R - R - R = 2R\left(\sqrt{2}-1\right)$

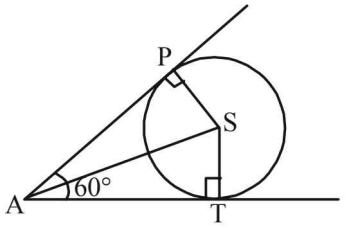
$\Rightarrow \quad 2r = 2R\left(\sqrt{2}-1\right) \Rightarrow r = R\left(\sqrt{2}-1\right)$

42. **(c)**



PQRS is a rectangle.

$\therefore \quad PQ = 10 + 20 + 10 = 40$ cm for AP,



$\angle A = 60°$, $\angle PST = 120°$

$\therefore \quad \angle PSA = \angle AST = \frac{120°}{2} = 60°$

and $\angle PAS = \angle SAT = 30°$

$\therefore$ In ΔPSA,

$\tan 30° = \frac{PS}{AP} = \frac{10}{AP}$

$\Rightarrow \quad AP = \frac{10}{\tan 30°} \Rightarrow AP = 10\sqrt{3}$

Similarly, $QC = 10\sqrt{3}$

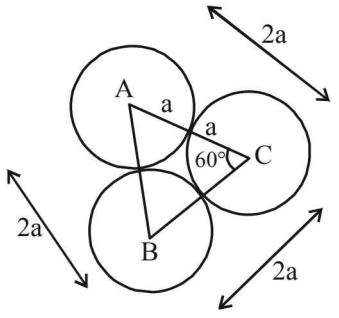
$\therefore \quad AC = PQ + AP + QC$

$= 40 + 10\sqrt{3} + 10\sqrt{3} = 20\left(2+\sqrt{3}\right)$ cm

$\therefore \quad AB = BC = AC = 20\left(2+\sqrt{3}\right)$

$\therefore$ Perimeter of $\Delta ABC = 60\left(2+\sqrt{3}\right)$ cm

43. **(d)**



Hence ABC is an equilateral triangle
AB = BC = AC = 2a cm

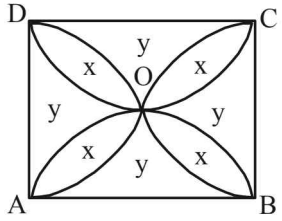
$$\text{Area of } \Delta ABC = \frac{\sqrt{3}}{4}(2a)^2 = \sqrt{3}a^2$$

$$\text{Area of 3 sectors} = 3 \times \frac{60^\circ}{360^\circ} \times \pi a^2 = \frac{\pi a^2}{2}$$

Area of shaded region = area of ΔABC – area of

$$3 \text{ sector} = \sqrt{3}a^2 - \frac{\pi a^2}{2} = \left(\frac{2\sqrt{3} - \pi}{2}\right) a^2 \text{cm}^2$$

44. (c)

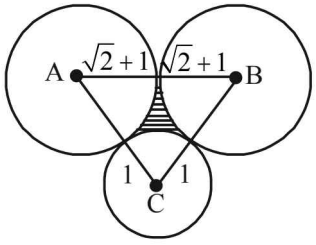


Let area of each shaded portion = x
and area of each unshaded portion = y
Total area of square = $(8)^2 = 64$ cm²
∴ $4(x + y) = 64 \Rightarrow x + y = 16$ ....(i)
Again in a semicircle, AOB

$$x + y + x = \frac{1}{2}\pi \times (4)^2 \Rightarrow 2x + y = 8\pi \ldots. \text{(ii)}$$

From (i) & (ii) we get,
$x = 8\pi - 16 = 8(\pi - 2)$
∴ Total area of shaded region
$= 32(\pi - 2)$ cm².

45. (d)



$AB = 2(\sqrt{2} + 1)$ and $AC = BC = 2 + \sqrt{2}$

$\therefore \quad AB^2 = 4(\sqrt{2} + 1)^2$

and $AC^2 = BC^2 = (2 + \sqrt{2})^2$

So, it is clear
$AB^2 = AC^2 + BC^2$
i.e. ABC is an isosceles right angled triangle.
∴ $\angle ACB = 90^\circ$ and $\angle CAB = \angle ABC = 45^\circ$
∴ Required perimeter

$$= 2\pi(\sqrt{2} + 1)\frac{45^\circ}{360^\circ} + 2\pi(\sqrt{2} + 1)\frac{45^\circ}{360^\circ}$$

$$+2\pi \times 1 \times \frac{90^\circ}{360^\circ} = 2 \times 2\pi(\sqrt{2} + 1)\frac{1}{8} + 2\pi\frac{1}{4}$$

$$= \frac{2\pi}{4}(\sqrt{2} + 1 + 1) = \frac{\pi}{2}(\sqrt{2} + 2) \text{ unit}$$

46. (d) Area of sector $OED = \pi r^2 \times \frac{\theta}{360}$

$$= \pi \times 4 \times 4 \times \frac{45}{360} = 2\pi \text{ m}^2$$

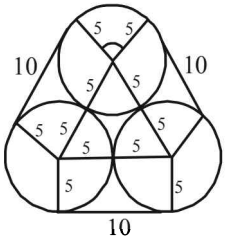
Area of the sector $OAB = \pi r^2 \times \frac{\theta}{360}$

$$= \pi \times 3 \times 3 \times \frac{45}{360} = \frac{9}{8}\pi \text{ m}^2$$

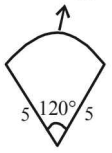
So, area of shaded portion = Area of OED – Area of OAB

$$= 2\pi - \frac{9}{8}\pi = \frac{16\pi - 9\pi}{8} = \frac{7}{8}\pi = \frac{11}{4}\text{m}^2$$

47. (b)

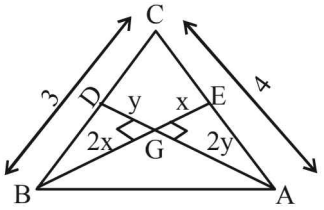


$$\frac{2\pi r \times 120}{360} = \frac{2\pi \times 5}{3} = \frac{10\pi}{3}$$



Length of the rubber band $= 3 \times \frac{10\pi}{3} + 3 \times 10$
$= 30 + 10\pi$ cm

48. (a)



In ΔDGB, $DG^2 + BG^2 = DB^2$

$$\Rightarrow y^2 + (2x)^2 = \left(\frac{3}{2}\right)^2$$

$$y^2 + 4x^2 = \frac{9}{4} \quad \ldots(i)$$

In ΔEGA, $EG^2 + GA^2 = EA^2$

$\Rightarrow (x)^2+(2y)^2=\left(\frac{4}{2}\right)^2$

$\Rightarrow x^2+4y^2=4$ ....(ii)

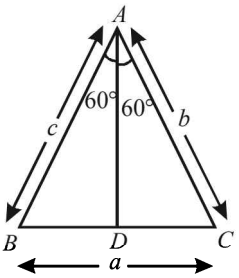
Adding (i) and (ii),

$5y^2+5x^2=\frac{9}{4}+4=\frac{9+16}{4}=\frac{25}{4}$

$AB^2=4x^2+4y^2=\frac{25\times4}{4\times5}=5$

$AB^2=5\Rightarrow AB=\sqrt{5}$ cm

49. (b)



Let $AD=h$(say),

then area of ΔABC

$=\frac{1}{2}BC\sin120°=\frac{\sqrt{3}}{4}BC$

Area of ΔBAD

$=\frac{1}{2}h\sin60°$

$=\frac{\sqrt{3}}{4}h$

and area of $\Delta CAD=\frac{1}{2}bh\sin60°=\frac{\sqrt{3}}{4}bh$

Now,

$ar(\Delta ACD)=ar(\Delta BAD)+ar(\Delta CAD)$

$\frac{\sqrt{3}}{4}bc=\frac{\sqrt{3}}{4}ch+\frac{\sqrt{3}}{4}bh$

$\Rightarrow bh=h(b+c)\Rightarrow h=\frac{bc}{b+c}$

50. (c) Area of $\Delta ABC=\frac{1}{2}\times15\times20=150$ sq. cm.

$AB=\sqrt{AC^2+BC^2}=\sqrt{225+400}=\sqrt{625}=25$

Area of $ABC=\frac{1}{2}\times25\times CD=150$;

$CD=12$ cm.

$PQ=$ Radius of incircle of $ACD+$ Radius of incirlce of BCD

Radius of incircle triangle ADC

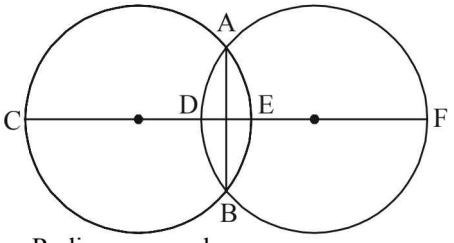
Area of $ADC=r\times s$

Area of $ADC=\frac{1}{2}\times12\times16=96$

$96=r\times\left(\frac{12+20+16}{2}\right)=24r\Rightarrow r=4$

$\therefore PQ=7$

51. (c)



$\because$ Radius are equal

Then, CE = DF

CD + DE = DE + EF

CD = EF

EF = 4.5 cm

52. (c) Circumference of the circular face of the cylinder $=2\pi r$

$\Rightarrow \quad 2\times\frac{22}{7}\times\frac{35}{100}=2.2$m

Number of revolutions required to lift the bucket by 11 m $=\frac{11}{2.2}=5$

53. (b) Area of the shaded portion = Area of quadrant ABC + Area of quadrant ACD – Area of square $ABCD$.

$=\frac{\pi}{2}\times4^2-4^2=\left(\frac{\pi}{2}-1\right)4^2=(\pi-2)8$

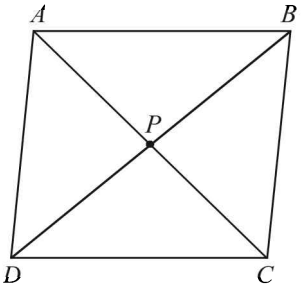
$=9.12$ sq. cm.

54. (b) Sum of interior angles of a hexagon = 720°

6 sectors with same radius $r=2$ full circles of same radius.

So area of shaded region $\Rightarrow 2\pi r^2$

55. (b) We know ratio of area of triangles divided by diagonals are same.



GP_3386

$$\frac{\text{Area of } \Delta APD}{\text{Area of } \Delta APB} = \frac{\text{Area of } \Delta DPC}{\text{Area of } \Delta CPB}$$

$$\Rightarrow \frac{27}{x} = \frac{x}{12}$$

$$\Rightarrow x^2 = 27 \times 12 = 3 \times 3 \times 3 \times 3 \times 2 \times 2$$

$$x = 18$$

56. (d) Let O'E be x. Then in ΔBO'E,

$BE^2 = 1^2 + x^2$ ....(i)

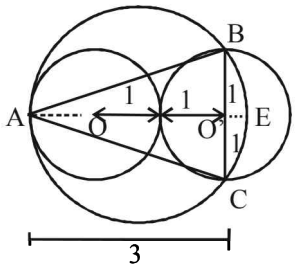
In ΔBAO', $AB^2 = 1^2 + 3^2 = 1 + 9 = 10$

Similarly, in ΔABE, $(3 + x)^2$

$= AB^2 + BE^2 = 10 + 1 + x^2$

$9 + x^2 + 6x = 11 + x^2$

$\Rightarrow 6x = 2$ or $x = \frac{1}{3}$



$\therefore$ Radius $= \frac{3 + 1/3}{2} = \frac{10}{6} = \frac{5}{3}$

57. (b) $$\frac{\text{Area of } \Delta DAE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2} \times DE \times AE}{\frac{1}{2} \times DE \times CE}$$

$$= \frac{AE}{CE} = \frac{(AD)^2}{(DC)^2} = \left(\frac{6}{8}\right)^2 = \frac{9}{16}$$

Similarly, in ΔABC,

$$\frac{\text{Area of } \Delta BCF}{\text{Area of } \Delta BFA} = \frac{9}{16}$$

$\therefore$ The area of shaded to unshaded region

$= \frac{16}{9}$

58. (d) $\pi r_1^2 + \pi r_2^2 = 180\pi$

$\Rightarrow r_1^2 + r_2^2 = 180$ ...(1)

and distance between centers i.e.

$= r_1 - r_2 = 6 \Rightarrow r_2 = r_1 - 6$

From the eq. (1), $r_1^2 + (r_1 - 6)^2 = 180$

$\Rightarrow r_1^2 + (r_1^2 - 12r_1 + 36) = 180$

$\Rightarrow 2r_1^2 - 12r_1 + 36 = 180$

$\Rightarrow 2r_1^2 - 12r_1 - 144 = 0$

$\Rightarrow (r_1 - 12)(r_1 + 6) = 0$.

Hence, $r_1 = 12$ cm and $d_1 = 24$ cm.

59. (d) $PQ = QR = RS = \frac{12}{3} = 4$ cm

Area of unshaded region $\Rightarrow \frac{\pi 6^2}{2} + \frac{\pi 4^2}{2}$

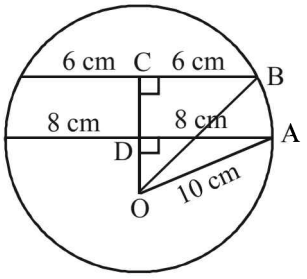
$\Rightarrow 18\pi + 8\pi \Rightarrow 26\pi$

Area of shaded region $\Rightarrow \frac{\pi 6^2}{2} - \frac{\pi 4^2}{2}$

$\Rightarrow 18\pi - 8\pi = 10\pi$

Ratio $= \frac{10\pi}{26\pi} \Rightarrow \frac{5}{13} \Rightarrow 5:13$

60. (b)



In ΔADO,

$OD = \sqrt{(AO)^2 - AD^2}$

$= \sqrt{100\text{cm}^2 - 64\text{cm}^2} = 6$ cm

In ΔBCO,

$OC = \sqrt{OB^2 - CB^2}$

$= \sqrt{100\text{cm}^2 - 36\text{cm}^2} = 8$ cm

distance between chords = OC – OD = 2 cm

CHAPTER

23 Mensuration-Solid Figure

SOLID

A **solid** body has three dimensions namely length, breadth (or width) and height (or thickness). The surfaces that bind it are called **faces** and the lines where faces meet are called **edges**.

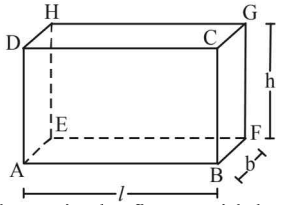
The area of the surface that binds the solid is called its **surface area**.

We measure the size of a solid body in terms of its volume. The amount of space that any solid body occupies is called its **volume**.

Surface areas are measured in square units and volumes are measured in cubic units.

Cube

A cuboid is like a three dimensional box. It is defined by its length (l), breadth (b) and height (h). A cuboid can also be visualised as a room. It has six rectangular faces. It is also called **rectangular parallelopiped**.



A cuboid is shown in the figure with length 'l', breadth 'b' and height 'h'. 'd' denotes the length of a diagonal (AG, CE, BH or DF) of the cuboid.

Total surface area of a cuboid = $2(lb + bh + hl)$

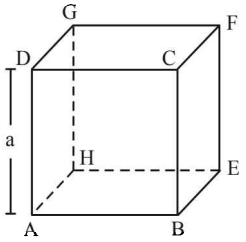
Lateral surface area (i.e., total area excluding area of the base and top) = $2h(l + b)$

Length of a diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

Volume of a cuboid = Space occupied by cuboid
= Area of base × height
= $(l \times b) \times h = lbh$

Cube

A cube is a cuboid whose all edges are equal i.e., length = breadth = height = a (say).



Area of each face of the cube is a^2 square units.

Total surface area of the cuboid

= Area of 6 square faces of the cube = $6 \times a^2 = 6a^2$

Lateral surface area of cube i.e., total surface area excluding top and bottom faces = $4a^2$

Length of diagonal of the cube = $\sqrt{a^2 + a^2 + a^2}$

$= \sqrt{3a^2} = a\sqrt{3}$

Volume of the cube (V) = Base area × Height = $a^2 \times a = a^3$

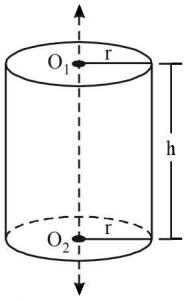
Note that If a cube of the maximum volume is inscribed in a sphere of radius 'r', then the edge of the cube = $\frac{2r}{\sqrt{3}}$.

Cylinder

A cylinder is a solid object with circular ends of equal radius and the line joining their centres perpendicular to them. This line is called **axis** of the cylinder. The length of axis between centres of two circular ends is called **height** of the cylinder.

GP_3386

In the figure, a cylinder with circular ends each of radius r and height h is shown.



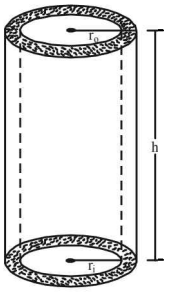
Curved surface area of a cylinder
= Circumference of base × height = $2\pi r \times h = 2\pi rh$
If cylinder is closed at both the ends then total surface area of the cylinder
= Curved surface area + Area of circular ends
= $2\pi rh + 2 \times \pi r^2 = 2\pi r(h + r)$
Volume of the cylinder (V)
= Base area × Height
= $\pi r^2 \times h = \pi r^2 h$

Note that

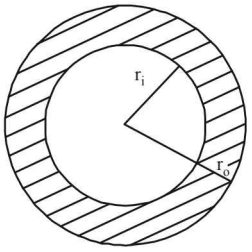
- a cylinder can be generated by rotating a rectangle by fixing one of its sides.
- the curved surface of a cylinder is also called **lateral surface.**

Hollow Cylinder

A hollow cylinder is like a pipe.



Inner radius = r_i and outer radius = r_o
Hence $r_o - r_i$ = thickness of material of the cylinder.
Let length or height of the cylinder = h,
Curved surface area (C.S.A) of the hollow cylinder
= Outer curved surface area of the cylinder
+ Inner curved surface area of the cylinder
= $2\pi r_o h + 2\pi r_i h = 2\pi h(r_o + r_i)$
Total surface area of hollow cylinder
= C.S.A. of hollow cylinder + Area of 2 circular end rings.

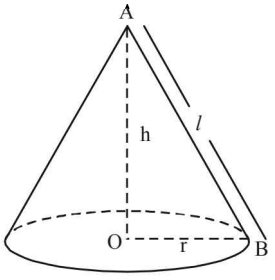


(one end of the pipe)

$= 2\pi h (r_o + r_i) + 2\pi (r_o^2 - r_i^2)$
$= 2\pi (r_o + r_i) (h + r_o - r_i)$
Volume of hollow cylinder
= Volume of the material used in making the cylinder
$= \pi (r_o^2 - r_i^2) h$

Cube

A cone is a solid obtained by rotating a strip in the shape of a right angled triangle about its height. It has a circular base and a slanting lateral curved surface that converges at a point. Its dimensions are defined by the radius of the base (r), the height (h) and slant height (l).
A structure similar to cone is the ice-cream cone.
Height (AO) of cone is always perpendicular to base radius (OB) of the cone.



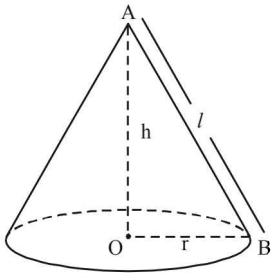
Slant height (l) = $\sqrt{h^2 + r^2}$

Volume of cone = $\frac{1}{3}$ × base area × height

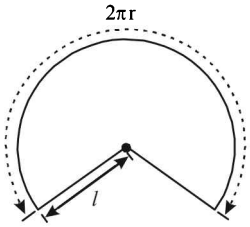
$= \frac{1}{3} \times \pi r^2 h$

Curved surface area (C.S.A.) = πrl
Total surface area (T.S.A.)
= C.S.A. + Base area
= $\pi rl + \pi r^2 = \pi r(l + r)$

When a conical cup of paper (hollow cone) is unrolled, it forms a sector of a circle.



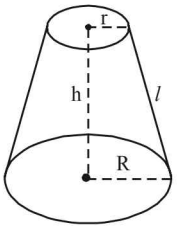
Conical cup of paper



Unrolled conical cup, which is a sector of a circle.
Radius of this sector is equal to slant height of the cone.
Length of curved edge of this sector is equal to the circumference of the base of the cone.

Frustum of a Cone

When top portion of a cone cut off by a plane parallel to the base of it, the left-over part is called the **frustum** of the cone.
In the figure, r and R are the radius of two ends, h is the height and l is the slant height of the frustum of cone.



Slant height, $l = \sqrt{(R-r)^2 + h^2}$

Curved surface area $= \pi(R+r)l$

Total surface area = (Curved surface area) + (Area of two circular ends)

$= \pi(R+r)l + \pi R^2 + \pi r^2 = \pi(Rl + rl + R^2 + r^2)$

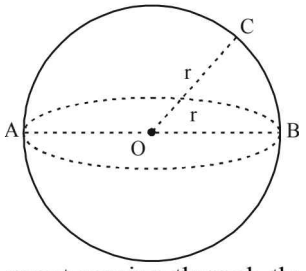
Height of the original cone $= \dfrac{Rh}{R-r}$

Volume of the frustum of cone

$= \dfrac{\pi h}{3}\left(R^2 + r^2 + Rr\right)$

Cube

A sphere is formed by revolving a semi-circle about its diameter. It has one curved surface which is such that all points on it are equidistant from a fixed point within it, called the **centre.**
Length of a line segment joining the centre to any point of the curved surface is called the **radius (*r*)** of the sphere.

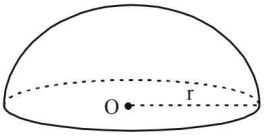


Any line segment passing through the centre and joining two points on the curved surface is called the **diameter (*d*)** of the sphere.
Centre = O ; Radius = $OC = OA = OB = r$,
Diameter = $AB = d = 2r$

Surface area = $4\pi r^2$; Volume (V) = $\dfrac{4}{3}\pi r^3$

Hemisphere

A plane through the centre of the sphere cuts the sphere into two equal parts. Each part is called a **hemisphere.**



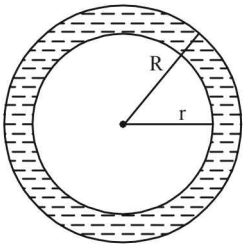
Volume of a hemisphere = $\dfrac{2}{3}\pi r^3$

Curved surface area (C.S.A.) of a hemisphere = $2\pi r^2$
Total surface area (T.S.A.) = C.S.A. + Base area
$= 2\pi r^2 + \pi r^2 = 3\pi r^2$
Note that if a sphere is inscribed in a cylinder then the volume of the sphere is $\dfrac{2}{3}$rd of the volume of the cylinder.

GP_3386

Hollow Sphere or Spherical Shell

A rubber ball is an *example* of hollow sphere. In the rubber ball, air is filled inside it. Thickness of the rubber in the ball is uniform. If outer and inner radii are R and r, then thickness of rubber or material used in hollow sphere $= R - r$.



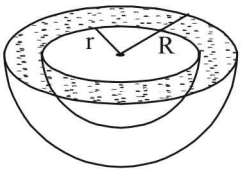
Volume of the rubber or material used in hollow sphere
= External volume – Internal volume

$$= \frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(R^3 - r^3)$$

External surface area $= 4\pi R^2$.
Internal surface area $= 4\pi r^2$

Hemispherical Bowl

When a spherical shell is cut off in two equal parts, then each part is called a **hemispherical bowl** as shown in the figure.



If R and r are external and internal radii of the hemisphere respectively, then
Volume of the material used in the hemispherical bowl
= External volume – Internal volume

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3)$$

External curved surface area $= 2\pi R^2$
Internal surface area $= 2\pi r^2$
Area of the cross-sectional ring $= \pi R^2 - \pi r^2$
$= \pi(R^2 - r^2)$
Total surface area = (External curved surface area) + (Internal curved surface area) + (Area of cross-sectional ring)

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2) = \pi(3R^2 + r^2)$$

Illustration – 1

If the radius of a sphere is increased by 2 cm, then its surface area increases by 352 cm^2. The radius of the sphere before the increase was:
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

Solution : (d) $4\pi(r+2)^2 - 4\pi r^2 = 352$

$$\Rightarrow (r+2)^2 - r^2 = \left(352 \times \frac{7}{22} \times \frac{1}{4}\right) = 28.$$

$$\Rightarrow (r+2+r)(r+2-r) = 28$$

$$\Rightarrow 2r+2 = \frac{28}{2} \Rightarrow 2r+2 = 14 \Rightarrow r = 6\text{ cm}$$

Illustration – 2

A cylindrical bucket of height 36 cm and radius 21 cm is filled with sand. The bucket is emptied on the ground and a conical heap of sand is formed, the height of the heap being 12 cm. The radius of the heap at the base is:
(a) 63 cm (b) 53 cm (c) 56 cm (d) 66 cm

Solution : (a) Volume of the bucket = Volume of the sand emptied
Volume of sand $= \pi(21)^2 \times 36$
Let r be the radius of the conical heap.

$$\therefore \frac{1}{3}\pi r^2 \times 12 = \pi(21)^2 \times 36 \Rightarrow r^2 = (21)^2 \times 9$$

$\Rightarrow r = 63$ cm

Illustration – 3

The length of the longest rod that can be placed in a room which is 24 m long, 18 m broad and 16 m high is
(a) 27 m (b) 39 m (c) 34 m (d) 33 m

Solution : (c) Required length = length of the diagonal of room

$$= \sqrt{24^2 + 18^2 + 16^2} = \sqrt{576 + 324 + 256} = \sqrt{1156} = 34\text{ m}$$

Illustration – 4

The internal measurements of a box with lid are 115 × 75 × 35 cm^3 and the wood of which it is made is 2.5 cm thick. Find the volume of wood.
(a) 82,125 cm^3 (b) 70,054 cm^3
(c) 78,514 cm^3 (d) None of these

Solution : (a) Internal volume = 115 × 75 × 35 = 3,01, 875 cm³ External volume
= (115 + 2 × 2.5) × (75 + 2× 2.5) × (35 + 2 × 2.5)
= 120 × 80 × 40 = 3,84,000 cm³
∴ Volume of wood = External volume – Internal volume
= 3,84,000 – 3,01,875 = 82,125 cm³

Illustration – 5

A rectangular tank is 225 m by 162 m at the base. With what speed must water flow into it through an aperture 60 cm by 45 cm that the level may be raised 20 cm in 5 hours ?

(a) 5000 m/hr **(b) 5400 m/hr**
(c) 5200 m/hr **(d) 5600 m/hr**

Solution : (b) Required speed of flow of water

$$= \frac{225\times162\times20}{5\times100} = \frac{60}{100}\times\frac{45}{100}\times h \Rightarrow h = 5400 \text{ m/ hr}$$

Illustration – 6

A metallic sheets is of rectangular shape with dimensions 48 cm × 36 cm. From each one of its corners, a square of 8 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box

(a) 5110 cm³ **(b) 5130 cm³**
(c) 5120 cm³ **(d) 5140 cm³**

Solution : (c) Volume of the box made of the remaining sheet

$$= 32\times20\times8 = 5120 \text{ cm}^3$$

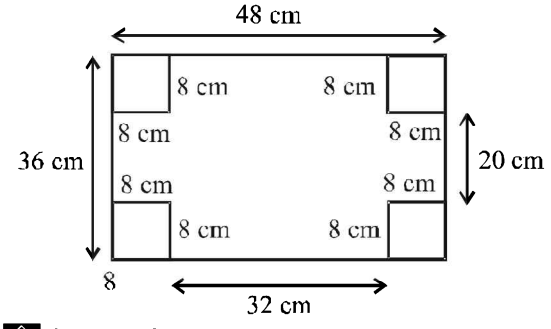


Illustration – 7

The capacity of a cylindrical tank is 246.4 litres. If the height is 4 metres, what is the diameter of the base?

(a) 1.4 m **(b) 2.8 m**
(c) 14 m **(d) None of these**

Solution : (d) Volume of the tank = 246.4 litres = 246400 cm³.
Let the radius of the base be r cm. Then,

$$\left(\frac{22}{7}\times r^2\times400\right) = 246400$$

$$\Rightarrow r^2 = \left(\frac{246400\times7}{22\times400}\right) = 196 \Rightarrow r = 14.$$

∴ Diameter of the base = $2r$ = 28 cm = 0.28 m

Illustration – 8

A cone, a hemisphere and a cylinder stand on equal bases and have the same height. What is the ratio of their volumes?

(a) 2 : 1 : 3 **(b) 2.5 : 1 : 3**
(c) 1 : 2 : 3 **(d) 1.5 : 2 : 3**

Solution : (c) As they stand on the same base so their radius is also same.

Then; volume of cone $= \frac{\pi r^2 h}{3}$

Volume of hemisphere $= \frac{2\pi r^2}{3}$

Volume of cylinder = $\pi r^2 h$

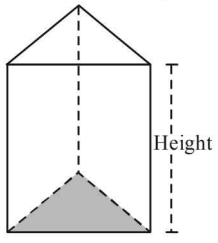
Ratio = $\frac{\pi r^2 h}{3} : \frac{2\pi r^3}{3} : \pi r^2 h$

$\Rightarrow \quad \frac{h}{3} : \frac{2r}{3} : h \Rightarrow h : 2r : 3h$

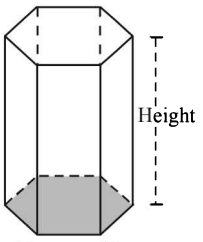
Radius of a hemisphere = Its height
So $h : 2h : 3h \Rightarrow 1 : 2 : 3$

Cube

A **'prism'** is a solid having identical and parallel top and bottom (or base) faces. These identical faces are regular polygon of any number of sides. The side faces of a prism are rectangular and are known as lateral faces. Number of lateral faces is equal to the number of sides in the base.
Here are some example of prisms



Triangular base prism



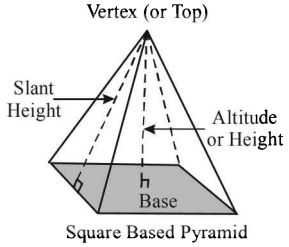
Hexagonal base prism

GP_3386

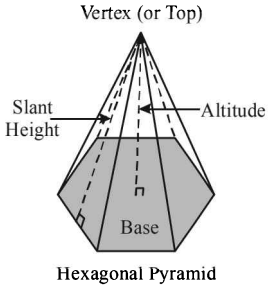
Lateral surface area of the prism
= (Perimeter of the base) × (Height)
Total surface area of the prism
= (Surface area of the top and bottom) + (Lateral surface area)
= 2 × Area of the base + Perimeter of base × Height
Volume of the prism = (Area of base) × (Height)
The actual formula used to find the surface area and volume will depend upon the number of sides in the base of the prism.

Cylinder

It is a three-dimensional body made up of a regular polygon shaped base and triangular lateral faces that meet at a point called **vertex**, which is also called the **apex** of the pyramid. The number of triangular faces is equal to the number of sides in the base.
For example: A pyramid with a square base has four triangular faces, whereas a pyramid with a hexagonal face is made up of six triangular faces, and so on.
Lower face is called the **base** and the perpendicular distance of the vertex (or top) from the base is called the **height** or **altitude** of the pyramid.
The altitude of a lateral face of a pyramid is the **slant height**, which is the perpendicular distance of the vertex (or top) from the mid-point of any side of the base.
The **lateral surface area** of a regular pyramid is the sum of the areas of its lateral faces.



Square Based Pyramid



Hexagonal Pyramid

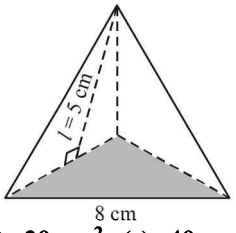
Lateral surface area of a pyramid = $\frac{1}{2}$ × (Area of the base) × (Slant height)

Total surface area of a pyramid
= $\frac{1}{2}$ × (Perimeter of the base) × (Slant height) + (Area of the base)

Volume of a pyramid = $\frac{1}{3}$ × Area of base × Height

Illustration – 9

Find the lateral surface area of a regular pyramid with triangular base, if each edge of the base measures 8 cm and slant height is 5 cm.



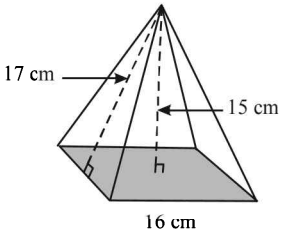
(a) 10 cm² (b) 20 cm² (c) 40 cm² (d) 60 cm²

Solution : (d) The perimeter of the base is the sum of the sides,

$p = 3.(8) = 24$ cm L.S.A. = $\frac{1}{2} \times (24) \times (5) = 60$ cm²

Illustration – 10

Find the total surface area of a pyramid with a square base if each side of the base measures 16 cm, the slant height of a side is 17 cm and the altitude is 15 cm.



(a) 600 cm² (b) 800 cm² (c) 400 cm² (d) 200 cm²

Solution : (b) The perimeter of the base,
$p = 4 \times 16 = 64$ cm
Area of the base = $16^2 = 256$ cm²

T.S.A. = $\frac{1}{2}(64)(17) + 256 = 544 + 256 = 800$ cm²

LEVEL - 1

1. The surface area of a cube is 150 m^2. The length of its diagonal is

(a) $5\sqrt{3}$ m (b) 5m (c) $\frac{10}{\sqrt{3}}$ m (d) 15m

2. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diamater 8 cm. The height of the cone is:

(a) 12 cm (b) 14 cm (c) 15 cm (d) 18 cm

3. The length of the longest rod that can be placed in a room which is 12 m long, 9 m broad and 8 m high is

(a) 27m (b) 19m (c) 17m (d) 13m

4. Water flows out through a circular pipe whose internal diameter is 2 cm, at the rate of 6 metres per second into a cylindrical tank, the radius of whose base is 60 cm. By how much will the level of water rise in 30 minutes?

(a) 2m (b) 4m (c) 3m (d) 5m

5. A cube of 384 cm^2 surface area is melt to make x number of small cubes each of 96 mm^2 surface area. The value of x is

(a) 80,000 (b) 8 (c) 8,000 (d) 800

6. A hollow spherical shell is made of metal of density 4.8 g/cm^3, If its internal and external radii are 10 cm and 12 cm respectively, find the weight of the shell

(a) 15.24 kg (b) 12.84 kg
(c) 14.64 kg (d) None of these

7. The edge of a cube is increased by 100%, the surface area of the cube is increased by :

(a) 100% (b) 200% (c) 300% (d) 400%

8. 250 men took a dip in a water tank at a time, which is 80 m × 50 m. What is the rise in the water level if the average displacement of 1 man is 4 m^3?

(a) 22 cm (b)25 cm (c) 18 cm (d) 30 cm

9. Three cubes of metal, whose edges are 3 cm, 4 cm and 5 cm respectively are melted to form a new cube. What is the surface area of the new cube?

(a) 216 cm^2 (b) 56 cm^2
(c) 36 cm^2 (d) None of these

10. The three-terminus edges of a rectangular solid are 36 cm, 75 cm and 80 cm respectively. Find the edge of a cube which will be of the same capacity :

(a) 60 cm (b) 52 cm
(c) 46 cm (d) None of these

11. The edge of a cube is doubled. What will be the new volume?

(a) 2 times (b) 3 times (c) 4 times (d) 8 times

12. A rectangular block has length 10 cm, breadth 8 cm and height 2 cm. From this block, a cubical hole of side 2 cm is drilled out. Find the volume and the surface area of the remaining solid :

(a) 152 cm^3, 512 cm^2 (b) 125 cm^3, 215 cm^2
(c) 152 cm^3, 240 cm^2 (d) 125 cm^3, 512 cm^2

13. If the length of diagonal of a cube is $6\sqrt{3}$ cm, then the length of its edge is:

(a) 2 cm (b) 3 cm (c) 6 cm (d) $\frac{36}{\sqrt{3}}$ cm

14. A school hall has the dimensions 30 m, 12 m by 6 m. Find the number of children,who can be accommodated, if each child should get 8 m^3 of space :

(a) 240 (b) 270 (c) 250 (d) 150

15. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the total surface areas of the three cubes :

(a) 5 : 7 (b) 7 : 9
(c) 9 : 7 (d) None of these

16. The radii of two cylinders are in the ratio of 3 : 5 and their heights are in the ratio 4 : 3. The ratio of their volumes is:

(a) 12 : 25 (b) 13 : 25 (c) 4 : 5 (d) 5 : 4

17. A hollow garden roller 42 cm wide with a girth of 132 cm is made of iron 3 cm thick. The volume of the iron of the roller is:

(a) 15544 cm^3 (b) 15444 cm^3
(c) 15545 cm^3 (d) None of these

18. If the base of right rectangular prism remains constant and the measures of the lateral edges are halved, then its volume will be reduced by:

(a) 50% (b) 33.33%
(c) 66.66% (d) None of these

GP_3386

19. The volume of a pyramid of base area 25 cm^2 and height 12 cm is:
(a) 200 cm^3 (b) 100 cm^3
(c) 400 cm^3 (d) 800 cm^3

20. If the height of a cone is half the radius of a sphere then the radius of the base of the cone, which has the same volume as a sphere of radius 7 cm is:
(a) 14m (b) $\frac{14}{\sqrt{2}}$cm
(c) $14\sqrt{2}$ cm (d) None of these

21. A solid metal ball of diameter 16 cm is melted and cast into smaller balls, each of radius 1 cm. The number of such balls is :
(a) 256 (b) 2048 (c) 512 (d) 4096

22. If a hemispherical dome has an inner radius 21 m then its volume (in m^3) is :
(a) 4910 m^3 (b) 18354 m^3
(c) 19404 m^3 (d) None of these

23. The radius of a copper sphere is 12 cm. The sphere is melted and drawn into a long wire of uniform circular cross- section. If the length of the wire is 144 cm, the radius of wire is :
(a) 1 cm (b) 2 cm
(c) 4 cm (d) None of these

24. A conical circus tent is to be made of canvas. The height of the tent is 35 m and the radius of the base is 84 m. If $\pi = \frac{22}{7}$, then the canvas required is :
(a) 24000 m^2 (b) 24004 m^2
(c) 24014 m^2 (d) 24024 m^2

25. A cone and a hemisphere have equal base diameter and equal volumes. The ratio of their heights is :
(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 3

26. If the surface areas of two spheres are in the ratio 4 : 9, then the ratio of their volumes is :
(a) 8 : 25 (b) 8 : 26 (c) 8 : 27 (d) 8 : 28

27. If the lateral surface of a right circular cone is 2 times its base, then the semi-verticle angle of the cone must be:
(a) 15° (b) 30° (c) 45° (d) 60°

28. If h, s, V be the height, curved surface area and volume of a cone respectively, then $(3\pi Vh^3 + 9V^2 - s^2h^2)$ is equal to:
(a) 0 (b) π (c) $\frac{V}{sh}$ (d) $\frac{36}{V}$

29. A sphere of 30 cm radius is dropped into a cylindrical vessel of 80 cm diameter, which is partly filled with water, then its level rises by x cm. find x :
(a) 27.5 cm (b) 22.5 cm
(c) 18.5 cm (d) None of these

30. The base of a right prism is a triangle whose perimeter is 28 cm and the inradius of the triangle is 4 cm. If the volume of the prism is 366 cc, then its height is
(a) 4 cm (b) 8 cm
(c) 6 cm (d) None of these

31. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and re-solidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches, in nearest to
(a) 3 (b) 3.5 (c) 4 (d) 4.5

32. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.
(a) 2 cm (b) 3 cm (c) 1 cm (d) 3.5 cm

33. If *P, R, T* are the area of a parallelogram, a rhombus and a triangle standing on the same base and between the same parallels, which of the following is true?
(a) $R < P < T$ (b) $P > R > T$
(c) $R = P = T$ (d) $R = P = 2T$

34. A metal pipe of negligible thickness has radius 21 cm and length 90 cm. The outer curved surface area of the pipe in square cm is
(a) 11880 (b) 11680
(c) 11480 (d) 10080

35. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. The ratio of their radii is
(a) 4 : 1 (b) 4 : 3 (c) 3 : 4 (d) 1 : 4

36. If a rectangular paper of length 6 cm. and width 3 cm. is rolled to form a cylinder with height equal to the width of the paper, then its base radius is–
(a) $\frac{6}{\pi}$cm (b) $\frac{3}{\pi}$cm (c) $\frac{3}{2\pi}$cm (d) $\frac{9}{2\pi}$cm

37. A trapezium based prism with two parallel sides 8 cm and 14 cm respectively and distance between two parallel sides is 8 cm. Find the height of the prism if the volume of the prism is 1056 cm^3?

(a) 11 cm (b) 10 cm (c) 9 cm (d) 12 cm

38. The sum of the radius of the base and the height of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 sq m, find the volume.
(a) 4620 cu m (b) 4630 cu m
(c) 4520 cu m (d) 4830 cu m

39. If the volume and surface area of a sphere are numerically the same, then its radius is
(a) 1 unit (b) 2 units (c) 3 units (d) 4 units

40. The sum of radii of two spheres is 10 cm and the sum of their volume is 880 cm^3. What will be the product of their radii?
(a) 21 (b) $26\frac{1}{3}$ (c) $33\frac{1}{3}$ (d) 70

41. The ratio of the surface area of a sphere to the curved surface area of the cylinder circumscribing the sphere is:
(a) 1 : 2 (b) 1 : 1 (c) 2 : 1 (d) 2 : 3

42. If the volume of a sphere is divided by its surface area, the result is 27 cms. The radius of the sphere is
(a) 9 cms (b) 27 cms
(c) 81 cms (d) 243 cms

43. How many spherical bullets can be made out of a lead cylinder 28 cm high and with base radius 6 cm, each bullet being 1.5 cm in diameter?
(a) 1845 (b) 1824
(c) 1792 (d) 1752

44. A spherical ball of lead, 3 cm in diameter, is melted and recast into three spherical balls. The diameter of two of these balls are 1.5 cm and 2 cm respectively. The diameter of the third ball is
(a) 2.5 cm (b) 2.66 cm
(c) 3 cm (d) 3.5 cm

45. A conical cavity is drilled in a circular cylinder of 15 cm height and 16 cm base diameter. The height and the base diameter of the cone are same as those of the cylinder. Determine the total surface area of the remaining solid.
(a) $440\pi\ cm^2$ (b) $215\pi\ cm^2$
(c) $542\pi\ cm^2$ (d) $376\pi\ cm^2$

46. The length of a cold storage is double its breadth. Its height is 3 metres. The area of its four walls (including the doors) is 108 m^2. Find its volume.
(a) 215 m^3 (b) 216 m^3
(c) 217 m^3 (d) 218 m^3

47. A wooden box of dimension 8 metre × 7 metre × 6 metre is to carry rectangular boxes of dimensions 8 cm × 7 cm × 6 cm. The maximum number of boxes that can be carried in 1 wooden box is
(a) 7500000 (b) 9800000
(c) 1200000 (d) 1000000

48. A rectangular piece of paper of dimensions 22 cm by 12 cm is rolled along its length to form a cylinder. The volume (in cm^3) of the cylinder so formed is $\left(\text{use } \pi = \frac{22}{7}\right)$
(a) 562 (b) 412
(c) 462 (d) 362

49. A godown is 15 m long and 12 m broad. The sum of the areas of the floor and the ceiling is equal to the sum of areas of the four walls. The volume (in m^3) of the godown is:
(a) 900 (b) 1200
(c) 1800 (d) 720

50. If the sum of the length, breadth and height of a rectangular parallelopiped is 24 cm and the length of its diagonal is 15 cm, then its total surface area is
(a) 351 cm^2 (b) 256 cm^2
(c) 265 cm^2 (d) 315 cm^2

51. A square of side 3 cm is cut off from each corner of a rectangular sheet of length 24 cm and breadth 18 cm and the remaining sheet is folded to form an open rectangular box. The surface area of the box is
(a) 423 cm^2 (b) 468 cm^2
(c) 396 cm^2 (d) 612 cm^2

GP_3386

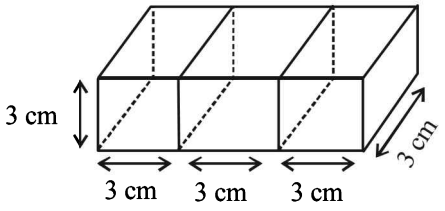
LEVEL - 2

1. The length, breadth and height of a room are X, Y and Z feet respectively. The cost of whitewashing the four walls of this room is Rs.2500. Find the cost of preparing another room whose length, breadth and height are double that of the previous room.
(a) 5000 (b) 12,500
(c) 10,000 (d) 20,000

2. The dimensions of a field are 20 m by 9 m. A pit 10 m long, 4.5 m wide and 3 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. What will be the rise in the height of field as a result of this operation?
(a) 1m (b) 2m
(c) 3m (d) 4m

3. The length of a room is double its breadth. The cost of colouring the ceiling at ₹ 25 per sq. m is ₹ 5,000 and the cost of painting the four walls at ₹ 240 per sq. m is ₹ 64,800. Find the height of the room.
(a) 4.5m (b) 4m
(c) 3.5m (d) 5m

4. Three cubes each of edge 3 cm long are placed together as shown in the adjoining figure. Find the surface area of the cuboid so formed :



(a) 182 sq. cm (b) 162 sq. cm
(c) 126 sq. cm (d) None of these

5. A hemispherical bowl of internal radius 6 cm contains alcohol. This alcohol is to be filled into cylindrical shaped small bottles of diameter 6 cm and height 1 cm. How many bottles will be needed to empty the bowl?
(a) 36 (b) 27 (c) 16 (d) 4

6. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted to form a solid cylinder of base diameter 8 cm. The height of the cylinder is approximately :
(a) 4.5 cm (b) 4.57 cm (c) 4.67 cm (d) 4.7 cm

7. If a cone is cut into two parts by a horizontal plane passing through the mid point of its axis, the ratio of the volumes of the upper part and the frustum is :
(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 7

8. The base of a pyramid is a rectangle 40 m. long and 20 m wide. The slant height of the pyramid from the mid-point of the shorter side of the base to the apex is 29 m. What is the volume of pyramid?
(a) $5600\,m^3$ (b) $400\,m^3$
(c) $6500\,m^3$ (d) $1753\sqrt{110}\ m^3$

9. The base of a right pyramid is an equilateral triangle of side $10\sqrt{3}$ cm. If the total surface area of the pyramid is $270\sqrt{3}$ sq. cm, its height is
(a) 12 cm (b) $12\sqrt{3}$ cm
(c) 10 cm (d) $10\sqrt{3}$ cm

10. A flask in the shape of a right circular cone of height 24 cm is filled with water. The water is poured in a right circular cylindrical flask whose radius is $\frac{1}{3}^{rd}$ of the radius of the base of the circular cone. Then the height of the water in the cylindrical flask is
(a) 32 cm (b) 24 cm
(c) 48 cm (d) 72 cm

11. A vessel is in the form of an inverted cone. Its height is 11 cm and radius of its top, which is open, is 2.5 cm. It is filled with water upto the rim. When lead shots, each of which is a sphere of radius 0.25 cm are dropped into the vessel, 2/5 of the water flows out. The number of lead shots dropped into the vessel is
(a) 880 (b) 440
(c) 220 (d) 110

12. 2 cm of rain has fallen on a sq. km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a

100 m × 10 m base, by what level would the water level in the pool have increased?
(a) 15m (b) 20m
(c) 10m (d) 25m

13. A right circular solid cylinder of base radius 4 cm and vertical height 22.5 cm is melted to form 8 equal solid spheres. If there is a process loss of 20% during such formation, then what is the radius of each of the solid sphere so formed?
(a) 2 cm (b) 3 cm (c) 2.5 cm (d) 3.5 cm

14. A cone, a hemisphere and a cylinder stand on equal bases and have the same height, the height being equal to the radius of the circular base. Their total surface areas are in the ratio:
(a) $(\sqrt{2}+1):3:4$ (b) $(\sqrt{3}+1):3:4$
(c) $\sqrt{2}:3:4$ (d) $\sqrt{3}:7:8$

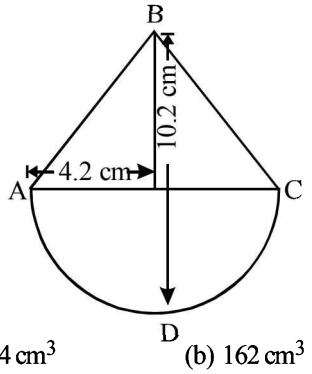
15. 4 pipes each of 3 inch diameter are to be replaced by a single pipe discharging the same quantity of water. What should be the diameter of the single pipe, if the speed of water is the same ?
(a) 6 inches (b) 3 inches
(c) 9 inches (d) 12 inches

16. A sphere is melted and half of the molten liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is
(a) $(4/3)^{1/3}$ (b) $(8/3)^{1/3}$
(c) $(3)^{1/3}$ (d) 2

17. A cube is inscribed in a hemisphere of radius R, such that four of its vertices lie on the base of the hemisphere and the other four touch the hemispherical surface of the half-sphere. What is the volume of the cube?
(a) $0.25R^3$ (b) $0.67\sqrt{\frac{2}{3}}R^3$
(c) $0.5\sqrt{\frac{2}{3}R^3}$ (d) $0.67R^3$

18. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if the height of conical part is 12 cm.
(a) 1440 cm^2 (b) 385 cm^2
(c) 1580 cm^2 (d) 770 cm^2

19. A solid wooden toy in the shape of a right circular cone is mounted on a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy.



(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

20. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.
(a) 2 cm (b) 3 cm
(c) 1 cm (d) 3.5 cm

21. The cost of the paint is ₹ 36.50 per kg. If 1 kg of paint covers 16 square feet, how much will it cost to paint outside of a cube having 8 feet each side?
(a) ₹ 692 (b) ₹ 768
(c) ₹ 876 (d) ₹ 972

22. What is the number of cones of semi-vertex angle α and having r as the radius of the mid-section which can be moduled out of a cylinder of base radius r and height $2r \cot \alpha$?
(a) 5 (b) 7
(c) 6 (d) 4

23. Find the radius of maximum size sphere which can be inscribed or put in a cone whose base radius and height are 6 cm and 8 cm respectively?
(a) 4 cm (b) 5 cm
(c) 3 cm (d) None of these

24. A vertical cone of volume V with vertex downward is filled with water up to half of its height. The volume of the water is :
(a) V/16 (b) V/8
(c) V/4 (d) V/2

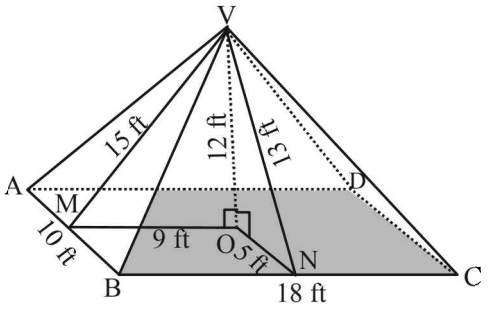
GP_3386

25. If the areas of 3 adjacent sides of a cuboid are x, y, z respectively, then the volume of the cuboid is:

(a) xyz (b) $\sqrt{xyz}$

(c) 3xyz (d) None of these

26. The diagram shows a pyramid with vertex V and a rectangular base ABCD. M is the mid-point of AB, N is the mid-point of BC and O is the point at the centre of the base.



AB = 10 ft BC = 18 ft
VO = 12 ft VM = 15 ft
VN = 13 ft
What is the total surface area of the pyramid?
(a) 532 ft² (b) 618 ft² (c) 564 ft² (d) 735 ft²

Hints and Solutions

Level 1

1. (a) In a cube,
Area $= 6\,(\text{side})^2$
or $150 = 6\,(\text{side})^2$

$\therefore$ side $= \sqrt{25} = 5$ m

Length of diagonal $= \sqrt{3} \times \text{side} = 5\sqrt{3}$ m

2. (b) Volume of material in the sphere

$$= \left[\frac{4}{3}\pi \times \left\{(4)^3 - (2)^3\right\}\right] \text{cm}^3 = \left(\frac{4}{3}\pi \times 56\right) \text{cm}^3.$$

Let the height of the cone be h cm.

Then, $\frac{1}{3}\pi \times 4 \times 4 \times h = \left(\frac{4}{3}\pi \times 56\right)$

$$\Rightarrow h = \left(\frac{4 \times 56}{4 \times 4}\right) = 14 \text{ cm.}$$

3. (c) Required length = length of the diagonal

$$= \sqrt{12^2 + 9^2 + 8^2}$$
$$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17 \text{m}$$

4. (c) Let h be the required height then,

$$\frac{22}{7} \times (60)^2 \times h$$
$$= 30 \times 60 \times \frac{22}{7} \times (1)^2 \times (600)$$
$$\Rightarrow \quad 60h = 30 \times 600$$
$$\Rightarrow \quad h = 300 \text{ cm} = 3 \text{ m}$$

5. (c) Let 'A' be the side of bigger cube and 'a' be the side of smaller cube
Surface area of bigger cube $= 6A^2$
or $384 = 6A^2$
$\therefore A = 8$ cm.
Surface area of smaller cube $= 6a^2$
$96 = 6a^2$
$\therefore a = 4$ mm $= 0.4$ cm
So, Number of small cube

$$= \frac{\text{Volume of bigger cube}}{\text{Volume of smaller cube}}$$

$$= \frac{(8)^3}{(0.4)^3} = \frac{512}{0.064} = 8,000$$

6. (c) Volume of spherical shell

$$= \frac{4\pi}{3}(R^3 - r^3) = \frac{4\pi}{3}(12^3 - 10^3)$$

$$= \frac{4}{3} \times \pi \times (12 - 10)(12^2 + 12 \times 10 + 10^2)$$

$$= \frac{4}{3} \times \pi \times 2 \times 364 \text{ cm}^3$$

Weight = volume × density

$$= \frac{4}{3} \times \pi \times 2 \times 364 \times 4.8 = 14.64 \text{ kg}$$

7. (c) Let each edge of smaller cube = 1 m
$\therefore$ Each edge of larger cube = 2 m
and Surface area of smaller cube
$= 6 \times (1)^2 = 6 \text{ m}^2$
$\therefore$ Surface area of larger cube
$= 6 \times (2)^2 = 24 \text{ m}^2$
$\therefore$ % increase in surface area

$$= \frac{24 - 6}{6} \times 100 = 300\%$$

Alternatively :

$$\frac{S_2}{S_1} = \left(\frac{e_2}{e_1}\right)^2 \Rightarrow \frac{S_2}{S_1} = \frac{4}{1}$$

where S = surface area, e = edge of cube.
$\therefore$ Percentage increase in surface area

$$= \frac{4 - 1}{1} \times 100 = 300\%$$

8. (b) Total volume of water displaced by 250 men
$= 250 \times 4 = 1000 \text{ m}^3$
$\therefore$ Rise in water level (h) $= \dfrac{\text{Volume}}{\text{Base area}}$

$$= \frac{1000}{80 \times 50} = 25 \text{ cm}$$

9. (a) Volume of new cube $= (3)^3 + (4)^3 + (5)^3$
$= 216 \text{ cm}^3$
$\therefore$ Each edge of the new cube = 6 cm
and hence, surface area $= 6 \times (a)^2$
$= 216 \text{ cm}^2$

10. **(a)** Volume of cube = Volume of cuboid

$a^3 = lbh$

$\Rightarrow a^3 = 36 \times 75 \times 80 = 216000$

$\Rightarrow a = 60$ cm

11. **(d)** $\frac{V_2}{V_1} = \left(\frac{e_2}{e_1}\right)^3 \quad \{\because V = (e)^3\}$

{e = edge of cube}

$\therefore \frac{V_2}{V_1} = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

$\therefore V_2 = 8V_1$

12. **(c)** Net volume $= (10 \times 8 \times 2) - (2 \times 2 \times 2)$

$= 152\ cm^3$

Net surface area $= 2(10 \times 8 + 8 \times 2 + 2 \times 10) + 4(2 \times 2) - 2(2 \times 2)$

$= 240\ cm^2$

13. **(c)** $a\sqrt{3} = 6\sqrt{3} \Rightarrow a = 6$ cm

14. **(b)** Number of children × Space required for one child = Volume of room

$\therefore$ Number of children $= \frac{30 \times 12 \times 6}{8} = 270$

15. **(b)** Let us consider that surface area of each face of the cube 1 cm^2.

$\therefore$ Total surface area of the cuboid = 14 cm^2

and Total surface area of the 3 cubes = 18 cm^2

Hence, required ratio = 14 : 18 = 7 : 9

16. **(a)** $\frac{V_1}{V_2} = \frac{\pi(r_1)^2 h_1}{\pi(r_2)^2 h_2} = \frac{3 \times 3 \times 4}{5 \times 5 \times 3} = \frac{12}{25}$

17. **(b)** h = 42 cm, $2\pi R = 132$ cm

$\Rightarrow R = 21$ cm

$\therefore r = 21 - 3 = 18$ cm

$\therefore$ Required volume $= \pi(R^2 - r^2)h$

$\frac{22}{7}\left[(21)^2 - (18)^2\right] \times 42$

$= 15444\ cm^3$

18. **(a)** Volume of prism = Base area × height

Since base area is constant and height is being halved therefore volume will also be halved. Hence, its volume will be reduced by 50%.

19. **(b)** Volume of pyramid

$= \frac{1}{3} \times$ base area $\times$ height

$= \frac{1}{3} \times 25 \times 12 = 100\ cm^3$

20. **(c)** Volume of cone = Volume of sphere

$\frac{1}{3}\pi r^2 \times \frac{7}{2} = \frac{4}{3}\pi(7)^3$

$\Rightarrow r = 14\sqrt{2}$ cm

21. **(c)** Since volume is constant

$\therefore n \times \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \times (8)^3$

$\Rightarrow n = 512$

22. **(c)** Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (21)^3 = 19404\ m^3$

23. **(c)** $\pi r^2 \times 144 = \frac{4}{3}\pi \times (12)^3$

($\because$ Volume remains constant)

$\Rightarrow r = 4$ cm

24. **(d)** $l = \sqrt{84^2 + 35^2} = 91$m

$\pi rl = \frac{22}{7} \times 84 \times 91 = 24024\ m^2$

25. **(b)** Let r be the radius of both cone and hemisphere. h be the height of cone.

$\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow \frac{h}{r} = \frac{2}{1}$

26. **(c)** $\frac{S_1}{S_2}=\left(\frac{r_1}{r_2}\right)^2=\frac{4}{9}\Rightarrow\frac{r_1}{r_2}=\frac{2}{3}$

$\therefore \frac{V_1}{V_2}=\left(\frac{r_1}{r_2}\right)^3=\left(\frac{2}{3}\right)^3=\frac{8}{27}$

27. **(b)** Let l and r be the slant height and radius of the cone.

$\therefore \pi rl=2\pi r^2$

$\Rightarrow \frac{r}{l}=\frac{1}{2}\Rightarrow \sin\theta=\frac{1}{2}$

$\Rightarrow \theta=30^o$

28. **(a)** $3\pi Vh^3+9V^2-s^2h^2$

$=3\pi\left(\frac{1}{3}\pi r^2h\right)h^3+9\left(\frac{1}{3}\pi r^2h\right)^2-\pi^2r^2$

$(r^2+h^2)h^2$

$=(\pi rh^2)^2+(\pi r^2h)^2-(\pi rh)^2(r^2+h^2)$

$=(\pi rh)^2(r^2+h^2)-(\pi rh)^2(r^2+h^2)=0$

29. **(b)** Volume of water displaced = Volume of sphere

$\pi\times(40)^2\times h=\frac{4}{3}\pi\times(30)^3$

$\Rightarrow h=\frac{90}{4}=22.5$ cm

Thus, the level of water rises by 22.5 cm.

30. **(d)** Area of base

$=\frac{1}{2}\times r\times a+\frac{1}{2}\times r\times b+\frac{1}{2}\times r\times c$

$=\frac{1}{2}r(a+b+c)$

$=r\times s=4\times14=56\ cm^2$

[where r = inradius, s = semi-perimeter]

volume of prism = area of base × height

$366=56\times h$

$h=6.5$ cm [approx]

31. **(b)** Volume of the given ice cuboid

$=8\times11\times2=176$

Let the length of the required rod is ℓ.

$\therefore \pi\ell\frac{8^2}{4}=176\ \ \therefore \ell=3.5$ inches

32. **(c)** Let the internal radius of the cylinder = r

Then, the volume of sphere = Volume of hollow cylinder

$\Rightarrow \frac{4\pi.6^3}{3}=\pi h\left(5^2-r^2\right)$

$\Rightarrow \frac{864\pi}{3}=32\pi\left(25-r^2\right)$

$\Rightarrow r^2=16\Rightarrow r=4$ cm

So thickness of the cylinder = 5 – 4 = 1 cm

33. **(d)** Parallelogram Area = l × b

Rhombus area = l × b

Triangle area $=\frac{l\times b}{2}$

Therefore R = P = 2T.

34. **(a)** Curved surface area of cylinder = $2\pi rh$

$=2\times\frac{22}{7}\times21\times90=11880$ sq.cm

35. **(a)** $C_1=2C_2$

$\pi r_1l_1=2\pi r_2l_2$

also, $l_2=2l_1$

$\pi r_1l_1=2\times2_6\,\pi r_2l_1$

$\frac{r_1}{r_2}=\frac{4}{1}$

36. **(b)** Circumference of base = $2\pi r=6$

$\therefore\ r=\frac{3}{\pi}$

37. **(d)** Volume = Area of the base × Height

$\Rightarrow 1056\ cm^3=\frac{1}{2}\times(8+14)\times8\times$ Height

$\Rightarrow$ Height $=\frac{1056}{22\times4}=12$ cm

38. **(a)** r + h = 37

TSA = $2\pi rh+2\pi r^2=2\pi r(h+r)=1628$

$\Rightarrow 2\pi r=\frac{1628}{37}\Rightarrow r=\frac{1628\times7}{37\times2\times22}=7$

h = 37 – 7 = 30

Volume = $\pi r^2h=\frac{22}{7}\times7\times7\times30=4620\ m^3$.

39. **(c)** Volume $=\frac{4}{3}\pi r^3$

Surface area = $4\pi r^2$

$\because$ Volume = Surface area of the sphere

GP_3386

$\Rightarrow \frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3$ units

40. (b) $R + r = 10$

$$\frac{4}{3}\pi R^3 - \frac{4}{3}\pi r^3 = 880 \text{ cm}^3$$

$$\frac{4}{3}\pi(R^3 - r^3) = 880 \Rightarrow R^3 - r^3 = 210$$

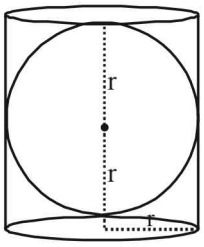
$\Rightarrow R^3 - r^3 = (R + r)(R^2 + r^2 - Rr)$

$\Rightarrow 210 = 10(R^2 + r^2 - Rr) \Rightarrow R^2 + r^2 - Rr = 21$

$\Rightarrow (R + r)^2 - 3Rr = 21 \Rightarrow 100 - 3Rr = 21$

$\Rightarrow 3Rr = 79 \Rightarrow Rr = \frac{79}{3} = 26\frac{1}{3}$

41. (b)



Height of the cylinder = Diameter of sphere.
CSA $= 2\pi rh = 2\pi r \cdot 2r = 4\pi r^2$
Total surface area of the sphere $= 4\pi r^2$
Required ratio $= 4\pi r^2 : 4\pi r^2 = 1 : 1$

42. (c) In a sphere, volume $= \frac{4}{3}\pi r^3$

and surface area $= 4\pi r^2$

According to question, $\frac{4}{3}\pi r^3 \div 4\pi r^2 = 27$

or $r = 27 \times 3 = 81$ cms

43. (c) Volume of cylinder $= (\pi \times 6 \times 6 \times 28)\text{ cm}^3$
$= (36 \times 28)\pi \text{ cm}^3$.

Volume of each bullet

$$= \left(\frac{4}{3}\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}\right)\text{cm}^3$$

$$= \frac{9\pi}{16}\text{cm}^3.$$

Number of bullets $=$

$$\frac{\text{Volume of cylinder}}{\text{Volume of each bullet}}$$

$$= \left[(36 \times 28)\pi \times \frac{16}{9\pi}\right] = 1792.$$

44. (a) Let radius of the 3rd spherical ball be R,

$\therefore$

$$\frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 + \frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi R^3$$

$$\Rightarrow R^3 = \left[\left(\frac{3}{2}\right)^3 - \left(\frac{3}{4}\right)^3\right] - 1^3$$

$$= \frac{27}{8} - \frac{27}{64} - 1 = \frac{125}{64} = \left(\frac{5}{4}\right)^3$$

$$\Rightarrow R = \frac{5}{4} = 1.25$$

$\therefore$ Diameter of the third spherical ball
$= 1.25 \times 2 = 2.5$ cm.

45. (a) Total surface area of the remaining solid = Curved surface area of the cylinder + Area of the base + Curved surface area of the cone

$= 2\pi rh + \pi r^2 + \pi r\ell$
$= 2\pi \times 8 \times 15 + \pi \times (8)^2 + \pi \times 8 \times 17$
$= 240\pi + 64\pi + 136\pi$
$= 440\,\pi \text{ cm}^2$

46. (b) Let L be the length and B be the breadth of cold storage.
$L = 2B, H = 3$ metres
Area of four walls $= 2[L \times H + B \times H] = 108$
$\Rightarrow 6BH = 108 \Rightarrow B = 6$
$\therefore L = 12, B = 6, H = 3$
Volume $= 12 \times 6 \times 3 = 216 \text{ m}^3$

47. (d) Maximum number of boxes

$$= \frac{800 \times 700 \times 600 \text{ cm}^3}{8 \times 7 \times 6 \text{ cm}^3} = 1000000$$

48. (c) $2\pi r = 22$ cm

$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2} \text{ cm}$$

Height, $h = 12$ cm
Volume of cylinder

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462 \text{ cm}^3$$

49. (b) If the height of the godown be h meter, then

$2(15 \times 12) = 2 \times h(15 + 12)$

$\Rightarrow 27h = 15 \times 12$

$\Rightarrow h = \frac{15 \times 12}{27} = \frac{20}{3}$ meter

$\therefore$ Volume of the godown

$= \frac{15 \times 12 \times 20}{3} = 1200$ cu.meter

50. (a) Let length = l, breadth = b, height = h.

$l + b + h = 24$ (given) ... (i)

Diagonal of parallellopiped = 15 cm

$\sqrt{l^2 + b^2 + h^2} = 15$ or $l^2 + b^2 + h^2 = 225$

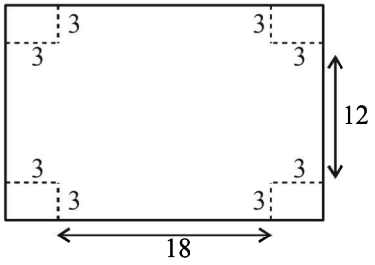
Squaring eqn. (i) on both sides

$l^2 + b^2 + h^2 + 2lb + 2bh + 2hl = 576$

$2(lb + bh + hl) = 576 - 225 = 351$

[$\because$ Surface area of parallellopiped = $2(lb + bh + hl)$]

51. (c)



$\ell = 18$ cm, $b = 12$ cm, $h = 3$ cm

$S = 2(\ell h + bh) + \ell b$ {Box is open from upper side}

$= 2(54 + 36) + 216$

$= 396$ cm^2

Level 1

1. (c) Surface area of the walls of the first room $= 2(XZ + YZ)$

Surface area of the walls of the second room $= 2(4XZ + 4YZ) = 8(XZ + YZ) = 4$ times the first area

$\Rightarrow$ Cost required $= 4 \times 2500 =$ ₹ 10,000.

2. (a) Volume of mud dugout $= 10 \times 4.5 \times 3 = 135$ m^3

Let the remaining ground rise by $= h$ m

Then $\{(20 \times 9) - (10 \times 4.5)\} h = 135$

$135h = 135 \Rightarrow h = 1$ m

3. (a) Let the length of the room be ℓ m

Then its, breadth = $\ell/2$

Therefore, $\ell \times \frac{\ell}{2} = \frac{5000}{25}$

or $\ell^2 = 400$

or $\ell = 20$ m

Also, $2\ell h + 2 \times \frac{\ell}{2} \times h = \frac{64800}{240} \Rightarrow h = 4.5$ m

4. (c) Required surface area

$= 2(9 \times 3 + 3 \times 3 + 3 \times 9)$

$= 126$ cm^2

5. (c) Number of bottles × Volume of each bottle = Volume of hemisphere

$n \times \pi \times (3)^2 \times 1 = \frac{2}{3}\pi \times (6)^3$

$\Rightarrow n = 16$

6. (c) $h = \frac{\frac{4}{3}\pi(4^3 - 2^3)}{\pi(4)^2} = 4.67$

7. (d) $\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi h(r^2 + 4r^2 + 2r^2)} = \frac{1}{7}$

8. (a) Height $= h = \sqrt{29^2 - 20^2} = 21$ m

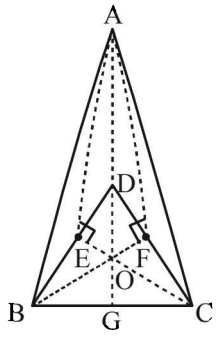
$V = \frac{1}{3} \times (40 \times 20) \times 21 = 5600$ m^3

9. (a) Now, T.S.A of pyramid

= ar (ΔABD) + ar (ΔADC) + ar (ΔABC) + ar (ΔBDC)

$\therefore$ T.S.A of pyramid

$= \frac{1}{2} \times BD \times AE + \frac{1}{2} \times DC \times AF + \frac{1}{2} \times BC \times AG + \frac{\sqrt{3}}{4} \times (\text{side})^2$



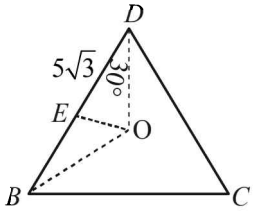
($\therefore AE = AF = AG$ = height of isosceles Δ (h))

$$\Rightarrow \quad 270\sqrt{3} = \frac{1}{2} \times h[BD + DC + BC] + \frac{\sqrt{3}}{4}(\text{side})^2$$

$$\Rightarrow \quad 270\sqrt{3} = \frac{1}{2} \times h\left[10\sqrt{3} + 10\sqrt{3} + 10\sqrt{3}\right] + \frac{\sqrt{3}}{4}(10\sqrt{3})^2$$

$$\Rightarrow \quad 270\sqrt{3} = 15\sqrt{3}h + 75\sqrt{3} \Rightarrow \text{h} = 13\,\text{cm} \qquad ...(1)$$

Now to find height of pyramid (H), we use



$$\text{In } \Delta ODE, \tan 30° = \frac{OE}{ED} = \frac{OE}{5\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} = \frac{OE}{5\sqrt{3}} \Rightarrow OE = 5\,\text{cm} \qquad ...(2)$$

From (1) & (2), we use pythagorals theorem, in ΔAEO

$(AE)^2 = (EO)^2 + (AO)^2$ or $h^2 = (OE)^2 + H^2$

$\Rightarrow \quad (13)^2 - (5)^2 = \text{H}^2 \Rightarrow 144 \Rightarrow \text{H} = 12\,\text{cm}$

10. (d) Let radius of base of cone be r and height of cylinder be h.

Vol. of cone = Vol. of cylinder

$$\frac{1}{3}\pi r^2 \times 24 = \pi\left(\frac{r}{3}\right)^2 \times h$$

$h = 72$ cm

11. (b) Vol. of cone $= \frac{1}{3}\pi \times (2.5\,\text{cm})^2 \times 11\,\text{cm}$

Vol. of one sphere $= \frac{4}{3}\pi(0.25\text{cm})^3$

Vol. of all spheres = Vol. of water flows out

$$n \times \frac{4}{3}\pi(0.25\text{cm})^3 = \frac{2}{5} \times \frac{\pi}{3} \times (2.5\text{cm})^2 \times 11\text{cm}$$

$$2n \times \frac{25}{100} \times \frac{25}{100} \times \frac{25}{100} = \frac{1}{5} \times \frac{25}{10} \times \frac{25}{10} \times 11$$

$n = 440$

12. (c) Volume of rain that is to be collected in a pool $= 2 \times 1 \times 10^{10} \times \frac{1}{2}$

$= 10^{10}\,\text{cm}^3 = 10^4$ meter^3

Volume of pool $= L \times B \times h$

$10^4 = 100 \times 10 \times h \Rightarrow h = 10$ m.

13. (b) (Volume of solid cylinder) $\times 0.8 = 8 \times$ Volume of each solid sphere.

$\therefore (\pi \times r_1^2 \times h) \times 0.8 = 8 \times (4/3) \times \pi \times r_2^3$

$\Rightarrow (4^2 \times 22.5 \times 0.8) = 8 \times (4/3) \times r_2^3$

$\Rightarrow r_2 = 3$ cm.

14. (a) $\pi r\,(r + l) : 3\pi r^2 : 2\,\pi r\,(r + h)$

$= \pi \times 1\ (1 + \sqrt{2}) : 3 \times \pi \times 1 : 2 \times \pi \times 1\,(1 + 1)$

$= (\sqrt{2} + 1) : 3 : 4$

15. (a) Let h be the length of water column discharged in 1 hour or 1 minute.

Volume discharged by the 4 pipe = Volumes discharged by the single pipe

$4 \times \pi \times (1.5)^2 \times h = \pi \times (r)^2 \times h$

$\therefore \ r^2 = 9 \ \therefore r = 3$

Diameter = 6 inches.

16. **(b)** As per the given conditions,

$$11a^3 = 7 \times \frac{4}{3} \times \pi \times r^3 \ \therefore \ \frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

17. **(b)** Let $ABCDEFGH$ be the cube of side a and O be the centre of the hemisphere.

$AC = \sqrt{2}a$;

$OD = OC = R$

Let P be the mid-point of AC

$OP = a$

In ΔAOC,

$$R^2 = a^2 + \frac{a^2}{2} \Rightarrow a = \sqrt{\frac{2}{3}}R$$

$$\therefore \quad \text{Volume} = a^3 = 0.67\sqrt{\frac{2}{3}}R^3$$

18. **(d)** Radius of cylinder, hemisphere and cone = 5 cm

Height of cylinder = 13 cm

Height of cone = 12 cm

$$\ell = \sqrt{h^2 + r^2} = \sqrt{12^2 + 5^2} = 13$$

$$\text{Surface area of toy} = 2\pi rh + \frac{4\pi r^2}{2} + \pi rl$$

$\Rightarrow$ $(2 \times 3.14 \times 5 \times 13) + (2 \times 3.14 \times 25) + (3.14 \times 5 \times 13)$

$= 770 \text{ cm}^2$

19. **(d)** Volume of the cone is given by $= 1/3 \times \pi r^2 h$

Here, $r = 4.2$ cm, $h = 10.2 - r = 6$ cm

Therefore the volume of the cone

$= 1/3 \ \pi \times (4.2)^2 \times 6 \text{ cm} = 110.88 \text{ cm}^3$

Volume of the hemisphere

$$= \frac{1}{2} \times \frac{4}{3}\pi r^3 = 155.23 \text{ cm}^3$$

Total volume = 110.88 + 155.232 = 266.112

20. **(c)** Let the internal radius of the cylinder = r

Then, the volume of sphere = Volume of hollow cylinder

$$\Rightarrow \quad \frac{4\pi.6^3}{3} = \pi h\left(5^2 - r^2\right)$$

$$\Rightarrow \quad \frac{864\pi}{3} = 32\pi\left(25 - r^2\right)$$

$\Rightarrow$ $r^2 = 16 = r = 4$ cm

So thickness of the cylinder = 5 – 4 = 1 cm

21. **(c)** Surface area of the cube = (6×8^2) sq. ft.

= 384 sq. ft.

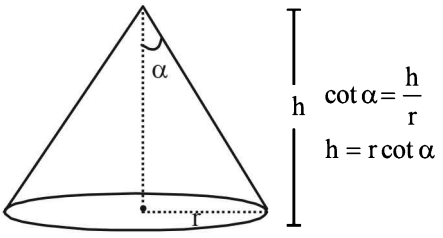
Quantity of paint required

$$= \left(\frac{384}{16}\right)\text{kg} = 24 \text{ kg.}$$

$\therefore$ Cost of painting = ₹ (36.50 × 24)

= ₹ 876.

22. **(c)**



$\cot\alpha = \frac{h}{r}$

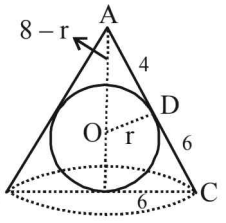
$h = r\cot\alpha$

Let number of cones be n .

n × (Volume of a cone) = Volume of the cylinder.

$$\Rightarrow \quad n \times \frac{1}{3}\pi r^2.r\cot\alpha = \pi r^2.2r\cot\alpha \Rightarrow n = 6.$$

23. **(c)**



$AC = \sqrt{6^2 + 8^2} \Rightarrow l = 10$

In ΔODA, $AO^2 = OD^2 + AD^2$

$\Rightarrow$ $(8 - r)^2 = r^2 + 4^2$

$\Rightarrow$ $64 + r^2 - 16r = r^2 + 16$

$\Rightarrow$ $16r = 48 \Rightarrow r = 3$ cm

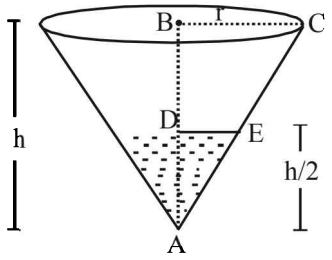
Alternate: –

Radius of maximum size sphere

$$= \frac{hr}{l + r} = \frac{8 \times 6}{10 + 6} = 3 \text{ cm}$$

GP_3386

24. (b)



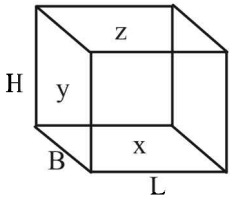
$\Delta ADE \sim \Delta ABC$; $\frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \quad \frac{\frac{h}{2}}{h} = \frac{DE}{r} \Rightarrow DE = \frac{r}{2}$

Volume of the water filled in the cone

$$= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \cdot \frac{h}{2} = \frac{1}{3}\frac{\pi r^2 h}{4.2} = \frac{V}{8}.$$

25. (b)



$L \times B = x$ ....(i)

$B \times H = y$ ...(ii)

$L \times H = z$ ....(iii)

(i) × (ii) × (iii),

$(L \times B \times H)^2 = xyz$

$v^2 = xyz$

$v = \sqrt{xyz}$

26. (c) Total surface area = 2 × area ΔABV + 2 × area ΔVBC + Area of ABCD

$$= 2 \times \frac{1}{2} \times 10 \times 15 + 2 \times \frac{1}{2} \times 18 \times 13 + 10 \times 18$$

$= 150 + 234 + 180 = 564 \text{ ft}^2$.

CHAPTER

24

Set Theory

(Including Venn Diagram)

SETS

A set is a well- defined collection of different objects. In everyday life, we often speak about the collection of objects of particular kind such as a cricket team, the rivers of India, the vowels in the English alphabet etc. Each of these collection is well-defined collection of objects in the sense that we can definitely decide whether a given particular object belongs to the given collection or not.

For example: We say that 10 does not belongs to the given collection of all odd natural numbers. On the other hand, 15 belongs to this given collection.

Note that

(i) Objects, elements and members of a set are synonymous terms.

(ii) Sets are usually denoted by capital letters *A, B, C, D, E, F,* etc.

(iii) The elements of a set are represented by small letters *a, b, c, d, e, f,* etc.

(iv) Each element in a set comes only once i.e. repetation of any element is not allowed.

If a is an element of a set A, we say that "a belongs to A". The Greek symbol $\in$ (epsilon) is used to denote the phrase 'belongs to'.

Thus, we write $a \in A$. If 'b' is not an element of a set A, we write $b \notin A$ and read "b does not belong to A".

If V be the set of vowels of English alphabet, then $a \in V$ but $b \notin V$. In the set P of prime factors of 30, $3 \in P$ but $15 \notin P$.

Representation of Sets

There are two methods of representing a set:

(i) Roster or tabular form **(ii)** Set-builder form.

Roster or Tabular Form

(i) In roster form, all the elements of a set are listed within a bracket { } and separated by commas.

For example: The set of all even positive integers less than 7 is described in roster form as {2, 4, 6}.

(ii) In roster form, the order in which the elements are listed is immaterial.

Set-builder Form

The set $\{a, e, i, o, u\}$ in roster form can be written as set in builder form as $\{x : x$ is a vowel of English alphabet$\}$. Here the set written in set builder form is read as 'x' is an element of the set such that x is a vowel of English alphabet'. Here the colon (:) read as 'such that'. In set-builder, a common property which posses all the elements of the set is written after colon (:).

	Statement	Roster form	Set-builder form
(1)	The set of currencies used in USA, England, Japan, Germany and Russia.	{Dollar, Pound Yen, Euro, Rouble}	{x:x is the currencies used in USA, England Japan, Germany and Russia}
(2)	The set of Capital of Kerala, Karnataka, Tamilnadu, Andhra Pradesh and Gujarat	{Tiruvananth apuram, Bangalore, Chennai, Hyderabad and Gandhi Nagar}	{x:x is the capitals of Kerala, Karnataka, Tamilnadu, Andhra Pradesh and Gujarat}

GP_3386

Standard Symbols of Some Special Sets

N : Set of all natural numbers
Z : Set of all integers
Q : Set of all rational numbers
R : Set of all real numbers
$\mathbf{Z^+}$: Set of all positive integers
$\mathbf{Q^+}$: Set of all positive rational numbers, and
$\mathbf{R^+}$: Set of all positive real numbers.

The symbols for the special sets given above will be referred throughout the chapter.

Types of sets

Empty Set

A set which does not contain any element is called an **empty set, null set** or **void set**.

The empty set is denoted by the symbol ϕ or $\{\}$.

Given below are few examples of empty sets.

(i) If A = $\{x : 1 < x < 2, x$ is a natural number$\}$, then A is the empty set, because there is no natural number between 1 and 2.

(ii) If B = $\{x : x^2 - 2 = 0$ and x is rational number$\}$, then B is the empty set, because the equation $x^2 - 2 = 0$ is not satisfied by any rational value of x.

(iii) If C = $\{x : x$ is an even prime number greater than 2$\}$, then C is the empty set, because 2 is the only even prime number.

Finite and Infinite Sets

A set which is empty or consists of a definite number of elements is called **finite** otherwise, the set is called **infinite**.

Consider some examples:

(i) Let W be the set of the days of the week, then W is the finite set.

(ii) Let S be the set of solutions of the equation $x^2 - 16 = 0$, then S is the finite set.

(iii) Let G be the set of points on a line, then G is infinite set.

Equal Sets

Two sets A and B are said to be equal if they have exactly the same elements.

For example : Let A = {1, 2, 3, 4} and B = {3, 1, 4, 2}. then A = B, because elements of both sets are the same. Only order of the elements in the two sets is different but it is not considered in a set.

Illustration – 1

State which of the following sets are finite and which are infinite:

(i) $A = \{x : x \in Z$ and $x^2 - 5x + 6 = 0\}$
(ii) $B = \{x : x \in Z$ and x^2 is even$\}$
(iii) $C = \{x : x \in Z$ and $x^2 = 36\}$
(iv) $D = \{x : x \in Z$ and $x > -10\}$

Solution : We have,

(i) $A = \{x : x \in Z$ and $x^2 - 5x + 6 = 0\} = \{2, 3\}$
So, A is a finite set

(ii) $B = \{x : x \in Z$ and x^2 is even$\}$
$= \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$
Clearly, B is an infinite set.

(iii) $C = \{x : x \in Z$ and $x^2 = 36\} = \{6, -6\}$
Clearly, C is a finite set.

(iv) $D = \{x : x \in Z$ and $x > -10\}$
$= \{-9, -8, -7, \ldots\}$
Clearly, D is an infinite set.

Illustration – 2

Find the pairs of equal sets, from the following sets, if any, giving reasons:
$A = \{0\}$, $B = \{x : x > 15$ and $x < 5\}$,
$C = \{x : x - 5 = 0\}$, $D = \{x : x^2 = 25\}$
$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$.

Solution : We have,

$A = \{0\}$,
$B = \{x : x > 15$ and $x < 5\} = \phi$,
$C = \{x : x - 5 = 0\} = \{5\}$,
$D = \{x : x^2 = 25\} = \{-5, 5\}$, and $E = \{5\}$.
Clearly, C = E.

SUBSETS

Set A is said to be a subset of a set B if every element of set A is also an element of set B. Here set B is called **superset** of set A.

For example : If A is the set of all divisors of 56 and B the set of all prime divisors of 56, then B is a subset of A.

UNIVERSAL SET

If there are some sets under consideration, and out of these sets, there is a set in which elements of all other sets under consideration are present, then this set is known as the universal set, denoted by U.

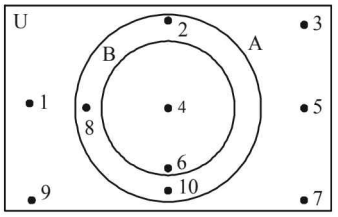
For example,

(i) In the context of human population studies, the universal set consists of all the people in the world.

(ii) If {1, 2, 3, 4}, {2, 5, 6}, {1, 3, 7, 8, 9} and {1, 2, 3, 4, 5, 6, 7, 8, 9} are the sets under consideration, then set {1, 2, 3, 4, 5, 6, 7, 8, 9} can be considered as universal set because elements of all other three sets are present in this set.

(F)VENN DIAGRAMS

In order to illustrate universal sets, subsets and certain operations on sets in a clear and simple way, we use geometric figures. These figures are called Venn-Diagrams. In Venn Diagrams, a universal set is represented by a rectangle and any other set is represented by a circle.



In the Venn-diagrams, the elements of the sets are written in their respective circles. Here, U = {1, 2, 3, ..., 10} is the universal set of which A = {2, 4, 6, 8, 10} and B = {4, 6} are subsets of U. Also B is the subset of A.

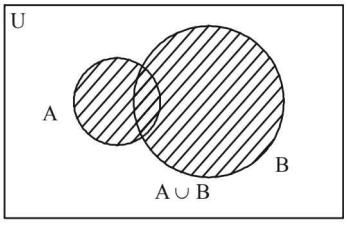
OPERATION ON SETS

Union of Sets

Union of two sets A and B is the set which consists of all those elements which are either in A or in B (including those which are in both sets A and B). In symbols, union of two sets A and B are represented by

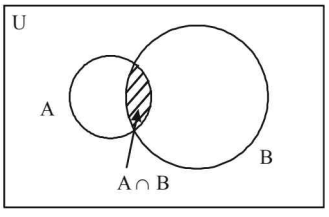
$$A \cup B$$

The union of two sets A and B can be represented by a Venn diagram as shown in figure by shaded portion



Intersection of Sets

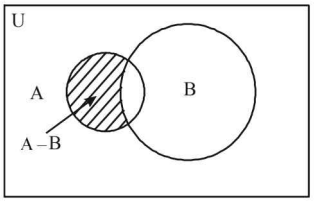
The intersection of two sets A and B is the set of all those elements which belong to both sets A and B. Symbolically, intersection of two sets A and B is denoted by $A \cap B$.



The shaded portion in figure indicates the intersection of sets A and B.

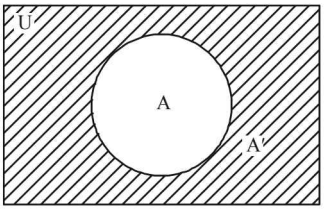
Difference of Sets

The difference of the sets A and B (in the order A minus B) is the set of elements which belong to A but not to B. The difference of two sets A and B (i.e. A – B) is represented in Venn diagram by shaded portion.



Complements of a Set

Let U be the universal set and A be a subset of U. Then the complement of A is the set of all elements of U which are not the elements of set A. Symbolically, we write A' or A^c to denote the complement of set A.



Complement of set A i.e. A' is represented in Venn diagram by shaded region.

Illustration – 3

Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4}, B = {2, 4, 6, 8} and C = {3, 4, 5, 6}. Find

(i) A' **(ii)** $(A \cup B)'$

(iii) $(A')'$ **(iv)** $(B - C)'$

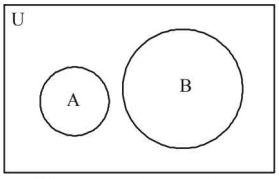
Solution :

(i) {5, 6, 7, 8, 9} (ii) {5, 7, 9}

(iii) A (iv) {1, 3, 4, 5, 6, 7, 9}

DISJOINT SETS

If A and B are two sets such that they have no common elements, then A and B are called **disjoint sets.For example,** let A = {2, 4, 6, 8} and B = {1, 3, 5, 7}. Here A and B are disjoint sets, because there is no element common to both sets A and B.



In the Venn diagram, A and B are disjoint sets.

GP_3386

NUMBER OF ELEMENTS IN A SET

Number of elements in a set A is represented by n(A). If A = {*a*, *b*, *c*, *d*, *e*, *f* }, then n(A) = 6

1. If A and B are finite sets then
   $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. If A, B and C are three finite sets, then
   $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

Illustration – 4

In a political survey, 78% of the politicians favour at least one proposal, 50% of them are in favour of proposal A, 30% are in favour of proposal B and 20% are in favour of proposal C. 5% are in favour of all three proposals. what is the percentage of people favouring more than one proposal?

(a) 16 **(b) 17**
(c) 18 **(d) 19**

Solution : (b)

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

or $78 = 50 + 30 + 20 - \Sigma n(A \cap B) + 5$

or $\Sigma n(A \cap B) = 27$

This includes $n(A \cap B \cap C)$ three times.

$\therefore$ Percentage of people favouring more than one proposal $= 27 - 5 \times 2 = 17$

Illustration – 5

If X and Y are two sets such that X ∪ Y has 50 elements, X has 28 elements and Y has 32 elements, how many elements does X ∩ Y have?

Solution : Given that $n(X \cup Y) = 50$,
$n(X) = 28$, $n(Y) = 32$, $n(X \cap Y) = ?$
By using the formula,
$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$,
$n(X \cap Y) = n(X) + n(Y) - n(X \cup Y) = 28 + 32 - 50 = 10$

Illustration – 6

In a class of 35 students, 24 like to play cricket and 16 like to play football. Also, each student likes to play at least one of the two games. How many students like to play both cricket and football?

Solution : Let X be the set of students who like to play cricket and Y be the set of students who like to play football. Then X ∪ Y is the set of students who like to play at least one of the two games, and X ∩ Y is the set of students who like to play both games.

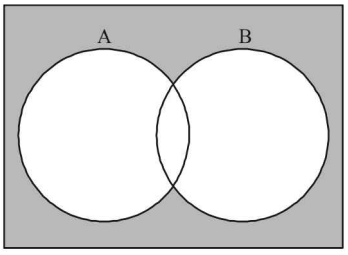
Given $n(X) = 24$, $n(Y) = 16$, $n(X \cup Y) = 35$, $n(X \cap Y) = ?$
$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$\Rightarrow 35 = 24 + 16 - n(X \cap Y)$
$n(X \cap Y) = 5$ i.e., 5 students like to play both games.

Illustration – 7

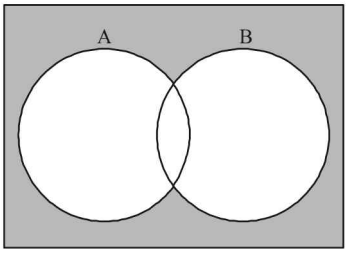
Draw the appropriate Venn diagram for each of the following:
(i) (A ∪ B)' (ii) A' ∩ B' (iii) (A ∩ B)'
(iv) A' ∪ B'

Solution :

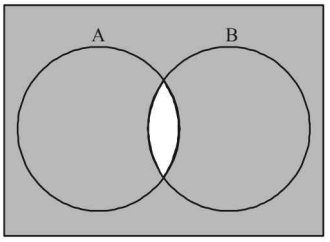
(i) (A ∪ B)' is represented by the shaded region.



(ii) A' ∩ B' is represented by the shaded region.



(iii) (A ∩ B)' is represented by the shaded region.



(iv) A' ∪ B' is represented by the shaded region.

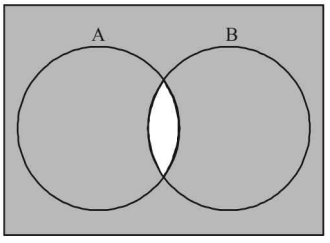


Illustration – 8

Out of 10000 people surveyed, 3700 liked city A, 4000 liked city B and 5000 liked city C. 700 people liked A and B 1200 liked A and C and 1000, liked B and C. Each person liked at least one city. Then find

(A) The number of people liking all the three cities.

(B) The number of persons liking at least two cities as a % of number of people liking exactly one city.

(C) The number of persons liking exactly two cities as a percentage of the number of people liking at least one city.

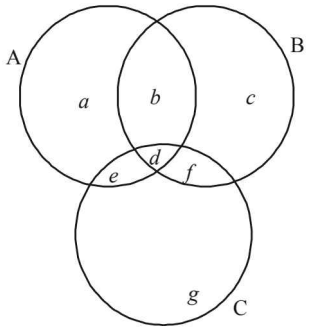
(D) The number of persons liking A and B but not C.

Solution : Refer the figure given

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B \cap C)$

$\Rightarrow 10000 = 3700 + 4000 + 5000 - 700 - 1000 - 1200 + d$

$\Rightarrow d = 200$



Once the value of d is known, all other values will be determined fastly.

e.g. $b + d = 700$ (given) $\Rightarrow b = 500$

Similarly $e = 1000, f = 800, a = 2000, c = 2500, g = 3000$

A. $d = 200$.

B. At least two cities $b + d + e + f = 2500$

Exactly one city $a + c + g = 7500$

$\Rightarrow \% = 2500/7500 \times 100\ \% = 33.33\%$

C. Exactly two cities $= b + e + f = 2300$

At least one city = 10000

$\Rightarrow$ Required % = 23 %.

D. $b = 500$.

Illustration – 9

In a survey of 100 students, the number of students studying the various languages were found to be: English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:

(i) How many students were studying Hindi?

(ii) How many students were studying English and Hindi?

Solution : We have, $a = 18$, $a + b = 23$,

$d + e = 8$, $a + b + d + e = 26$,

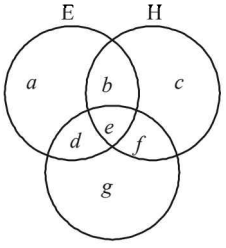
$d + e + f + g = 48$, $e + f = 8$,

$a + b + c + d + e + f + g = 100 - 24 = 76$

$\therefore a = 18, b = 0, c = 10, d = 5, e = 3, f = 5$ and $g = 35$

(i) $n(H) = b + c + e + f = 18$

(ii) $n(H \cap E) = b + e = 3$



GP_3386

LEVEL - 1

1. In a group of students, 100 students know Hindi, 50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?
(a) 100 (b) 115 (c) 110 (d) 125

2. If X and Y are two sets such that n (X) = 17, $n(Y) = 23$ and $n(X \cup Y) = 38$, then $n(X \cap Y)$
(a) 2 (b) 1 (c) 3 (d) 4

3. Let $S = \{0, 1, 5, 4, 7\}$. Then the total number of subsets of S is
(a) 64 (b) 32 (c) 40 (d) 20

4. If A and B are two disjoint sets, then which one of the following is correct?
(a) $A - B = A - (A \cap B)$
(b) $B - A' = A \cap B$
(c) $A \cap B = (A - B) \cap B$
(d) All of these

5. If A and B are disjoint sets, then $A \cap (A' \cup B)$ is equal to which one of the following?
(a) ϕ (b) A
(c) $A \cup B$ (d) $A - B'$

6. In a certain office, 72% of the workers prefer tea and 44% prefer coffee. If each of them prefers tea or coffee and 40 like both, the total number of workers in the office is :
(a) 200 (b) 240 (c) 250 (d) 320

7. If A and B are any two sets, then what is $A \cap (A \cup B)$ equal to?
(a) Complement of A (b) Complement of B
(c) B (d) A

Directions (8–11): 5% of the passengers do not like coffee, tea and lassi and 10% like all the three, 20% like coffee and tea, 25% like lassi and coffee and 25% like lassi and tea. 55% like coffee, 50% like tea, and 50% like lassi.

8. The passengers who like only coffee is greater than the passengers who like only lassi by
(a) 25% (b) 10% (c) 75% (d) 0%

9. The percentage of passengers who like both tea and lassi but not coffee, is
(a) 15 (b) 25 (c) 40 (d) 75

10. The percentage passengers who like at least 2 of the coffee, tea and lassi, is
(a) 30 (b) 45 (c) 50 (d) 60

11. If the number of passengers is 180, then the number of passengers who like lassi only, is
(a) 10 (b) 18 (c) 27 (d) 36

12. If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$, then what is the value of $(A' \cap B')$?
(a) 100 (b) 200 (c) 300 (d) 400

13. The number of the proper subset of {a, b, c} is:
(a) 3 (b) 8 (c) 6 (d) 7

14. If A = {1, 2, 5, 6} and B = {1, 2, 3}, then $(A \times B) \cap (B \times A)$ is equal to?
(a) {(1, 1), (2, 1), (6, 1), (3, 2)}
(b) {(1, 1), (1, 2), (2, 1), (2, 2)}
(c) {(1, 1), (2, 2)}
(d) {(1, 1), (1, 2), (2, 5), (2, 6)}

15. Which one of the following is a null set ?
(a) {0} (b) {{{}}}
(c) {{}} (d) $\{x | x^2 + 1 = 0, x \in R\}$

16. A survey show that 63% of the Indians like cheese whereas 76% like apples. If x% of the Indians like both cheese and apples, then find the range of x.
(a) $0 \le x \le 23\%$ (b) $0 \le x \le 39\%$
(c) $4 \le x \le 35\%$ (d) $6 \le x \le 33\%$

17. $A \equiv \{x | x$ is a prime number $\le 100\}$
$B \equiv \{x | x$ is an odd number $\le 100\}$
What is the ratio of the number of subsets of set A to set B ?
(a) 2^{25} (b) 2^{-25}
(c) 2 (d) $\frac{50^2}{25^2}$

18. If A and B are two sets such that A has 12 elements, B has 17 elements, and $A \cup B$ has 21 elements, how many elements does $A \cap B$ have?
(a) 7 (b) 8 (c) 9 (d) 10

19. In a town three newspapers A, B and C are published. 42% of the people in that town read A, 68% read B, 51% read C, 30% read A and B, 28% read B and C, 36% A and C and 18% do not read any paper. Find the % of population of town that reads all the three.
(a) 15% (b) 25% (c) 20% (d) 35%

LEVEL - 2

1. In an examination out of 100 students, 75 passed in English 60 passed in Mathematics and 45 passed in both English and Mathematics. What is the number of students passed in exactly one of the two subjects?
(a) 45 (b) 60 (c) 75 (d) 90

2. Let T be the set of integers $\{3, 11, 19, 27 .....451, 459, 467\}$ and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is
(a) 32 (b) 28 (c) 29 (d) 30

3. In a car agency one day 120 cars were decorated with three different accessories *viz.*, power window, AC and music system. 80 cars were decorated with power windows, 84 cars were decorated with AC and 80 cars were decorated with music systems. What is the minimum and maximum number of cars which were decorated with all of three accessories?
(a) 10, 61 (b) 10, 45
(c) 25, 35 (d) None of these

4. In an examination 70% of the candidates passed in English, 65% in Mathematics, 27 % failed in both the subjects. If 248 candidates passed in both the subjects, then find the total number of candidates.
(a) 200 (b) 400 (c) 300 (d) 100

Directions (5–8): In a city, three daily newspapers A, B, C are published, 42% read A; 51% read B; 68% read C; 30% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers.

5. What is the percentage of persons who read all the three papers?
(a) 20% (b) 25% (c) 30% (d) 40%

6. What is the percentage of persons who read only two papers?
(a) 19% (b) 31%
(c) 44% (d) None of these

7. What is the percentage of persons who read only one paper?
(a) 38% (b) 48%
(c) 51% (d) None of these

8. What is the percentage of persons who read only A but neither B nor C?
(a) 4% (b) 3%
(c) 1% (d) None of these

9. A survey among 151 persons is conducted regarding their favourite channel of radio – Radio city, Radio Mirchi and Radio life. It was found that every listener of Radio Mirchi also listen either Radio city or Radio life. The number of persons listening all the radio channels is the same as the number of persons who listen none of the channels. 55 persons listen exactly two channels and 70 persons listen only one channel. The number of people who listen all the three channels?
(a) 16 (b) 13
(c) 9 (d) data insufficient

10. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total 64 played both basketball and hockey, 80 played cricket and basketball and 40 played cricket and hockey, 24 played all the three games. The number of boys who did not play any game is :
(a) 128 (b) 216 (c) 240 (d) 160

11. From 50 students taking examination in Mathematics, Physics and Chemistry, each of the students has passed in at least one of the subject, 37 passed Mathematics, 24 Physics and 43 Chemistry. Atmost 19 passed Mathematics and Physics, atmost 29 Mathematics and Chemistry and atmost 20 Physics and Chemistry. Then, the largest numbers that could have passed all three examinations, are
(a) 12 (b) 14 (c) 15 (d) 16

12. A class has 175 students. The following data shows the number of students opting one or more subjects. Maths–100, Physics–70, Chemistry–40, Maths and Physics–30, Maths and Chemistry–28, Physics and Chemistry–23, Maths, Physics and Chemistry–18. How many have offered Maths alone?
(a) 35 (b) 48 (c) 60 (d) 22

13. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers like product P_1 and 1450 consumers like product P_2. What is the least number that must have liked both the products?
(a) 1150 (b) 2000 (c) 1170 (d) 2500

GP_3386

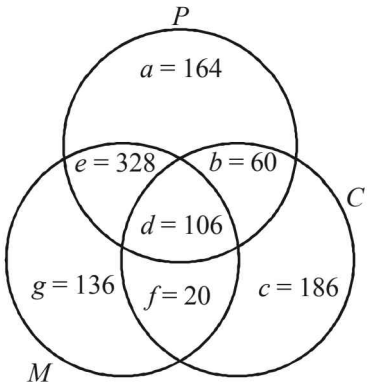
14. In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all of three newspapers, then the number of families which buy A only, is
(a) 4400 (b) 3300 (c) 2000 (d) 500

15. A survey of 500 television viewers produced the following information, 285 watch football, 195 watch hockey, 115 watch basket-ball, 45 watch football and basket ball, 70 watch football and hockey, 50 watch hockey and basket ball, 50 do not watch any of the three games. The number of viewers, who watch exactly one of the three games are
(a) 325 (b) 310 (c) 405 (d) 372

Directions (Qs. 16–18) : The result of an exam is given below. Out of 1000 students who appeared

(i) 658 failed in Physics
(ii) 166 failed in Physics and Chemistry
(iii) 372 failed in Chemistry, 434 failed in Physics and Maths
(iv) 590 failed in Maths, 126 failed in Maths & Chemistry.

Find the number of people who failed in (assuming that none is passed in all subjects).



We have the following equations :
$a+b+c+d+e+f+g=1000$
$a+b+d+e=658, b+d=166$
$b+d+c+f=372.$
$d+e=434$ as in the figure.
$d+e+f+g=590, d+f=126.$ Find the values.

16. Chemistry but not in Physics.
(a) 318 (b) 198 (c) 213 (d) 206

17. Physics or Maths but not in Chemistry.
(a) 558 (b) 718
(c) 628 (d) None of these

18. Physics but neither Chemistry nor Maths.
(a) 164 (b) 228
(c) 196 (d) None of these

19. 70 per cent of the employees in a multinational corporation have VCD players, 75 per cent have microwave ovens, 80 per cent have ACs and 85 per cent have washing machines. At least what percentage of employees has all four gadgets?
(a) 15 (b) 5
(c) 10 (d) Cannot be determined

20. Let $A=\{(n,2n): n \in N\}$ and $B=\{(2n,3n): n \in N\}$. Then $A \cap B$ equal to ?
(a) $\{(n,6n): n \in N\}$ (b) $\{(2n,6n): n \in N\}$
(c) $\{(n,3n): n \in N\}$ (d) ϕ

21. If $n(A)=115, n(B)=326, n(A-B)=47$, then what is $n(A \cup B)$ equal to?
(a) 373 (b) 165 (c) 370 (d) 394

Directions (Qs. 22 -25) : In a survey of 1000 boys conducted in an area, it is found that 65% play Cricket, 48% play Football and 40% play Hockey. Of the total, 30% play both Football and Cricket, 25% play Football and Hockey, while 24% play Cricket and Hockey. Only 5% do not play any of the three games.

22. Find the number of players who play Football but not Hockey.
(a) 180 (b) 230
(c) 350 (d) Can't be determined

23. How many play all the three above-mentioned games?
(a) 180 (b) 240 (c) 230 (d) 210

24. How many play Hockey but neither Cricket nor Football?
(a) 140 (b) 320
(c) 120 (d) Can't be determined

25. Find the percentage of players who play only Football.
(a) 12% (b) 14% (c) 32% (d) 18%

Hints and Solutions

Level 1

1. (d) Total number of students = 100 + 50 – 25 = 125

2. (a) $n(X \cap Y) = n(X) + n(Y) - n(X \cup Y)$
$= 17 + 23 - 38 = 40 - 38 = 2$

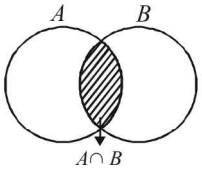
3. (b) $\because n(S) = 5$
Total number of subsets of s = $2^5 = 32$

4. (d) Since, A and B are two disjoints therefore
$A \cap B = \phi$
$\therefore A - B = A - (A \cap B)$

5. (a) $A \cap (A' \cup B) = A \cap A' = \phi$

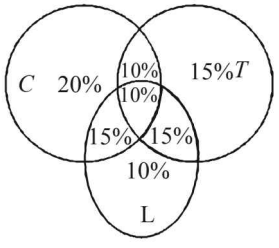
6. (c) Let total number be x. Then
$$n(A) = \frac{72}{100}x = \frac{18x}{25}, \quad n(B) = \frac{44}{100}x = \frac{11x}{25}$$
and $n(A \cap B) = 40$
$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$
$$\Rightarrow x = \frac{18x}{25} + \frac{11x}{25} - 40 \Rightarrow x = 250$$

7. (d)



$A \cap (A \cup B)$
$= (A \cap A) \cup (A \cap B)$
$= A \cup (A \cap B)$
$= A$ (By diagram)
Thus, $A \cap (A \cup B) = A$

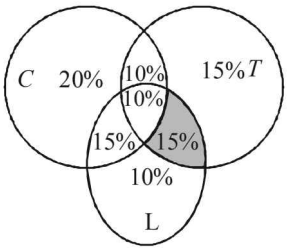
(8 – 11)



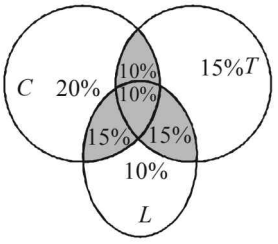
Where C = Coffee, T = Tea and L = Lassi

8. (b) The passengers who like only coffee = 20%
and the passengers who like only lassi = 10%
Required passengers = 10%

9. (a) It can be seen that the percentage of passengers who like both tea and lassi but not coffee = 15%. This is the figure representing this area



10. (c) The percentage of passengers who like at least 2 of the coffee, tea and lassi can be seen in the below figure:



11. (b) 10% of the people like only lassi. So, the number of persons = 18

12. (c) From the given data
$n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$.
We know that,
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 200 + 300 - 100 = 400$
Now, $n(A' \cap B') = n(U) - n(A \cup B)$
$= 700 - 400 = 300$

13. (c) The number of proper subsets of {1, 2, 3, ... n} is $2^n - 2$.
Hence the number of proper subset of {a, b, c} is
$2^3 - 2 = 6$

14. (b) Let $A = \{1, 2, 5, 6\}$ and $B = \{1, 2, 3\}$
$\therefore A \times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),$
$(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$
and $B \times A = \{(1,1),(1,2),(1,5),(1,6),(2,1),(2,2),$
$(2,5),(2,6),(3,1),(3,2),(3,5),(3,6)\}$
$(A \times B) \cap (B \times A) = \{(1,1),(1,2),(2,1),(2,2)\}$

GP_3386

15. (d) Consider the set given in option 'd'.
$\{x \mid x^2+1=0, x \in R\}$
Let $x^2+1=0 \Rightarrow x^2=-1 \Rightarrow x=\pm i$ which is complex.
But $x \in R$. Hence for, any $x \in R$, x^2+1 can not be zero.

16. (b) $n(C)=63\%; n(A)=76\%$
$n(C \cup A)=n(C)+n(A)-n(C \cap A)$
$100\%=63\%+76\%-x\%$
$x\%=39\%$

17. (b) We know that there are 25 prime number below 100.
$n(A)=25$
The total number of subsets of sets $A=2^{25}$
There are 50 odd numbers below 100.
The total number of subsets of $B=2^{50}$
Required ratio $=\dfrac{2^{25}}{2^{50}}=2^{-25}$.

18. (b) $n(A)=12, n(B)=17, n(A \cup B)=21$
$n(A \cup B)=n(A)+n(B)-n(A \cap B)$
$21=12+17-n(A \cap B)$
or $n(A \cap B)=8$
$\Rightarrow A \cap B$ has 8 elements.

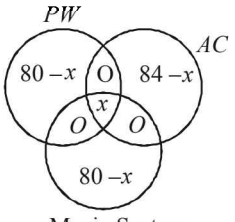
19. (a) $n(A \cup B \cup C)=n(A)+n(B)+n(C)-n(A \cap B)$
$-n(A \cap C)-n(B \cap C)+n(A \cap B \cap C)$
$\Rightarrow 100-18=42+68+51-30-28-36+x \Rightarrow x=15$

Level 2

1. (a) Total number of students = 100
Let E denote the students who have passed in English.
Let M denote the students who have passed in Maths.
$\therefore n(E)=75, n(M)=60$ and $n(E \cap M)=45$
we know $n(E \cup M)=n(E)+n(M)-n(E \cap M)$
$=75+60-45=90$
Required number of students $=90-45=45$

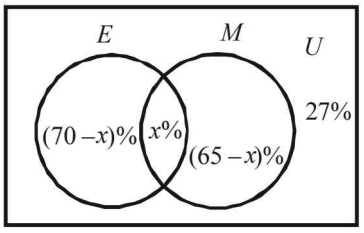
2. (d) $T=\{3, 11, 19, 27 \ldots .467\}$ is an AP with $a=3$ and $d=8$. To find number of terms, we use the formula for nth term :
$a+(n-1)d=3+(n-1)8=467.$
Hence, $n=59$. $S=$ subset in which not sum of two elements = 470 .
So, S can be a set in which either the first half or the second half of the terms are present. So number of maximum possible elements in $S=\dfrac{59}{2}=29.5=30.$

3. (d) 80 cars were decorated with power windows it means at least 40 cars were decorated with AC or music system or both.
84 cars were decorated with ACs, it means at least 36 cars were decorated with power windows or music systems.
80 cars were decorated with music system it means at least 40 cars were decorated with power windows or ACs.
It means if there is no intersection in these three, then at most $40+36+40=116$ cars had been decorated with one or two accessories.
Hence at least 4 cars would have been decorated with all three accessories.
For maximum value of x:



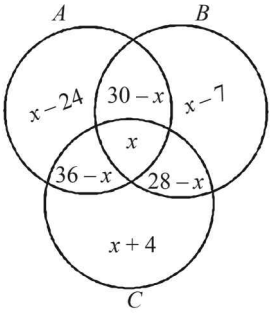
Total number of cars $=(80-x)+(84-x)+(80-x)+x$
$120=244-2x$
$\Rightarrow 2x=124 \Rightarrow x=62$
minimum $\rightarrow$ 4 cars and maximum $\rightarrow$ 62 cars

4. (b) Let the set E and M represent students who passed in English and Mathematics respectively .
$n(E \cup M)=(100-27)\%=73\%$
$n(E \cup M)=n(E)+n(M)-n(E \cap M)$
$73\%=70\%+65\%-x\% \Rightarrow x\%=62\%$
Now. $62\% \equiv 248$



Total number of candidates $=\dfrac{248 \times 100}{62}=400$

(5-8).



Let x% people read all the three newspapers.
Since 8% people do not read any newspapers.
$\therefore (x-24)+(x-7)+(x+4)+(30-x)+(36-x)+(28-x)+x=92 \Rightarrow x=92-67=25$

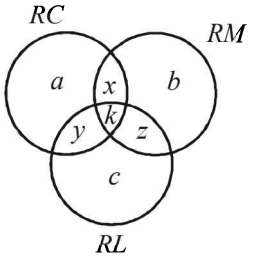
5. (b) Hence people who read all the three newspapers = 25%

6. (a) $(30-x)+(36-x)+(28-x)=94-3x$
$=94-3\times 25=19$
Hence percentage of people who read only two newspapers =19%

7. (b) $(x-24)+(x-7)+(x+4)=3x-27$
$=3\times 25-27=48$
Hence percentage of people who read only one newspaper = 48%

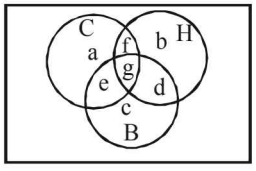
8. (c) $x-24=25-24=1$
Hence percentage of people who read only Newspaper A but neither B nor C = 1%

9. (b) $\beta=(x+y+z)=55$
$\alpha=(a+b+c)=70$ and $\gamma=k$



Let m people listen none of the three channels, then $m=\gamma=k$
$(\alpha+\beta+\gamma)+m=151 \Rightarrow \alpha+\beta+\gamma+\gamma=151$
$\Rightarrow (55+70)+2\gamma=151 \Rightarrow \gamma=13$
Hence, there are 13 people listen all three channels.

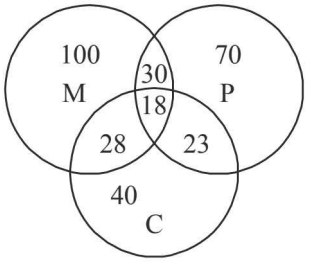
10. (d) $a+e+f+g=224$
$b+d+f+g=240$



$c+d+e+g=336$
$d+g=64$, $e+g=80$, $f+g=40$, $g=24$
$\Rightarrow d=40$, $e=56$, $f=16$
$a=128$, $b=160$, $c=216$
$\therefore$ Boys who did not play any game
$= 800-(a+b+c+d+e+f+g)=160$

11. (b) Let M be the set of students passing in Mathematics, P be the set of students passing in Physics and C be the set of students passing in Chemistry.
Now, $n(M \cup P \cup C) = 50$, $n(M) = 37$, $n(P) = 24$, $n(C) = 43$
$n(M \cap P) \le 19$, $n(M \cap C) \le 29$, $n(P \cap C) \le 20$ [given]
$n(M \cup P \cup C)=n(M)+n(P)+n(C)-n(M \cap P)-n(M \cap C)-n(P \cap C)+n(M \cap P \cap C) \le 50$
$\Rightarrow 37+24+43-19-29-20+n(M \cap P \cap C) \le 50$
$\Rightarrow n(M \cap P \cap C) \le 50-36$
$\Rightarrow n(M \cap P \cap C) \le 14$
Thus, the largest possible number that could have passed all the three examinations, is 14.

12. (c) $n(M \cap P' \cap C')$
$= n(M) - [n(M \cap P) + n(M \cap C) - n(M \cap C \cap P)]$
$= 100 - 30 - 28 + 18 = 60$
[This can be solved directly by seeing the Venn Diagram]



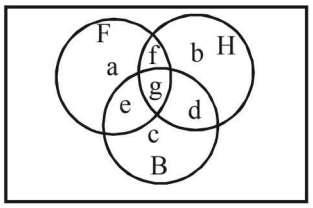
13. (c) Let U be the set of all consumers who were questioned, A be the set of consumers who liked product P_1 and B be the set of consumers who liked product P_2.
It is given that $n(U) = 2000$, $n(A) = 1720$, $n(B) = 1450$, $n(A \cup B)$
$= n(A) + n(B) - n(A \cap B)$
$= 1720 + 1450 - n(A \cap B)$
$= 3170 - n(A \cap B)$
Since, $A \cup B \subseteq U$
$\therefore \quad n(A \cup B) \le n(U)$
$\Rightarrow \quad 3170 - n(A \cap B) \le 2000$
$\Rightarrow \quad 3170 - 2000 \le n(A \cap B)$
$\Rightarrow \quad n(A \cap B) \ge 1170$
Thus, the least value of $n(A \cap B)$ is 1170.
Hence, the least number of consumers who liked both the products is 1170.

14. (b) $n(A) = 40\%$ of $10000 = 4000$, $n(B) = 2000$, $n(C) = 1000$, $n(A \cap B) = 500$, $n(B \cap C) = 300$, $n(C \cap A) = 400$, $n(A \cap B \cap C) = 200$

$\therefore \quad n(A \cap B' \cap C') = n\left\{A \cap (B \cup C)'\right\}$

$= n(A) - n\{A \cap (B \cup C)\}$
$= n(A) - n(A \cap B) - n(A \cap C)$
$+ n(A \cap B \cap C)$
$= 4000 - 500 - 400 + 200 = 3300.$

15. (a)



$a + e + f + g = 285$, $b + d + f + g = 195$
$c + d + e + g = 115$, $e + g = 45$, $f + g = 70$, $d + g = 50$
$a + b + c + d + e + f + g = 500 - 50 = 450$
As in previous question, we obtain
$a + f = 240$, $b + d = 125$, $c + e = 65$
$a + e = 215$, $b + f = 145$; $b + c + d = 165$
$a + c + e = 255$; $a + b + f = 335$
On solving we get, On $b = 95$, $c = 40$, $a = 190$, $d = 30$, $e = 25$, $f = 50$ and $g = 20$
Desired quantity $= a + b + c = 325$

16. (d) Chemistry but not Physics $= c + f = 206$.

17. (c) Physics and Maths but not Chemistry $= a + e + g = 628$.

18. (a) Physics but neither Maths nor Chemistry $= a = 164$.

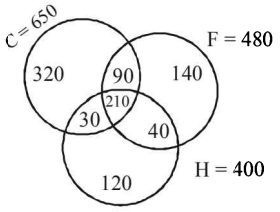
19. (c) Employees who doesn't have VCD $= 100 - 70 = 30\%$
Employees who doesn't have MWO $= 100 - 75 = 25\%$
Employees who doesn't have AC $= 100 - 80 = 20\%$
Employees who doesn't have WM $= 100 - 85 = 15\%$
$\therefore$ Total employees who doesn't have atleast one of the four equipments $= 30 + 25 + 20 + 15 = 90\%$
$\therefore$ Percentage of employees having all four gadgets $= 100 - 90 = 10\%$.

20. (d) $A = \{(n, 2n) : n \in N\}$ and $B = \{(2n, 3n)\} : n \in N$
Listing few members of each set
$A = \{(1, 2), (2, 4), (3, 6), ...\}$
$B = \{(2, 3), (4, 6), (6, 9).....\}$
There is no member common to both these sets, hence, $A \cap B = \phi$

21. (a) We know, for two sets A and B
$A - B = A - (A \cap B)$
$\therefore\ n(A - B) = n(A) - n(A \cap B)$
Given, $n(A) = 115$, $n(B) = 326$ and $n(A - B) = 47$
$\Rightarrow 47 = 115 - n(A \cap B)$
$\Rightarrow n(A \cap B) = 68$
Consider $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 115 + 326 - 68 = 373$

(22-25) :

$n(C) = 650$, $n(F) = 480$, $n(H) = 400$
$n(C \cap F) = 300$, $n(F \cap H) = 250$, $n(C \cap H) = 240$
and $n(C \cup F \cup H) = (100 - 5)\%$ of $1000 = 950$
Since, $n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$
$\Rightarrow n(C \cap F \cap H) = 210$



22. (b) 230
23. (d) 210
24. (c) 120
25. (b) 14%

CHAPTER

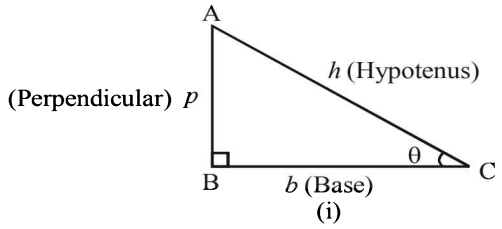
25

Trigonometry

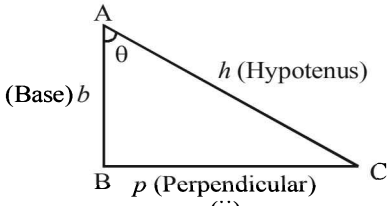
(Including Height and Distances, Second, Minute, Digree, Radians, Relation $\theta = \frac{l}{r}$, Minimum and Maximum Value of Trigonometric Functions)

TRIGONOMETRIC RATIO

Consider a ΔABC right angled at B i.e., $\angle ABC = 90°$. The side opposite to $\angle B$ i.e., longest side AC is called hypotenus (h).



(i)



(ii)

In trigonometry, to find the perpendicular (p) and base (b), first of all we have to consider any of the two acute angles A and C. If we consider $\angle C$, then the side AB opposite to $\angle C$ will be perpendicular (p) and the remaining side BC will be base (b) as shown in the figure (i). But if we consider $\angle A$, then the side BC opposite to $\angle A$ is will be the perpendicular (p) and the remaining side AB will be the base (b) as shown in the figure (ii).

There are six ratios formed from three sides h, p and b. Each one of these six ratios is given a specific name.

If we consider $\angle C = \theta$, as shown in the figure (i), then the six ratios between h, p and b for this angle are named as

$$\boxed{\sin\theta = \frac{p}{h} = \frac{AB}{AC}}, \quad \boxed{\cos\theta = \frac{b}{h} = \frac{BC}{AC}}, \quad \boxed{\tan\theta = \frac{p}{b} = \frac{AB}{BC}}$$

$$\boxed{\text{cosec}\,\theta = \frac{h}{p} = \frac{AC}{AB}}, \quad \boxed{\sec\theta = \frac{h}{b} = \frac{AC}{BC}}, \quad \boxed{\cot\theta = \frac{b}{p} = \frac{BC}{AB}}$$

If we consider $\angle A = \theta$ as shown in the figure (ii), then for this angle

$$\sin\theta = \frac{p}{h} = \frac{BC}{AC}, \cos\theta = \frac{b}{h} = \frac{AB}{AC}, \tan\theta = \frac{p}{b} = \frac{BC}{AB}$$

$$\text{cosec}\,\theta = \frac{h}{p} = \frac{AC}{BC}, \sec\theta = \frac{h}{b} = \frac{AC}{AB}, \cot\theta = \frac{b}{p} = \frac{AB}{BC}$$

Clearly $\sin\theta$ & $\text{cosec}\,\theta$ are reciprocal to each other, $\cos\theta$ & $\sec\theta$ are reciprocal to each other and $\tan\theta$ and $\cot\theta$ are reciprocal to each other.

i.e.,
$$\boxed{\begin{aligned} &\sin\theta = \frac{1}{\text{cosec}\,\theta}, \text{cosec}\,\theta = \frac{1}{\sin\theta} \\ &\cos\theta = \frac{1}{\sec\theta}, \quad \sec\theta = \frac{1}{\cos\theta} \\ &\tan\theta = \frac{1}{\cot\theta}, \quad \cot\theta = \frac{1}{\tan\theta} \end{aligned}}$$

In other words,

$$\boxed{\sin\theta.\,\text{cosec}\,\theta = 1,\ \cos\theta.\sec\theta = 1,\ \tan\theta\cot\theta = 1}$$

We can also see that

GP_3386

$$\tan\theta = \frac{p}{b} = \frac{\frac{p}{h}}{\frac{b}{h}} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{b}{p} = \frac{\frac{b}{h}}{\frac{p}{h}} = \frac{\cos\theta}{\sin\theta}$$

i.e., $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}}$

By Pythagorus theorem,

$p^2 + b^2 = h^2$ ...(i)

By dividing h^2 on both sides of equation (i), we get

$$\frac{p^2}{h^2} + \frac{b^2}{h^2} = 1 \Rightarrow \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = 1$$

$$\Rightarrow (\sin\theta)^2 + (\cos\theta)^2 = 1$$

Note that: we write $(\sin\theta)^2$ as $\sin^2\theta$ and

$(\cos\theta)^2 = \cos^2\theta$

Therefore $\sin^2\theta + \cos^2\theta = 1$

Similarly on dividing by b^2 on both sides of equation (i), we get $\tan^2\theta + 1 = \sec^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

and on dividing by p^2 on both sides of equations (i), we get $1 + \cot^2\theta = \text{cosec}^2\theta$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1$

Thus

$\boxed{\sin^2\theta + \cos^2\theta = 1, \sec^2\theta - \tan^2\theta = 1, \text{cosec}^2\theta - \cot^2\theta = 1}$

From the above formule, we can easily derived

$\boxed{\sin\theta = \sqrt{1-\cos^2\theta}, \cos\theta = \sqrt{1-\sin^2\theta}}$
$\boxed{\tan\theta = \sqrt{\sec^2\theta - 1}, \sec\theta = \sqrt{1+\tan^2\theta}}$
$\boxed{\cot\theta = \sqrt{\text{cosec}^2\theta - 1}, \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}}$

VALUE OF TRIGONOMETRIC RATIOS FOR SOME SPECIFIC ANGLES

Trigonometrical ratio (θ)	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\text{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined

Trigonometric Ratios for Complementary Angles

$\sin(90°-\theta)=\cos\theta$, $\cos(90°-\theta)=\sin\theta$,
$\tan(90°-\theta)=\cot\theta$, $\cot(90°-\theta)=\tan\theta$,
$\operatorname{cosec}(90°-\theta)=\sec\theta$ $\sec(90°-\theta)=\operatorname{cosec}\theta$ ....

Illustration – 1

Evaluate $\dfrac{\cos 43°}{\sin 47°}+\dfrac{\sec 32°}{\operatorname{cosec} 58°}$

Solution : We know that $\cos(90°-\theta)=\sin\theta$

$\sin 47°=\sin(90°-\theta)=\cos 43°$

Also, $\operatorname{cosec} 58°=\operatorname{cosec}(90°-32°)=\sec 32°$

$\therefore \dfrac{\cos 43°}{\sin 47°}+\dfrac{\sec 32°}{\operatorname{cosec} 58°}=\dfrac{\cos 43°}{\cos 43°}+\dfrac{\sec 32°}{\sec 32°}=1+1=2$

Illustration – 2

Evaluate $\dfrac{\sec^2 54°-\cot^2 36°}{\operatorname{cosec}^2 57°-\tan^2 33°}+2\sin^2 38°\sec^2 52°$

$-\sin^2 45°+\dfrac{2}{\sqrt{3}}\tan 17°\tan 60°\tan 73°$.

Solution : The given expression is

$$\frac{\sec^2 54°-\cot^2 36°}{\operatorname{cosec}^2 57°-\tan^2 33°}+2\sin^2 38°\sec^2 52°-\sin^2 45°+\frac{2}{\sqrt{3}}\tan 17°\tan 60°\tan 73°$$

$$=\frac{\sec^2(90°-36°)-\cot^2 36°}{\operatorname{cosec}^2(90°-33°)-\tan^2 33°}+2\sin^2 38°\sec^2(90°-38°)-\sin^2 45°+\frac{2}{\sqrt{3}}\tan(90°-73°)\tan 73°\tan 60°$$

$$=\frac{1}{1}+2\sin^2 38°\times\frac{1}{\sin^2 38°}-\frac{1}{2}+\frac{2}{\sqrt{3}}\times\frac{1}{\tan 73°}\times\tan 73°\times\sqrt{3}$$

$[\because \operatorname{cosec}^2\theta-\cot^2\theta=1,\ \sec^2\theta-\tan^2\theta=1]$

$$=1+2-\frac{1}{2}+2=5-\frac{1}{2}=\frac{9}{2}$$

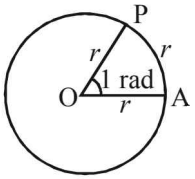
SYSTEM OF MEASUREMENT OF ANGLES

There are two system which are commonly use for measuring angles

(1) Sexagesimal or English system :

1 right angle = 90 degree (= 90°)
1° = 60 minutes (= 60')
1' = 60 second (= 60")

(2) Circular system : The measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle is called one radian.

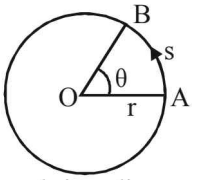


Consider a circle of radius r having centre at O. Let A be a point on the circle. Now cut off an arc AP whose length is equal to the radius r of the circle. Then by the definition the measure of $\angle$ AOP is 1 radian (= 1^c)

π radians = 180°

RELATION BETWEEN ARC, RADIUS AND CENTRAL ANGLE

If s is the length of an arc of a circle of radius r, then the angle θ (in radians) subtended by this arc at the centre of the circle is given by $\theta=\dfrac{s}{r}$ or $s=r\theta$.



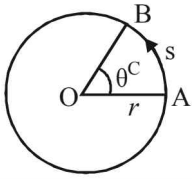
i.e., Arc = radius × angle in radians

Note that the unit of *s* and *r* will be always same in this formula

Area of a Sector

OAB be a sector having central angle θ (in radians) and radius *r*. Area of the sector OAB is given by $\dfrac{\pi}{2}r^2\theta$.

GP_3386



IMPORTANT RESULTS

1. $\tan(45+\theta)=\dfrac{1-\tan\theta}{1+\tan\theta}=\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

2. (a) If $\sin\theta+\operatorname{cosec}\theta=2$, then $\sin\theta=\operatorname{cosec}\theta=1$

(b) If $\cos\theta+\sec\theta=2$, then $\cos\theta=\sec\theta=1$

(c) If $\tan\theta+\cot\theta=2$, then $\tan\theta=\cot\theta=1$

3. (a) If $\sin A+\cos B=2$, then $A=90°$ and $B=0°$

(b) If $\sin A+\cos B=0$, then $A=0°$ and $B=90°$

4. If $p\sin\theta+q\cos\theta=r$ then $\tan\theta=\dfrac{p}{q}$

5. If $a\sin\theta+b\cos\theta=m$ and $b\sin\theta-a\cos\theta=n$ then $a^2+b^2=m^2+n^2$

6. (a) If $\sec\theta+\tan\theta=x$, then $\sec\theta-\tan\theta=\dfrac{1}{x}$

(b) If $\operatorname{cosec}\theta-\cot\theta=x$, then $\operatorname{cosec}\theta+\cot\theta=\dfrac{1}{x}$

7. (a) If $a\sec\theta-b\tan\theta=c$ and $b\sec\theta-a\tan\theta=d$, then $a^2-b^2=c^2-d^2$

(b) If $a\sec\theta+b\tan\theta=c$ and $b\sec\theta+a\tan\theta=d$, then $a^2-b^2=c^2-d^2$

8. If $A+B=45°$ or $225°$, then

(a) $(1+\tan A)(1+\tan B)=2$

(b) $(1-\cot A)(1-\cot B)=2$

9. If $4\theta<60°$, then

(a) $\sin\theta.\sin2\theta.\sin4\theta=\dfrac{1}{4}\sin3\theta$

(b) $\cos\theta.\cos2\theta.\cos4\theta=\dfrac{1}{4}\cos3\theta$

(c) $\tan\theta.\tan2\theta.\tan4\theta=\tan3\theta$

10. For all value of θ

(a) $\sin(60-\theta).\sin\theta.\sin(60+\theta)=\dfrac{1}{4}\sin3\theta$

(b) $\cos(60-\theta).\cos\theta.\cos(60+\theta)=\dfrac{1}{4}\cos3\theta$

(c) $\tan(60-\theta).\tan\theta.\tan(60+\theta)=\tan3\theta$

11. Maximum and Minimum Value

	Minimum	Maximum
$\sin\theta$ (odd power)	-1	$+1$
$\cos\theta$ (odd power)	-1	$+1$
$\sin^2\theta$ (even power)	0	$+1$
$\cos^2\theta$ (even power)	0	$+1$
$\tan\theta$ (odd power)	$-\infty$	$+\infty$
$\cot\theta$ (odd power)	$-\infty$	$+\infty$
$\tan^2\theta$ (even power)	0	$+\infty$
$\cot^2\theta$ (even power)	0	$+\infty$
$\sec\theta$ (odd power)	$-\infty$	$+\infty$
$\operatorname{cosec}\theta$ (odd power)	$-\infty$	$+\infty$
$\sec^2\theta$ (even power)	1	∞
$\operatorname{cosec}^2\theta$ (even power)	1	∞

Note that :

(i) The value of $\sec\theta$ and $\operatorname{cosec}\theta$ can be any thing from $-\infty$ to $+\infty$, except between -1 and 1.

(ii) When value of θ increases from $0°$ to $90°$, then

* value of $\sin\theta$ increases from 0 to 1.
* value of $\cos\theta$ decreases from 1 to 0.
* value of $\tan\theta$ increases from 0 to ∞

(iii) If $a > b$, then

- maximum value of $a\sin^2\theta + b\cos^2\theta = a$
- minimum value of $a\sin^2\theta + b\cos^2\theta = b$

If $b > a$, then

- Maximum value of $a\sin^2\theta + b\cos^2\theta = b$
- Minimum value of $a\sin^2\theta + b\cos^2\theta = a$

(iv) Maximum value of $a\sin\theta \pm b\cos\theta = \sqrt{a^2 + b^2}$

Minimum value of $a\sin\theta \pm b\cos\theta = -\sqrt{a^2 + b^2}$

(v) Maximum value of $\sin^{2n}\theta + \cos^{2n}\theta = 1$

Minimum value of $\sin^{2n}\theta + \cos^{2n}\theta$

$= \sin^{2n} 45^\circ + \cos^{2n} 45^\circ$

(vi) Maximum value of $\sin^n\theta + \cos^n\theta = \dfrac{1}{2^n}$

Minimum value of

$$\sin^n\theta + \cos^n\theta = \begin{cases} 0, \text{ when } n \text{ is even} \\ \dfrac{-1}{2^n}, \text{ when } n \text{ is odd} \end{cases}$$

(vii) Maximum value of $a\tan^2\theta + b\cot^2\theta = \infty$

Minimum value of $a\tan^2\theta + b\cot^2\theta = 2\sqrt{ab}$

(viii) If $a \le b$, then minimum value of

$a\sin^2\theta + b\operatorname{cosec}^2\theta = a + b$

If $a \ge b$, then minimum value of

$a\sin^2\theta + b\operatorname{cosec}^2\theta = 2\sqrt{ab}$

(ix) If $a \le b$, then minimum value of

$a\cos^2\theta + b\sec^2\theta = a + b$

If $a \ge b$, then minimum value of $a\cos^2\theta + b\sec^2\theta = 2\sqrt{a+b}$

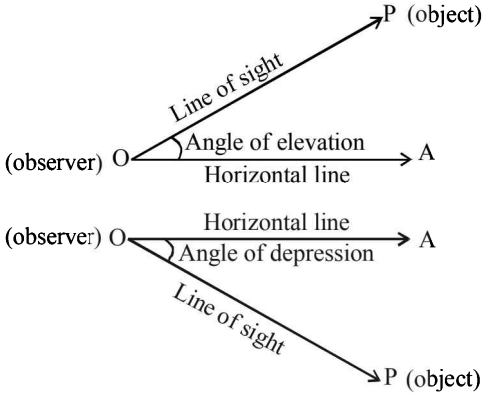
(x) Minimum value of $a\sec^2\theta + b\operatorname{cosec}^2\theta$

$= \left(\sqrt{a} + \sqrt{b}\right)^2$

Maximum value of $a\sec^2\theta + b\operatorname{cosec}^2\theta = \infty$

ANGLE OF ELEVATION AND ANGLE OF DEPRESSION

Let an observer at the point O is observing an object at the point P. The line OP is called the LINE OF SIGHT of the point P. Let OA be the horizontal line passing through O. O, A and P should be in the same vertical plane. If object P be above the line of sight OA, then the acute angle AOP, between the line of sight and the horizontal line is known as ANGLE OF ELEVATION of object P. If the object P is below the horizontal line OA then the angle AOP, between the line of sight and horizontal line is known as ANGLE OF DEPRESSION of object P.



TO FIND THE HEIGHT AND THE DISTANCE OF AN INACCESSIBLE TOWER STANDING ON A HORIZONTAL PLANE

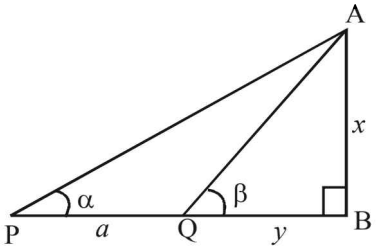
Let AB be a tower and B be its foot. On the horizontal line through B, take two points P and Q. Measure the length PQ.

Let $PQ = a$.

Let the angles of elevation of the top A of the tower as seen from P and Q be respectively α and β ($\beta > \alpha$) then $\angle APB = \alpha$, $\angle AQB = \beta$. Let $AB = x$, $BQ = y$.

From right angled Δ ABP, $\tan\alpha = \dfrac{AB}{PB} = \dfrac{x}{a+y}$

$\therefore a + y = x\cot\alpha$. .....(i)



From right angled ΔABQ, $\tan\beta = \dfrac{AB}{BQ} = \dfrac{x}{y}$

GP_3386

$\therefore y = x\cot\beta$ .....(ii)

From equations (i) and (ii),

$\therefore a = x\cot\alpha - x\cot\beta$.

$\Rightarrow \quad x = \dfrac{a}{\cot\alpha - \cot\beta}$

Also $y = x\cot\alpha - a \Rightarrow y = \dfrac{a\cot\alpha}{\cot\alpha - \cot\beta} - a$

$\Rightarrow y = \dfrac{a\cot\alpha - a(\cot\alpha - \cot\beta)}{\cot\alpha - \cot\beta}$

$\Rightarrow y = \dfrac{a\cot\beta}{\cot\alpha - \cot\beta}$

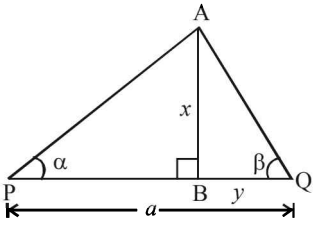
In the above case, P and Q are on the same side of the tower. If the two points are on the opposite sides of the tower then from the adjoining figure, we get

$\tan\alpha = \dfrac{x}{PB}$ *or* $PB = x\cot\alpha$

and $\tan\beta = \dfrac{x}{BQ}$ *or* $BQ = x\cot\beta$.

$\therefore a = PB + BQ = x(\cot\alpha + \cot\beta)$

$\therefore x = \dfrac{a}{\cot\alpha + \cot\beta}$



and $y = BQ = x\cot\beta$

Note : Here, all the lines AP, AQ, AB are in the same plane.

Illustration – 3

A man is standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

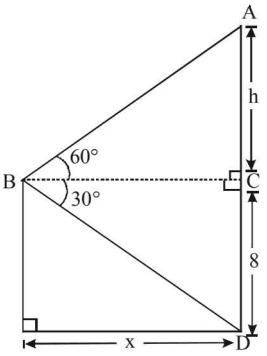
Solution : Let x be the distance of hill from man and $h + 8$ be height of hill which is required.

In rt. ΔACB,

$\tan 60° = \dfrac{AC}{BC} = \dfrac{h}{x} \Rightarrow \sqrt{3} = \dfrac{h}{x}$

In rt. ΔBCD,

$\tan 30° = \dfrac{CD}{BC} = \dfrac{8}{x}$



$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{8}{x} \quad \Rightarrow \quad x = 8\sqrt{3}$

$\therefore$ Height of hill $= h + 8 = \sqrt{3}x + 8$

$= (\sqrt{3})(8\sqrt{3}) + 8 = 32$ m

Distance of ship from hill $= x = 8\sqrt{3}$ m

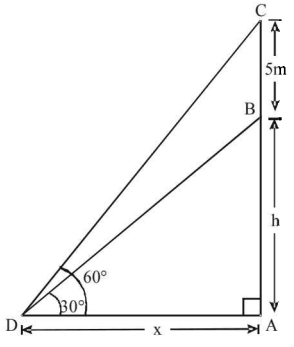
Illustration – 4

A vertical to stands on a horizontal plane and is surmounted by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.

Solution : Let AB be the tower of height h meter and BC be the height of flag staff surmounted on the tower.

Let the point of the plane be D at a distance m meter from the foot of the tower.

In ΔABD,



$$\tan 30° = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \quad \text{..... (i)}$$

In Δ ADC,

$$\tan 60° = \frac{AC}{AD} \Rightarrow \sqrt{3} = \frac{5+h}{x} \Rightarrow x = \frac{5+h}{\sqrt{3}} \quad \text{..... (ii)}$$

From (i) and (ii), $\sqrt{3}h = \frac{5+h}{\sqrt{3}} \Rightarrow 3h = 5 + h$

$\Rightarrow 2h = 5 \Rightarrow h = \frac{5}{2} = 2.5$ m

So, the height of tower = 2.5 m

Illustration – 5

The angles of depressions of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of multistoried building and the distance between the two buildings.

Solution : Let AB be the multistoried building of height h and let the distance between two buildings be x meters.

$\angle XAC = \angle ACB = 45°$ (Alternate angles)

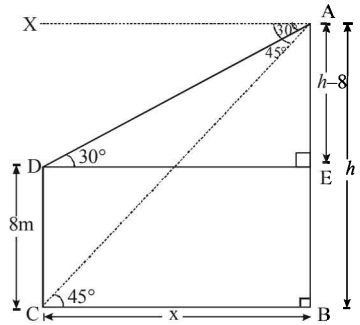
$\angle XAD = \angle ADE = 30°$ (Alternate angles)

In ΔADE, $\tan 30° = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$ $[\because CB = DE = x]$

$\Rightarrow x = \sqrt{3}(h-8)$ ..... (i)

In ΔACB,

$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h$ ..... (ii)



From (i) and (ii),

$\sqrt{3}(h-8) = h \Rightarrow \sqrt{3}h - 8\sqrt{3} = h$

$\Rightarrow \sqrt{3}h - h = 8\sqrt{3} \Rightarrow h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$

$\Rightarrow h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow h = 4\sqrt{3}(\sqrt{3}+1) \Rightarrow h = 4(3+\sqrt{3})$ metres

From (ii), $x = h$

So, $x = 4(3+\sqrt{3})$ metres

Hence,

Height of multistoried building = $4(3+\sqrt{3})$ metres

distance between two building = $4(3+\sqrt{3})$ metres.

Illustration – 6

The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 sec, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Solution : Let the point on the ground is E which is y metres from point B and let after 15 sec. flight it covers x metres distance

In ΔAEB, $\tan 45° = \frac{AB}{EB}$

$\Rightarrow 1 = \frac{3000}{y} \Rightarrow y = 3000$m ..... (i)

In ΔCED, $\tan 30° = \frac{CD}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3000}{x+y}$ $(\because AB = CD)$

$\Rightarrow x + y = 3000\sqrt{3}$ ..... (ii)

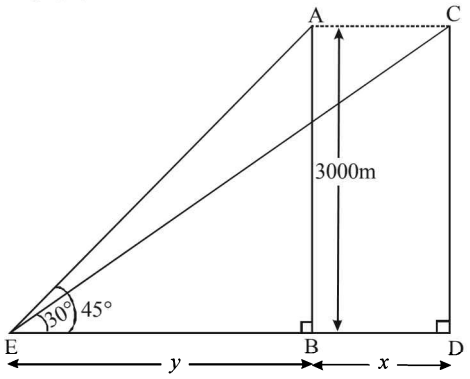
From eqs. (i) and (ii)

$x + 3000 = 3000\sqrt{3} \Rightarrow x = 3000\sqrt{3} - 3000$

$\Rightarrow x = 3000(\sqrt{3}-1) \Rightarrow x = 3000 \times (1.732 - 1)$

$\Rightarrow x = 3000 \times 0.732 \Rightarrow x = 2196$ m

Speed of aeroplane = $\frac{\text{Distance covered}}{\text{Time taken}}$



$$= \frac{2196}{15}\text{m/sec} = 146.4\text{ m/sec}$$

$$= \frac{2196}{15} \times \frac{18}{5}\text{km/hr} = 527.04\text{ km/hr}$$

Hence, the speed of aeroplane is 527.04 km/hr.

Illustration – 7

A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

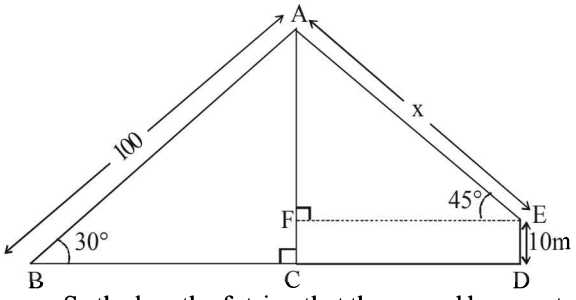
Solution : Let the length of second string be x m.

In ΔABC, $\sin 30° = \frac{AC}{AB}$ or $\frac{1}{2} = \frac{AC}{100} \Rightarrow AC = 50\text{m}$

In ΔAEF, $\sin 45° = \frac{AF}{AE} \Rightarrow \frac{1}{\sqrt{2}} = \frac{AF - FC}{x}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{50-10}{x} \quad [\because AC = 50\text{m}, FC = ED = 10\text{m}]$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{40}{x} \Rightarrow x = 40\sqrt{2}\text{m}$$



So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$m

GP_3386

LEVEL - 1

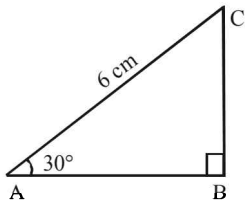
1. In the adjoining figure, the length of BC is

(a) $2\sqrt{3}$ cm

(b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm

(d) 3 cm



2. If the length of the shadow of a tower is $\sqrt{3}$ times that of its height, then the angle of elevation of the sun is

(a) 15° (b) 30° (c) 45° (d) 60°

3. The angle of elevation of the top of a tower at point on the ground is 30°. If on walking 20 metres towards the tower, the angle of elevation become 60°, then the height of the tower is

(a) 10 metre (b) $\frac{10}{\sqrt{3}}$ metre

(c) $10\sqrt{3}$ metre (d) None of these

4. If $\sec\theta + \tan\theta = x$, then $\sec\theta = ?$

(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2+1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{x^2-1}{x}$

5. If angles A, B, C of a ΔABC form an increasing AP, then $\sin B = ?$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$

6. $\sec^4 A - \sec^2 A$ is equal to

(a) $\tan^2 A - \tan^4 A$ (b) $\tan^4 A - \tan^2 A$

(c) $\tan^4 A + \tan^3 A$ (d) $\tan^2 A + \tan^4 A$

7. If θ & $2\theta - 45°$ are acute angles such that $\sin\theta = \cos(2\theta - 45°)$, then $\tan\theta$ is equal to

(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

8. The value of $\tan^6 20° - 33\tan^4 20° + 27\tan^2 20°$ is :

(a) 2 (b) 3 (c) 4 (d) 5

9. The value of $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 2

10. If $\frac{x\,\text{cosec}^2 30°.\sec^2 45°}{8\cos^2 45°.\sin^2 60°} = \tan^2 60° - \tan^2 30°$, then $x = ?$

(a) 1 (b) -1

(c) 2 (d) 0

11. If $x\sin^3\theta + y\cos^3\theta = \sin\theta.\cos\theta.$ and $x\sin\theta - y\cos\theta = 0$, find the value of $x^2 + y^2$

(a) 0 (b) 1 (c) 2 (d) 3

12. What is $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ equal to?

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$

(c) $\text{cosec}\theta + \cot\theta$ (d) $\text{cosec}\theta - \cot\theta$

13. What is $\log(\tan 1°) + \log(\tan 2°) + \log(\tan 3°) + \ldots\ldots + \log(\tan 89°)$ equal to?

(a) 0 (b) 1 (c) 2 (d) -1

14. If $\sin 7x = \cos 11x$, then the value of $\tan 9x + \cot 9x$ is

(a) 1 (b) 2 (c) 3 (d) 4

15. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is:

(a) 0 (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

16. If $\sin\theta + \sin^2\theta = 1$. Find the value of $\cos^2\theta + \cos^4\theta$

(a) 0 (b) 2 (c) 1 (d) 3

17. How many degrees are there in an angle which equals two third of its complement?

(a) 36° (b) 45° (c) 48° (d) 60°

18. For any real value of θ, $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = ?$

(a) $\cot\theta - \text{cosec}\,\theta$ (b) $\sec\theta - \tan\theta$

(c) $\text{cosec}\,\theta - \cot\theta$ (d) $\tan\theta - \sec\theta$

19. $\tan 5x \tan 3x \tan 2x = ?$

(a) $\tan 5x - \tan 3x - \tan 2x$ (b) 0

(c) $\frac{\sin 6x - \sin 3x - \sin 2x}{\cos 5x - \cos 3x - \cos 2x}$ (d) $\tan 9x$

20. Find the maximum and minimum value of $16^{\sin\theta}.4^{\sin\theta}.2^{\cos\theta}.128^{\cos\theta}$

(a) 10, –10 (b) 5, –5 (c) 1, –1 (d) 0, –1

GP_3386

21. Find the maximum and minimum value of $15\sin^2\theta + 10\cos^2\theta$.
(a) 25, 15 (b) 20, 10 (c) 15, 10 (d) 30, 20

22. If $P = \sin^2 20° + \sin^2 40° + \sin^2 50° + \sin^2 70°$, then.
(a) $0 < P < 1$ (b) $1 < P < 1.5$
(c) $1.5 < P < 2$ (d) $P = 2$

23. If $3\sin\theta + 4\cos\theta = 5$, then what is $3\cos\theta - 4\sin\theta$ equal to?
(a) 0 (b) 3 (c) 4 (d) 5

24. If $\tan\alpha, \tan\beta$ are the roots of $x^2 + px + q = 0$ $(p \neq q)$ then $\tan(\alpha + \beta) = ?$
(a) $p - 1$ (b) $\frac{p}{q-1}$
(c) $2q + p$ (d) None of these

25. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^2\theta + \tan^4\theta$ is
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) 0

26. The value of $\tan 4°.\tan 43°.\tan 47°.\tan 86°$ is
(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

27. The minimum value of $\cos 2\theta + \cos\theta$ for real values of θ is–
(a) $-9/8$ (b) 0
(c) -2 (d) None of these

28. If $5\tan\theta - 4 = 0$, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta} = ?$
(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{6}$

29. The value of $\tan^6 20° - 33\tan^4 20° + 27\tan^2 20°$ is :
(a) 2 (b) 3 (c) 4 (d) 5

30. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = ?$
(a) $\frac{5}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{12}$ (d) $\frac{3}{4}$

31. If θ is an acute angle such that $\tan^2\theta = \frac{8}{7}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is
(a) $\frac{7}{8}$ (b) $\frac{8}{7}$ (c) $\frac{7}{4}$ (d) $\frac{64}{49}$

32. If $3\cos\theta = 5\sin\theta$, then the value of $\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is equal to
(a) $\frac{271}{979}$ (b) $\frac{376}{2937}$
(c) $\frac{542}{2937}$ (d) None of these

33. If $\frac{x\operatorname{cosec}^2 30°.\sec^2 45°}{8\cos^2 45°.\sin^2 60°} = \tan^2 60° - \tan^2 30°$, then $x = ?$
(a) 1 (b) -1 (c) 2 (d) 0

34. If θ is an acute angle such that $\sec^2\theta = 3$, then $\frac{\tan^2\theta - \operatorname{cosec}^2\theta}{\tan^2\theta + \operatorname{cosec}^2\theta}$ is
(a) $\frac{4}{7}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

35. If θ & $2\theta - 45°$ are acute angles such that $\sin\theta = \cos(2\theta - 45°)$, then $\tan\theta$ is equal to
(a) 1 (b) -1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

36. If 5θ & 4θ are acute angles satisfying $\sin 5\theta = \cos 4\theta$ then $2\sin 3\theta - \sqrt{3}\tan 3\theta = ?$
(a) 1 (b) 0 (c) -1 (d) $\frac{1}{\sqrt{3}}$

37. If $\sec\theta + \tan\theta = x$, then $\sec\theta = ?$
(a) $\frac{x^2+1}{x}$ (b) $\frac{x^2+1}{2x}$ (c) $\frac{x^2-1}{2x}$ (d) $\frac{x^2-1}{x}$

38. $\frac{\tan\theta}{\sec\theta - 1} + \frac{\tan\theta}{\sec\theta + 1}$ is equal to
(a) $2\tan\theta$ (b) $2\sec\theta$
(c) $2\operatorname{cosec}\theta$ (d) $2\tan\theta.\sec\theta$.

39. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then $a^2 + b^2 =$
(a) $m^2 - n^2$ (b) $m^2 n^2$ (c) $n^2 - m^2$ (d) $m^2 + n^2$

40. If $\sin\alpha + \cos\beta = 2$ $(0° \leq \beta < \alpha \leq 90°)$, then $\sin\left(\frac{2\alpha + \beta}{3}\right) =$
(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{3}$ (c) $\sin\frac{\alpha}{3}$ (d) $\cos\frac{2\alpha}{3}$

41. If $\sin\alpha\sec(30° + \alpha) = 1$ $(0 < \alpha < 60°)$, then the value of $\sin\alpha + \cos 2\alpha$ is

(a) 1 (b) $\frac{2+\sqrt{3}}{2\sqrt{3}}$ (c) 0 (d) $\sqrt{2}$

42. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is
(a) 0 (b) 3 (c) 2 (d) 1

43. If cosec 39° = x, the value of $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \frac{1}{\sin^2 51° \sec^2 39°}$ is

(a) $\sqrt{x^2-1}$ (b) $\sqrt{1-x^2}$ (c) x^2-1 (d) $1-x^2$

44. If $\tan 2\theta \tan 4\theta = 1$, then the value of $\tan 3\theta$ is

(a) $\sqrt{3}$ (b) 0 (c) 1 (d) $\frac{1}{\sqrt{3}}$

45. If $\sec\theta = x + \frac{1}{4x}$ $(0° < \theta < 90°)$, then $\sec\theta + \tan\theta$ is equal to

(a) $\frac{x}{2}$ (b) 2x (c) x (d) $\frac{1}{2x}$

46. If $x = a\sec\theta\cos\phi$, $y = b\sec\theta\sin\phi$, $z = c\tan\theta$, then, the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$ is :

(a) 1 (b) 4 (c) 9 (d) 0

47. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = \frac{5}{3}$, then $\sin\theta$ is equal to :

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

48. If $0 \le \theta \le \frac{\pi}{2}$ and $\sec^2\theta + \tan^2\theta = 7$, then θ is

(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian

(c) $\frac{\pi}{5}$ radian (d) $\frac{\pi}{6}$ radian

49. The simplest value of
$\sin^2 x + 2\tan^2 x - 2\sec^2 x + \cos^2 x$ is
(a) 1 (b) 0 (c) –1 (d) 2

50. If $x = a\sin\theta$ and $y = b\tan\theta$ then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2}$ is

(a) 1 (b) 2 (c) 3 (d) 4

51. The radian measure of $63°14'51''$ is

(a) $\left(\frac{2811\pi}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$

(c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

52. $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$ $\left(\text{where } \theta \ne \frac{\pi}{2}\right)$ is equal to

(a) $\frac{1+\sin\theta}{\cos\theta}$ (b) $\frac{1-\sin\theta}{\cos\theta}$

(c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1+\cos\theta}{\sin\theta}$

53. The simplified value of $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\theta)$ is
(a) –2 (b) 2 (c) 1 (d) –1

54. The value of $\frac{\sin 53°}{\cos 37°} \div \frac{\cot 65°}{\tan 25°}$ is
(a) 2 (b) 1 (c) 3 (d) 0

55. The value of $\frac{\cos 60° + \sin 60°}{\cos 60° - \sin 60°}$ is

(a) –1 (b) $\sqrt{3}+2$

(c) $-(2+\sqrt{3})$ (d) $\sqrt{3}-2$

56. In a triangle, the angles are in the ratio 2 : 5 : 3. What is the value of the least angle in the radian ?

(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{10}$ (c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{5}$

57. If $\tan\alpha + \cot\alpha = 2$, then the value of $\tan^7\alpha + \cot^7\alpha$ is
(a) 2 (b) 16 (c) 64 (d) 128

58. From 125 metre high towers, the angle of depression of a car is 45°. Then how far the car is from the tower ?
(a) 125 metre (b) 60 metre
(c) 75 metre (d) 95 metre

59. If $\sin 17° = \frac{x}{y}$, then $\sec 17° - \sin 73°$ is equal to

(a) $\frac{y}{\sqrt{y^2-x^2}}$ (b) $\frac{y^2}{\left(x\sqrt{y^2-x^2}\right)}$

(c) $\frac{x}{\left(y\sqrt{y^2-x^2}\right)}$ (d) $\frac{x^2}{\left(y\sqrt{y^2-x^2}\right)}$

GP_3386

60. If θ is a positive acute angle and $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$, then the value of $\operatorname{cosec}\theta$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) 1

61. If $\cos\alpha + \sec\alpha = \sqrt{3}$, then the value of $\cos^3\alpha + \sec^3\alpha$ is

(a) 2 (b) 1 (c) 0 (d) 4

62. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\cot\theta$ is

(a) $\sqrt{2}+1$ (b) $\sqrt{2}-1$ (c) $\sqrt{3}-1$ (d) $\sqrt{3}+1$

63. The value of $\sin^2 1° + \sin^2 2° + \sin^2 3° + ... + \sin^2 89°$ is

(a) 22 (b) 44 (c) $22\frac{1}{2}$ (d) $44\frac{1}{2}$

64. If $\tan\theta = \frac{x\sin\phi}{1-x\cos\phi}$, then $\cot\theta + \cot\phi =$

(a) $\frac{\sin\phi}{\sin\theta}$ (b) $\frac{1}{x\sin\phi}$

(c) $\frac{\sin\theta}{1-\cos\theta}$ (d) $\frac{\sin\theta}{1-\cos\phi}$

65. If $\cos(81° + \theta) = \sin\left(\frac{k}{3} - \theta\right)$, then $k =$

(a) 9° (b) 30° (c) 27° (d) 45°

66. If $\sin(A-B) = \frac{1}{2}$ and $\cos(A+B) = \frac{1}{2}$, then

(a) $A = 60°, B = 45°$ (b) $A = 30°, B = 15°$

(c) $A = 45°, B = 15°$ (d) $A = 60°, B = 30°$

67. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with level ground such that $\tan\theta = \frac{15}{8}$, how high is the kite ?

(a) 75 m (b) 78.05 m

(c) 226 m (d) None of these

68. A ladder is resting against a wall at height of 10m. If the ladder is inclined with the ground at an angle of 30°, then the distance of the foot of the ladder from the wall is

(a) $10\sqrt{3}$ m (b) $20\sqrt{3}$ m (c) $\frac{10}{\sqrt{3}}$ m (d) $\frac{20}{\sqrt{3}}$ m

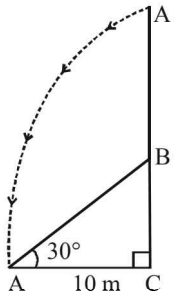
69. The value of the expression

$[\operatorname{cosec}(75° + \theta) - \sec(15° - \theta) - \tan(55° + \theta) + \cot(35° - \theta)]$

(a) –1 (b) 0 (c) 1 (d) $\frac{3}{2}$

LEVEL - 2

1. The top of a broken tree has its top touching the ground (shown in the adjoining figure) at a distance of 10 m from the bottom. If the angle made by the broken part with ground is 30°, then the length of the broken part is



(a) $10\sqrt{3}$ cm (b) $\frac{20}{\sqrt{3}}$ m (c) 20 cm (d) $20\sqrt{3}$ m

2. An aeroplane at a height of 600 m passes vertically above another aeroplane at an instant when their angles of elevation at the same observing point are 60° and 45° respectively. How many metres higher is the one from the other ?

(a) 286.53 m (b) 274.53 m

(c) 253.58 m (d) 263.83 m

3. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is.

(a) $\frac{17}{13}$ (b) $\frac{13}{17}$ (c) $\frac{1}{13}$ (d) $\frac{1}{17}$

4. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°; when he reaches 20 m far (away) from the bank, he finds the angle to be 30°. The height of the tree and the breadth of the river are :

(a) $10\sqrt{3}$ m, 10 m (b) 10, $10\sqrt{3}$ m

(c) 20 m, 30 m (d) None of these

5. If $\sin\theta + \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta - \cos\theta = ?$

(a) $\pm\frac{\sqrt{3}}{2}$ (b) $\pm\frac{\sqrt{7}}{2}$ (c) $\pm\frac{7}{2}$ (d) $\pm\sqrt{7}$

6. If $2\sin\alpha + 15\cos^2\alpha = 7, (0° < \alpha < 90°)$, find the value of $\cot\alpha$

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

7. If $\sin x + \sin y = a$ and $\cos x + \cos y = b$, what is $\sin x . \sin y + \cos x . \cos y$ equal to?

(a) $a + b - ab$ (b) $a + b + ab$

(c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2 + b^2 - 2}{2}\right)$

8. If $\sin x + \cos x = c$, then $\sin^6 x + \cos^6 x$ is equal to.

(a) $\frac{1 + 6c^2 - 3c^4}{16}$ (b) $\frac{1 + 6c^2 - 3c^4}{4}$

(c) $\frac{1 + 6c^2 + 3c^4}{16}$ (d) $\frac{1 + 6c^2 + 3c^4}{4}$

9. If A, B, C and D are the successive angles of a cyclic quadrilateral, then what is $\cos A + \cos B + \cos C + \cos D$ equal to?

(a) 4 (b) 2 (c) 1 (d) 0

10. If $1 + \sin x + \sin^2 x + \sin^3 x + ...\infty = 4 + 2\sqrt{3}$, find the value of x

(a) 60^o (b) 30^o (c) 90^o (d) 0^o

11. In a $\Delta ABC, \angle B = \frac{\pi}{3}, \angle C = \frac{\pi}{4}$ and D divides BC internally in the ratio 1 : 3, then $\frac{\sin\angle BAD}{\sin\angle CAD}$ is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{6}$

12. Find the minimum & maximum value of $\sin^6\theta + \cos^6\theta$.

(a) $\frac{1}{4}, 1$ (b) $\frac{1}{4}, \frac{1}{2}$ (c) $\frac{3}{4}, 1$ (d) $\frac{1}{4}, \frac{3}{4}$

13. If $\tan\theta = 1$, then find the value of

$$\frac{8\sin\theta + 5\sin\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$$

(a) 2 (b) $2\frac{1}{2}$ (c) 3 (d) $\frac{4}{5}$

14. If $\tan 15° = 2 - \sqrt{3}$, the value of $\tan 15°. \cot 75° + \tan 75° . \cot 15°$ is.

(a) 14 (b) 12 (c) 10 (d) 8

15. The value of $(\sin^2 1° + \sin^2 3° + \sin^2 5° + ...+ \sin^2 85° + \sin^2 87° + \sin^2 89°)$

(a) $21\frac{1}{2}$ (b) 22

(c) $22\frac{1}{2}$ (d) $23\frac{1}{2}$

16. If $y = \frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha}$ then $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is equal to

(a) 1/y (b) y (c) 1 – y (d) 1 + y

17. If $1 + \sin^2 A = 3 \sin A \cos A$, then what are the possible values of tan A?

(a) 1/4, 2 (b) 1/6, 3
(c) 1/2, 1 (d) 1/8, 4

18. If $\theta + \phi = \frac{\pi}{6}$, what is the value of $(\sqrt{3} + \tan\theta)(\sqrt{3} + \tan\phi)$?

(a) 1 (b) – 1 (c) 4 (d) – 4

19. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is 60°)?

(a) 20 feet (b) 30 feet

(c) 40 feet (d) $20\sqrt{2}$ feet

20. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is $\frac{1}{2}$. What is the height of the pole ?

(a) 110m (b) 200m
(c) 120m (d) 150m

21. The correct value of the parameter 't' of the identity $2(\sin^6 x + \cos^6 x) + t(\sin^4 x + \cos^4 x) = -1$ is:

(a) 0 (b) –1 (c) –2 (d) –3

GP_3386

22. If $a\cos\theta - b\sin\theta = c$, then $a\cos\theta + b\sin\theta = ?$

(a) $\pm\sqrt{a^2+b^2+c^2}$ (b) $\pm\sqrt{a^2+b^2-c^2}$

(c) $\pm\sqrt{c^2-a^2-b^2}$ (d) None of these

23. The angular elevation of a tower CD at a place A due south of it is 60° ; and at a place B due west of A, the elevation is 30°. If AB = 3 km, the height of the tower is

(a) $2\sqrt{3}$ km (b) $3\sqrt{6}$ km

(c) $\frac{3\sqrt{3}}{2}$ km (d) $\frac{3\sqrt{6}}{4}$ km

24. If $\tan(\theta_1+\theta_2) = \sqrt{3}$ and $\sec(\theta_1-\theta_2) = \frac{2}{\sqrt{3}}$, then the value of $\sin 2\theta_1 + \tan 3\theta_2$ is equal to (assume that $0 < \theta_1 - \theta_2 < \theta_1 + \theta_2 < 90°$)

(a) 0 (b) 3 (c) 1 (d) 2

25. A kite is flying at a height of 50 metre. If the length of string is 100 metre then the inclination of string to the horizontal ground in degree measure is

(a) 90° (b) 60° (c) 45° (d) 30°

26. From the top of a light-house at a height 20 metres above sea-level, the angle of depression of a ship is 30°. The distance of the ship from the foot of the light-house is

(a) 20m (b) $20\sqrt{3}$ m

(c) 30m (d) $30\sqrt{3}$ m

27. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then the relation between x and y is

(a) $2x^2+y^2=2$ (b) $x^2+4y^2=4$

(c) $x^2+4y^2=1$ (d) $4x^2+y^2=4$

28. If $\sec\theta + \tan\theta = \sqrt{3}$, then the positive value of $\sin\theta$ is

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) 1

29. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha}$ is

(a) 4 (b) 0 (c) $\frac{1}{8}$ (d) 1

30. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft respectively are complementary angles. Then the height of the tower is

(a) 9 ft (b) 12 ft

(c) 16 ft (d) 144 ft

31. If $\sin^2\alpha = \cos^3\alpha$, then the value of $(\cot^6\alpha - \cot^2\alpha)$ is

(a) 1 (b) 0 (c) –1 (d) 2

32. The value of

$$\frac{\cot 5°.\cot 10°.\cot 15°.\cot 60°.\cot 75°.\cot 80°.\cot 85°}{(\cos^2 20° + \cos^2 70°)+2}$$ is

(a) $\frac{9}{\sqrt{3}}$ (b) $\frac{1}{9}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{9}$

33. If $x = a\cos\theta - b\sin\theta$, $y = b\cos\theta + a\sin\theta$, then find the value of $x^2 + y^2$.

(a) a^2 (b) b^2

(c) $\frac{a^2}{b^2}$ (d) $a^2 + b^2$

34. The value of $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$ is equal to

(a) –1 (b) 1 (c) 2 (d) 0

35. The shadow of a tower standing on a level plane is found to be 30 m longer when the Sun's altitude changes from 60° to 45°. The height of the tower is

(a) $15\left(3+\sqrt{3}\right)$m (b) $15\left(\sqrt{3}+1\right)$m

(c) $15\left(\sqrt{3}-1\right)$m (d) $15\left(3-\sqrt{3}\right)$m

36. If $x = r\sin\alpha\cos\beta$, $y = r\sin\alpha\sin\beta$ and $z = r\cos\alpha$, then

(a) $x^2+y^2+z^2=r^2$

(b) $x^2+y^2-z^2=r^2$

(c) $x^2-y^2+z^2=r^2$

(d) $x^2+y^2-z^2=r^2$

37. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1, \frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1,$ then

(a) $x^2+y^2=a^2+b^2$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

(c) $a^2x^2+b^2y^2=1$ (d) None of these

38. An aeroplane flying horizontally 1 km. above the ground is observed at an elevation of 60° and after 10 seconds the elevation is observed to be 30°. The uniform speed of the aeroplane in km/h is

(a) 240 (b) $240\sqrt{3}$

(c) $60\sqrt{3}$ (d) None of these

39. A 25 m ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4m, then the foot of the ladder will slide

(a) 5m (b) 8m (c) 9m (d) 15m

40. The angles of elevation of the top of a tower from two points at distances m and n metres are complementary. If the two points and the base of the tower are on the same straight line, then the height of the tower is

(a) $\sqrt{mn}$ (b) mn

(c) $\frac{m}{n}$ (d) None of these

41. The top of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with the horizontal, then the length of the wire is

(a) 12m (b) 10m (c) 8m (d) 6m

42. Two men standing on opposite sides of a flag-staff measure the angles of the top of the flag-staff is 30° and 60°. If the height of the flagstaff is 20 m, distance between the men is

(a) 46.19m (b) 40m

(c) 50m (d) 30m

43. If $x\sin\theta+y\cos\theta=\sqrt{x^2+y^2}$ and $\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}=\frac{1}{x^2+y^2}$, then the correct relation is

(a) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (b) $\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (d) $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

44. If $x=p\sec\theta$ and $y=q\tan\theta$ then

(a) $x^2-y^2=p^2q^2$

(b) $x^2q^2-y^2p^2=pq$

(c) $x^2q^2-y^2p^2=\frac{1}{p^2q^2}$

(d) $x^2q^2-y^2p^2=p^2q^2$

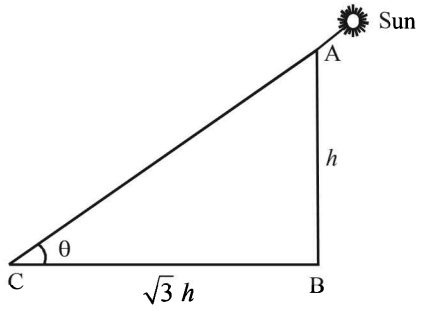
GP_3386

Hints and Solutions

Level 1

1. (d) $\sin 30° = \frac{BC}{AC} \Rightarrow \frac{1}{2} = \frac{BC}{6\text{ cm}} \Rightarrow BC = 3\text{ cm}.$

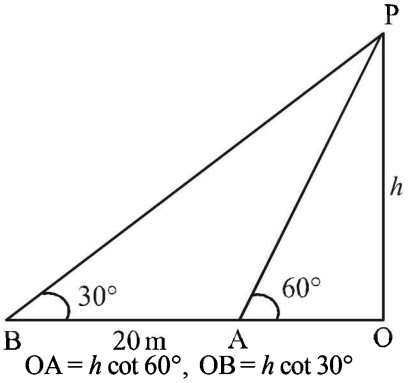
2. (b) Let height of tower (AB) be h metres, then length of its shadow $(BC) = \sqrt{3}\,h$ metres.



Let angle of elevation be θ, then

$$\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

3. (c)



$OA = h\cot 60°$, $OB = h\cot 30°$

$OB - OA = 20 = h(\cot 30° - \cot 60°)$

$$\Rightarrow h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{20\sqrt{3}}{2} = 10\sqrt{3}\text{ m}$$

4. (b) Given, $\sec\theta + \tan\theta = x$ ...(i)

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\text{or } \sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{x} \quad ...(ii)$$

Adding (i) & (ii), we get

$$2\sec\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$$

$$\sec\theta = \frac{x^2+1}{2x}$$

5. (b) Given, angles A, B, C are increasing AP.

$B - A = C - B$

$$B = \frac{A+C}{2} \Rightarrow A + C = 2B \quad .....(i)$$

A, B, C are the angles of ΔABC,

$A + B + C = 180°$

$\Rightarrow B + (A + C) = 180°$ [from (i), $A + C = 2B$]

$\Rightarrow B + 2B = 180°$

$B = 60°$

$$\therefore \sin B = \sin 60° = \frac{\sqrt{3}}{2}$$

6. (d) $\sec^4 A - \sec^2 A = \sec^2 A(\sec^2 A - 1)$

$= (1 + \tan^2 A) \times \tan^2 A$ [$\because 1 + \tan^2 A = \sec^2 A$]

$= \tan^2 A + \tan^4 A$

7. (a) $\sin\theta = \cos(2\theta - 45°)$

or $\cos(90° - \theta) = \cos(2\theta - 45°)$

$\Rightarrow 90° - \theta = 2\theta - 45°$

$\Rightarrow \theta = 45°$

$\therefore \tan\theta = \tan 45° = 1$

8. (b) $\sqrt{3} = \tan 60° = \tan(3 \times 20°)$

$$= \frac{3\tan 20° - \tan^3 20°}{1 - 3\tan^2 20°}$$

Squaring, $3 = \frac{9t^2 + t^6 - 6t^4}{1 + 9t^4 - 6t^2}$, $\tan 20° = t$

$\Rightarrow t^6 - 33t^4 + 27t^2 = 3$

$\Rightarrow \tan^6 20° - 33\tan^4 20° + 27\tan^2 20° = 3$

9. (c) $\cos 20° = \cos(90° - 70°) = \sin 70°$

$\cos 70° = \sin 20°$

$$\therefore \frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$$

$$= \frac{\sin^3 70° - \sin^3 20°}{\sin^3 70° - \sin^3 20°} = 1$$

10. (a) $\dfrac{x\times 2^2.(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2}=(\sqrt{3})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

or, $\dfrac{x\times 4\times 2}{8\times\frac{1}{2}\times\frac{3}{4}}=3-\frac{1}{3}\Rightarrow\frac{8x}{3}=\frac{9-1}{3}$

or, $\frac{8}{3}x=\frac{8}{3}$ $\boxed{x=1}$

11. (b) $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$

$x\sin\theta-y\cos\theta=0$

$\Rightarrow x\sin\theta=y\cos\theta$

$\Rightarrow x.\sin\theta.\sin^2\theta+y\cos^3\theta=\sin\theta.\cos\theta$

$\Rightarrow y\cos\theta.\sin^2\theta+y\cos^3\theta=\sin\theta.\cos\theta$

$\Rightarrow y\cos\theta(\sin^2\theta+\cos\theta)=\sin\theta\cos\theta$

$\Rightarrow y\cos\theta=\sin\theta\cos\theta$

$y=\sin\theta$

Similarly, $x=\cos\theta$

$x^2+y^2=\sin^2\theta+\cos^2\theta=1.$

12. (b) $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}}=\sqrt{\dfrac{(1+\sin\theta)\times(1+\sin\theta)}{(1-\sin\theta)\times(1+\sin\theta)}}$

$=\sqrt{\dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}}=\sqrt{\dfrac{(1+\sin\theta)^2}{\cos^2\theta}}$

$=\dfrac{1+\sin\theta}{\cos\theta}=\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}$

$=\sec\theta+\tan\theta.$

13. (a) $\log(\tan 1°)+\log(\tan 2°)+\log(\tan 3°)+$ ..... $+\log(\tan 89°)$

$=\log(\tan 1°.\tan 2°.\tan 3°\ldots\ldots.\tan 89°)$

$=\log(\tan 1°.\tan 89°.\tan 2°.\tan 88°.$ ..... $\tan 45°)$

$[\tan\theta.\tan(90-\theta)=\tan\theta.\cot\theta=1].$

$=\log(1\times 1\times 1\times 1\times 1\ldots\ldots\ldots\times 1)$

$=\log 1=0.$

14. (b) $\sin 7x=\cos 11x$

$\Rightarrow \sin 7x=\sin(90-11x)$

$\Rightarrow 7x=90-11x$

$\Rightarrow 7x+11x=90$

$x=\dfrac{90}{18}=5$, So, $\tan 9x+\cot 9x$

$=\tan 45°+\cot 45°=1+1=2.$

15. (c) $\cos^4\theta-\sin^4\theta=\dfrac{2}{3}$

$(\cos^2\theta)^2-(\sin^2\theta)^2=\dfrac{2}{3}$

$\Rightarrow (\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)=\dfrac{2}{3}$

$\Rightarrow \cos^2\theta-\sin^2\theta=\dfrac{2}{3}$

$\Rightarrow \cos^2\theta-(1-\cos^2\theta)=\dfrac{2}{3}$

$\Rightarrow \cos^2\theta-1+\cos^2\theta=\dfrac{2}{3}\Rightarrow 2\cos^2\theta-1=\dfrac{2}{3}$

16. (c) $\sin\theta+\sin^2\theta=1\Rightarrow\sin\theta=1-\sin^2\theta$

$\Rightarrow \sin\theta=\cos^2\theta$ Now,

$\cos^2\theta+\cos^4\theta=\cos^2\theta+(\cos^2\theta)^2$

$=\sin\theta+\sin^2\theta=1.$

17. (a) Let the required angle be x,

then complement of $x=90-x.$

According to question,

$x=\dfrac{2}{3}(90-x)$

$\Rightarrow 3x=180-2x\Rightarrow 5x=180$

$x=\dfrac{180}{5}=36°$

18. (c) $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}=\sqrt{\dfrac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}}=\sqrt{\dfrac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}}$

$=\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}=\sqrt{\dfrac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$

$=\sqrt{\dfrac{(1-\cos\theta)^2}{1+\cos^2\theta}}=\sqrt{\dfrac{(1-\cos\theta)^2}{\sin^2\theta}}=\dfrac{1-\cos\theta}{\sin\theta}$

$=\dfrac{1}{\sin\theta}-\dfrac{\cos\theta}{\sin\theta}=\operatorname{cosec}\theta-\cot\theta$

19. (a) We know, $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A.\tan B}$

$\Rightarrow \tan(5x)=\dfrac{\tan 3x+\tan 2x}{1-\tan 3x.\tan 2x}$

GP_3386

$\Rightarrow \tan 5x - \tan 5x \tan 3x . \tan 2x = \tan 3x + \tan 2x$

$\Rightarrow \tan 5x \tan 3x . \tan 2x = \tan 5x - \tan 3x - \tan 2x.$

20. (a) $16^{\sin\theta} . 4^{\sin\theta} . 2^{\cos\theta} . 128^{\cos\theta}$

$= 2^{4\sin\theta} . 2^{2\sin\theta} . 2^{\cos\theta} . 2^{7\cos\theta}$

$= 2^{6\sin\theta + 8\cos\theta}$

$\max 6\sin\theta + 8\cos\theta$

[Minimum value of $a\sin\theta + b\cos\theta$ $= -\sqrt{a^2+b^2}$ Maximum value of $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}$]

$= \sqrt{6^2+8^2} = 10$

Min $6\sin\theta + 8\cos\theta$

$= -\sqrt{6^2+8^2} = -10$

Maximum value of the expression $= 2^{10}$

Minimum value of the expression $= 2^{-10}$

21. (c) $15\sin^2\theta + 10\cos^2\theta$

$= 15\sin^2\theta + 10(1 - \sin^2\theta)$

$= 15\sin^2\theta + 10 - 10\sin^2\theta$

$= 5\sin^2\theta + 10$

Minimum value of $\sin^2\theta = 0$ and Max value of $\sin^2\theta = 1$

Hence,

Minimum $5\sin^2\theta + 10 = 5 \times 0 + 10 = 10$

Maximum $5\sin^2\theta + 10 = 5 \times 1 + 10 = 15.$

22. (d) $P = \sin^2 20° + \sin^2 40° + \sin^2 50° + \sin^2 70°.$

$= \sin^2 20° + \sin^2 70° + \sin^2 40° + \sin^2 50°$

$= \sin^2 20° + \sin^2 (90 - 20) + \sin^2 40° + \sin^2(90-40)$

$= \sin^2 20° + \cos^2 20° + \sin^2 40° + \cos^2 40°$

$= 1 + 1 = 2.$

23. (a) $3\sin\theta + 4\cos\theta = 5$ ...(i)

$3\cos\theta - 4\sin\theta = k$ ...(ii)

$(i)^2 + (ii)^2$

$9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta = 25$

$$\frac{9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta = k^2}{9(\sin^2\theta + \cos^2\theta) + 16(\cos^2\theta + \sin^2\theta) = 25 + k^2}$$

$\Rightarrow 9 + 16 = 25 + k^2$

$k = 0.$

24. (b) $\tan\alpha + \tan\beta = -p$

$\tan\alpha . \tan\beta = q$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta}$$

$$= \frac{-p}{1-q} = \frac{p}{q-1}$$

25. (b) Given, $\cos^2\theta + \cos^4\theta = 1$

or, $\cos^4\theta = 1 - \cos^2\theta$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\cos^4\theta = \sin^2\theta.$

$$\text{or, } 1 = \frac{\sin^2\theta}{\cos^2\theta} . \frac{1}{\cos^2\theta}$$

$\Rightarrow \tan^2\theta . \sec^2\theta = 1$

or, $\tan^2\theta . (1 + \tan^2\theta) = 1$

$[\because \sec^2\theta - \tan^2\theta = 1]$

or, $\boxed{\tan^2\theta + \tan^4\theta = 1}$

26. (b) $\tan 4° . \tan 43° . \tan 47° . \tan 86°$

$= \tan 4° . \tan 43° . \tan(90° - 43°) \tan(90° - 4°)$

$= \tan 4° . \tan 43° . \cot 43° \cot 4°$

$[\because \tan(90 - \theta) = \cot\theta]$

$$= \tan 4° \times \tan 43° \times \frac{1}{\tan 43°} \times \frac{1}{\tan 4°}$$

$$\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$= 1$

27. (a) Let $S = \cos 2\theta + \cos\theta$

$= 2\cos^2\theta - 1 + \cos\theta$

$$= -1 + 2\left(\cos^2\theta + \frac{1}{2}\cos\theta + \frac{1}{16}\right) - \frac{1}{8}$$

$$= -\frac{9}{8} + 2\left(\cos\theta + \frac{1}{4}\right)^2 \geq -\frac{9}{8}$$

So, the minimum value $S = -9/8$

28. (c) Given, $5\tan\theta - 4 = 0$

$$\Rightarrow \tan\theta = \frac{4}{5}$$

$$\text{Expression} = \frac{\frac{(5\sin\theta - 4\cos\theta)}{\cos\theta}}{\frac{(5\sin\theta + 4\cos\theta)}{\cos\theta}} = \frac{5\tan\theta - 4}{5\tan\theta + 4}$$

$$= \frac{5 \times \frac{4}{5} - 4}{5 \times \frac{4}{5} + 4} = \frac{4-4}{4+4} = \frac{0}{8} = 0$$

29. (b) $\sqrt{3} = \tan 60° = \tan(3 \times 20°)$

$$= \frac{3\tan 20° - \tan^3 20°}{1 - 3\tan^2 20°}$$

Squaring, $3=\frac{9t^2+t^6-6t^4}{1+9t^4-6t^2}$, $\tan 20^\circ = t$

$\Rightarrow t^6-33t^4+27t^2=3$

$\Rightarrow \tan^6 20^\circ-33\tan^4 20^\circ+27\tan^2 20^\circ=3$

30. (d) Given, $\tan\theta=\frac{1}{\sqrt{7}}$

$$\sec\theta=\sqrt{1+\tan^2\theta}=\sqrt{1+\left(\frac{1}{\sqrt{7}}\right)^2}=\sqrt{\frac{8}{7}}$$

$$\text{cosec}\,\theta=\frac{\sec\theta}{\tan\theta}=\frac{\sqrt{\frac{8}{7}}}{\sqrt{\frac{1}{7}}}=\sqrt{8}$$

$$\therefore \quad \frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta}=\frac{\left(\sqrt{8}\right)^2-\left(\sqrt{\frac{8}{7}}\right)^2}{\left(\sqrt{8}\right)^2+\left(\sqrt{\frac{8}{7}}\right)^2}$$

$$=\frac{8-\frac{8}{7}}{8+\frac{8}{7}}=\frac{8\left(1-\frac{1}{7}\right)}{8\left(1+\frac{1}{7}\right)}=\frac{\frac{6}{7}}{\frac{8}{7}}=\frac{6}{8}=\frac{3}{4}$$

31. (a) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$=\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\tan^2\theta}=\frac{1}{\frac{8}{7}}=\frac{7}{8}.$$

32. (a) Given, $3\cos\theta=5\sin\theta \Rightarrow \tan\theta=\frac{3}{5}$.

$$\sec\theta=\sqrt{1+\tan^2\theta}=\sqrt{1+\left(\frac{3}{5}\right)^2}$$

$$=\sqrt{\frac{25+9}{25}}=\frac{\sqrt{34}}{5}.$$

In expression, dividing the numerator & denominator by $\cos\theta$,

$$=\frac{5\tan\theta-2\sec^4\theta+2}{5\tan\theta+2\sec^4\theta-2}$$

$$=\frac{5\times\frac{3}{5}-2\times\left(\frac{\sqrt{34}}{5}\right)^4+2}{5\times\frac{3}{5}+2\times\left(\frac{\sqrt{34}}{5}\right)^4-2}$$

$$=\frac{3-2\times\frac{1156}{625}+2}{3+2\times\frac{1156}{625}-2}=\frac{5-\frac{2312}{625}}{1+\frac{2312}{625}}$$

$$=\frac{813}{2937}=\frac{271}{979}$$

33. (a) $\frac{x\times 2^2.(\sqrt{2})^2}{8\times\left(\frac{1}{\sqrt{2}}\right)^2\times\left(\frac{\sqrt{3}}{2}\right)^2}=(\sqrt{3})^2-\left(\frac{1}{\sqrt{3}}\right)^2$

or, $\frac{x\times 4\times 2}{8\times\frac{1}{2}\times\frac{3}{4}}=3-\frac{1}{3} \Rightarrow \frac{8x}{3}=\frac{9-1}{3}$

or, $\frac{8}{3}x=\frac{8}{3}$ $\boxed{x=1}$

34. (d) $\sec^2\theta=3$

$\tan^2\theta=\sec^2\theta-1=3-1=2$

$$\text{cosec}^2\theta=\frac{\sec^2\theta}{\tan^2\theta}=\frac{3}{2}$$

Now, $\frac{\tan^2\theta-\text{cosec}^2\theta}{\tan^2\theta+\text{cosec}^2\theta}=\frac{2-\frac{3}{2}}{2+\frac{3}{2}}=\frac{\frac{1}{2}}{\frac{7}{2}}=\frac{1}{7}$

35. (a) $\sin\theta=\cos(2\theta-45^\circ)$

or, $\cos(90^\circ-\theta)=\cos(2\theta-45^\circ)$

$\Rightarrow 90^\circ-\theta=2\theta-45^\circ$

$\Rightarrow \theta=45^\circ$

$\therefore \tan\theta=\tan 45^\circ=1$

36. (b) Given, $\sin 5\theta=\cos 4\theta=\sin(90^\circ-4\theta)$

$\Rightarrow 5\theta=90^\circ-4\theta$

$\theta=10^\circ$

GP_3386

$2\sin 3\theta - \sqrt{3}\tan 3\theta$

$= 2\sin 30° - \sqrt{3}\tan 30°$

$= 2\times\frac{1}{2} - \sqrt{3}\times\frac{1}{\sqrt{3}} = 1-1 = 0.$

37. (b) Given, $\sec\theta + \tan\theta = x$ .....(i)

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or $\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{x}$ .....(ii)

Adding (i) & (ii), we get

$2\sec\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$

$\sec\theta = \frac{x^2+1}{2x}$

38. (c) $\tan\theta\left[\frac{1}{\sec\theta-1} + \frac{1}{\sec\theta+1}\right]$

$= \tan\theta\left[\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta-1)(\sec\theta+1)}\right]$

$= \tan\theta\left[\frac{2\sec\theta}{\sec^2\theta-1}\right]$

$= \tan\theta\times\frac{2\sec\theta}{\tan^2\theta} = \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta\times\frac{\sin\theta}{\cos\theta}}$

$= \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta.$

39. (d) $(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

$= m^2 + n^2.$

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta.\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta.\cos\theta$

$= m^2 + n^2.$

or $a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= m^2 + n^2.$

or $a^2 + b^2 = m^2 + n^2$

40. (b) $\sin\alpha + \cos\beta = 2$

$\because$ Maximum value of $\cos\beta$ and $\sin\alpha$ is 1.

$\therefore \sin\alpha = 1$ and $\cos\beta = 1$

$\Rightarrow \alpha = 90°;\ \beta = 0°$

$\therefore \sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180°}{3}\right)$

$= \sin 60° = \frac{\sqrt{3}}{2}$

$\cos\frac{\alpha}{3} = \cos 30° = \frac{\sqrt{3}}{2}$

41. (a) $\frac{\sin\alpha}{\cos(30°+\alpha)} = 1$

$\Rightarrow \frac{\sin\alpha}{\sin(90°-30°-\alpha)} = 1$

$\Rightarrow \frac{\sin\alpha}{\sin(60°-\alpha)} = 1$

$\Rightarrow \sin\alpha = \sin(60°-\alpha)$

$\Rightarrow \alpha = 60° - \alpha$

$\Rightarrow 2\alpha = 60° \Rightarrow \alpha = 30°$

$\therefore \sin\alpha + \cos 2\alpha$

$= \sin 30° + \cos 60° = \frac{1}{2} + \frac{1}{2} = 1$

42. (c) $2\sin^2\theta + 3\cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$

$= 2 + \cos^2\theta$

$\therefore$ Minimum value of $\cos^2\theta = 0$

$\therefore$ Required minimum value $= 2 + 0 = 2$

43. (c) $\frac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51°$

$-\frac{1}{\sin^2 51°.\sec^2 39°}$

$= \sin^2 51° + \sin^2 39° + \tan^2(90°-39°)$

$-\frac{1}{\sin^2(90°-39°).\sec^2 39°}$

$= \cos^2 39° + \sin^2 39° + \cot^2 39°$

$-\frac{1}{\cos^2 39°.\sec^2 39°}$

$[\because \sin(90°-\theta) = \cos\theta, \tan(90°-\theta) = \cot\theta]$

$= 1 + \cot^2 39° - 1$

$= \text{cosec}^2 39° - 1 = x^2 - 1$

44. (c) $\tan 2\theta = \frac{1}{\tan 4\theta} = \cot 4\theta$

$\Rightarrow \tan 2\theta = \tan(90°-4\theta)$

$\Rightarrow 2\theta = 90° - 4\theta$

$\Rightarrow 6\theta = 90° \Rightarrow \theta = 15°$

$\therefore \tan 3\theta = \tan 45° = 1$

45. (b) $\sec\theta = \dfrac{4x^2+1}{4x}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$$= \sqrt{\left(\frac{4x^2+1}{4x}\right)^2 - 1} = \sqrt{\frac{\left(4x^2+1\right)^2 - (4x)^2}{(4x)^2}}$$

$$= \frac{(2x+1)(2x-1)}{4x} = \frac{4x^2-1}{4x}$$

$$\therefore \sec\theta + \tan\theta = \frac{4x^2+1}{4x} + \frac{4x^2-1}{4x}$$

$$= \frac{4x^2+1+4x^2-1}{4x} = \frac{8x^2}{4x} = 2x$$

46. (a) $x = a\sec\theta.\cos\phi;\ y = b\sec\theta.\sin\phi;\ z = c\tan\theta$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$$

$$= \frac{a^2\sec^2\theta\cos^2\phi}{a^2} + \frac{b^2\sec^2\theta\sin^2\phi}{b^2} - \frac{c^2\tan^2\theta}{c^2}$$

$= \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$
$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$
$= \sec^2\theta - \tan^2\theta = 1$

47. (a) $\dfrac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \dfrac{5}{3}$

$\Rightarrow 5\sec\theta - 5\tan\theta = 3\sec\theta + 3\tan\theta$
$\Rightarrow 2\sec\theta = 8\tan\theta$

$$\Rightarrow \frac{\tan\theta}{\sec\theta} = \frac{2}{8} = \frac{1}{4} \Rightarrow \frac{\sin\theta}{\cos\theta}\times\cos\theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{4}$$

48. (b) $\sec^2\theta + \tan^2\theta = 7$
$1 + \tan^2 + \tan^2\theta = 7$
$(\because\ 1 + \tan^2\theta = \sec^2\theta)$

$$\tan^2\theta = \frac{6}{2} = 3$$

$\tan\theta = \pm\sqrt{3}$
$\tan\theta = \sqrt{3}$ or $\tan\theta = -\sqrt{3}$
As $0 \le \theta \le \pi/2$

$\therefore\ \theta = \tan^{-1}\sqrt{3}$

$$\theta = \frac{\pi}{3}$$

49. (c) $\sin^2 x + 2\tan^2 x - 2\sec^2 x + \cos^2 x$
$\sin^2 x + \cos^2 x - 2(\sec^2 x - \tan^2 x)$
$1 - 2(1) = -1$

50. (a) $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = \dfrac{a^2}{a^2\sin^2\theta} - \dfrac{b^2}{b^2\tan^2\theta}$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$

51. (a) $63^\circ\ 14'\left(\dfrac{51}{60}\right)'$ [1 minute = 60 seconds]

$$\Rightarrow 63^\circ\left[14 + \frac{17}{20}\right]' \Rightarrow 63^\circ\left[\frac{297}{20}\right]' \Rightarrow 63^\circ + \frac{297}{20\times 60}$$

[1 degree = 60 minutes]

$$\Rightarrow \left(\frac{75897}{1200}\right)^\circ \Rightarrow \frac{75897}{1200}\times\frac{\pi}{180}\text{radian}$$

$$\Rightarrow \left(\frac{2811}{8000}\pi\right)^c$$

52. (a) $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

Dividing Numerator and Denominator by $\cos\theta$

$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}} \Rightarrow \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$$

$$\Rightarrow \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$\Rightarrow \frac{(\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$$

$\Rightarrow \tan\theta + \sec\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} \Rightarrow \frac{1+\sin\theta}{\cos\theta}$$

53. (b) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$\Rightarrow \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$\Rightarrow \left(\frac{\sin\theta + \cos\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

54. (b) $\dfrac{\sin 53^\circ}{\cos 37^\circ} \div \dfrac{\cot 65^\circ}{\tan 25^\circ}$

$$\frac{\sin 53^\circ}{\cos 37^\circ} \times \frac{\tan 25^\circ}{\cot 65^\circ} \Rightarrow \frac{\sin 53^\circ}{\cos(90^\circ-53^\circ)} \times \frac{\tan 25^\circ}{\cot(90^\circ-25^\circ)}$$

$$\Rightarrow \quad \frac{\sin 53^\circ}{\sin 53^\circ} \times \frac{\tan 25^\circ}{\tan 25^\circ} = 1$$

[$\because \cos(90^\circ-\theta) = \sin\theta$ and $\cot(90^\circ-\theta) = \tan\theta$]

55. (c) $$\frac{\cos 60^\circ+\sin 60^\circ}{\cos 60^\circ-\sin 60^\circ} = \frac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{\frac{1}{2}-\frac{\sqrt{3}}{2}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$\Rightarrow \frac{(1+\sqrt{3})^2}{1^2-(\sqrt{3})^2} = \frac{1+3+2\sqrt{3}}{1-3} = \frac{4+2\sqrt{3}}{-2}$$

$$\Rightarrow \frac{2(2+\sqrt{3})}{-2} = -(2+\sqrt{3})$$

56. (d) Let angles are $2x$, $5x$ and $3x$.

$2x+5x+3x = 180^\circ$

(sum of interior angle of triangles is 180°)

$10x = 180^\circ$

$x = 18^\circ$

$\therefore$ Least angle in degree $= 2x = 2\times 18 = 36^\circ$

$$\text{In radian} = \frac{\pi}{180^\circ} \times 36^\circ = \frac{\pi}{5}$$

57. (a) $\tan\alpha + \cot\alpha = 2$

$$\tan\alpha + \frac{1}{\tan\alpha} = 2 \Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$$

$$\Rightarrow \quad \tan^2\alpha - 2\tan\alpha + 1 = 0$$

$$\Rightarrow \quad \tan^2\alpha - \tan\alpha - \tan\alpha + 1 = 0$$

$$\Rightarrow \quad \tan\alpha(\tan\alpha-1) - 1(\tan\alpha-1) = 0$$

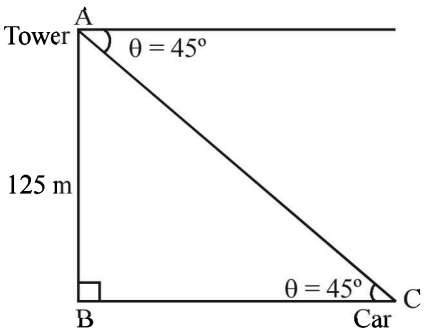
$$(\tan\alpha-1)(\tan\alpha-1) = 0$$

$$\therefore \quad \tan\alpha = 1$$

Now, $\tan^7\alpha + \cot^7\alpha = (\tan\alpha)^7$

$$+\frac{1}{(\tan\alpha)^7} = 1+1 = 2$$

58. (a)



In ΔABC

$$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 45^\circ = \frac{125}{BC} \Rightarrow 1 = \frac{125}{BC}$$

$BC = 125$ m

Hence, car is 125 m far from the tower.

59. (d) $\sin 17^\circ = \dfrac{x}{y}$

$$\cos 17^\circ = \sqrt{1-\frac{x^2}{y^2}} = \frac{\sqrt{y^2-x^2}}{y}$$

Now, $\sec 17^\circ - \sin 73^\circ = \sec 17^\circ - \cos 17^\circ$

$$= \frac{y}{\sqrt{y^2-x^2}} - \frac{\sqrt{y^2-x^2}}{y}$$

$$= \frac{y^2-y^2+x^2}{y\sqrt{y^2-x^2}} = \frac{x^2}{y\sqrt{y^2-x^2}}$$

60. (c) $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$

$$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \sqrt{3}$$

$$\frac{1+\cos\theta}{\sin\theta} = \sqrt{3}$$

$$\frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \sqrt{3}$$

$$\cot\frac{\theta}{2} = \sqrt{3}$$

$$\tan\frac{\theta}{2} = \frac{1}{\sqrt{3}}; \quad \frac{\theta}{2} = 30^\circ; \quad \theta = 60^\circ$$

$$\operatorname{cosec}\theta = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

61. **(c)** $\cos\alpha + \sec\alpha = \sqrt{3}$

taking cube both sides

$\cos^3\alpha + \sec^3\alpha + 3\cos\alpha\sec\alpha(\cos\alpha + \sec\alpha)$

$= 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha + 3\sqrt{3} = 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha = 0$

62. **(a)** $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\sin\theta = (\sqrt{2}-1)\cos\theta$

$\cot\theta = \dfrac{1}{\sqrt{2}-1}$

$\cot\theta = \dfrac{1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = \sqrt{2}+1$

63. **(d)** $(\sin^2 1° + \sin^2 89°) + (\sin^2 2° + \sin^2 88°) + \ldots$
$+ (\sin^2 44° + \sin^2 48°) + \sin^2 45°$

$= (\sin^2 1° + \cos^2 1°) + (\sin^2 2° + \cos^2 2°) + \ldots$
$+ (\sin^2 44° + \cos^2 44°) + \sin^2 45°$

$= 1 + 1 + \ldots + 1\,(44 \text{ times}) + \dfrac{1}{2}$

$= 44\dfrac{1}{2}$

64. **(b)** We have, $\tan\theta = \dfrac{x\sin\phi}{1 - x\cos\phi}$

$\Rightarrow \dfrac{1 - x\cos\phi}{x\sin\phi} = \dfrac{1}{\tan\theta}$

$\Rightarrow \dfrac{1}{x\sin\phi} - \cot\phi = \cot\theta$

$\Rightarrow \dfrac{1}{x\sin\phi} = \cot\theta + \cot\phi$

65. **(c)** We have, $\cos(81° + \theta) = \sin\left(\dfrac{k}{3} - \theta\right)$

$\Rightarrow 81° + \theta + \dfrac{k}{3} - \theta = 90°$

$[\because \cos\alpha = \sin\beta$, then $\alpha + \beta = 90°]$

$\Rightarrow \dfrac{k}{3} = 90° - 81° \Rightarrow k = 27°$

66. **(c)** $\sin(A - B) = \dfrac{1}{2}$ and $\cos(A + B) = \dfrac{1}{2}$

$\Rightarrow \sin(A - B) = \sin 30°$ and $\cos(A + B) = \cos 60°$

$\Rightarrow (A - B) = 30°$ ..... (1)

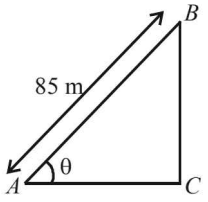
and $(A + B) = 60°$ ..... (2)

Solving (1) and (2), we get

$A = 45°$ and $B = 15°$

67. **(a)** Length of the string of the kite AB = 85 m

$\tan\theta = \dfrac{15}{8}$



$\Rightarrow \cot\theta = \dfrac{8}{15}$

$\Rightarrow \operatorname{cosec}^2\theta - 1 = \dfrac{64}{225}$

$\Rightarrow \operatorname{cosec}^2\theta = 1 + \dfrac{64}{225} = \dfrac{289}{225}$

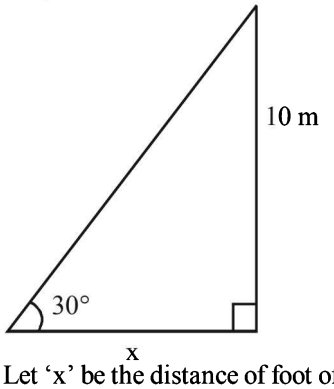
$\Rightarrow \operatorname{cosec}\theta = \sqrt{\dfrac{289}{225}} = \dfrac{17}{15} \Rightarrow \sin\theta = \dfrac{15}{17}$

In ΔABC, $\sin\theta = \dfrac{BC}{AB}$

$\Rightarrow \dfrac{15}{17} = \dfrac{BC}{85} \Rightarrow BC = 75\text{m}$

$\therefore$ Height of kite = 75 m

68. **(a)**



Let 'x' be the distance of foot of ladder

$\tan 30° = \dfrac{10}{x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{10}{x} \Rightarrow x = 10\sqrt{3}\text{ m}$

69. **(b)** $\sec(15° - \theta) = \sec(90° - 75° - \theta)$

$= \sec(90° - (75° + \theta) = \operatorname{cosec}(75° + \theta)$

GP_3386

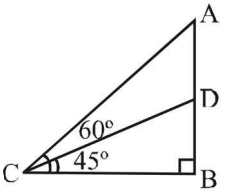
$\cot(35° - \theta) = \cot(90° - 55° - \theta)$
$= \cot(90° - (55° + \theta) = \tan(55° + \theta)$
$\therefore$ given expression = $\operatorname{cosec}(75° + \theta) - \operatorname{cosec}$
$(75° + \theta) - \tan(55° + \theta) + \tan(55° + \theta)$
$= 0$

Level 2

1. (b) $\cos 30° = \frac{AC}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{10\text{ m}}{AB}$

$\Rightarrow AB = \frac{20}{\sqrt{3}}$ m.

2. (c) Let the aeroplanes are at point A and D respectively. Aeroplane A is flying 600 m above the ground.



So, AB = 600.
$\angle ACB = 60°, \angle DCB = 45°$

From ΔABC, $\frac{AB}{BC} = \tan 60°$

$\Rightarrow BC = \frac{600}{\sqrt{3}} = 200\sqrt{3}$.

From ΔDCB, $\frac{DB}{BC} = \tan 45° \Rightarrow DB = 200\sqrt{3}$.

So, the distance AD = AB – BD
$= 600 - 200\sqrt{3}$
$= 200(3 - \sqrt{3}) = 200(3 - 1.7321) = 253.58$ m.

3. (a) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$
$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta \cdot \cos\theta + (\sin^2\theta$
$+ \cos^2\theta) - 2\sin\theta \cdot \cos\theta = 1 + 1 = 2$
So, $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

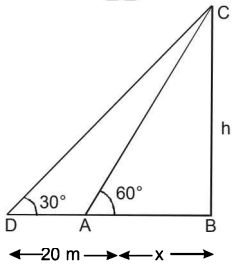
or, $(\sin\theta + \cos\theta)^2 + \left(\frac{7}{13}\right)^2 = 2$

or, $(\sin\theta + \cos\theta)^2 = 2 - \frac{49}{169} = \frac{289}{169}$

$\sin\theta + \cos\theta = \sqrt{\left(\frac{17}{13}\right)^2} = \frac{17}{13}$.

4. (a) From right angled Δs ABC and DBC, we have

$\tan 60° = \frac{BC}{AB}$ and $\tan 30° = \frac{BC}{BD}$



$\Rightarrow \sqrt{3} = \frac{h}{x}$

and $\frac{1}{\sqrt{3}} = \frac{h}{x + 20}$

$\Rightarrow h = x\sqrt{3}$

and $h = \frac{x + 20}{\sqrt{3}}$

$\Rightarrow x\sqrt{3} = \frac{x + 20}{\sqrt{3}} \Rightarrow 3x = x + 20 \Rightarrow x = 10$ m

Putting $x = 10$ in $h = \sqrt{3}x$, we get $h = 10\sqrt{3}$ m

Hence, height of the tree = $10\sqrt{3}$ m and the breadth of the river = 10 m.

5. (b) $\sin\theta + \cos\theta = \frac{1}{2}$, ...(i)

$\sin\theta - \cos\theta = k$ (Let) ....(ii)
$(i)^2 + (ii)^2$,

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = k^2$

$2\left(\sin^2\theta + \cos^2\theta\right) = \frac{1}{4} + k^2$

$\Rightarrow 2 = \frac{1}{4} + k^2 \Rightarrow k^2 = 2 - \frac{1}{4} \Rightarrow k^2 = \frac{7}{4}$

$k = \pm\frac{\sqrt{7}}{2}$

6. (a) $2\sin\alpha + 15\cos^2\alpha = 7$
$\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$
$\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$
$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$
$\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$
$3\sin\alpha + 2 = 0$
$\sin\alpha = -2/3$ or $5\sin\alpha - 4 = 0$

$\sin\alpha = \frac{4}{5}$

Since, α lies between $0° - 90°$. So, $\sin\alpha$ is positive.

$$\sin\alpha = \frac{4}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\text{Base} = \sqrt{(5)^2 - (4)^2} = 3$$

$$\therefore \quad \cot\alpha = \frac{\text{Base}}{\text{Perpendicular}} = \frac{3}{4}$$

7. (d) $\sin x + \sin y = a$ ...(i)

$\cos x + \cos y = b$ ....(ii)

$(i)^2 + (ii)^2$

$\sin^2x + \sin^2y + \cos^2x + \cos^2y + 2\sin x \sin y + 2\cos x \cos y = a^2 + b^2$

$\Rightarrow \sin^2x + \cos^2x + \sin^2y + \cos^2y + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$

$\Rightarrow 1 + 1 + 2(\sin x \sin y + \cos x \cos y) = a^2 + b^2$

$$\Rightarrow \sin x \sin y + \cos x \cos y = \left(\frac{a^2 + b^2 - 2}{2}\right)$$

8. (b) $\sin x + \cos x = c$

$\Rightarrow (\sin x)^2 + (\cos x)^2 + 2\sin x.\cos x = c^2$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x.\cos x = c^2$

$\Rightarrow 1 + 2\sin x \cos x = c^2$

$\Rightarrow 2\sin x \cos x = c^2 - 1$

$$\sin x \cos x = \frac{c^2 - 1}{2}$$

$\sin^6x + \cos^6x = 1 - 3\sin^2x \cos^2 x$

$$= 1 - 3\left(\frac{c^2 - 1}{2}\right)^2 = 1 - 3\left(\frac{c^4 + 1 - 2c^2}{4}\right)$$

$$= \frac{4 - 3c^4 - 3 + 6c^2}{4} = \frac{1 - 3c^4 + 6c^2}{4}$$

Note that:

$$(\sin^2 x + \cos^2 x)^3 = (\sin^2 x)^3 + (\cos^2 x)^3 + 3\sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$$

$$\Rightarrow (\sin^2 x + \cos^2 x)^3 = \sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x$$

$$\Rightarrow 1 = \sin^6 x + \cos^6 x + 3\sin^2 x \cos^2 x$$

$$\Rightarrow 1 - 3\sin^2 x \cos^2 x = \sin^6 x + \cos^6 x$$

9. (d) $A + C = B + D = 180°$ [$\because$ sum of opposite angles of cyclic quadrilateral is 180°]

$\cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B + \cos(180 - A) + \cos(180 - B)$

$= \cos A + \cos B - \cos A - \cos B = 0.$

10. (a) $a = 1, r = \sin x$

$$S_\infty = \frac{a}{1 - r}$$

$$\frac{1}{1 - \sin x} = 4 + 2\sqrt{3}$$

$$\Rightarrow \frac{1}{1 - \sin x} = 4 + 2\sqrt{3} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$\Rightarrow \frac{1}{1 - \sin x} = \frac{4}{4 - 2\sqrt{3}}$$

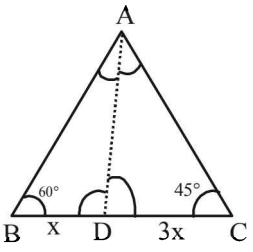
$$\Rightarrow \frac{1}{1 - \sin x} = \frac{\frac{4}{4}}{\frac{4}{4} - \frac{2\sqrt{3}}{4}}$$ [Dividing numerator and denominator by 4]

$$\Rightarrow \frac{1}{1 - \sin x} = \frac{1}{1 - \frac{\sqrt{3}}{2}}$$

Comparing both sides

$$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 60°.$$

11. (c)



$$\frac{\sin 60°}{AD} = \frac{\sin\angle BAD}{BD} \quad ...(i)$$

$$\frac{\sin 45°}{AD} = \frac{\sin\angle CAD}{DC} \quad ...(ii)$$

From (i) and (ii),

$$\frac{\sin 60^o}{\sin 45^o} = \frac{3x\sin\angle BAD}{x\sin\angle CAD}$$

GP_3386

$$\Rightarrow \quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \frac{3.\sin\angle BAD}{\sin\angle CAD}$$

$$\Rightarrow \quad \frac{\sin\angle BAD}{\sin\angle CAD} = \frac{\sqrt{3}}{\sqrt{2}\times 3} = \frac{1}{\sqrt{6}}$$

12. (a) $\sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta.\cos^2\theta\,(\sin^2\theta + \cos^2\theta)$

$= 1 - 3\sin^2\theta\cos^2\theta$

$$= 1 - \frac{3}{4}\times 4\sin^2\theta.\cos^2\theta$$

$$= 1 - \frac{3}{4}(2\sin\theta.\cos\theta)^2 = 1 - \frac{3}{4}(\sin 2\theta)^2$$

Min $\sin 2\theta = 0$

$$\text{So, Max } (\sin^6\theta + \cos^6\theta) = 1 - \frac{3}{4}(0)$$

$= 1 - 0 = 1$

Max $\sin 2\theta = 1$

$$\text{So, min } (\sin^6\theta + \cos^6\theta) = 1 - \frac{3}{4}(1) = \frac{1}{4}$$

13. (a) $\tan\theta = 1$

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+1} = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\text{Now, } \frac{8\sin\theta + 5\sin\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$$

$$= \frac{8\times\frac{1}{\sqrt{2}} + 5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3 - 2\times\left(\frac{1}{\sqrt{2}}\right)^3 + 7\times\frac{1}{\sqrt{2}}}$$

$$= \frac{\frac{8+5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}} = \frac{\frac{(8+5)}{\sqrt{2}}}{\frac{1-2+14}{2\sqrt{2}}}$$

$$= \frac{13\times 2}{13} = 2$$

14. (a) $\tan 15^\circ . \cot 75^\circ + \tan 75^\circ . \cot 15^\circ$

$= \tan 15 . \cot(90^\circ - 15^\circ) + \tan(90^\circ - 15^\circ)\cot 15^\circ$

$= \tan 15^\circ . \tan 15^\circ + \cot 15^\circ . \cot 15^\circ$

$= (\tan 15^\circ)^2 + (\cot 15^\circ)^2$

$$= (\tan 15^\circ)^2 + \frac{1}{(\tan 15^\circ)^2}$$

Putting the value of $\tan 15^\circ = 2 - \sqrt{3}$

$$= (2-\sqrt{3})^2 + \left(\frac{1}{2-\sqrt{3}}\right)^2$$

$$= (2-\sqrt{3})^2 + \left[\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}\right]^2$$

$$= (2-\sqrt{3})^2 + \left(\frac{2+\sqrt{3}}{4-3}\right)^2$$

$$= (2-\sqrt{3})^2 + (2+\sqrt{3})^2$$

$$= 2\left[2^2 + (\sqrt{3})^2\right] \quad [\because (a+b)^2 + (a-b)^2$$

$= 2(a^2 + b^2)]$

$= 2(4+3) = 2\times 7 = 14$

15. (c) To find total number of terms

First term = 1, last term = 89, common diff = 2.

$a_n = a_1 + (n-1)d$

$89 = 1 + (n-1)2$

$\Rightarrow 88 = (n-1)2$

$\Rightarrow n - 1 = 44$

$\Rightarrow$ 45 terms.

Now, $\sin^2 1^\circ + \sin^2 3^\circ + \sin^2 5^\circ + \ldots\ldots + \sin^2 85^\circ + \sin^2 87^\circ + \sin^2 89^\circ$

$= (\sin^2 1^\circ + \sin^2 89^\circ) + (\sin^2 3^\circ + \sin^2 87^\circ) + \ldots\ldots 22 \text{ terms} + \sin^2 45^\circ$

$= (\sin^2 1^\circ + \cos^2 1^\circ) + (\sin^2 3^\circ + \cos^2 3^\circ) + \ldots\ldots 22 \text{ terms} + \left(\frac{1}{\sqrt{2}}\right)^2$

$= (1 + 1 + \ldots\ldots 22 \text{ terms}) + \frac{1}{2}$

$= 22 + \frac{1}{2} = 22\frac{1}{2}$

16. (b) $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} =$

$\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} \cdot \frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \frac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{(1+\sin^2\alpha+2\sin\alpha)-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

17. (c) In the given equation,
$1 + \sin^2 A = 3 \sin A \cos A$
Dividing both sides by $\cos^2 A$,

We get $\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} = 3.\frac{\sin A}{\cos A}$

$\Rightarrow \sec^2 A + \tan^2 A = 3\tan A$

$\Rightarrow 1 + \tan^2 A + \tan^2 A = 3\tan A$

$\Rightarrow 2\tan^2 A - 3\tan A + 1 = 0$

$\Rightarrow 2\tan^2 A - 2\tan A - \tan A + 1 = 0$

$\Rightarrow 2\tan A(\tan A - 1) - 1(\tan A - 1) = 0$

$\Rightarrow (2\tan A - 1)(\tan A - 1) = 0$

$\Rightarrow \tan A = \frac{1}{2}, 1$

18. (c) Given that $\theta + \phi = \frac{\pi}{6}$

$\Rightarrow \tan(\theta + \phi) = \tan\frac{\pi}{6}$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{1}{\sqrt{3}}$

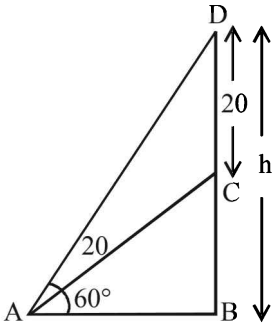
$\Rightarrow \sqrt{3}\tan\theta + \sqrt{3}\tan\phi = 1 - \tan\theta\tan\phi$ ...(1)

$(\sqrt{3} + \tan\theta)(\sqrt{3} + \tan\phi)$

$= 3 + \sqrt{3}\tan\theta + \sqrt{3}\tan\phi + \tan\theta\tan\phi$

$= 3 + 1 - \tan\theta\tan\phi + \tan\theta\tan\phi = 4$

19. (b) In ΔABD,



$\tan 60° = \frac{BD}{AB}$

$\Rightarrow \sqrt{3} = \frac{h}{AB}$

$\Rightarrow AB = \frac{h}{\sqrt{3}}$

Now, in ΔABC
$AC^2 = AB^2 + BC^2$

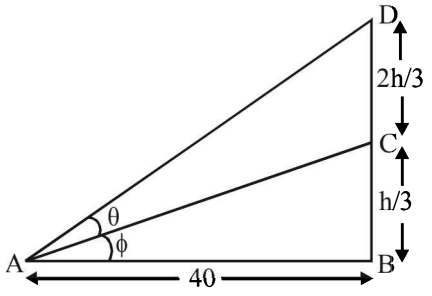
$\Rightarrow 20^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (h-20)^2$

$\Rightarrow h^2 + 3h^2 - 120h = 0$
$\Rightarrow 4h^2 - 120h = 0$
$\Rightarrow h(h-30) = 0$
$h = 0$ or 30
$h = 0$ not possible
$\Rightarrow h = 30$ ft

20. (c) Let h be the height of pole, upper portion CD subtend angle θ at A.

Then, $\tan\theta = \frac{1}{2}$

Let lower part BC subtend angle ϕ at A, then
In ΔABC,



GP_3386

$$\tan\phi = \frac{BC}{AB} = \frac{h/3}{40} = \frac{h}{120}$$

In ΔABD,

$$\tan(\theta+\phi) = \frac{BD}{AB}$$

$$\Rightarrow \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi} = \frac{h}{40}$$

$$\Rightarrow \frac{\frac{1}{2}+\frac{h}{120}}{1-\frac{h}{240}} = \frac{h}{40} \Rightarrow \frac{2(60+h)}{(240-h)} = \frac{h}{40}$$

$\Rightarrow 80(60+h) = 240h - h^2$

$\Rightarrow 4800 + 80h = 240h - h^2$

$\Rightarrow h^2 - 160h + 4800 = 0$

$\Rightarrow (h-120)(h-40) = 0$

$\Rightarrow h = 120$

[$h = 40$ is discarded, since $h > 100$ is given]

21. (d) Given identity

$2(\sin^6x + \cos^6x) + t(\sin^4x + \cos^4x) = -1$

$\Rightarrow 2[(\sin^2x + \cos^2x)^3 - 3\sin^2x\cos^2x(\sin^2x + \cos^2x)] + t[(\sin^2x + \cos^2x)^2 - 2\sin^2x\cos^2x] = -1$

[$\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$ and $(a+b)^2 = a^2 + b^2 + 2ab$ where $a = \sin^2x, b = \cos^2x$]

$\Rightarrow 2[1 - 3\sin^2x\cos^2x] + t[1 - 2\sin^2x\cos^2x] = -1$

$\Rightarrow 2 - 6\sin^2x\cos^2x + t - 2t\sin^2x\cos^2x = -1$

$= t(1 - 2\sin^2x\cos^2x) = -3(1 - 2\sin^2x\cos^2x)$

$\Rightarrow t = -3.$

22. (b) $(a\cos\theta - b\sin\theta)^2 + (a\cos\theta + b\sin\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta.\sin\theta.$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2 \times 1 + b^2 \times 1$

$= a^2 + b^2.$

$\therefore (a\cos\theta - b\sin\theta)^2 + (a\cos\theta + b\sin\theta)^2$

$= a^2 + b^2.$

$\Rightarrow c^2 + (a\cos\theta + b\sin\theta)^2 = a^2 + b^2$

$\Rightarrow a\cos\theta + b\sin\theta = \pm\sqrt{a^2 + b^2 - c^2}$

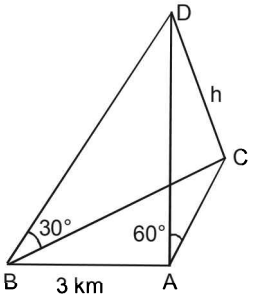
23. (d) In ΔACD, we get $AC = h\cot 60° = h.\left(1/\sqrt{3}\right)$,

In ΔBCD, $BC = h\cot 30° = h\sqrt{3}$.

Therefore, from right-angled triangle BAC, we have

$BC^2 = AB^2 + AC^2$

$\Rightarrow \left(h\sqrt{3}\right)^2 = (3)^2 + \left(\frac{h}{\sqrt{3}}\right)^2$



$\Rightarrow 3h^2 = 9 + \frac{h^2}{3} \Rightarrow \frac{8}{3}h^2 = 9$

$\Rightarrow h^2 = \frac{27}{8} \Rightarrow h = \frac{3\sqrt{3}}{2\sqrt{2}}\text{ km} = \frac{3\sqrt{6}}{4}\text{ km}$

24. (d) $\tan(\theta_1 + \theta_2) = \sqrt{3} = \tan 60°$

$\Rightarrow \theta_1 + \theta_2$

$= 60°$ and $\sec(\theta_1 - \theta_2) = \frac{2}{\sqrt{3}} = \sec 30°$

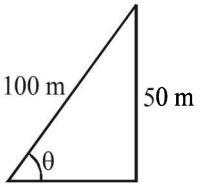
$\Rightarrow \theta_1 - \theta_2 = 30°$

$\therefore \theta_1 = 45°$ and $\theta_2 = 15°$

$\therefore \sin 2\theta_1 + \tan 3\theta_2 = \sin 90° + \tan 45°$

$= 1 + 1 = 2$

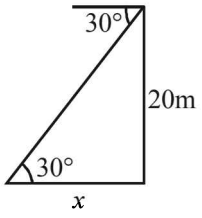
25. (d)



$$\sin\theta = \frac{50\text{ m}}{100\text{ m}} = \frac{1}{2}$$

$$\theta = 30°$$

26. (b) $\tan 30° = \frac{20}{x}$



$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$

$$x = 20\sqrt{3}\text{m}$$

27. (b) $2y\cos\theta = x\sin\theta$

$$\Rightarrow \sin\theta = \frac{2y}{x}\cos\theta$$

And $2x\sec\theta - y\,\text{cosec}\,\theta = 3$

$$\Rightarrow 2x\sec\theta - \frac{y}{\sin\theta} = 3$$

$$\Rightarrow \frac{2x}{\cos\theta} - \frac{yx}{2y\cos\theta} = 3$$

$$\Rightarrow 3\cos\theta = \frac{3}{2}x \Rightarrow \cos\theta = \frac{x}{2}$$

Now $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow y^2 + \frac{x^2}{4} = 1$$

$$\Rightarrow 4y^2 + x^2 = 4$$

28. (b) $\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$$\sqrt{3}(\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$$

...(1)

$\sec\theta + \tan\theta = \sqrt{3}$ (Given) ...(2)

Adding equations (1) and (2)

$$2\sec\theta = \sqrt{3} + \frac{1}{\sqrt{3}} \Rightarrow 2\sec\theta = \frac{4}{\sqrt{3}} \Rightarrow \sec\theta = \frac{2}{\sqrt{3}}$$

$$\therefore \cos\theta = \frac{\sqrt{3}}{2} \qquad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

Therefore, $\sin\theta = \sqrt{1-\cos^2\theta}$

$$\Rightarrow \sqrt{1-\frac{3}{4}} = \frac{1}{2}$$

29. (d) $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$

$\Rightarrow \cos^4\alpha \sin^2\beta + \sin^4\alpha \cos^2\beta$
$= \cos^2\beta \sin^2\beta$
$\Rightarrow \cos^4\alpha(1-\cos^2\beta) + \cos^2\beta(1-\cos^2\alpha)^2$
$= \cos^2\beta(1-\cos^2\beta)$
$\Rightarrow \cos^4\alpha - \cos^4\alpha\cos^2\beta + \cos^2\beta - 2\cos^2\alpha$
$\cos^2\beta + \cos^4\alpha\cos^2\beta = \cos^2\beta - \cos^4\beta$
$\Rightarrow \cos^4\alpha - 2\cos^2\alpha\cos^2\beta + \cos^4\beta = 0$
$\Rightarrow (\cos^2\alpha - \cos^2\beta)^2 = 0$
$\Rightarrow \cos^2\alpha = \cos^2\beta$
$\Rightarrow \sin^2\alpha = \sin^2\beta$

Then, $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha}$

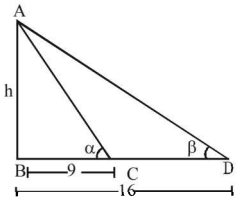
$$\Rightarrow \frac{\cos^2\beta\cos^2\alpha}{\cos^2\alpha} + \frac{\sin^2\beta\sin^2\alpha}{\sin^2\alpha}$$

$\Rightarrow \cos^2\beta + \sin^2\beta = 1$

30. (b) In ΔABC

$$\tan\alpha = \frac{h}{9} \qquad ...(1)$$

In ΔABD



$$\tan\beta = \frac{h}{16}$$

$\alpha + \beta = 90°$ (given)
$\beta = 90 - \alpha$

since $\tan\beta = \frac{h}{16}$

$$\tan(90-\alpha)=\frac{h}{16}\Rightarrow \cot\alpha$$

$$=\frac{h}{16}\text{ or }\tan\alpha=\frac{16}{h} \quad ...(2)$$

From eqs. (1) and (2)

$$\frac{h}{9}=\frac{16}{h}\Rightarrow h^2=16\times 9\Rightarrow h=12\text{ feet.}$$

31. (a) If $\sin^2\alpha=\cos^3\alpha$

$$\tan^2\alpha=\cos\alpha \quad ...(1)$$

Now consider, $\cot^6\alpha-\cot^2\alpha$

$$=\frac{1}{\tan^6\alpha}-\frac{1}{\tan^2\alpha}\left[\text{Since},\cot\alpha=\frac{1}{\tan\alpha}\right]$$

Substituting $\tan^2\alpha=\cos\alpha$, equation will be

$$=\frac{1}{\cos^3\alpha}-\frac{1}{\cos\alpha}=\frac{1-\cos^2\alpha}{\cos^3\alpha}=\frac{\sin^2\alpha}{\cos^3\alpha}$$

$$=\frac{\tan^2\alpha}{\cos\alpha}=1$$

32. (d) $$\frac{\cot 5^\circ.\cot 10^\circ.\cot 15^\circ.\cot 60^\circ.\cot 75^\circ.\cot 80^\circ.\cot 85^\circ}{(\cos^2 20^o+\cos^2 70^\circ)+2}$$

$$\Rightarrow \frac{\cot(90^\circ-85^\circ).\cot(90^\circ-80^\circ).\cot(90^\circ-75^\circ).\cot 60^\circ.\cot 75^\circ.\cot 80^\circ\cot 85^\circ}{(\cos^2(90^\circ-70^\circ)+\cos^2 70^\circ)+2}$$

$$\Rightarrow \frac{\cot 60^\circ}{(1+2)}=\frac{\frac{1}{\sqrt{3}}}{3}=\frac{1}{3\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{9}$$

33. (d) $x=a\cos\theta-b\sin\theta$

$y=b\cos\theta+a\sin\theta$

$x^2+y^2=(a\cos\theta-b\sin\theta)^2+(b\cos\theta+a\sin\theta)^2$

$\Rightarrow a^2\cos^2\theta+b^2\sin^2\theta-2ab\cos\theta\sin\theta+b^2\cos^2\theta+a^2\sin^2\theta+2ab\cos\theta\sin\theta$

$\Rightarrow (a^2+b^2)\cos^2\theta+(a^2+b^2)\sin^2\theta$

$\Rightarrow (a^2+b^2)(\cos^2\theta+\sin^2\theta)$

$\Rightarrow a^2+b^2.(1)\Rightarrow a^2+b^2$

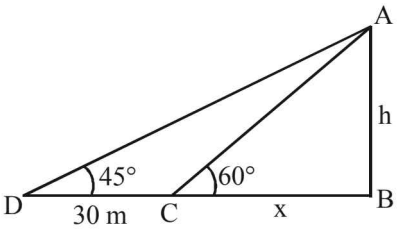
34. (c) $$\frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta)}{(\cos\theta+\sin\theta)}$$

$$+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta)}{(\cos\theta-\sin\theta)}$$

$=2\cos^2\theta+2\sin^2\theta-\sin\theta\cos\theta+\sin\theta\cos\theta$

$=2$

35. (a)



In ΔABC, $\tan 60^\circ=\frac{h}{x}$

$$x=\frac{h}{\sqrt{3}} \quad ...(1)$$

In ΔABD, $\tan 45^\circ=\frac{h}{30+x}$

$$1=\frac{h}{30+x}\text{ or }h=30+x$$

Putting value of x from (1)

$$h=30+\frac{h}{\sqrt{3}}$$

$$\text{or } h\frac{(\sqrt{3}-1)}{\sqrt{3}}=30\Rightarrow h=15\ (3+\sqrt{3})\text{ m}$$

36. (a) $x=r\sin\alpha\cos\beta, y=r\sin\alpha\sin\beta, z=r\cos\alpha$

$\therefore$ $x^2+y^2+z^2=r^2\sin^2\alpha\cos^2\beta+r^2\sin^2\alpha\sin^2\beta+r^2\cos^2\alpha$

$=r^2\sin^2\alpha(\cos^2\beta+\sin^2\beta)+r^2\cos^2\alpha$

$=r^2\sin^2\alpha+r^2\cos^2\alpha=r^2$

37. (b) $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1, \frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$

Squaring and adding, we get

$$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2+\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)^2$$

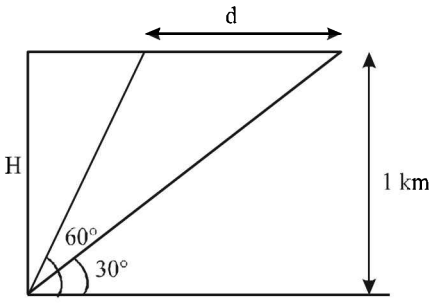
$=1+1$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=2$$

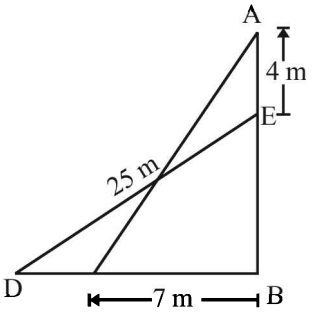
38. (b) $d = H\cot 30° - H\cot 60°$

Time taken = 10 second

$$\text{speed} = \frac{\cot 30° - \cot 60°}{10}\times 60\times 60 = 240\sqrt{3}$$



39. (b) In ΔABC,



$AB = \sqrt{25^2-7^2} = 24$ m

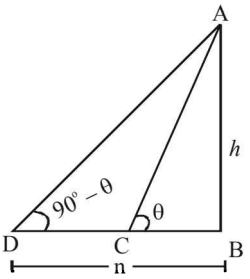
$\therefore$ EB = 24 – 4 = 20 m

In ΔBDE,

$BD = \sqrt{25^2-20^2} = 15$ m

DC = 15 – 7 = 8 m

40. (a) In ΔABC,



$h = m\tan\theta$ ... (1)

In ΔABD,

$h = n\tan(90° - \theta)$ ... (2)

Multiplying (1) and (2),

$h^2 = m\tan\theta \times n\tan(90° - \theta)$

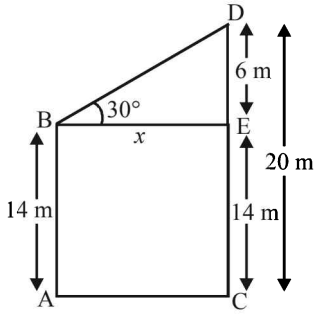
$= mn\tan\theta\cot\theta$

$= mn$

$\Rightarrow h = \sqrt{mn}$

41. (a) Here, CD = 20 m [Height of big pole]

AB = 14 m [Height of small pole]



DE = CD – CE $\Rightarrow$ DE = CD – AB [$\because$ AB = CE]

$\Rightarrow$ DE = 20 – 14 = 6 m

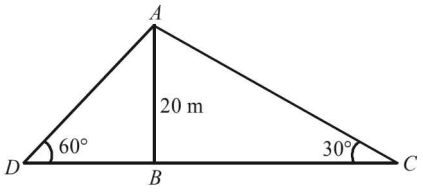
In ΔBDE, $\sin 30° = \frac{DE}{BD}$

$\Rightarrow \frac{1}{2} = \frac{6}{BD} \Rightarrow BD = 12$ m

$\therefore$ Length of wire = 12 m

42. (a) C and D be the position of the men and AB is the height of flagstaff.

GP_3386



In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{20}{BC} \Rightarrow BC = 20\sqrt{3}$

In ΔABD, $\tan 60° = \frac{AB}{BD}$

$\Rightarrow \quad \sqrt{3} = \frac{20}{BD} \Rightarrow BD = \frac{20}{\sqrt{3}}$

Distance between the men, CD = BC + BD

$= 20\sqrt{3} + \frac{20}{\sqrt{3}} + \frac{60+20}{\sqrt{3}} = \frac{80}{\sqrt{3}}$

$= \frac{80\sqrt{3}}{3} = \frac{80 \times 1.73}{3} = 46.19 \text{ m}$

43. (d) $x \sin\theta + y \cos\theta = \sqrt{x^2 + y^2}$

Put $x = \sin\theta$

$y = \cos\theta$ in the above equation, we have

$\sin^2\theta + \cos^2\theta = \sqrt{\sin^2\theta + \cos^2\theta}$

$\Rightarrow 1 = 1$

$\Rightarrow x = \sin\theta$ & $y = \cos\theta$ is the solution of above equation. Now, on using $x = \sin\theta$ & $y = \cos\theta$ in

$$\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{x^2 + y^2}$$

$$\Rightarrow \frac{y^2}{a^2} + \frac{x^2}{b^2} = \frac{1}{\sin^2\theta + \cos^2\theta}$$

$$\Rightarrow \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

44. (d) We know $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p}$,

$$\tan\theta = \frac{y}{q}$$

$\therefore \quad x^2q^2 - p^2y^2 = p^2q^2$

CHAPTER

26

Logarithm

DEFINITION

If $x = a^m$, then $\log_a x = m$, where 'a' and 'x' both are positive real numbers but 'a' is not equal to 1.
Here log is the short form of logarithm.
$\log_a x$ is read as log of x to the base a.

For example:

(i) Since, $10 = 10^1, 100 = 10^2, 1000 = 10^3$, etc.
Hence, $\log_{10}10 = 1, \log_{10}100 = 2, \log_{10}1000 = 3$, etc.

(ii) Since, $8 = 2^3, 16 = 2^4, 32 = 2^5$, etc.
Hence, $\log_2 8 = 3, \log_2 16 = 4, \log_2 32 = 5$, etc.

(iii) Since, $\frac{1}{8} = (2)^{-3}, \frac{1}{16} = (2)^{-4}$, etc.

Hence, $\log_2\left(\frac{1}{8}\right) = -3, \log_2\left(\frac{1}{16}\right) = -4$, etc.

(iv) Since, $0.01 = (10)^{-2}, 0.001 = (10)^{-3}$, etc.
Hence $\log_{10}(0.01) = -2, \log_{10}(0.001) = -3$, etc.

Note that if base of log is not mentioned, then we assume the base as 10.
$\therefore$ log m = $\log_{10}$m
log on base 10 is called *common log* 10.

LAWS OF LOGARITHM

(i) $\log_a(m \times n) = \log_a m + \log_a n$
In general,
$\log_a (m \times n \times p \times ....)$
$= \log_a m + \log_a n + \log_a p + ...$

For example:

$\log_2 (4 \times 5 \times 6) = \log_2 4 + \log_2 5 + \log_2 6$

Note that $\log_a m + \log_a n \neq \log_a (m + n)$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

For example:

$\log_4\left(\frac{8}{15}\right) = \log_4 8 - \log_4 15$

Note that $\log_a m - \log_a n \neq \log_a (m - n)$

(iii) $\log_a(m)^n = n \log_a m$

For example: $\log_3 (5)^4 = 4 \log_3 5$

(iv) $\log_b a = \frac{\log_c a}{\log_c b}$ [Change of base rule]

For example:

$\log_5 20 = \frac{\log_2 20}{\log_2 5} = \frac{\log_4 20}{\log_4 5} = \frac{\log_7 20}{\log_7 5} = ...$ etc.

(v) $\log_b a = \frac{1}{\log_a b}$

For example: $\log_{10} 100 = \frac{1}{\log_{100} 10}$

(vi) $\log_b a \,.\, \log_c b = \log_c a$ [Chain Rule]
In general,
$\log_b a \,.\, \log_c b \,.\, \log_d c.....\log_n m = \log_n a$

For example:

$\log_{24} 256 \,.\, \log_{10} 24 \,.\, \log_2 10 = \log_2 256$

Illustration – 1

$\log_a 4 + \log_a 16 + \log_a 64 + \log_a 256 = 10$. Then a = ?

(a) 4 **(b) 2** **(c) 8** **(d) 5**

Solution : (a) The given expression is:
$\log_a (4 \times 16 \times 64 \times 256) = 10$
i.e. $\log_a 4^{10} = 10$
Thus, $a = 4$.

GP_3386

Illustration – 2

Find x if $\log x = \log 1.5 + \log 12$

(a) 12 (b) 8 (c) 18 (d) 15

Solution : (c) $\log x = \log 18 \Rightarrow x = 18$

Illustration – 3

Find x, if $\log (2x - 2) - \log (11.66 - x) = 1 + \log 3$

(a) 452/32 (b) 350/32

(c) 11 (d) 11.33

Solution : (c) $\log (2x-2)/(11.66-x) = \log 30$

$\Rightarrow (2x-2)/(11.66-x) = 30$

$2x - 2 = 350 - 30x$

Hence, $32x = 352 \Rightarrow x = 11$.

Illustration – 4

Solve for x:

$$\log\frac{75}{35} + 2\log\frac{7}{2} - \log\frac{105}{x} - \log\frac{13}{25} = 0$$

(a) 90 (b) 65 (c) 13 (d) 45

Solution : (c) $(75/35) \times (49/25) \times (x/105) \times (25/13) = 1$

$\Rightarrow x = 13$

Some Important properties

(i) Since, $10000 = (100)^2 = (10)^4$

$\log_{100} 10000 = 2$, $\log_{10} 10000 = 4$

Thus value of log of a number on different bases is different i.e., value of log of a number depends on its base.

(ii) (a) Since, $a = a^1$, hence $\log a\, a = 1$

For example: $\log_5 5 = 1$, $\log_{10} 10 = 1$

Thus log of any number to the same base is always 1.

(b) Since, $1 = a^0$, hence, $\log_a 1 = 0$

For example: $\log_8 1 = 0$

Thus log of 1 to any base always equal to 0.

Illustration – 5

If $\log 3 = 0.4771$, find $\log (0.81)^2 \times \log\left(\frac{27}{10}\right)^{\frac{2}{3}} \div \log 9$.

(a) 2.689 (b) – 0.0552

(c) 2.2402 (d) 2.702

Solution : (b) $2\log (81/100) \times 2/3 \log (27/10) \div \log 9$

$= 2\,[\log 34 - \log 100] \times 2/3\,[(\log 33 - \log 10)] \times 2 \log 3$

$= 2\,[\log 34 - \log 100] \times 2/3\,[3\log 3 - 1] \times 2 \log 3$

$= 2[4\log 3 - \log(10)^2] \times \frac{2}{3}[3\log 3 - 1] \div 2\log 3$

$= 2[4\log 3 - 2\log 10] \times \frac{2}{3}[3\log 3 - 1] \div 2\log 3$

$= 2[4\log 3 - 2] \times \frac{2}{3}[3\log 3 - 1] \div 2\log 3$

$= -0.0552$ [on substitutions $\log 3 = 0.4771$]

LEVEL - 1

1. Find the value of
$\log_{10}100 + \log_{10}1000 + \log_{10}10000$.
(a) 2 (b) 9 (c) 24 (d) 100

2. If $a = \log_8 225$ and $b = \log_2 15$, then a, in terms of b, is :
(a) $b/2$ (b) $2b/3$ (c) b (d) $3b/2$

3. If $\log x - 5 \log 3 = -2$, then x equals –
(a) 1.25 (b) 0.81 (c) 2.43 (d) 0.8

4. Find the value of
$\log_{20}100 + \log_{20}1000 + \log_{20}10000$
[Assume that $\log 2 = 0.3$]
(a) 90/13 (b) 80/13 (c) 110/13 (d) 70/13

5. Find the value of $(\log_a b)(\log_b c)(\log_c a)$
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) abc

6. Find the value of $\{\log (a^p)\ (b^q)\}\{\log(b^p)(c^q)\}$ $\{\log(c^q)(d^p)\}\ \{\log (d^q)\ (a^p)\}$
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) abcd (d) None of these

7. Find the value of $\log_3\log_5 15625$
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$
(c) 1 (d) None of these

8. $\log_3\log_3\log_3 k = p$, then which one of the following is a correct relation between p and k?
(a) $k = 3^{3^p}$ (b) $k = 3^{3^{3^p}}$
(c) $p = 3^{3^k}$ (d) $p = 3^{3^{3^k}}$

9. Find x if
$\log x + \log x^2 + \log x^3 + \ldots . \log x^{10} = 11 \log 32$
(a) 2 (b) 4 (c) 8 (d) $\sqrt{2}$

10. If $a = \log_2 4$, $b = \log_4 16$, $c = \log_8 64$
Then find $\frac{abc}{a+b+c}$
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$

11. Solve for x: $8^{\log_6 x} - 36^{\log_x 6} = 58$
(a) 6 (b) 8 (c) 36 (d) 64

12. If $\log 2 = 0.3010$ then the arithmetic mean of log 40 and log 5 is
(a) 2.3010 (b) 30.10 (c) 3.0103 (d) 1.1505

13. If $\log_{10}x$, $\log_{10}y$, $\log_{10}z$ are in AP, then x, y, z are in :
(a) AP (b) GP
(c) HP (d) None of these

14. $\log_2 7$ is :
(a) an integer
(b) a prime number
(c) a rational number
(d) an irrational number

15. If a, b, c are three consecutive integers, then log (ac + 1) has the value :
(a) log b (b) $(\log b)^2$
(c) 2 log b (d) log 2 b

16. $\log_y x = ?$
(a) $x \log_e y$ (b) $\frac{x}{\log_e y}$
(c) $\frac{\log_e x}{\log_e y}$ (d) $\frac{\log_e y}{\log_e x}$

17. The value of $(\log_2 8 + \log_3 9 + \log_5 25)$ is :
(a) 5 (b) 6
(c) 7 (d) None of these

18. The value of $\log_2 \log_2 \log_3 \log_5 (125)^3$ is :
(a) 0 (b) 1 (c) 2 (d) 3

19. The value of $216^{\log_6 49}$ is :
(a) 117694 (b) 117649
(c) 216649 (d) None of these

20. The value of $\log_3 5 \times \log_{25} 9$ is :
(a) 0 (b) 1 (c) 2 (d) 4

21. Find x, if $\log x^3 - \log 3x = 2 \log 2 + \log 3$:
(a) 3 (b) 6
(c) 9 (d) None of these

22. The set of all values of x satisfying $x^{\log_x |3-x|^2} = 4$:
(a) $(\sqrt{3}, 2)$ (b) $(1, 5)$
(c) $[1, \infty)$ (d) $\{1, 5\}$

GP_3386

23. If $A = \frac{(\log_3 2187)}{5}$ and $B = \log_{243} 2187$, then which one of the following is correct?

(a) $A < B$
(b) $A = B$
(c) $A > B$
(d) can't be determined

24. Find the value of $\log_{64}(1/16)$.

(a) $-2/3$ (b) $2/3$ (c) $3/2$ (d) $-3/2$

25. What is the value of $\log_{32}27 \times \log_{243}8$?

(a) $\frac{\log 9}{\log 4}$ (b) $\frac{\log 3}{\log 2}$

(c) $\log 27$ (d) None of these

26. Find the value of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$.

(a) $\log 2$ (b) $\log 3$
(c) 1 (d) None of these

27. What is the value of x if

$\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$?

(a) 9 (b) 27
(c) 81 (d) None of these

28. $\log_5 P \times \log_P x = 2$. What is the value of x?

(a) 125 (b) P^2 (c) 25 (d) $5^{1/2}$

29. Find the value of

$\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25$.

(a) 5/2 (b) 5 (c) 2 (d) $\log\left(\frac{5}{2}\right)$

30. If $\log_3 a = 4$, find value of a.

(a) 27 (b) 3 (c) 9 (d) 81

31. Find the value of $\log\frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4}$

(a) 0 (b) 1 (c) 3 (d) $\log(3/4)$

32. If $\log_2[\log_3(\log_2 x)] = 1$, then x is equal to

(a) 512 (b) 128 (c) 12 (d) 0

33. Find the value of $\log_{27}\frac{1}{81}$

(a) $-4/3$ (b) -3 (c) -1 (d) $-1/3$

34. $\log 216\sqrt{6}$ to the base 6 is equal to

(a) 3 (b) 3/2
(c) 7/2 (d) None of these

35. If $\log_5\left[\log_3(\log_2 x)\right] = 1$, then x is

(a) 2^{234} (b) 243
(c) 2^{243} (d) None of these

36. If $\log_7\log_5(\sqrt{x} + 5 + \sqrt{x}) = 0$, find the value of x.

(a) 1 (b) 0
(c) 2 (d) None of these

37. If $\log_3[\log_3[\log_3 x]] = \log_3 3$, then what is the value of x?

(a) 3 (b) 27 (c) 3^9 (d) 3^{27}

38. What is $\log\left(a + \sqrt{a^2+1}\right) + \log\left(\frac{1}{a + \sqrt{a^2+1}}\right)$

is equal to?

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

LEVEL - 2

1. If $a = \log_{24} 12$, $b = \log_{36} 24$ and $c = \log_{48} 36$, then the value of $1 + abc$ would be
(a) $2ac$ (b) $2bc$ (c) bc (d) $2ab$

2. If $a = b^2 = c^3 = d^4$, then the value of $\log_a(abcd)$ would be –
(a) $\log_a 1 + \log_a 2 + \log_a 3 + \log_a 4$
(b) $\log 24$
(c) $1 + 1/2 + 1/3 + 1/4$
(d) All of these

3. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12$ equals:
(a) $(a+b)/(1+a)$ (b) $(2a+b)/(1+a)$
(c) $(a+2b)/(1+a)$ (d) $(2a+b)/(1-a)$

4. Simplify: $\log\frac{21}{8} + 2\log\frac{6}{7} + \log\frac{7}{27}$
(a) $\log\frac{1}{2}$ (b) $\log 2$ (c) $\log\frac{1}{3}$ (d) $\log\frac{1}{7}$

5. $\log_{\sqrt{a}} b^2 \times \log_{\sqrt{b}} c^2 \times \log_{\sqrt{c}} d^2 \times \ldots$
$\ldots \times \log_{\sqrt{y}} z^2 \times \log_{\sqrt{z}} a^2$ is
(a) 2^{27} (b) 2^{52} (c) 2^{54} (d) 2^{26}

6. If $\log(x+y) - \log 2 = \log_k k$ and
$\log(x+y) - \log xy = \log\frac{5}{16}$
Find the value of $\log_x y + \log_y x$
(a) 5/3 (b) 6/7 (c) 20/3 (d) 10/3

7. Find the number of digits in $6^6 \times 8^8 \times 10^{10} \times 12^{12}$ given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$
(a) 23 (b) 32
(c) 34 (d) None of these

8. If $p = \log_{12} 18$, $q = \log_{24} 54$ then the value of $pq + 5(p-q)$ is
(a) 0 (b) 4 (c) 1 (d) 9

9. If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by
(a) 10 (b) 1/100
(c) 1/1000 (d) None of these

10. If $x \geq y$ and $y > 1$, then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be
(a) -1 (b) -0.5 (c) 0 (d) 1

11. If $\log_2[\log_7(x^2 - x + 37)] = 1$, then what could be the value of x?
(a) 3 (b) 5
(c) 4 (d) None of these

12. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$, then which of the following is equal to 1?
(a) $x + y + z$
(b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$
(c) xyz
(d) None of these

13. If log 3, log $(3^x - 2)$ and log $(3^x + 4)$ are in arithmetic progression, then x is equal to
(a) 8/3 (b) $\log_3 8$ (c) $\log_2 3$ (d) 8

14. The value of (log tan 1° + log tan 2° + ... + log tan 89°) is :
(a) -1 (b) 0 (c) $\frac{1}{\sqrt{2}}$ (d) 1

15. The value of x for which
$\log_9 x - \log_9\left(\frac{x}{10} + \frac{1}{9}\right) = 1$ is :
(a) 2 (b) 4 (c) 9 (d) 10

16. If $x = \log_c b + \log_b c$, $y = \log_a c + \log_c a$ and $z = \log_b a + \log_a b$, then $x^2 + y^2 + z^2$ is equal to :
(a) 1 (b) 2 (c) xyz (d) xyz + 4

17. If $3^{x+1} = 6^{\log_2 3}$, then x is
(a) 2 (b) 3
(c) $\log_3 2$ (d) $\log_2 3$

18. If $\log_{10}\left[1 - \left\{1 - (1-x^2)^{-1}\right\}^{-1}\right]^{-1/2} = 1$, then which of the following is the value of x.
(a) 1 (b) 2
(c) 10 (d) None of these

19. If $\log_4 5 = a$ and $\log_5 6 = b$ then what is the value of $\log_3 2$?

GP_3386

(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$

(c) $2ab+1$ (d) $\frac{1}{2ab-1}$

20. If [N] = the greatest integer less than or equal to N, then $[\log_{10} 6730.4]$ is equal to

(a) 6 (b) 4

(c) 5 (d) None of these

21. N = n !, where n > 2. Find the value of $(\log_2 N)^{-1} + (\log_3 N)^{-1} + (\log_4 N)^{-1} + \ldots\ldots (\log_n N)^{-1}$.

(a) 0 (b) 1 (c) 10 (d) N

22. Evaluate : $3^{2-\log_3 5}$

(a) $\frac{9}{5}$ (b) 45 (c) 5/9 (d) $9 \log_{35}$

23. The value of

$$\left[\frac{1}{\log_{xy}(xyz)}+\frac{1}{\log_{yz}(xyz)}+\frac{1}{\log_{zx}(xyz)}\right]$$

is equal to

(a) 1 (b) 2 (c) 3 (d) 4

24. Find the value of $\frac{8\log_8 8}{2\log_{\sqrt{8}} 8}$

(a) 1 (b) 2 (c) 3 (d) 4

25. If $\log_3 (5+x) + \log_8 8 = 2^2$, find value of x.

(a) 22 (b) 33 (c) 11 (d) 44

26. The value of

$$\left[3\log\left(\frac{81}{80}\right)+5\log\left(\frac{25}{24}\right)+7\log\left(\frac{16}{15}\right)\right] \text{ is}$$

(a) log 3 (b) log 5 (c) log 7 (d) log 2

27. If $\log 3 = 0.4771$, find $\log (0.81)^2 \times \log\left(\frac{27}{10}\right)^{\frac{2}{3}} \div \log 9$.

(a) 2.689 (b) −0.0552

(c) 2.2402 (d) 2.702

28. What is the value of x in the following expression $\log_2 \log_2 (x^2+7) \log_{1/4} (x^2+7)^{-1} = -3$?

(a) +3 (b) −3

(c) ±3 (d) None of these

29. $\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ac)+1}+\frac{1}{(\log_c ab)+1}$ is equal to

(a) 1 (b) 2

(c) 0 (d) abc

30. If $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$, then antilog $(pa + qb - rc)$ = ?

(a) $\frac{pxqy}{rz}$ (b) $px + qy - rz$

(c) $\frac{x^p y^q}{z^r}$ (d) $x^p y^q z^r$

31. What is the value of $\frac{\log_{27} 9 \times \log_{16} 64}{\log_4 \sqrt{2}}$?

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) 8 (d) 4

Hints and Solutions

Level -1

1. (b) Consider $\log_{10}100 = 2$, $\log_{10}1000 = 3$ and $\log_{10}10000 = 4$
Hence $\log_{10}100 + \log_{10}1000 + \log_{10}10000 = 2 + 3 + 4 = 9$

2. (b) $a = \log_8 225 = \log_8 15^2 = (2/3)\log_2 15 = (2/3)b$
Since $b = \log_2 15$

3. (c) Given equation is $\log x - 5\log 3 = -2$
or, $\log x = 5\log 3 - 2$
or, $\log x = \log (3)^5 - \log 100$
or, $\log x = \log 243 - \log 100$
or, $\log x = \log 243/100$
or, $\log x = \log 2.43$
$\Rightarrow x = 2.43$

4. (a) Since $\log 2 = 0.3$ hence $\log 20 = 1.3$
$\log_{20}100 + \log_{20}1000 + \log_{20}10000$
$= [(\log 100)/(\log 20) + (\log 1000)/(\log 20) + (\log 10000)/(\log 20)]$
$= (2 + 3 + 4)/1.3 = 90/13$

5. (c) Since $(\log_a b)(\log_b c)(\log_c a)$
$= \frac{\log b}{\log a} \times \frac{\log c}{\log b} \times \frac{\log a}{\log c} = 1$

6. (d) $\{\log(a^p)(b^q)\}\{\log(b^p)(c^q)\}\{\log(c^q)(d^p)\}\{\log(d^q)(a^p)\}$
$= \{(q/p)(\log b)/(\log a)\} \times \{(q/p)(\log c)/(\log b)\}$
$\{(p/q)(\log d)/(\log c)\}\{(p/q)(\log a)/(\log d)$
$= \left\{\frac{q}{p}\times\frac{q}{p}\times\frac{p}{q}\times\frac{p}{q}\right\} \times \left\{\frac{\log b}{\log a}\times\frac{\log c}{\log b}\times\frac{\log d}{\log c}\times\frac{\log a}{\log d}\right\} = 1$

7. (c) $\log_3\log_5 15625 = \log_3(\log_5(5^6))$
$= \log_3 (6/2)(\log_5 5)]$
$= \log_3(3) = 1$

8. (b) Since $\log_3\log_3\log_3 k = p$
or $\log_3\log_3 k = 3^p$ or $\log_3 k = 3^{3^p}$
or $k = 3^{3^{3^p}}$

9. (a) L.H.S $= \log\left(x.x^2.x^3 \ldots x^{10}\right)$
$= \log\left(x^{1+2+3+\ldots+10}\right) = \log x^{55} = 55\log x$
$= 11\log x^5 = 11\log 32$
or $x^5 = 32$ or $x = 2$

10. (c) $a = \log_2 4 = 2\log_2 2 = 2$
$b = \log_4 16 = 2\log_4 4 = 2$
$c = \log_8 64 = 2\log_8 8 = 2$
hence $\frac{abc}{a+b+c} = \frac{8}{6} = \frac{4}{3}$

11. (c) This question should be solve by eliminating options, The easiest option to eliminate is $x = 6$ as it will give us $\log_6 x = 1$, we will find that it will not satisfy the given condition, then next option is $x = 36$ and it will satisfy the equation hence $x = 36$.

12. (d) $\text{AM} = \frac{\log 40 + \log 5}{2} = \frac{1}{2}\log 200$
$= \frac{1}{2}(\log 100 + \log 2) = \frac{1}{2}(2 + 0.3010) = 1.1505$

13. (b) $\log_{10} x$, $\log_{10} y$, $\log_{10} z$ are in AP
$\therefore$ $2\log_{10} y = \log_{10} x + \log_{10} z$
$\Rightarrow$ $\log_{10} y^2 = \log_{10}(xz)$
$\Rightarrow$ $y^2 = xz$
$\Rightarrow$ x, y, z are in GP

14. (d) Let $\log_2 7$ be a rational
$\log_2 7 = \left(\frac{p}{q}\right)$
$\Rightarrow$ $2^{(p/q)} = 7 \Rightarrow 2^p = 7^q$
Here p and q being the integers cannot satisfy the above relation.
Hence $\log_2 7$ is an irrational number.

15. (c) a, b, c are consecutive integers
$\therefore$ $b = a + 1$ and $c = a + 2$
$\therefore$ $\log(ac + 1) = \log[a(a + 2) + 1]$
$= \log[(b - 1)(b - 1 + 2) + 1]$
$(\because a = b - 1)$
$= \log b^2 = 2\log b$

16. (c) $\log_y x = \frac{\log_e x}{\log_e y}$

17. (c) $\log_2 2^3 + \log_3 3^2 + \log_5 5^2 = 3 + 2 + 2 = 7$

18. (a) $\log_2\log_2\log_3\log_5 (125)^3$
$= \log_2\log_2\log_3\log_5 5^9$
$= \log_2\log_2\log_3 9$
$= \log_2\log_2(2) = \log_2 1 = 0$

GP_3386

19. **(b)** $216^{\log_6 49} = 6^{3\log_6 49}$

$= 6^{\log_6 (49)^3} = (49)^3 = 117649$

20. **(b)** $\log_3 5 \times \log_{25} 9 = \frac{\log 5}{\log 3} \times \frac{\log 3^2}{\log 5^2}$

$= \frac{\log 5}{\log 3} \times \frac{2\log 3}{2\log 5} = 1$

21. **(b)** $\log x^3 - \log 3x = 2\log 2 + \log 3$

$\Rightarrow \log \frac{x^2}{3} = \log 12$

$\Rightarrow \frac{x^2}{3} = 12 \Rightarrow x = \pm 6$

But at $x = -6$, $\log 3x$ and $\log x^3$ are not defined.
Hence, $x = 6$ is the only correct answer

22. **(d)** $x^{\log_x |3-x|^2} = 4$

$\Rightarrow |3-x|^2 = 4$

$\Rightarrow |3-x| = 2$ (–2 is inadmissible)

$\Rightarrow (3-x) = 2$ or $-(3-x) = 2$

$\Rightarrow x = 1$ or $x = 5$

23. **(b)** $A = \frac{\log_3 2187}{5} = \frac{1}{5}\log_3 2187$

$= \frac{1}{5}\log_3 (3^7) = \frac{7}{5}\log_3 3 = \frac{7}{5}$

and $B = \log_{243} 2187 = \frac{\log_3 2187}{\log_3 243}$

$= \frac{\log_3 3^7}{\log_3 3^5} = \frac{7}{5}$

$\therefore A = B$

24. **(a)** $\log_{64} (1/16) = \log_{4^3} 4^{-2} = \frac{-2}{3}$

25. **(d)** $\log_{32} 27 \times \log_{243} 8 = \log_{2^5} 3^3 \times \log_{3^5} 2^3$

$= \frac{3}{5}\log_2 3 \times \frac{3}{5}\log_3 2$

$= \left(\frac{3}{5}\right)^2 \log_2 3 \times \log_3 2 = \frac{9}{25}$

26. **(a)** $\log\left(\frac{81}{80}\right)^3 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{16}{15}\right)^7$

$= \log\left[\frac{3^{12} \times 5^{10} \times 2^{28}}{2^{12} \times 5^3 \times 2^{15} \times 3^5 \times 3^7 \times 5^7}\right] = \log 2$

27. **(b)** $\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$

$\Rightarrow \log_3 x + \frac{1}{2}\log_3 x + \frac{1}{3}\log_3 x + \frac{1}{4}\log_3 x = \frac{25}{4}$

$\Rightarrow (\log_3 x)[1 + 1/2 + 1/3 + 1/4] = \frac{25}{4}$

$\Rightarrow \frac{25}{12} \times \log_3 x = \frac{25}{4}$

$\Rightarrow \log_3 x = 3 \Rightarrow x = 27$

28. **(c)** $\log_5 P \times \log_P x = 2$

$\Rightarrow \log_5 x = 2 \Rightarrow x = 5^2 = 25$

29. **(c)** $\log_5 10 \times \log_{10} 15 \times \log_{15} 20 \times \log_{20} 25.$

$= (\log 10/\log 5) \times (\log 15/\log 10) \times (\log 20/\log 15) \times (\log 25/\log 20)$

$= \log 25/\log 5 = 2 \log 5/\log 5 = 2.$

30. **(d)** $\because \log_3 a = 4 \ \therefore \ 3^4 = a \ \Rightarrow a = 81$

31. **(a)** Given $\log\frac{9}{8} - \log\frac{27}{32} + \log\frac{3}{4}$

$= \log\left(\frac{9}{8} \div \frac{27}{32}\right) + \log\frac{3}{4}$

$= \log\left(\frac{9}{8} \times \frac{32}{27} \times \frac{3}{4}\right) = \log 1 = 0$

$[\because \log_a 1 = 0]$

32. **(a)** $\log_2 [\log_3 (\log_2 x)] = 1 = \log_2 2$

$\Rightarrow \log_3 (\log_2 x) = 2$

$\Rightarrow \log_2 x = 3^2 = 9 \Rightarrow x = 2^9 = 512$

33. **(a)** Let $\log_{27}\left(\frac{1}{81}\right) = x$

$\therefore (27)^x = \frac{1}{81}$

$\therefore 3^{3x} = 3^{-4} \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$

34. **(c)** $\log_6 216\sqrt{6} = \log_6 (6)^3 (6)^{1/2}$

$= \log_6 (6)^{7/2}$

$= \frac{7}{2}\log_6 6 = \frac{7}{2}$ $(\because \log_a a = 1)$

35. **(c)** $\log_5 [\log_3 (\log_2 x)] = 1 = \log_5 5$

$\Rightarrow \log_3 (\log_2 x) = 5 = \log_3 3^5$

$\Rightarrow \log_2 x = 3^5 = 243$

$\Rightarrow 2^{243} = x$

36. (b) $\log_7 \log_5(\sqrt{x}+5+\sqrt{x}) = 0$

use $\log_a x = b$

$\Rightarrow a^b = x$

$\therefore \log_5(\sqrt{x}+5+\sqrt{x}) = 7^0 = 1$

$\sqrt{x}+5+\sqrt{x} = 5^1 = 5 \Rightarrow 2\sqrt{x} = 0 \therefore x = 0$

37. (d) Consider $\log_3[\log_3[\log_3 x]] = \log_3 3$

$\Rightarrow \log_3[\log_3 x] = 3$

$\Rightarrow \log_3 x = 3^3$

$\Rightarrow \log_3 x = 27 \Rightarrow x = 3^{27}$

38. (b) Let $\log(a+\sqrt{a^2+1}) + \log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$

$= \log(a+\sqrt{a^2+1}) + \log 1 -$

$\log(a+\sqrt{a^2+1})$

$= \log(a+\sqrt{a^2+1}) - \log(a+\sqrt{a^2+1})$

$= 0$

Level -1

1. (b) $A = \log_{24} 12$, $b = \log_{36} 24$, $c = \log_{48} 36$

We can write, $\log_a b = \log b / \log a$

$1 + abc = 1 + (\log_{24}12)(\log_{36}24)(\log_{48}36)$

$= 1 + (\log12/\log24)(\log24/\log36)$

$(\log36/\log48)$

$= 1 + (\log12/\log48)$

$= (\log48 + \log12)/\log48$

$[\because \log a + \log b = \log(ab)]$

$= \log(48 \times 12) / \log48 = \log(576)/\log(48)$

$= \log(24^2)/\log(48) = 2(\log24)/\log(48)$

$= 2\log_{36} 24.\log_{48} 36 = 2bc.$

2. (c) $a = b^2 = c^3 = d^4 = k$ [constant]

So, $a = k^1$, $b = k^{1/2}$, $c = k^{1/3}$, $d = k^{1/4}$

$\log_a(abcd) = \log(abcd)/\log a$

[we can write, $\log_a b = \log b/\log a$]

$= \log(k^1 k^{1/2} k^{1/3} k^{1/4})/\log(k)$

$= \log(k^{1+1/2+1/3+1/4})/\log k$

$= (1 + 1/2 + 1/3 + 1/4)(\log k/\log k)$

[we know $\log x^a = a \log x$]

$= (1 + 1/2 + 1/3 + 1/4).1$ [$\because \log a/\log a = 1$]

$= (1 + 1/2 + 1/3 + 1/4).$

3. (d) $\log_5 12 = \log_5(3 \times 4) = \log_5 3 + \log_5 4$

$= \log_5 3 + 2\log_5 2 = \{(\log_{10}3)/(\log_{10}5)\}$

$+ \{(2\log_{10}2)/(\log_{10}5)\}$

$= [(\log_{10}3)/\{(\log_{10}10) - (\log_{10}2)\}]$

$+ [(2\log_{10}2)/\{(\log_{10}10) - (\log_{10}2)\}]$

$= \{b/(1-a)\} + \{2a/(1-a)\} = (2a+b)/(1-a)$

4. (a) Since $\log\frac{21}{8} = \log 21 - \log 8$

$= \log 3 + \log 7 - 3\log 2$

$2\log\frac{6}{7} = 2\log 6 - 2\log 7$

$= 2\log 2 + 2\log 3 - 2\log 7$

$\log\frac{7}{27} = \log 7 - \log 27 = \log 7 - 3\log 3$

Hence final value is

$(\log 3 + \log 7 - 3\log 2) + (2\log 2 + 2\log 3 - 2\log 7)$

$+ (\log 7 - 3\log 3)$

$= -\log 2 = \log 2^{-1} = \log\frac{1}{2}$

5. (c) Since $\log_{\sqrt{a}} b^2 = 4\log_a b = \frac{4\log b}{\log a}$

Similarly $\log_{\sqrt{b}} c^2 = \frac{4\log c}{\log b}$

Hence given expression is equal to

$4^{27} \times \frac{\log b}{\log a} \times \frac{\log c}{\log b} \times \frac{\log d}{\log c} \times \ldots \times \frac{\log z}{\log y} \times \frac{\log a}{\log z}$

$= 4^{27} = 2^{54}$

6. (d) Since $\log(x+y) - \log 2 = \log_k k = 1 = \log 10$

$\log\frac{x+y}{2} = \log 10$

$x + y = 20$ ...(i)

From 2nd information

$\log(x+y) - \log xy = \log\frac{5}{16}$

$\log\frac{x+y}{xy} = \log\frac{5}{16}$

Or $\frac{x+y}{xy} = \frac{5}{16} \Rightarrow xy = 64$ ...(ii)

From eqs (i) and (ii) $x - y = 12$

Hence $x = 16$ and $y = 4$ (or vice, versa)

$\log_x y + \log_y x = \frac{\log y}{\log x} + \frac{\log x}{\log y}$

GP_3386

$$= \frac{\log 4}{\log 64} + \frac{\log 64}{\log 4} = \frac{2}{6}$$

$$= \frac{2}{6} + \frac{6}{3} = \frac{1}{3} + 3 = 10/3$$

7. (d) Since let $N = 6^6 \times 8^8 \times 10^{10} \times 12^{12}$

$= 2^{64} \times 3^{18} \times 5^{10}$

Since $\log 2 = 0.3010$ hence $\log 5 = 0.699$

Taking log both the side,

$\log N = 64\log 2 + 18\log 3 + 10\log 5$

$= (64 \times 0.3010) + (18 \times 0.4771) + (10 \times 0.699)$

$= 34.8418$, hence number of digits in N is $34 + 1 = 35$

8. (c) We have

$$p = \log_{12} 18 = \frac{\log_2 18}{\log_2 12} = \frac{1 + 2\log_2 3}{2 + \log_2 3}$$

$$q = \log_{24} 54 = \frac{\log_2 54}{\log_2 54} = \frac{1 + 3\log_2 3}{3 + \log_2 3}$$

Putting $x = \log_2 3$, we have

$$pq + 5(p - q) = \frac{1+2x}{2+x} \cdot \frac{1+3x}{3+x} + 5\left\{\frac{1+2x}{2+x} - \frac{1+3x}{3+x}\right\}$$

$$= \frac{6x^2 + 5x + 1 + 5(-x^2 + 1)}{(x+2)(x+3)} = \frac{x^2 + 5x + 6}{(x+2)(x+3)} = 1$$

9. (b) Let $\log_{10} x - \log_{10} \sqrt{x} = 2\log_x 10$

$$\Rightarrow \frac{1}{2}\log_{10} x = 2\log_x 10$$

$$\Rightarrow \log_{10} x = \log_x 10^4$$

$$\Rightarrow \frac{\log_{10} x}{\log_x 10} = 4 \Rightarrow (\log_{10} x)^2 = 4$$

$\Rightarrow \log_{10} x = \pm 2$

$\Rightarrow x = 10^2$ or 10^{-2}

10. (d) Let $A = \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$

$= \log_x x - \log_x y + \log_y y - \log_y x$

$\Rightarrow A = 2 - (\log_y x + \log_y x)$

$$= 2 - \left(\log_y x + \frac{1}{\log_y x}\right)$$

We know, $a + \frac{1}{a} > 1$

$\Rightarrow A = 2 -$ (a real no more than one)

or $A < 1$

So, A can now be 1.

11. (c) Given $\log_2[\log_7(x^2 - x + 37)] = 1$

$\because \log_p x = y \Rightarrow p^y = x$

$\therefore 2 = \log_7(x^2 - x + 37) \Rightarrow 7^2 = x^2 - x + 37$

$\Rightarrow 49 = x^2 - x + 37 \Rightarrow x^2 - x - 12 = 0$

$\Rightarrow (x - 4)(x + 3) = 0 \Rightarrow x = 4, -3$

$\therefore x = 4$

12. (b) $x = \log_a bc \therefore 1 + x = \log_a abc$

$y = \log_b ca \therefore 1 + y = \log_b abc$

$z = \log_c bc \therefore 1 + z = \log_c abc$

$$\therefore \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

13. (b) Since log3, log $3^x - 2$ and log$3^x + 4$ are in AP, therefore log $3^x - 2$ will be the AM of log 3 and log $3^x + 2$.

i.e., $2(\log 3^x - 2) = \log 3 + \log 3^x + 4$

$\Rightarrow \log(3^x - 2)^2 = \log 3(3^x + 4)$

$\Rightarrow (3^x - 2)^2 = 3(3^x + 4)$

$\Rightarrow (k - 2)^2 = 3(k + 4)$ (assuming, $3^x = k$)

$\Rightarrow k^2 - 7k - 8 = 0$

$\Rightarrow (k - 8)(k + 1) = 0$

$\therefore k = 8$ or $k = -1$

Now, when $k = 8$, $3^x = 8 \Rightarrow \log_3 8 = x$

and when $k = -1$, $3^x = -1$. This is an infeasible relation as RHS is negative but LHS is positive irrespective of the value of x. Therefore, there is only one possible value of x which is $\log_3 8$.

Hence, choice (b) is the correct answer.

Alternatively : When log a, log b and log c are in AP, then a, b and c must be in GP. Therefore , 3, $3^x - 2$ and $3^x + 4$ are in GP. That means, $(3^x - 2)^2 = 3(3^x + 4)$. Now, you can proceed like the above solution.

Alternatively :

log 3, log $3^x - 2$, log $3^x + 4$

$\Rightarrow$ log 3, log $(3^{\log_3 8} - 2)$, log $(3^{\log_3 8} + 4)$

$\Rightarrow$ log 3, log $(8 - 2)$, log $(8 + 4)$

$\Rightarrow$ $\log 3, \log 6, \log 12$

Since 3, 6 and 12 are in GP, therefore log 3, log 6 and log 12 will be in AP.

It proves that the choice (b) is correct.

14. (b) $\log \tan 1° + \log \tan 2° + ... + \log \tan 89°$

$= (\log \tan 1° + \log \tan 89°) + (\log \tan 2° + \log \tan 88°) + ... + \log \tan 45°$

$= \log (\tan 1°. \tan 89°) + \log (\tan 2°. \tan 88°) + ... \log 1$

$= \log (\tan 1° . \cot 1°) + \log (\tan 2° . \cot 2°) + ... \log 1$

$= \log 1 + \log 1 + ... + \log 1 = 0$

$(\because \tan (90 - \theta) = \cot \theta$ and $\tan 45° = 1)$

15. (d) $\log_9 x - \log_9 \left(\frac{x}{10} + \frac{1}{9}\right) = 1 = \log_9 9$

$\Rightarrow \log_9 \frac{x}{\left(\frac{x}{10} + \frac{1}{9}\right)} = \log_9 9$

$\Rightarrow \frac{x}{\left(\frac{x}{10} + \frac{1}{9}\right)} = 9 \Rightarrow \frac{x \times 90}{9x + 10} = 9$

$\Rightarrow 10x = 9x + 10 \quad \Rightarrow x = 10$

16. (d) Let $a = 10, b = 100$ and $c = 1000$ then

$x = \log_{1000}(100) + \log_{100}(1000) = \frac{13}{6}$

$y = \log_{10} 1000 + \log_{1000} 10 = \frac{10}{3}$

$z = \log_{100} 10 + \log_{10} 100 = \frac{5}{2}$

$\therefore x^2 + y^2 + z^2 = \left(\frac{13}{6}\right)^2 + \left(\frac{10}{3}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{794}{36}$

and $xyz = \frac{13}{6} \times \frac{10}{3} \times \frac{5}{2} = \frac{650}{36}$

$\therefore \quad xyz + 4 = \left(\frac{650}{36}\right) + 4 = \frac{794}{36}$

Hence choice (d) is correct.

17. (d) $3^{x+1} = (3 \times 2)^{\log_2 3}$

$\Rightarrow 3^{x+1} = 3^{\log_2 3} \times 2^{\log_2 3}$

$\Rightarrow 3^{x+1} = 3^{\log_2 3^2} \times 3$

$\Rightarrow 3^x = 3^{\log_2 3} \Rightarrow x = \log_2 3$

18. (c) $\log_{10}\left[1 - \left\{1 - \left(1 - x^2\right)^{-1}\right\}^{-1}\right]^{-1/2} = 1$

$\Rightarrow \log_{10}\left[1 - \left\{1 - \frac{1}{1 - x^2}\right\}^{-1}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[1 - \left\{\frac{-x^2}{1 - x^2}\right\}^{-1}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \left[\frac{1}{x^2}\right]^{-\frac{1}{2}} = 10 \Rightarrow x = 10$

19. (d) $\log_4 5 = a$ and $\log_5 6 = b$

$\Rightarrow \log_4 5 \times \log_5 6 = ab$

$\Rightarrow \log_4 6 = ab \Rightarrow \frac{1}{2} \log_2 6 = ab$

$\Rightarrow (1 + \log_2 3) = 2ab$

$\therefore \log_2 3 = 2ab - 1 \Rightarrow \log_3 2 = \frac{1}{2ab - 1}$

20. (d) $[\log_{10} 6730.4] = [\log_{10} 6.7304 \times 10^3]$

$= [3 + \log_{10} 6.7304]$

$= 3$ [Value of $\log_{10} n$ will lie in between 0 and 1 when $1 < n < 10$]

21. (b) $N = n!$ (given)

$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + ... \frac{1}{\log_n N}$

$= \log_N 2 + \log_N 3 + \log_N 4 + ... + \log_N n$

$= \log_N n\,! = \log_N N = 1$

22. (a) Given : $3^{2 - \log_3 5} = 3^2 . 3^{-\log_3 5}$

$(\because a^{m+n} = a^m . a^n)$

$= 9 . 3^{\log_3 5^{-1}} = 9 \times 5^{-1} = \frac{9}{5}$

23. (b) Given expression

$= \log_{xyz} (xy) + \log_{xyz} (yz) + \log_{xyz} (zx)$

$= \log_{xyz} (xy \times yz \times zx) = \log_{xyz} (xyz)^2$

$= 2 \log_{xyz} (xyz) = 2 \times 1 = 2$

24. (b) $\frac{8 \log_8 8}{2 \log_{\sqrt{8}} 8} = \frac{8 \times 1}{2 \log_{\sqrt{8}} \left(\sqrt{8}\right)^2} = \frac{8}{4 \log_{\sqrt{8}} \sqrt{8}}$

$= \frac{8}{4} = 2$

GP_3386

25. **(a)** $\log_3(5+x)+\log_8 8=2^2$

$\log_3(5+x)+1=4$

$\log_3(5+x)=3$

$3^3 = 5+x$

$5+x = 27 \Rightarrow x=27-5=22.$

26. **(d)** $3\log\frac{81}{80}+5\log\frac{25}{24}+7\log\frac{16}{15}$

$$= \log\left[\left(\frac{81}{80}\right)^3\times\left(\frac{25}{24}\right)^5\times\left(\frac{16}{15}\right)^7\right]$$

$$= \log\left(\frac{3^{12}\times5^{10}\times2^{28}}{2^{12}\times5^3\times2^{15}\times3^5\times3^7\times5^7}\right)$$

$= \log 2$

27. **(b)** $2\log(81/100)\times 2/3\log(27/10)\div\log 9$

$=2[\log 3^4-\log 100]\times 2/3[(\log 3^3-\log 10)]\div 2\log 3$

$=2[\log 3^4-\log 100]\times 2/3[(3\log 3-1)]\div 2\log 3$

Substitute $\log 3 = 0.4771 = -0.0552$

28. **(c)** $\log_2\left[\log_2\left(x^2+7\right)\log_{1/4}\left(x^2+7\right)^{-1}\right]=3$

$$\Rightarrow \log_2\left[\log_2\left(x^2+7\right)\log_{2^{-2}}\left(x^2+7\right)^{-1}\right]=3$$

$$\Rightarrow \log_2\left[\frac{1}{2}\left\{\log_2\left(x^2+7\right)\right\}^2\right]=3$$

$$\Rightarrow \left\{\log_2\left(x^2+7\right)\right\}^2=8\times2$$

$$\Rightarrow \log_2\left(x^2+7\right)=4$$

$$\Rightarrow x^2+7=16 \Rightarrow x^2=9 \Rightarrow x=\pm3$$

29. **(a)** $\frac{1}{(\log_a bc)+1}+\frac{1}{(\log_b ac)+1}+\frac{1}{(\log_c ab)+1}$

$$=\frac{1}{\log_a bc+\log_a a}+\frac{1}{\log_b ac+\log_b b}+\frac{1}{\log_c ab+\log_c c}$$

$$=\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$$

$$=\log_{abc} a+\log_{abc} b+\log_{abc} c$$

$\log_{abc} abc = 1$

30. **(c)** $(pa + qb - rc) = p\log_{10}x + q\log_{10}y - r\log_{10}z$

$$=\log_{10}\left(x^p\right)+\log_{10}\left(y^q\right)-\log_{10}\left(z^r\right)$$

$$=\log_{10}\left(\frac{x^p y^q}{z^r}\right)$$

$$\Rightarrow \text{antilog}(pa+qb-rc)=\frac{x^p y^q}{z^r}$$

31. **(d)** The given logarithm expression

$$\frac{\log_{27}9\log_{16}64}{\log_4\sqrt{2}}$$

is simplified as :

$$\frac{\log 9}{\log 27}\times\frac{\log 64}{\log 16}\times\frac{\log 4}{\log\sqrt{2}}$$

$$=\frac{2\log 3}{3\log 3}\times\frac{6\log 2}{4\log 2}\times\frac{2\log 2}{\frac{1}{2}\log 2}$$

$$=\frac{2}{3}\times\frac{6}{4}\times4=4$$

CHAPTER

27

Data Analysis and Data Interpretation

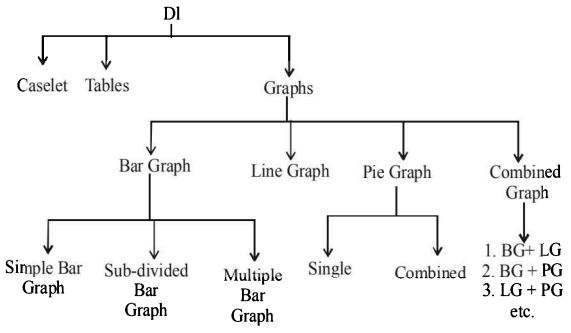
(Including Caselet, Table, Line Graph, Bar Graph, Mixed Bar)

(A) DATA ANALYSIS

In Data Analysis we have to compare the data of the given question. In this type of question, mostly two values have different codes (i.e.-Quantity$_1$ & Quantity$_2$ or x & y) and after the calculation of exact value, one can easily compare the values of there two quantities. This topic is related to all the topics of Arithmetic, Algebra, Mensuration, etc.

(B) DATA INTERPRETATION

Data Interpretation questions are based on the information given in the tables and graphs.



CASELETS

Caselets are generally known as mathematical puzzle. In the caselet problems, informations about one or more things are given. Using this we have to solve the problems based on it.

Normally five questions are asked in a caselet.

CASELETS

A table is one of the easiest way for summarising data. A statistical table is the logical listing of related quantitative data in vertical columns and horizontal rows of numbers with sufficient explanatory and qualifying words, phrases and statements in the form of titles, heading and notes to make clear the meaning of data.

REMEMBER

- $\text{Average} = \dfrac{\text{Sum of all items}}{\text{Total number of items}}$
- % change (increase or decrease)

$$= \left| \frac{\text{Final value} - \text{Initial value}}{\text{Initial value}} \times 100 \right|$$

GRAPHS

Graphs are a convenient way to represent information. The graphs should be labelled properly to show what part of the graphs shows what a value.

1. Bar Graph

Bar diagram consists of a number of equidistant rectangles. One for each category of the data in which the magnitudes are represented by the length or height of rectangle, whereas width of rectangles are immaterial. Thus, a bar is just one dimensional as only the length of the bar is to be considered and not the width. All the bars drawn in a diagram are generally of uniform width which depends on the number of bars to be constructed and the availability of the space.

Types of Bar Graphs are–

(i) Simple Bar Graph

It is used to represent only one dependent variable.

(ii) Sub-divided Bar Graphs

These are used to represent the break down of a total into its component bars. A bar is divided into different segments, each segment respresents a given component. Different

GP_3386

shades, colours, designs etc. are used to distinguish the various components. An index is given to represent the various components. An index is given to represent the various components. To compare, the order of various components in the different bars is same.

(iii) Multiple Bar Graph (MBG)

When a combination of inter-related variables are to be represented graphically, multiple bar diagrams are used. These are extended form of simple bar diagrams. In M.B.G. many aspects of the data are presented simultaneously with separated bars or various shades of colours. An index is given to explain the shades or colours used.

2. Line Graph (LG)

LG are used to show how a quantity changes, very often the quantity is measured as time changes. If the line goes up, the quantity is increasing and the line goes down, the quantity is decreasing. If the line is horizontal, the quantity is not changing.

3. Ple Graph (PG)

It is s a pictorial representation of numerical data by non-intersecting adjacent sectors of a circle sector's area of each sector is proportional to the magnitude of the data represented by the sector.

$$1\% \text{ of total value} = \frac{360}{100} = 3.6^\circ$$

The % of components parts can be converted to degrees by multiplying 3.6°.

Degree of any component part

$$= \frac{\text{component value}}{\text{total value}} \times 360 .$$

LEVEL - 1

1. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Respective ratio of Meera and her mother is 3:8. 8 years hence age of Meera's brother is half of her mother. If Meera is 6 years younger than her brother than ;
X=Sum of ages of meera and her brother
Y=age of mother

2. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A person is 6 years elder than his wife. Age of person is 5 times of that of son and age of mother is 4 time of daughter then what is age of person?
x = sum of ages of father and daughter
y = sum of ages of mother and son

3. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Two cars are moving with uniform speeds from A and B respectively. Distance between A and B is 500 km. If both cars reach their destination (B and A) in 16 hrs. and 25 hrs. after meeting then
X = speed of car moving from A to B
Y = speed of car moving from B to A

4. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A starts a business by investing some amount. B joins here after 4 months investing 50% more amount than A. C joins them for last 4 months by investing amount equal to 60% of total amount of A and B together. If they got annual profit of Rs 100000,
X=difference between shares of A and B
Y =difference between shares of B and C

5. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d)If $x \geq y$ or $x \leq y$ (e) Can't be determine
A and B started a business together. They invested 150000 and 120000 respectively. After 4 months B invested another 25% and A withdraws 25% if they got annual profit of 212000 then
X=share of A in profit
Y=share of B in profit

6. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A can complete a piece of work in 50 days. A is 25% more efficient than B. If A and B can complete the same work in 10 days with the help of C then
X = number of days required for B to complete whole work alone
Y = number of days required for C to complete whole work alone

7. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Three persons A, B and C are working together. A and B completed $\frac{7}{15}$ of the whole work and B and C completed $\frac{11}{15}$ of whole work together then;
X= working efficiency of A
Y= working efficiency of C

8. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine

Speed of boat in still water is 2 times to speed of current. If a person covers 50 km in 10 hrs in upstream then;
X= time required to cover 40 km in downstreem
Y = time required to cover 30 km in upstreem

9. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A, B and C can complete a piece of work in 50 days. A is 25% more work efficient than B and C can complete the work in 60 days alone then;
X = Number of days required for C to complete the whole work alone.
Y = Number of days required for B to complete the whole work alone.

10. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
X = Square of least two digits number completely divisible by 6, 5, 10,9 and 15
Y = square of highest two digits number which is completely divisible by 30,15, 18 and 45.

11. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \leq y$ or $x \leq y$ (e) Can't be determine
Two trains are moving in same direction. First train crosses a 120m platform in 20 sec and second train crosses a 100m long platform in 30 sec.
X = speed of first train.
Y = speed of second train.

12. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \leq y$ or $x \leq y$ (e) Can't be determine
First train crosses a 150m long platform in 25 sec and second train crosses a person standing on same platform in 20 sec. If speed of both trains are 60 km/h and 45 km/h respectively then
X = length of first train.
Y = length of second train.

13. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A person sells a T.V and a music system. T.V is sold by him in 20000 and cost price of music system is 15000. He got 20% profit on music system after 10% discount and he allow 25% discount on T.V.
X = Marked price of music system.
Y = selling price of T.V.

14. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
X=number of sides of a regular polygon which each exterior angle is 18°
Y=number of sides of a regular polygon which has 170 diagonals

15. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Area of a rectangle is 1200 sq m and respective ratio of its length and breadth is 3:4 Area of a square is 25 sq m more than that of rectangle then
X=perimeter of rectangle
Y=perimeter of square

16. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
First train crosses a 200m long platform in 15 sec and 400m long bridge in 25 second. Another train crosses a person standing on same platform in 20 sec. If speed of second train is 36 km/h then;
X=length of first train
Y=length of second train

17. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Marked price of a Fan is 50%more than its cost price and it is sold with 30% discount then

seller's profit is ₹ 50.A bookself is sold at 10% profit for Rs.1100 then;
X=cost price of fan
Y=cost price of bookself

18. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
X=Area of incircle of square which side is 20 cm
Y=Area of square which each side is 12 cm

19. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
Two trains are moving with uniform speeds from A and B respectively. Distance between A and B is 1000 km. If both trains reach their destination (B and A) in 36 hrs. and 64 hrs. after meeting then
X=speed of car moving from A to B
Y=speed of car moving from B to A

20. In the question given below some date is given and two different values are coded as x and y. Find the value of x and y and answer as
(a) If $x > y$ (b) If $x < y$ (c) If $x = y$
(d) If $x \geq y$ or $x \leq y$ (e) Can't be determine
A starts a business with a investment of 75000.Capital of A is 25% less than that of B who joins 2 month after starting.C got $\frac{11}{30}$ of total profit.C joined business with A then
X=Capital of C
Y = Capital of A

Directions (Qs. 21-25): Study the following information carefully and answer the given questions.
Among 800 cricket players, 45% played in IPL –1 and 6.25% played only in IPL –1. Again, 57.5% players played in IPL –2 and 11.25% players played only in IPL –2. Again, 72.5% players played in IPL –3 and 27.5% players played only in IPL–3. Twenty per cent players played in all three IPL tournaments.

21. How many players are there who played in IPL –1 and IPL –2 but not in IPL –3?
(a) 210 (b) 190 (c) 120 (d) 220

22. What is the percentage of players who played in IPL –2 and IPL –3 but not in IPL –1?
(a) 65% (b) 56.25% (c) 75% (d) 55%

23. What is the percentage of the number of players who played in at least two IPL tournaments?
(a) 30% (b) 35% (c) 45% (d) 55%

24. The number of players who played only in either IPL –1 or IPL –2 is what percentage of number of players who played in all the three IPL?
(a) 72% (b) 75% (c) 87% (d) 85%

25. The number of players who played in at most one IPL tournament is what percentage more/ less than the number of players who played in at least one IPL?
(a) 55% more (b) 55% less
(c) 40% more (d) 40% less

Directions (Qs. 26-30): These questions are based on the following data. Study it carefully and answer the questions that follow.
In a school having 18000 students boys and girls are in the ratio 3 : 5 respectively. The students like Tea, Coffee or both. 12% of the boys like only Tea, 22% of the girls like only Coffee. 24% of the total students like only Tea and the number of boys liking both the beverages is six times the number of boys liking only Tea.

26. Total how many boys like Tea?
(a) 4260 (b) 5670 (c) 5975 (d) 4235

27. How many girls like only Tea?
(a) 3485 (b) 3050 (c) 3864 (d) 3510

28. Total how many students like Coffee?
(a) 13680 (b) 17110 (c) 12025 (d) 12053

29. Number of girls liking only Tea is what percent of the total number of students liking only Tea?
(a) 68.25 (b) 71.50 (c) 81.25 (d) 78.50

30. What is the respective ratio between the number of boys and girls liking both the beverages?
(a) 23 : 25 (b) 12 : 25 (c) 12 : 13 (d) 25 : 13

Directions (Qs. 31-35): Study the following information carefully to answer the questions that follow:
A company produces 4 different products, viz AC, fans, refrigerators and ovens, each product of two different qualities i.e, Quality A and Quality B. The company produces a total of 500 products. One-fifth of total number of products are fans, out of which 35% are of Quality B. Fifteen percent of the total number of products are AC. Two-thirds of the ACs are of Quality A. Twenty five per cent of the total number of products are refrigerators, out of which 40 are of quality B. Ten per cent of the number of ovens are of Quality B.

31. What is the total number of AC and ovens of Quality B and fans and refrigerators of Quality A together made by the company?
(a) 165 (b) 205 (c) 155 (d) 195

GP_3386

32. What is the average number of products of Quality A made by the company?
(a) 90 (b) 75 (c) 80 (d) 95

33. What is the ratio of the number of ovens of Quality B to the number of fans of Quality A?
(a) 5 : 2 (b) 4 : 13 (c) 5 : 13 (d) 4 : 9

34. What is the difference between the number of ACs of Quality A and Quality B?
(a) 25 (b) 50 (c) 35 (d) 40

35. The number of refrigerators of Quality A is **approximately** what percentage of the total number of ovens (both Quality A and B together)?
(a) 39 (b) 31 (c) 35 (d) 43

Directions (Qs. 36-40): Study the information carefully to answer the questions that follow.
A company produced five different products, viz mobile phone, pen drive, calculator, television and washing machine. The total number of all the five products is 1650. 24% of the total number of products is mobile phones. One-sixth of the total number of products is pen drives. 14% of the total number of products is calculators. Remaining products are either television or washing machine. The number of washing machines is 50 more than the number of televisions produced.

36. What is the ratio of the number of washing machines to the number of calculators produced by the company?
(a) 17 : 11 (b) 19 : 11 (c) 11 : 17 (d) 19 : 13

37. If 24 per cent of the pen drives are defective, what is the number of pen drives which are not defective?
(a) 209 (b) 215 (c) 219 (d) 225

38. The number of televisions produced is approximately what per cent of the total number of calculators and washing machines produced together?
(a) 63 (b) 55 (c) 59 (d) 51

39. What is the difference between the total number of televisions and mobile phones together and the number of calculators produced?
(a) 534 (b) 524 (c) 514 (d) 523

40. What is the total number of pen drives, calculators and washing machines produced by the company?
(a) 907 (b) 917 (c) 925 (d) 905

Directions (Qs. 41-45): Study the information carefully to answer the question that follows:
In an annual match, 1008 students participated in a College. The ratio of the number of girls to the number of boys is 5:3 respectively. Out of the total girls, 20% participated in cricket and the remaining girls participated in hockey, football and tennis in the ratio of 2:3:4 , Two –thirds of the total boys participated in football and the remaining boys participated in hockey and cricket in the ratio of 4:5.

41. What is the approximate percentage of the boys who have participated in cricket out of the total number of boys?
(a) 18.5% (b) 23% (c) 16% (d) 27%

42. What is the approximate percentage of the girls who participated in hockey out of the total number of participants?
(a) 11% (b) 15% (c) 6% (d) 20%

43. What is total number of girls who have participated in football and tennis together?
(a) 492 (b) 392 (c) 384 (d) 368

44. What is the ratio of number of boys to the number of girls who have participated in hockey?
(a) 1 : 2 (b) 2 : 1 (c) 4 : 3 (d) 3 : 2

45. What is the difference between the number of boys and that of girls who have participated in cricket?
(a) 63 (b) 35 (c) 56 (d) 126

Directions (Qs. 46-50): These questions are based on the following data. Study it carefully and answer the questions that follow.
In a school having 400 students, boys and girls are in the ratio of 3 : 5. The students speak Hindi, English or both the languages. 12% of the boys speak only Hindi. 22% of the girls speak only English. 24% of the total students speak only Hindi and the number of boys speaking both the languages is six times the number of boys speaking only Hindi.

46. How many boys speak Hindi?
(a) 18 (b) 126 (c) 108 (d) 26

47. How many girls speak only Hindi?
(a) 55 (b) 117 (c) 96 (d) 78

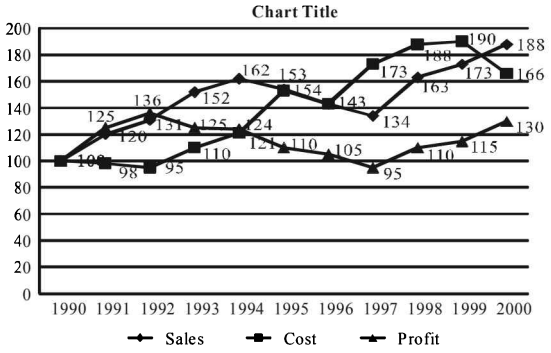
48. How many students speak English?
(a) 304 (b) 79 (c) 225 (d) 117

49. The number of girls speaking only Hindi is what per cent of the total number of students speaking only Hindi?
(a) 38.2% (b) 71.8%
(c) 31.2% (d) None of these

50. What is the ratio of the number of boys to the number of girls speaking both the languages?
(a) 23 : 25 (b) 12 : 25 (c) 12 : 13 (d) 25 : 13

Directions (Qs. 51-56): Study the following graph to answer these questions.

Index of sales, cost, and profit



Profit = Sales – Cost – Tax

Gross Profit = Sales – Cost

51. What is the profit in the year 1998?
(a) ₹ 110 (b) ₹ 143
(c) ₹ 95 (d) cannot be determined

52. What is the gross profit in 1993?
(a) ₹ 27 (b) ₹ 30
(c) ₹ 35 (d) None of these

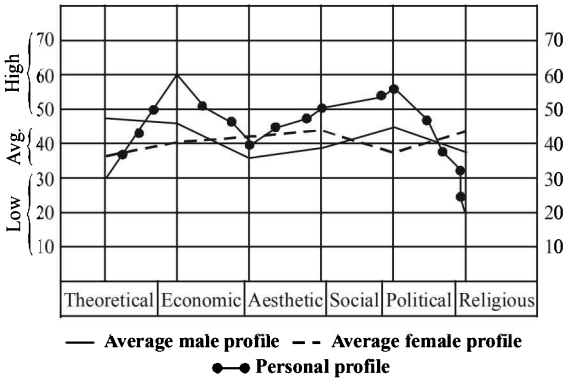
53. If the profit was ₹600 in 1993, then what was the profit in 1990?
(a) ₹ 441 (b) ₹ 395 (c) ₹ 480 (d) ₹ 545

54. If the base of the profit index is changed to 1993 = 100, then what would be the index of profit for the year 2000?
(a) 104 (b) 100
(c) 109 (d) None of these

55. If sales were ₹1200 crore in 1990, then what were the total sales in the period 1990-1995?
(a) ₹ 8628 crore (b) ₹ 9828 crore
(c) ₹ 9156 crore (d) ₹ 8136 crore

56. What was the difference between the average sales index and the average cost index?
(a) 7.3 (b) 7.7 (c) 7.5 (d) 7.9

Directions (Qs. 57-61): Use the graph given below to answer these questions : Given is graph is the profile of values of a college student marked as personal profile. The normative profiles are given as average male profile and average female profile.



57. Compare the three and state which of the given values is the highest in the personal profile of the student?
(a) Theoretical (b) Religious
(c) Social (d) Economic

58. In the given personal profile, which is the value with the lowest score?
(a) Theoretical (b) Religious
(c) Social (d) Aesthetic

59. In which values score, there exists maximum difference between average female profiles and personal profile?
(a) Theoretical (b) Religious
(c) Economic (d) Political

60. In which value score, there exists convergence between personal profile and average female profile?
(a) Theoretical (b) Social
(c) Aesthetic (d) None of these

61. In which value score, there exists a no difference state between the personal profile and average male profile?
(a) Economic (b) Social
(c) Aesthetic (d) None of these

Direction (Qs. 62–66): Study the following table carefully to answer the questions that follow. Monthly rent (in ₹ thousand) at five different places in six different years

Years	Place				
	Churchgate	Dadar	Kandivali	Borivali	Virar
2005	5.3	3.8	1.5	2.7	1.1
2006	12.5	8.3	3.4	4.8	2.1
2007	16.7	11.7	5.5	6.6	1.8
2008	20.9	13.6	9.8	12.7	3.6
2009	25.8	14.5	11.5	14.1	5.5
2010	30.3	20.9	15.6	15.9	7.8

62. In which place did the monthly rent not increase consistently from year 2005 to 2010?
(a) Churchgate (b) Dadar
(c) Kandivali (d) Virar

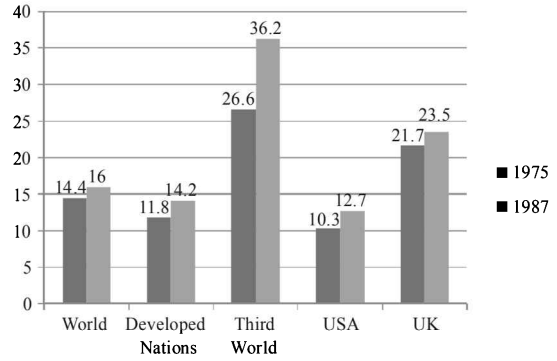
63. In which year at Churchgate, the monthly rent increased more than 100 per cent from the previous year ?
(a) 2006 (b) 2007 (c) 2008 (d) 2009

64. What is the difference between the monthly rent at Dadar in the year 2009 and Borivali in the year 2007?
(a) ₹ 7,600 (b) ₹ 7,900 (c) ₹ 8,100 (d) ₹ 8,600

65. Monthly rent at Kandivali in the year 2008 was approximately what per cent of the total monthly rent at Virar over all the years together ?
(a) 30 (b) 38 (c) 42 (d) 45

66. Which city was most expensive in terms of rent ?
(a) Churchgate (b) Dadar
(c) Kandivali (d) Borivali

Directions (Qs. 67-70): The following bar graph depicts the annual rates of Inflation in percentages for 1975 and 1987



67. From 1975 to 1987, inflation rate increased in the third world countries approximately by :
(a) 10% (b) 20% (c) 30% (d) 35%

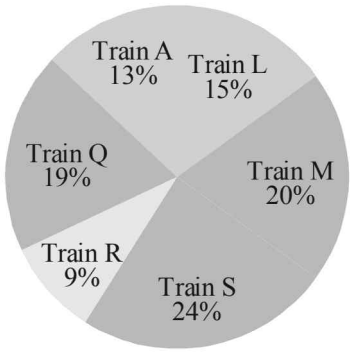
68. In the year 1975,the lowest rate of inflation was in the
(a) Developed Nations(b) UK
(c) USA (d) Third World

69. In the year 1987,the inflation rate in the third world countries vis-à-vis the whole world jumped approximately by :
(a) 50% (b) 100% (c) 125% (d) 200%

70. Comparing the figures for the USA vis-à-vis the developed nations,it can be concluded that :
(a) USA had better control on inflation.
(b) Developed Nations had better control on inflation,
(c) The control on inflation rate continues to be the same for the USA and for the developed nations.
(d) No conclusion can be drawn.

Directions (Qs. 71-75): Study the following pie-chart carefully to answer these questions.
Total Number of passengers = 8500

Percentage of Passengers



71. What was the approximate average number of passenger in Train-S, Train-M and Train-L together?
(a) 1521 (b) 1641 (c) 1651 (d) 1671

72. If in Train-R 34 percent of the passengers are females and 26 percent are children, what is the number of males in that train?
(a) 306 (b) 316 (c) 308 (d) 318

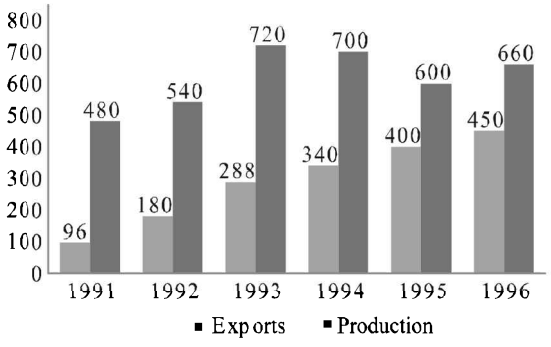
73. The number of passengers in Train-Q is approximately what percentage of the total number of passengers in train-A and Train-R?
(a) 90 (b) 70
(c) 75 (d) None of these

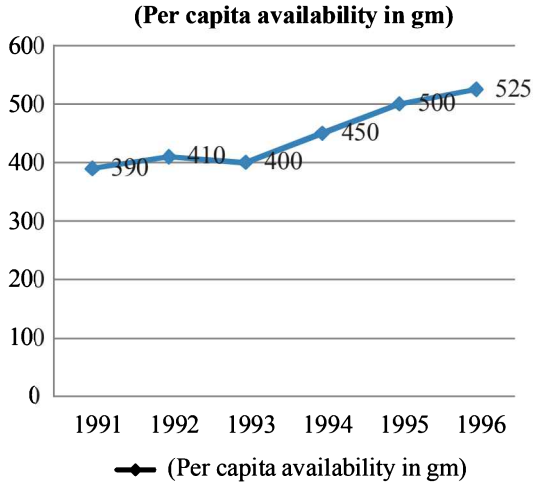
74. Which train has the second highest number of passengers?
(a) A (b) Q (c) S (d) M

75. How many more per cent (approximately) number of passengers are there in Train-M as compared to the number of passengers in Train-L?
(a) 29 (b) 49 (c) 43 (d) 33

Directions (Qs. 76-85): Study the following graph and answer these questions given below it.

Tea in India (In Million kg)





76. Which year shows the maximum percentage of export with respect to production?
(a) 1992 (b) 1993 (c) 1996 (d) 1995

77. The population of India in 1993 was:
(a) 800 million (b) 1080 million
(c) 985 million (d) 900 million

78. If the area under tea production was less by 10% in 1994 than in 1993, then the approximate rate of increase in productivity of tea in 1994 was:
(a) 97.22 (b) 3
(c) 35 (d) Cannot be determined

79. The average proportion of tea exported to the tea produced over the period is:
(a) 0.87 (b) 0.47 (c) 0.48 (d) 0.66

80. What is the first half decade's average per capita availability of tea?
(a) 475g (b) 535g (c) 446g (d) 430g

81. In which year was the per capita availability of tea minimum?
(a) 1996 (b) 1994
(c) 1991 (d) None of these

82. In which year was there minimum percentage of export with respect to production?
(a) 1991 (b) 1992 (c) 1993 (d) 1994

83. In which year we had maximum quantity of tea for domestic consumption?
(a) 1994 (b) 1991 (c) 1993 (d) 1996

84. What approximately was the average quantity of tea available for domestic consumption during the period?
(a) 324.3 million Kg (b) 400 million kg
(c) 410.3 million kg (d) 320.3 million kg

85. What was approximately the average population during the period?
(a) 625 million (b) 624 million
(c) 600 million (d) 757 million

Directions (Qs. 86-90) : Study the following table carefully and answer the given questions.
The table shows the percentage of 25000 people who are involved in different professions and the percentage of female and male professionals among them.

Profesions	Percentage of people	Percentage of females	Percentage of males
Banking	20	40	–
Law	15	20	–
Teaching	30	–	40
Engineering	25	–	30
Medical	10	60	–

86. The total number of people in the Teaching profession is what percentage of the total number of people in the Medical profession?
(a) 175% (b) 225% (c) 300% (d) 140%

87. What is the ratio of the total number of males in the Medical and Banking professions together to the total number of females in the same profession together?
(a) 3 : 5 (b) 7 : 5 (c) 8 : 7 (d) 7 : 8

88. The number of females in the Engineering profession are **approximately** what percent of the number of males in the Banking profession ?
(a) 35% (b) 125% (c) 146% (d) 153%

89. What is the ratio of the total number of males in the Banking and Medical professions together to the total number of females in the Law and Teaching professions together ?
(a) 4 : 5 (b) 3 : 7 (c) 16 : 21 (d) 21 : 16

90. The total number of females in the Engineering profession is **approximately** what percentage more than the number of males in the Law profession?
(a) 46% (b) 51% (c) 37% (d) 54%

Direction (Qs. 91-95): Study the following table carefully and answer the given questions.
Table shows the data related to different given products.

Name of product	Cost Price (in ₹)	Selling Price (in ₹)	Cost of Transportation (in ₹)	Profit (in ₹)	Loss (in ₹)	Profit or Loss%
Bike	1,50,000	–	15,000	–	–	Profit%= 10%
Computer	–	25,500	0	–	2500	
Laptop	27,000	–	5,000	3,000	–	
Camera	40,000	35,000	4,200	–	–	
Mobile	40,000	–	2,000	3,000	–	
Hard disk	2,000	–	500	–	–	

GP_3386

91. If the profit on Bike is 10%, then what is its selling price?
(a) ₹ 2,00,500 (b) ₹ 1,81,500
(c) ₹ 1,65,000 (d) ₹ 1,71,000

92. The selling price of Laptop is what percentage of the cost price of Computer?
(a) 200% (b) 175% (c) 150% (d) 125%

93. What is the ratio between the loss on Camera and Computer?
(a) 19 : 25 (b) 92 : 25 (c) 39 : 28 (d) 27 : 38

94. What is the difference between the selling price of Mobile and Laptop?
(a) ₹ 7,000 (b) ₹ 9,000 (c) ₹ 10,000 (d) ₹ 12,000

95. If the loss on Hard disk is 5%, then its selling price is **approximately** what percentage less than the selling price of Computer?
(a) 80% (b) 98 % (c) 85% (d) 91%

LEVEL - 2

Direction (Qs. 1-5): In the given paragraph some information's are given about some different schools. Two values are coded as x and y in each of the given questions. Study the given paragraph carefully for the value of x and y and answer the questions. As:
(a) If X > Y
(b) If X < Y
(c) If X = Y
(d) If X ≥ Y or X ≤ Y
(e) Can't be determined

Respective ratio of the number of boys and girls in school A is 5 : 7 Average number of students in schools A, B, C, D and E is 4800. Total number of students in school E is 25% of total number of students in all schools and number of girls is 40% of total number of students in this school. Number of students in school B is 20% less than the number of students in school E and respective ratio of number of boys and girls in school B is 9 : 7. Respective ratio of number of students is school B and school D is 5 : 4 and number of boys in school B is 10% less than that of school D. Number of girls is school C is equal to number of girls in school A and respective ratio of number of students in school A and school C is 20 : 19.

1. X = Number of student in school A
Y = number of students in school B

2. X = Number of boys in school C
Y = Number of girls in school E

3. X = Average number of boys in all schools
Y = Average number of girls in all schools

4. X = Average number of boys in schools A, C and E
Y = Average number of girls in school B and D

5. X = Difference between number of students in schools A and C
Y = Difference between number of students in schools E and B

Direction (Qs. 6-10): Some information are given in the paragraph Two values are coded as x and y. Study the paragraph carefully for value of x and y and answer as:
(a) If X > Y
(b) If X < Y
(c) If X = Y
(d) If X ≥ Y or X ≤ Y
(e) Can't be determined

Train A is running with a speed of 60 km/h. Distance covered by train D is 40% more than distance covered by train E and speed of train E is 20% more than train A. Train C starts at 7AM and reaches its destination at 5 PM. Its speed is 80% of the speed of train D which speed is 50% more than the speed of train A. Respective ratio of distance covered by trains A, B and C is 5 : 7 : 3. Time taken by train A is equal to the time taken by train E and 20% more than train D speed of train B is 4 times to number of hrs taken by E.

6. X = Time taken by A
Y = Time taken by B

7. X = Speed of C
Y = Speed of E

8. X = Average distance covered by trains A, C and E
Y = Average distance covered by trains B and D

9. X = total distance covered by A and C together
Y = total distance covered by B and D together

10. X = Speed of C
Y = total number of hrs taken by A, B, C and E together

Direction (Qs. 11-15): In the paragraph given below some information's are given about the selling of some items. Two values are coded as x and y. Study the given information for the value of x and y and answer as:
(a) If X > Y
(b) If X < Y
(c) If X ≥ Y
(d) If X ≤ Y
(e) If X = Y or any other options

A shopkeeper has some electronic goods in his shop. Cost price of T.V is 40% less than the selling price of A.C which is sold at 20% profit after a discount of 20%. Selling price of cooler is 50% less than the cost price of T.V and it is sold at 10% profit with a discount of 20%. T.V is sold at 20% profit with a discount of 40% and amount of discount on T.V is 24000. Laptop is sold at 30% discount and its cost price is two times to selling price of Mobile which is sold at 50% gain and 25% discount. Selling price of Laptop is 50% more than the selling price of A. C and person got 20% profit on it.

11. X = Average cost price of A.C and Mobile
Y = cost price of Laptop

12. X = Average profit amount of all items
Y = average discount amount of all items.

13. X = cost price of Cooler
Y = profit amount of Laptop

14. X = Profit amount of Mobile
Y = Discount amount of Mobile

15. X = Marked price of Laptop
Y = Marked price of A. C

Direction (Qs. 16-20): In the paragraph some information are given about the income expenditure and savings of some different persons. Study the given information's carefully and answer the related questions as below where two values are coded as x and y

(a) If $X>Y$
(b) If $X<Y$
(c) If $X=Y$
(d) If $X \leq Y$
(e) If $X \geq Y$

Five persons A, B, C, D and E are working in 5 different fields. Salary of D is 20% less than that of B and 50% less than that of A. Expenditure of C is 80% of the expenditure of E who spends 80% of his salary. Respective ratio of savings of A and E is 8 : 5 and A saves 25% of his salary. Savings of B and D are 40% and 20% of their salaries respectively. Salary of C is 20% less than average salary of all others. Saving of E is 10000.

16. X= Salary of B
Y= Expenditure of A

17. X = Average salary of all persons together
Y = Salary of D

18. X = Total expenditure of all persons together
Y = Total salary of A, D and E together

19. X = Saving of A
Y = Saving of C

20. X = Total saving of all persons together
Y = Salary of A

Direction (Qs. 21-25): For solving these questions we summarise the given information as below:
Certain number of students passed from five colleges P, Q, R, S and T. Total students passed in College P is 18% of total passed students from all colleges. Ratio of total passed students from College Q to total passed students from College R is 11:10 and their average value is 840. From College S 600 students passed which is 15% of total passed students and students passed in College T is 5/4th of number of total passed students of College R. Ratio of boys to girls passed from College P is 13 : 5. Number of passed boys in College Q is 240% of number of passed girls from College P. Ratio of total passed in College R to number of girls passed in College R is 20:11. Number of girls passed in College S is 40% more than the number of passed boys in College S and 25% of total students passed in College T. Average of boys passed in all five colleges is 462.

21. What is the average of total girls passed in all colleges?
(a) 274 (b) 372 (c) 338 (d) 369

22. What is the ratio of passed boys and girls of College T?
(a) 5 : 3 (b) 4 : 3 (c) 7 : 3 (d) 7 : 5

23. Number of boys passed in College Q is how much percentage more than the number of passed girls?
(a) 20% (b) 30% (c) 40% (d) 45%

24. Number of girls passed in College P is what percent of number of passed boys in College S?
(a) 60% (b) 70% (c) 80% (d) 40%

25. In which college the percentage of girls passed with respect to its total passed students is maximum?
(a) P (b) Q (c) R (d) S

Directions (Qs. 26-30): Study the following information carefully and answer the given questions.
In a UN conference there were 400 delegates. Out of them 45% delegates speak English. The ratio between number of people speaking French and number of people speaking Latin is 5 : 7. Difference between the number of people speaking English and number of people speaking Latin is 10% of the total number of delegates. Among these three languages English speaking people are maximum. 30 delegates are there who can speak English and French but cannot speak Latin and it is 25% more than the number of people who can speak French and Latin. 5% of the delegates can speak all three languages. Number of people who can speak English and Latin but cannot speak French is 5.

26. How many delegates are there who can speak only one language out of English, French and Latin?

(a) 224 (b) 248 (c) 272 (d) 280

27. How many delegates are there who cannot speak any of three language out of English, French, Latin?
(a) 64 (b) 48 (c) 44 (d) 36

28. How many delegates can speak at least two languages out of English, French, and Latin?
(a) 64 (b) 80 (c) 88 (d) 104

29. How many delegates are there who can speak exact two languages out of English, French, and Latin?
(a) 24 (b) 36 (c) 40 (d) 44

30. How many delegates are there who can speak at most two languages?
(a) 316 (b) 380 (c) 248 (d) 296

Directions (Qs.31-35): Study the following information and answer the questions that follow:
The premises of a bank are to be renovated. The renovation is in terms of flooring. Certain areas are to be floored either with marble or wood. All rooms/halls and pantry are rectangular. The area to be renovated comprises a hall for customer transaction measuring 23m by 29m, the branch manager's room measuring 13m by 17m, a pantry measuring 14m by 13m, a record keeping-cum-server room measuring 21m by 13 m and locker area measuring 29m by 21m. The total area of the bank is 2000 square metres. The cost of wooden flooring is ₹170 per square metre and the cost of marble flooring is ₹ 190 per square metre. The locker area, record keeping-cum-server room and pantry are to be floored with marble. The branch manager's room and the hall for customer transaction are to be floored with wood. No other area is to be renovated in terms of flooring.

31. What is the ratio of the total cost of wooden flooring to the total cost of marble flooring?
(a) 1879 : 2527 (b) 1887 : 2386
(c) 1887 : 2527 (d) 1829 : 2527

32. If the four walls and ceiling of the branch manager's room (the height of the room is 12 metres) are to be painted at the cost of ₹190 per square metre, how much will be the total cost of renovation of the branch manager's room, including the cost of flooring?
(a) ₹ 1,36,800 (b) ₹ 2,16,360
(c) ₹ 1,78,790 (d) ₹ 2,11,940

33. If the remaining area of the bank is to be carpeted at the rate of ₹110 per square metre, how much will be the increment in the total cost of renovation of bank premises?
(a) ₹ 5,280 (b) ₹ 4,848 (c) ₹ 3,689 (d) ₹ 6,690

34. What is the percentage area of the bank that is not to be renovated?
(a) 2.2% (b) 2.4% (c) 4.2% (d) 4.4%

35. What is the total cost of renovation of the hall for customer transaction and the locker area?
(a) ₹ 2,29,100 (b) ₹ 2,30,206
(c) ₹ 2,16,920 (d) ₹ 2,42,440

Directions (Qs. 36-40): Study the following information carefully to answer the questions that follow:
There are two trains, Train A and Train B. Both trains have four different types of coaches, viz General, Sleeper, First Class and AC. In Train A, there are total 700 passengers. Train B has thirty per cent more passengers than Train A. Twenty per cent of the passengers of Train A are in General Coach. One-fourth of the total number of passengers of Train A are in AC coach. Twenty three per cent of the passengers of Train A are in Sleeper Coach. Remaining passengers of Train A are in First Class Coach. The total number of passengers in AC Coach in both the trains together is 480. Thirty per cent of the numbers of passengers of Train B are in Sleeper Coach. Ten per cent of the total passengers of Train B are in First Class Coach. The remaining passengers of Train B are in General Coach.

36. What is the ratio of the number of passengers in First Class Coach of Train A to the number of passengers in Sleeper Coach of Train B?
(a) 13 : 7 (b) 7 : 13
(c) 32 : 39 (d) Data Inadequate

37. What is the total number of passengers in the General Coach of Train A and the AC Coach of Train B together?
(a) 449 (b) 459 (c) 435 (d) 445

38. What is the difference between the number of passengers in the AC Coach of Train A and the total number of passengers in Sleeper and First Class Coach together of Train B?
(a) 199 (b) 178
(c) 187 (d) None of these

39. The total number of passengers in General Coaches of both the trains together is approximately what percentage of the total number of passengers in Train B?
(a) 35 (b) 42 (c) 46 (d) 38

40. If the cost per ticket of First Class coach is ₹ 450, what will be the total amount generated from First Class Coach of Train A?
(a) ₹ 1,00,080 (b) ₹ 1,08,000
(c) ₹ 1,00,800 (d) ₹ 10,800

Directions (Qs. 41-45): The following table gives the quarterly output of three production units of machine manufacturing company.

Year	1st Quarter			2nd Quarter			3rd Quarter			4th Quarter		
Unit	1	2	3	1	2	3	1	2	3	1	2	3
1998	30	34	38	40	44	42	36	37	39	34	35	37
1999	34	45	42	52	54	56	50	49	48	44	45	48
2000	35	45	43	58	53	58	54	56	56	48	49	43
2001	54	58	57	76	77	79	68	69	70	62	64	66
2002	80	82	87	92	97	93	86	88	87	82	84	83

41. Which of the following statement is correct ?
(i) The performance of Unit 1 has been the lowest during the 5 years period
(ii) The performance of Unit 2 has been the highest during the 5 years period
(iii) The performance of Unit 3 has been the highest during the 5 years period
(iv) The performance of Unit 2 has been the lowest during the 5 years period
(a) (i) (b) (iii) and (iv)
(c) (i) and (iii) (d) None of these

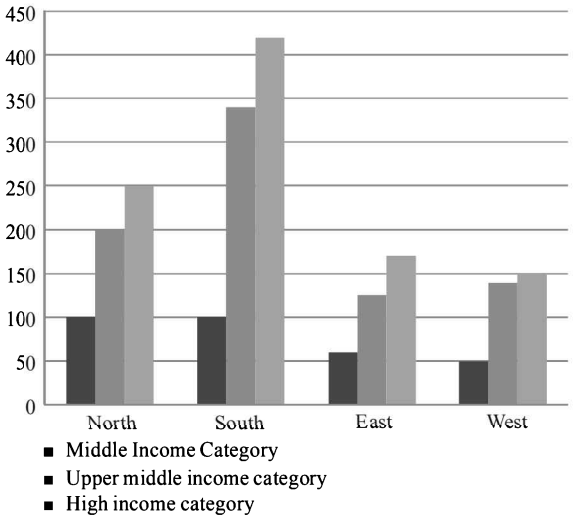
42. Incentives based on the production are given on quarterly basis. In which quarter would the workers get maximum incentive ?
(a) 1st Quarter (b) 2nd Quarter
(c) 3rd Quarter (d) 4th Quarter

43. Which unit has shown the least growth in production during the period 1998 – 2002 ?
(a) Unit 1 (b) Unit 2
(c) Unit 3 (d) Cannot say

44. During 1998-2002 the highest quarterly production has been achieved by Unit ... in the ..... quarter of 2002 ?
(a) 1, 1st (b) 2, 2nd
(c) 3, 3rd (d) 2, 4th

45. The highest increase in the yearly average has been over the period :
(a) 1998-1999 (b) 2001-2002
(c) 1999-2000 (d) 2000-2001

Directions (Qs. 46-48): Answer the questions based on the following information.
The following bar chart gives the growth percentage in the number of households in middle, upper-middle and high income categories in the four regions for the period between 1987-88 and 1994-95.



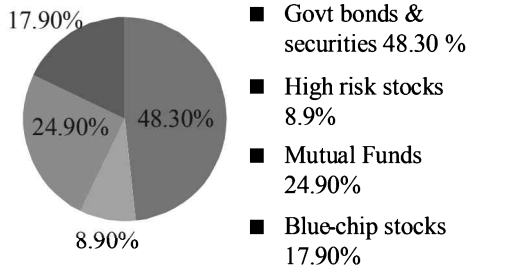
	Number of households in 1987-88 (in thousands)	Average household income in 1987-88	Growth in average household income (1994-95 over 1987-88)
Middle - income	40	₹ 30,000	50%
Upper - income	10	₹ 50,000	60%
High - income	5	₹ 75,000	90%

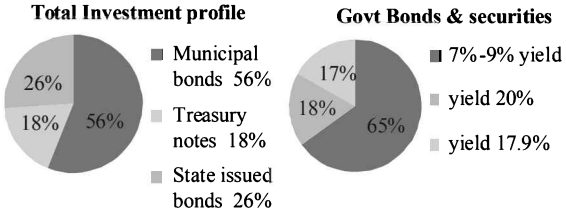
46. Which region showed the highest growth in number of households in all the income categories for the period?
(a) North (b) South
(c) West (d) None of these

47. What was the total household income in northern region for upper-middle class?
(a) ₹ 50 lakh (b) ₹ 500 million
(c) ₹ 300 million (d) Cannot be determined

48. What is the percentage increase in total number of households for the northern region (upper-middle) over the given period?
(a) 100% (b) 200%
(c) 240% (d) Cannot be determined

Directions (Qs. 49-52): These questions are based on the following pie-charts.
Total Investment Funds = ₹11 crore 5 lakh.





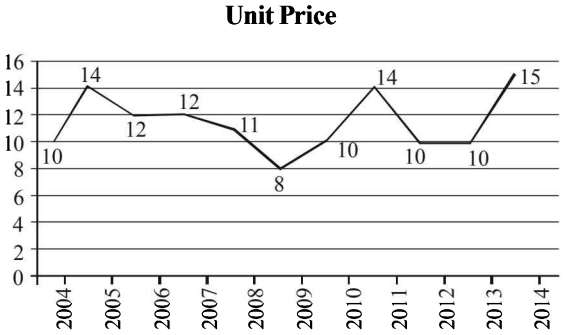
49. According to these graphs, approximately how much money from the investment portfolio was invested in high-risk stocks?
(a) ₹ 98,00,000 (b) ₹ 10,10,000
(c) ₹ 9,00,000 (d) None of these

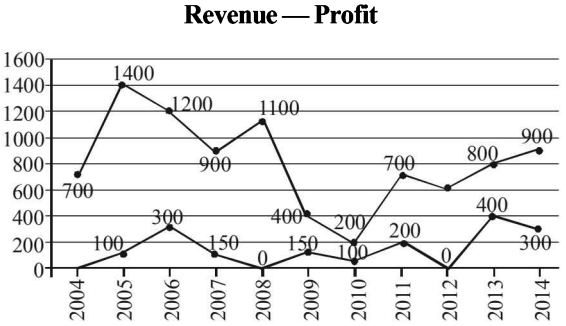
50. Approximately how much money belonging to the investment portfolio was invested in State-issued bonds?
(a) ₹ 4,50,00,000 (b) ₹ 3, 39, 50,000
(c) ₹ 2,87,00,000 (d) None of these

51. Which of the following earned the least amount of money for the investment portfolio?
(a) Government bonds and securities
(b) State-issued bonds
(c) Municipal bonds
(d) None of the above

52. Which of the following was the greatest?
(a) The amount of money invested in high-risk stocks
(b) The amount of money invested in State-issued bonds
(c) The amount of money invested in municipal bonds which yielded between 7% and 9%
(d) The amount of money invested in municipal bonds which yielded over 9%

Directions (Qs. 53-57): Answer the questions based on the following two graphs, assuming that there is not fixed cost component and all the units produced are sold in the same year.





53. In which year per unit cost in highest?
(a) 2006 (b) 2005
(c) 2009 (d) 2011

54. What is the approximate average quantity sold during the period 2004-2014?
(a) 64 units (b) 70 units
(c) 77 units (d) 81 units

55. If volatility of a variable during 2000-2014 is defined as $\frac{\text{Maximum value} - \text{Minimum value}}{\text{Average value}}$, then which of the following is true?
(a) Price per unit has highest volatility
(b) Cost per unit has highest volatility
(c) Total profit has highest volatility
(d) Revenue has highest volatility

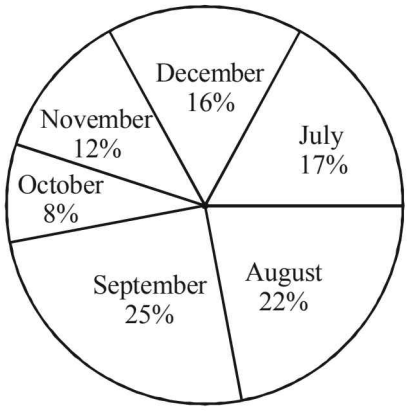
56. If the price per unit decrease by 20% during 2004-2008 and cost per unit increase by 20% during 2009-2014, then during how many number of years there is loss?
(a) 3 years (b) 4 years
(c) 5 years (d) 7 years

57. If the price per unit decrease by 20% during 2000-2004 and cost per unit increase by 20% during 2005-2010, then the cumulative profit for the entire period 2000-2010 decrease by
(a) ₹1650 (b) ₹1550
(c) ₹1300 (d) ₹1250

Directions (Qs. 58-62) : Study the following pie-chart and table carefully and answer the questions given below :

Percentage wise distribution of the number of mobile phones sold by a shopkeeper during six months

Total number of mobile phones sold = 45,000



The respective ratio between the number of mobile phones sold of company A and company B during six months

Month	Ratio
July	8:7
August	4:5
September	3:2
October	7:5
November	7:8
December	7:9

58. What is the respective ratio between the number of mobile phones sold of company B during July and those sold during December of the same company?
(a) 119:145 (b) 116:135 (c) 119 :135 (d) 119:130

59. If 35% of the mobile phones sold by company A during November were sold at a discount, how many mobile phones of company A during that month were sold without a discount?
(a) 882 (b) 1635 (c) 1638 (d) 885

60. If the shopkeeper earned a profit of ₹433/- on each mobile phone sold of company B during October, what was his total profit earned on the mobile phones of that company during the same month?
(a) ₹6,49,900/- (b) ₹6,45,900/-
(c) ₹6,49,400/- (d) ₹6,49,500/-

61. The number of mobile phones sold of company A during July is approximately what percent of the number of mobile phones sold of company A during December ?
(a) 110 (b) 140 (c) 150 (d) 130

62. What is the total number of mobile phones sold of company B during August and September together ?
(a) 10,000 (b) 15,000 (c) 10,500 (d) 9,500

Directions (Qs. 63–67): Study the following table carefully and answer the given questions.

Data related to number of candidates appeared and qualified in a competitive exam from two states during five years.

	State - A		State - B	
Years	Number of appeared candidates	Percentage of appeared candidates who qualified	Number of appeared candidates	Percentage of appeared candidates who qualified
2011	900	60%	-	30%
2012	1200	43%	-	45%
2013	-	60%	560	60%
2014	960	70%	1100	50%
2015	760	-	800	-

Note : Few value are missing in the table (indicated by A candidate is expected to calculate the missing value, if it is required to answer the given questions on the basis of given data and information.)

63. Out of the number of qualified candidates from state A in 2013, the respective ratio of male and female candidates is 11 : 7. If the number of female qualified candidates from state A is 252, then what is the number of appeared candidates (both male and female) from state A in 2013 ?
(a) 930 (b) 1080 (c) 1570 (d) 1690

64. The number of appeared candidates from state B is increased by 100% from 2011 to 2012. If the total number of qualified candidates from state B in 2011 and 2012 together is 816, then what is number of appeared candidates from state B in 2011?
(a) 780 (b) 560 (c) 680 (d) 640

65. What is the difference between the number of qualified candidates from state A in 2011 and 2012 ?
(a) 24 (b) 22 (c) 34 (d) 28

66. If the average number of qualified candidates from state B in 2013, 2014 and 2015 is 420, then what is the number of qualified candidates from state B in 2015?
(a) 384 (b) 395 (c) 483 (d) 374

67. If the respective ratio between the number of qualified candidates from state A in 2014 and

GP_3386

2015 is 14: 9, then what is the number of qualified candidates from state A in 2015?
(a) 352 (b) 407 (c) 432 (d) 534

Directions (Qs. 68-71): Given below is the percent of number of students from 5 different colleges attended different number of seminars.

	No. of seminars Attended-1	No. of seminars Attended-2	No. of seminars Attended-3	No. of seminars Attended-4	No. of seminars Attended-5	No. of seminars Attended-6
P	20	15	24	8	–	15
Q	–	16	25	–	–	18
R	12	20	25	–	–	–
S	–	–	25	15	22	12
T	15	–	24	–	32	5

68. If the number of students from college R who attended at most 3 seminars is equal to number of students from college S who attended at least 3 seminars and the total number of students from College S is 114. Then find the number of students from college R who attended 3 seminars.
(a) 37 (b) 41 (c) 45 (d) 48

69. Total no. of students from college P who attended atmost 2 seminars is equal to the sum of number of students from college T who attended 1 seminar and the number of students from the same college who attended 6 seminars. Then the total number of students from college P is what percent of total no. of student from college T ?
(a) $57\frac{1}{7}\%$ (b) 45% (c) 49% (d) 60%

70. The number of students from college P who attended more than 2 seminars is approximate what percent less than the number of students from college S who attended at least 3 seminars if the number of student who attended 2 seminars from college P is 48. And the number of students of college S who attended 6 seminars is 14 more than the number of students from college P, who attended 1 seminar ?
(a) 57% (b) 44% (c) 60% (d) 48%

71. If the difference between number of students from college T who attended 3 seminars and students who attended 5 seminars is 22, and the total students from college R is 60% of the total students from college T then find the number of students of college R who are attending 2 seminars.
(a) 32 (b) 35 (c) 33 (d) 38

Directions (Qs. 72-76): Study the following table carefully and answer the given question.
Table shows the MP, SP, Profit, Profit% and Discount% of five different products sold. Some values are missing in the table. Find out the values and use it to answer the given questions.

72. What is the difference between the MP of products T and P?
(a) ₹ 780 (b) ₹ 830 (c) ₹ 800 (d) ₹ 700

73. Find the difference between the percentage discounts given for products T and discounts given for products Q and R together.
(a) 49% (b) 36% (c) 33% (d) 45%

74. What is the total of MP of product S and SP of product Q?
(a) ₹ 642 (b) ₹ 1232.42
(c) ₹ 628 (d) ₹ 1286.44

75. If a person buys products P and T for their respective selling prices, then what is the resultant discount% given to him?
(a) 50.70% (b) 60.20% (c) 48.06% (d) 67.62%

76. Find the profit% given for product U.
(a) 16% (b) 12% (c) 18% (d) 15%

Directions (Qs. 77–81): Study the following table carefully and answer the given questions.
Table shows the number of different types of products sold in five months, also the average number of all the products sold in respective months and average number of product sold in different months. Some values are missing in the table. Find the values on the basis of given information and answer accordingly.

Product	January	February	March	April	May	Average
TV	2050	2200		2300		2300
Fridge		2350	2250	2500		2404
Mobile	2250		1950		2230	2200
Laptop	2100	2540		2450	2070	2242
Computer		2650	2300		2400	2310
Average	2150	2392	2190	2350	2374	

77. Find the average number of Mobile sold in February, TV sold in April and Computer sold in January?
(a) 2800 (b) 2190 (c) 2280 (d) 2130

78. The total number of Fridge sold in January and April together are **approximately** how much percentage more than the total number of Laptop sold in March and April together?
(a) 8% (b) 5% (c) 9% (d) 7%

79. If the number of TV sold in June is equal to the average number of Fridge and Computer sold in May, then find the average number of TV sold from January to June?
(a) 2675 (b) 2725 (c) 2335 (d) 2635

80. In which month, there is the highest number of Fridge sold?

(a) May (b) April (c) January (d) March

81. The number of Laptop sold in March is **approximately** what percentage the total number of TV sold in February and May together?
(a) 35% (b) 32% (c) 54% (d) 43%

Directions (Qs. 82-86): Read the following table carefully and answer the given questions.
Table shows the performance of six batsmen

Name of Batsman	Number of matches played by batsman in the tournament	Average run scored by batsman in the tournament	Total balls faced in the tournament	Strike Rate
A	20	-	-	160
B	16	55	-	-
C	-	60	400	120
D	-	-	-	80
E	10	70	800	-
F	-	-	-	70

(i) Strike rate = (Total runs scored/total balls faced)×100
(ii) All the batsman could bat in all the matches played
(iii) You have to calculate the missing value and give the answer accordingly.

82. If the respective ratio between the balls faced by D and F is 4 : 5, then by what percent did F score more than D?
(a) 5.27% (b) 9.675% (c) 8.57% (d) 9.375%

83. If the runs scored by E in the last three matches of the tournament are not considered, his average decreases by 15. If the runs scored by E in 8th and 9th match are below 100 and no two scores among these three scores are equal, then is the minimum possible run scored by E in the 10th match?
(a) 115 (b) 116 (c) 117 (d) 118

84. Total balls faced by A is 600 less than the total runs made by him, then what is the average of A in the tournament?
(a) 70 (b) 75 (c) 80 (d) 85

85. B faces equal number of balls in first eight matches and the last eight matches in the tournament. If his strike rate in first eight and last eight matches of the tournament are 80 and 96 respectively, then what is the total number of balls faced by him in the tournament?
(a) 500 (b) 400 (c) 1000 (d) 800

86. What is the number of match played by C in the tournament?
(a) 10 (b) 12 (c) 8 (d) 6

Direction (Qs. 87-91): Study the following table carefully and answer the given questions.
Table shows the percentage marks obtained in five different subjects by five different students and maximum marks of each subjects respectively

Student	Hindi (80)	English (60)	History (50)	Science (100)	Math (150)
J	80	75	–	75	80
K	70	80	60	65	–
L	–	70	70	60	85
M	55	–	80	–	90
N	–	60	90	80	70

Note: Some data are missing. If require then find them first and then solve the given questions.

87. If average mark obtained by all the five students in English is 70%, then find the missing data in English
(a) 70% (b) 65%
(c) 75% (d) 80%

86. If marks obtained by J in History is 20% less than the mark obtained by K in the same subject, then find the mark obtained by J in History.
(a) 25 (b) 35
(c) 30 (d) 24

89. If N obtained 10% less mark in Hindi than that of K in the same subject, then find the **approx** average mark obtained by N in all subjects.
(a) 57 (b) 63
(c) 71 (d) 75

90. If total marks obtained by M in all the five subjects is 338 and marks obtained in English is 20% less than the marks obtained by J in English, then find the mark obtained by M in Science.
(a) 75 (b) 80
(c) 83 (d) 70

91. What is the total marks obtained by all the students in Math, if marks obtained by J in Science is 20% less than that of marks obtained by K in Math?
(a) 547.5 (b) 562.5
(c) 550 (d) 438.75

GP_3386

Hints and Solutions

Level 1

1. (b) Let age of Meera and her mother be 3x years and 8x years respectively 8 years hence age of mother = 8x + 8 years

8 years hence age of Meera's brother

$= \frac{8x+8}{2} = 4x + 4$ years

8 years hence age of Meera = 3x + 8

According to the question

$4x + 4 - 6 = 3x + 8 \quad (\therefore x = 10 \text{ years})$

$x = 3 \times 10 + 3 \times 10 + 6 = 66$ years

$y = 8 \times 10 = 80$ years

Hence $x < y$

2. (d) Let age of person be x years

age of his wife = x – 6 years

age of son $= \frac{x}{5}$

age of daughter $= \frac{x-6}{4}$

Hence, cant be determined

3. (a) Respective ratio of speeds of cars

$= \frac{1}{\sqrt{\text{Respective ratio of time taken by cars after meeting}}}$

$= \frac{1}{\sqrt{\frac{16}{25}}} = 5:4$

Hence $x > y$

4. (b) Let capital of A be x

capital of B $= x \times \frac{150}{100} = 1.5x$

capital of C $= (x + 1.5x) \times \frac{60}{100}$

$= 2.5x \times \frac{60}{100} = 1.5x$

Profit of A : B : C $= x \times 12 : 1.5x \times 8 : 1.5x \times 4$

$= 12x : 12x : 6x = 2 : 2 : 1$

$x = 2 - 2 = 0$

$y = 2 - 1 = 1$

Hence $x < y$

5. (b) Profit share of A : B $= (150000 \times 4 + 150000 \times \frac{75}{100} \times 8) : (120000 \times 4 + 120000 \times \frac{125}{100} \times 8)$

$= 600000 + 900000 : 480000 + 1200000$

$= 1500000 : 1680000 = 25 : 28$

Hence $x < y$

6. (a) Work completed by A in 1 day $= \frac{1}{50}$

Work completed by B 1 day

$= \frac{1}{50} \times \frac{125}{100} = \frac{1}{40}$

Time of B = 40 days

work completed by C in 1 day

$= \frac{1}{10} - \left(\frac{1}{50} + \frac{1}{40}\right) = \frac{1}{10} - \frac{9}{200} = \frac{11}{200}$

x = 40 days

$y = \frac{200}{11}$ days

Hence $x > y$

7. (b) Work completed by C $= 1 - \frac{7}{15} = \frac{8}{15}$

work completed by B $= \frac{11}{15} - \frac{8}{15} = \frac{3}{15}$

work completed by A $= 1 - \frac{11}{15} = \frac{4}{15}$

$x = \frac{4}{15}, y = \frac{8}{15}$

Hence $x < y$

8. (b) Speed of boat : Speed of current = 2 : 1

spced of person in upstreem $= \frac{50}{10} = 5$ km/h

speed of boat $= \frac{5 \times 2}{1} = 10$ km/h

speed of current $= \frac{5}{1} = 5$ km/h

$x = \frac{40}{10+5} = \frac{40}{15} = \frac{8}{3}$ hrs

$$y = \frac{30}{10-5} = 6 \text{ hrs}$$

Hence $x < y$

9. (b) Let B can complete the whole work in x days

$$\text{work completed by B in 1 day} = \frac{1}{x}$$

$$\text{work completed by A in 1 day} = \frac{1}{x} \times \frac{125}{100} = \frac{5}{4x}$$

According to question

$$\frac{5}{4}x = \frac{1}{50} - \frac{1}{60}$$

$$\frac{5}{4x} = \frac{1}{300}$$

$$x = \frac{300 \times 5}{4} = 375$$

$$x = \frac{375 \times 60}{315} = \frac{500}{7} \text{ days}$$

$y = 375$ days

Hence $x < y$

10. (c) $x = (\text{L.C.M of } 6, 5, 10, 9, 15)^2 = (90)^2 = 8100$

$y = (\text{L.C.M of } 30, 15, 18, 15)^2 = (90)^2 = 8100$

Hence $x = y$

11. (e) Here length or speeds of trains are not given hence, can't be determined.

12. (a) $x = 60 \times \frac{5}{18} \times 25 - 150 = \frac{1250}{3} - 150 = \frac{800}{3} \text{m}$

$$y = 45 \times \frac{5}{18} \times 20 = 250 \text{ m}$$

Hence $x > y$

13. (c) $x = 20000$

$$y = 15000 \times \frac{120}{90} = 20000$$

Hence $x = y$

14. (c) $x = \frac{360}{18} = 20$

$$y \Rightarrow \frac{n(n-3)}{2} = 170$$

$x^2 - 3x = 340$

$\Rightarrow x^2 - 3x - 340 = 0$

$\Rightarrow x^2 - 20x + 17x - 340 = 0$

$\Rightarrow x(x-20) + 17(x-20) = 0$

$\Rightarrow (x+17)(x-20) = 0$

$\Rightarrow \therefore x = -17, 20$

Hence $y = 20$

$x = y$

15. (c) Let length and breadth of rectangle be 3x and 4x respectively.

$3x \times 4x = 1200$ given

$12x^2 = 1200 \Rightarrow x^2 = 100 \quad \therefore x = 10$

area of square $= 1200 + 25 = 1225$

side of square $= \sqrt{1225} = 35$

$x = 2(3 \times 10 + 4 \times 10) = 140$

$y = 4 \times 35 = 140$

Hence $x = y$

16. (b) Let length of first train be x m

$$\frac{x+200}{15} = \frac{x+400}{25}$$

$25x + 5000 = 15x + 6000 \Rightarrow 10x = 1000$

$\therefore x = 100$

$x = 100$ m

$$y = 36 \times \frac{5}{18} \times 20 = 200 \text{ m}$$

Hence $x < y$

17. (c) Let cost price of fan be x

$$x \times \frac{150}{100} \times \frac{70}{100} - x = 50$$

$$\frac{105}{100}x - x = 50$$

$$\frac{5x}{100} = 50 \qquad \therefore x = 1000$$

$x = 1000$

$$y = \frac{1100 \times 100}{110} = 1000$$

Hence $x = y$

18. (a) $x = \pi \times \left(\frac{20}{2}\right)^2 = 100\pi$

$y = (12)^2 = 144$

Hence $x > y$

19. (a) Respective ratio of speeds of cars

$$= \frac{1}{\sqrt{\text{Respective ratio of times after meeting}}}$$

$$= \frac{1}{\sqrt{\frac{36}{64}}} = 4:3$$

Hence $x > y$

20. **(a)** Capital of A = 75000

$$\text{Capital of B} = 75000 \times \frac{100}{75} = 10000$$

Respective ratio of share of A + B : C

$$= \frac{19}{30} : \frac{11}{30} = 19 : 11$$

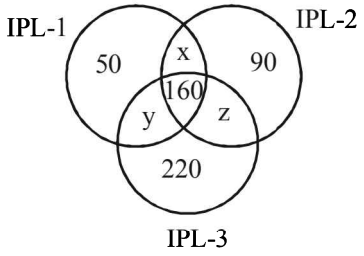
$$A : B = 75000 \times 12 : 100000 \times 10 = 9 : 10$$

$$A : B : C = 19 \times \frac{9}{19} : 19 \times \frac{10}{19} : 11 = 9 : 10 : 11$$

Hence x > y

Solution (21-25):

Among 800 cricket players, 45% played in IPL–1 and 6.25% played only in IPL–1. Again, 57.5% players played in IPL–2 and 11.25% players played only in IPL–2. Again, 72.5% players played in IPL–3 and 27.5% players played only in IPL–3. Twenty per cent players played in all three IPL tournaments.



$x + y = (360 - 210) = 150$ ...(i)

$x + z = (460 - 250) = 210$ ...(ii)

$y + z = (580 - 380) = 200$ ...(iii)

$\Rightarrow$ adding (i) + (ii) + (iii), we get

$2(x + y + z) = 560$

$x + y + z = 280$

x = 80, y = 70 and z = 130

21. **(d)** Required number of players = 50 + 90 + 80 = 220

22. **(d)** Required number of players = 90 + 130 + 220 = 440

Required % = 440/800 × 100 = 55%

23. **(d)** Number of players who played at least two IPL

= 80 + 70 + 130 + 160 = 440

Required % = 440/800 × 100 = 55%

24. **(d)** Only IPL 1 = 50, only IPL 2 = 90

Total = 140

Required % = 140/160 × 100 = 87.5%

25. **(b)** At least one IPL = 800

At most one IPL = 50 + 90 + 220 = 360

Required less % = (800 – 360)/800 × 100

= 55% less

Solution (26-30)

	Boys	Girls	Total
Tea	810	3510	4320
Coffee	1080	2475	3555
Both	4860	5265	10125
Total	6750	11250	18000

26. **(b)** Boys who like tea = 810 + 4860 = 5670

27. **(d)** Total girls like only tea = 3510

28. **(a)** Students who like coffee = 3555 + 10125 = 13680

29. **(c)** Required percentage = 3510/4320×100 = 81.25

30. **(c)** Required ratio = 4860: 5265 = 12 : 13

31. **(d)** Required total of Fans, ACs, Ovens and Refrigerators = 65 + 25 + 20 + 85 = 195

32. **(d)** Required average no. of products

$$= \frac{(50 + 65 + 85 + 180)}{4} = \frac{380}{4} = 95$$

33. **(b)**

34. **(a)**

35. **(d)** Required percentage

$$= \frac{85}{200} \times 100 = 43$$

Solution (36-40):

$$\text{Number of mobile phones} = \frac{1650 \times 24}{100} = 396$$

$$\text{Number of pen drives} = 1650 \times \frac{1}{6} = 275$$

$$\text{Number of pen drives} = 1650 \times \frac{14}{100} = 231$$

Number of televisions and washing machines = 748

Number of washing machines = T + 50

T + W = 748

T + T + 50 = 748

2T = 748 – 50 = 698

$$\therefore T = \frac{698}{2} = 349$$

Washing machines = 349 + 50 = 399

36. (b) Ratio $= \frac{399}{231} = \frac{133}{77} = \frac{19}{11} = 19:11$

37. (a) Number of pen drives which are not defective

$$= 275 - 275 \times \frac{24}{100} = 275 - 66 = 209$$

38. (b) Required % $= \frac{349}{231+399} \times 100 = \frac{349}{630} \times 100$

$= 55.39\% = 55\%$

39. (c) Difference = 349 + 396 – 231 = 745 – 231 = 514

40. (d) Total number of pen drives, calculators and washing machines = 275 + 231 + 399 = 905

Solutions (41-45):

41. (a) Required percentage of boys

$$= \frac{\frac{1}{3} \times \frac{3}{8} \times \frac{5}{9} \times 1008}{\frac{3}{8} \times 1008} \times 100 = 18.5$$

42. (a) Required percentage

$$= \frac{\frac{2}{9} \text{ of } 80\% \text{ of } \frac{3}{8} \times \frac{5}{8} \text{ of } 1008}{1008} \times 100 = 11\%$$

43. (b) Required number of girls $= \frac{7}{9}$ of 80% of $\frac{5}{8}$ of 1008 = 392

44. (a) Required ratio

$$= \frac{1}{3} \times \frac{3}{8} \times \frac{4}{9} \times 1008 : \frac{2}{9} \times 80\% \text{ of } \frac{5}{8} \times 1008 = 1:2$$

45. (c) Required Number

$$= \frac{5}{8} \times 1008 \times \frac{1}{5} - \frac{1}{3} \times \frac{3}{8} \times \frac{5}{9} \times 1008 = 56$$

Solution (46-50):

	Eng	Hindi	Both	Total
Boys	24	18	108	150
Birls	55	78	117	250
Total	79	96	225	400

46. (b)

47. (d)

48. (a)

49. (d)

50. (c)

51. (a) If we follow the graph we can clearly understand the profit is 110 in 1998

52. (d) Lets follow the formula, gross profit = sales – cost = 152 – 110 = 42

53. (d) We can solve it by ratio, if the profit of 1993 was 600 and cost = 110. Let the profit be x for 1990 and the cost was 100, then $\frac{600}{110} = \frac{x}{100}$ [assuming the ratio of profit and cost be same] then x = 545.

54. (a) Profit index = 104

55. (b) Total sales from 1990 – 1995 = 819. If the sales of ₹100 becomes ₹1200 crore then sales of ₹ 819 becomes, then $\frac{1200}{100} \times 819$ = 9828 crore

56. (c) Average sales index – average cost index = 147.27 – 139.73 = 7.54

57. (d) As we follow the graph we can clearly observe that personal profile is the highest.

58. (b)

59. (b)

60. (c)

61. (d)

62. (d) As we follow the table, we can observe that the rent of all the cities gradually increases except virar, whose rent fluctuates from 2005 to 2010.

63. (a) If we observe that the Churchgate, monthly rent increases more than 100% in 2006.

64. (b)

65. (d) In 2008 Kandiwali was 9.8. Virar = 21.9, required % = [9.8/21.9] × 100 = 44.7% or 45%

66. (a) from the table we can observe that Cruchgate was most expensive.

67. (d) 36.2 – 26.6 = 9.6; $\frac{9.6}{26.6} \times 100 = 36.09$ i.e. approx.36%

68. (c) It is visible that USA has the lowest rate of inflation.

GP_3386

69. (c) $36.2 - 16 = 20.2$; $\frac{20.2}{16} \times 100 = 126.25\%$.
Since the only option nearest to it is 125% so the answer is (c) .

70. (c) The control on inflation rate continues to be same for both developed nations & USA i.e. $14.2 - 11.8 = 2.4$ and $12.7 - 10.3 = 2.4$

71. (d) The average number of passengers is
$\frac{2040 + 1700 + 1275}{3} = 1671.66$

72. (a) In Train R, number of females are 34 % of $765 = 260$ and children are 26% of $765 = 199$.
Therefore, number of males
$= 756 - (260 + 199) = 306$.

73. (d) Number of passengers in A and R
$1105 + 765 = 1870$. Q is $\frac{1615}{1870} \times 100$
$= 86.36\%$ of A and R together.

74. (d) The second highest number of passengers is in M.

75. (d) Train M has 425 more passengers than Train L. Therefore, $\frac{425}{1275} \times 100 = 33.33\%$

76. (c) 1996 shows maximum percentage of 68.18% export with respect to production.

77. (b) Tea available in India $1993 = 720 - 288 = 432$ and per capita availability in $1993 = 0.4$kg. therefore, the population in India is
$\frac{432}{0.4} = 1080$

78. (d) Cannot be determined since there is no data given about area.

79. (b) Tea exported over the period $= 96 + 180 + 288 + 340 + 400 + 450 = 1754$ million kg and tea produced over the period
$= 480 + 540 + 720 + 700 + 600 + 660 = 3700$ million kg. The average proportion
$= \frac{1754}{3700} = 0.47$

80. (b) The average per capita availability of tea is
$\frac{390 + 410 + 400 + 450 + 500 + 525}{5} = 535g$

81. (c) It is clearly evident from the graph that 1991 has the minimum per capita availability of tea.

82. (a) There was minimum percentage of export with respect to production is in 1991.

83. (c) Quantity of tea for domestic consumption in the following years:-
$1991 \Rightarrow 480 - 96 = 384$
$1993 \Rightarrow 720 - 288 = 432$
$1994 \Rightarrow 700 - 340 = 360$
$1996 \Rightarrow 660 - 450 = 210$

84. (a) The average quantity of tea available for domestic consumption is
$384 + 432 + 360 + 210 + 360 + 200$
$= \frac{1946}{6} = 324.3$ million kg

85. (d) Population of all the years :-
$1991 \Rightarrow \frac{480 - 96}{0.39} = 985$ million
$1992 \Rightarrow \frac{540 - 180}{0.41} = 878$ million
$1993 \Rightarrow \frac{720 - 288}{0.4} = 1080$ million
$1994 \Rightarrow \frac{700 - 340}{0.45} = 800$ million
$1995 \Rightarrow \frac{600 - 400}{0.5} = 400$ million
$1996 \Rightarrow \frac{660 - 450}{0.525} = 400$ million
Therefore, the average population
$\frac{985 + 878 + 1080 + 800 + 400 + 400}{6} = 757$ million

Solution (86-90):

86. (c) Number of people in Teaching profession
$\frac{30}{100} \times 25000 = 7500$
Number of people in Medical profession
$= \frac{10}{100} \times 25000 = 2500$
Required $\% = \frac{7500}{2500} \times 100 = 300\%$

87. (c) Total numbers of males in Banking and Medical professions
$= 25000 \times \frac{20}{100} \times \frac{60}{100} + 25000 \times \frac{10}{100} \times \frac{40}{100}$
$= 3000 + 1000 = 4000$
The total number of females in Medical and Banking profession = 10% of 60% of $25000 + 20\%$ of 40% of $25000 = 1500 + 2000 = 3500$

So, required ratio $= \frac{4000}{3500} = \frac{8}{7} = 8:7$

88. (c) Females in Engineering professions

$$25000 \times \frac{25}{100} \times \frac{70}{100} = 4375$$

Males in Banking profession

$$25000 \times \frac{20}{100} \times \frac{60}{100} = 3000$$

Required %

$$= \left(\frac{4375}{3000} \times 100\right)\% = 145.83 \approx 146\%$$

89. (c) Number of males in Banking and Medical = 20% of 60% of 25000 + 10% of 40% of 25000 = 3000 + 1000 = 4000

Number of females in Law and Teaching

$$\frac{15}{100} \times \frac{20}{100} \times 25000 + \frac{30}{100} \times \frac{60}{100} \times 25000 = 5250$$

Required ratio $\frac{4000}{5250} = \frac{16}{21} = 16:21$

90. (a) Number of females in Engineering profession = 25% of 70% of 25000 = 4375

Number of males in Law profession = 15% of 80% of 25000 = 3000

Required %

$$\left(\frac{4375 - 3000}{3000} \times 100\right)\% = 46\% \text{ (approx)}$$

91. (b) Total CP of Bike = 150000 + 15000 = ₹ 1,65,000

$$SP = 165000 \times \frac{110}{100} = 1,81,500$$

92. (d) S.P of Laptop = 27000 + 5000 + 3000 = ₹ 35,000

C.P. of computer = 25500 + 2500 = ₹ 28,000

$$\therefore \text{ Required } \% = \left(\frac{35000}{28000} \times 100\right)\% = 125\%$$

93. (b) Loss on Camera = 40000 + 4200 – 35000 = ₹ 9200

Loss on Computer = ₹ 2500

So required ratio = 9200 : 2500 = 92 : 25

94. (c) S.P. of Mobile = 40000 + 2000 + 3000 = ₹45,000

S.P. of Laptop = 27000 + 5000 + 3000 = ₹35,000

So required difference = 45000 – 35000 = ₹ 10,000

95. (d) Total C. P of Hard disk = 2000 + 500 = ₹ 2500

So S. P. of Hard disk

$$= 2500 \times \frac{95}{100} = ₹2375$$

S. P of Computer = ₹ 25,500

So required less%

$$= \left(\frac{(25500 - 2375)}{25500} \times 100\right)$$

$$= 90.68\ \% \approx 91\%$$

Level-2

Solution (1-5):

	No. of Students	No. of Boys	No. of Girls	Others
A	$9360 \times \frac{20}{39} = 4800$	$4800 \times \frac{5}{12} = 2000$	2800	
B	$6000 \times \frac{80}{100} = 4800$	$4800 \times \frac{9}{16} = 2700$	2100	
C	4560	1760	2800	
D	$4800 \times \frac{4}{5} = 3840$	$2700 \times \frac{100}{90} = 3000$	840	
E	$4800 \times 5 \times \frac{25}{100} = 6000$	3600	$6000 \times \frac{40}{100} = 2400$	

GP_3386

1. **(c)** $X = 4800$

$Y = 4800$

Hence, $X = Y$

2. **(b)** $X = 1760$

$Y = 2400$

Hence, $X < Y$

3. **(a)** $X = \dfrac{2000 + 2700 + 1760 + 3000 + 3600}{5}$

$= \dfrac{13060}{5} = 2612$

$Y = 4800 - 2612 = 2188$

Hence, $X > Y$

4. **(a)** $X = \dfrac{2000 + 1760 + 3600}{3}$

$= \dfrac{7360}{3} = 2453$

$Y = \dfrac{2100 + 840}{2} = 1470$

Hence, $X > Y$

5. **(b)** $X = 4800 - 4560 = 240$

$Y = 6000 - 4800 = 1200$

Hence, $X < Y$

Solution (6-10):

	SPEED	TIME	DISTANCE
A	60 km/h	20 hrs	$720 \times \frac{5}{3} = 1200\,km$
B	80 km/h	21 hrs	$720 \times \frac{7}{3} = 1680\,km$
C	$90 \times \frac{80}{100} = 72\,km/h$	10 hrs	720 km
D	$60 \times \frac{150}{100} = 90\,km/h$	$20 \times \frac{100}{120} = \frac{50}{3}\,hrs$	$90 \times \frac{50}{3} = 1500\,km$
E	$\frac{7500}{7} \times \frac{1}{20} = \frac{375}{7}$ km/hr	20 hrs	$1500 \times \frac{100}{140} = \frac{7500}{7}\,km$

6. **(b)** $X = 20$ hrs

$Y = 21$ hrs

Hence, $X < Y$

7. **(b)** $X = 72$ km/h

$Y = \dfrac{375}{7} = 53\dfrac{4}{7}\,km/h$

Hence, $X > Y$

8. **(b)** $X =$

$$1200 + 720 + \frac{7500}{7} = \left(\frac{8400 + 5040 + 7500}{7}\right) \times \frac{1}{3}$$

$$= \frac{20940}{3 \times 7} = \frac{6980}{7} = 997\frac{1}{7}$$

$$y = \frac{1680 + 1500}{2} = \frac{3180}{2} = 1590\,km$$

Hence, $X < Y$

9. **(b)** $X = 1200 + 720 = 1920$

$Y = 1680 + 1500 = 3180$

Hence, $X < Y$

10. **(a)** $X = 72$

$Y = 20 + 21 + 10 + 20 = 71$

Hence, $X > Y$

Solution (11-15):

T.V.	$36000\times\frac{100}{120}=30000$	36000	$24000\times\frac{100}{40}=60000$	6000	24000
A.C.	$50000\times\frac{100}{120}=\frac{125000}{3}$	$30000\times\frac{100}{60}=50000$	$50000\times\frac{100}{80}=62500$	$\frac{25000}{3}$	12500
COOLER	$15000\times\frac{100}{110}=\frac{150000}{11}$	$30000\times\frac{50}{100}=15000$	$15000\times\frac{100}{80}=18750$	$\frac{15000}{11}$	3750
LAPTOP	$75000\times\frac{100}{120}=62500$	$50000\times\frac{150}{100}=75000$	$75000\times\frac{100}{70}=\frac{75000}{7}$	12500	$\frac{225000}{7}$
MOBILE	$31250\times\frac{100}{150}=\frac{62500}{3}$	$\frac{62500}{2}=31250$	$31250\times\frac{100}{75}=\frac{1250000}{3}$	$\frac{31250}{3}$	$\frac{31250}{3}$

11. (b) $X=\left(\frac{125000}{3}+\frac{62500}{3}\right)\div 2=31250$

$Y=62500$

Hence, $X<Y$

12. (b) $X=\frac{6000+\frac{25000}{3}+\frac{15000}{11}+12500+\frac{31250}{3}}{5}$

$=\frac{38613.6}{5}$

$y=\frac{24000+12500+3750+\frac{22500}{7}+\frac{31250}{3}}{5}$

$=\frac{82809.5}{5}$

Hence, $X<Y$

13. (b) $X=\frac{150000}{11}$

$Y=\frac{225000}{7}$

Hence, $X<Y$

14. (c) $X=\frac{31250}{3}$

$Y=\frac{31250}{3}$

Hence, $X=Y$

15. (a) $X=\frac{750000}{7}$

$Y=62500$

Hence, $X>Y$

Solution (16-20):

Persons	Salary	Saving	Expenditure	Note
A	$16000 \times \frac{100}{25}$ $=64000$	$10000 \times \frac{8}{5}$ $=16000$	$64000-16000$ $=48000$	Respective ratio of savings of A and E is 8 : 5 A saves 25% of his salary.
B	$32000 \times \frac{100}{80}$ $=40000$	$40000 \times \frac{40}{100}$ $=16000$	$40000-16000$ $=24000$	Salary of D is 20% less than that of B Savings of B is 40% of his salary.
C	$\left(\begin{matrix}64000+40000+\\32000+50000\end{matrix}\right)\frac{80}{400}$ $=\frac{186000}{5}=37200$	$37200-32000$ $=5200$	$40000 \times \frac{80}{100}$ $=32000$	Expenditure of C is 80% of the expenditure of E Salary of C is 20% less than the average salary of all others.
D	$64000 \times \frac{50}{100}$ $=32000$	$32000 \times \frac{20}{100}$ $=6400$	$32000-6400$ $=25600$	Salary of D is 50% less than that of A saving of D is 20% of his salary
E	$10000 \times \frac{100}{20}=50000$	10000	$10000 \times \frac{80}{20}$ $=40000$	Savings of E is 10000 E spends 80% of his salary

16. (b) $X=40000$
$Y=48000$
Hence, $X>Y$

17. (a) $X=\frac{64000+40000+37200+32000+50000}{5}$

$=\frac{223200}{5}=44640$

$Y=32000$
Hence, $X>Y$

18. (a) $X=48000+24000+32000+25600+40000$
$=159600$
$Y=64000+32000+50000=146000$
Hence, $X>Y$

19. (a) $X=16000$
$Y=5200$
Hence, $X<Y$

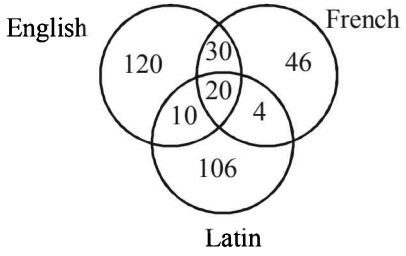
20. (b) $X=16000+16000+52000$
$+6400+10000=53600$
$Y=64000$
Hence, $X<Y$

Solutions (21 – 25)

College	Total passed	Boys	Girls
P	720	520	200
Q	880	480	400
R	800	360	440
S	600	250	350
T	1000	700	300
Total	4000	2310	1690

21. (c) Required average = 1690/5 =338
22. (c) Required ratio = 700/300 = 7 : 3
23. (a) Required percentage = (480 – 400)/400 ×100 = 20%
24. (c) Number of girls passed in College P = 200
Number of passed boys in College S = 250
Required percentage = 200/250 ×100 = 80%
25. (d) College S college the percentage of girls passed with respect to its total passed students is maximum.

Solution (26-30):



26. (c) Required delegates = 120 + 46 + 106 = 272
27. (a) Required delegates = 64
28. (a) Required delegates = 30 + 10 + 20 + 4 = 64
29. (d) Required delegates = 30 + 10 + 4 = 44
30. (b) Required delegates = 400 - 20 = 380

Solution (31-35):

A		B
50	AC	25
65	Fans	35
85	Re frigerators	40
180	Oven	20

Total no. of Fans = 100
Total no. of ACs = 75
Total no. of Refrigerators = 125
Total no. of Ovens = 200

31. (c) Total flooring area with marble
= locker area + record keeping + pantry
= 182 + 273 + 609 = 1064 sqm
Cost of flooring = 1064 × 190
Total flooring area with wooden
= Branch Manager room + Hall = 221 + 667
= 888 sqm
Cost of flooring = 888 × 170
Ratio = 888 × 170 : 1064 × 190
= 888 × 17 : 1064 × 19
= 15096 : 20216
= 1887 : 2527

32. (b) Cost of flooring of branch manager room
= 221 × 170 = ₹ 37570
Cost of painting
= [2(17 × 12 + 13 × 12) + 13 × 17] × 190
= [2(204 + 156) + 221] × 190 = (2 × 360 + 221) × 190
= (720 + 221) × 190 = 941 × 190 = ₹ 178790
Total cost = 178790 + 37570 = ₹ 216360

33. (a) Total area of bank = 2000 sq m
Total flooring area = 1952 sq m
Remaining area = 2000 – 1952 = 48 sq m
So Cost of carpeting = 48 × 110 = ₹ 5280

34. (b) Area not to be renovated = 48 sq m

$$\therefore \text{Regd}\% = \frac{48}{2000} \times 100 = 2.4\%$$

35. (a) Cost of renovation of hall + locker area
= 667 × 170 + 609 × 190
= 113390 + 115710 = ₹ 229100

Solution (36-40):

	Train A (700)	Train B + (910)
General coaches	140	241
Sleeper coaches	161	273
First Class	224	91
AC coaches	175	205

36. (c) **37. (d)** **38. (e)** **39. (b)** **40. (c)**

41. (c) Following table provides the production of all the units year wise.

	1998	1999	2000	2001	2002	Total
Unit 1	140	180	195	260	340	1115
Unit 2	150	193	203	268	351	1165
Unit 3	156	194	200	272	350	1172

it is clear that performance of unit 1 is the lowest and that of unit 3 is the highest during the 5 yrs period.

42. (b) Table below give full idea

Total production in 1st quarter	**764**
Total production in 2nd quarter	971
Total production in 3rd quarter	893
Total production in 4th quarter	824
Total	3452

43. (c) From the table in question 150 growth unit wise during (19982002) is given below

$$\text{Unit 1} = \frac{340-140}{140} \times 100 = 142\,\%$$

$$\text{Unit 2} = \frac{351-150}{150} \times 100 = 134\,\%$$

$$\text{Unit 3} = \frac{350-156}{156} \times 100 = 124\,\%$$

44. (b)

45. (d) below the table give us clear idea.

Year	Average Production	Growth
1998	111.5	—
1999	141.5	27.13%
2000	149.5	5.46%
2001	200	33.11%
2002	260.25	30.12%

Solution (46-48):

46. (b) It can be seen from the graph that the southern region showed the highest growth in number of households in all the income categories for the period.

47. (d) We only know the total number of households for all four regions combined. Nowhere have they given the region-wise break-up of this value. In the light of this, the given question cannot be answered.

48. (b) It is very clear from the graph that the percentage increase in total number of households for the northern region for upper middle income category is 200%.

Solution (49-50):

49. (a) Required investment in high-risk stock

$$= \frac{11,0500000 \times 8.9}{100} = ₹\,98,34500$$

50. (d) Required investment

$$= 11,05,00,000 \times \frac{48.3}{100} \times \frac{26}{100}$$

$= ₹1,38,76,590$

51. (d) High-risk stocks

52. (c)

Solution (53-57):

53. (b) Suppose x units are produced each year.
So, in the year 2006,
Total revenue = 1200
$\Rightarrow 12 \times x = 1200 \Rightarrow x = 100$
Prifit = 300
$\therefore$ Cost price = 1200 – 300 = 900

$$\therefore \text{Cost per unit} = \frac{900}{100} = 9$$

In the year 2005,
Total revenue = 1400
$\Rightarrow 14 \times x = 1400$
$x = 100$
Profit = 100
$\therefore$ Cost price = 1400 – 100 = 1300

$$\therefore \text{Cost per unit} = \frac{1300}{100} = 13$$

In the year 2009,
Total revenue = 400
$\Rightarrow 8 \times x = 400$
$\Rightarrow x = 50$
Profit = 150
$\therefore$ Cost price = 400 – 150 = 250

$$\therefore \text{Cost per unit} = \frac{250}{50} = 5$$

In the year 2011,
Total revenue = 700
$\Rightarrow 14 \times x = 700 \Rightarrow x = 50$
Profit = 200
$\therefore$ Cost price = 700 – 200 = 500

$\therefore$ Cost per unit $= \frac{500}{50} = 10$

Thus, per unit cost is highest in the year 2005.

54. (b)

Years	Unit price	Revenue	Total units $= \frac{\text{Revenue}}{\text{Unit price}}$
2000	10	700	70
2001	14	1400	100
2002	12	1200	100
2003	12	900	75
2004	11	1100	100
2005	8	400	50
2006	10	200	20
2007	14	700	50
2008	10	600	60
2009	10	800	80
2010	15	900	60
			Total = 765

$\therefore$ Average units $= \frac{765}{11} \simeq 70$ units

55. (c) Checking option (a),

$$\text{Volatility per unit} = \frac{\frac{15-8}{126}}{11} = \frac{77}{126} = 0.611$$

Checking of option (b),
Total cost = Revenue – Profit
$= (700+1400+1200+900+1100+400+200+700+600+800+900) - (0+100+300+150+0+150+100+200+0+400+300)$
$= 8900 - 1700 = 7200$

$$\text{Average cost per unit} = \frac{7200}{11}$$

Volatility cost per unit

$$= \frac{(1400-100)-(200-100)}{7200}$$

$$= \frac{1300-100}{7200} \times 11 = 1.833$$

Checking option (c),

$$\text{Average profit} = \frac{1700}{11}$$

$\therefore$ Profit volatility

$$= \frac{\frac{400-0}{1700}}{11} = \frac{44}{17} = 2.588$$

Checking option (d),

$$\text{Average revenue} = \frac{8600}{11}$$

$$\therefore \text{Revenue volatility} = \frac{\frac{1400-200}{8600}}{11}$$

$$= \frac{132}{86} = 1.535$$

$\therefore$ Total profit has highest volatility.

56. (c)

Year	New revenue	Total cost = Old revenue – Profit
2004	80% of 700 – 560	700
2005	80% of 1400 = 1120	1300
2006	80% of 1200 = 960	900
2007	80% of 900 = 720	750
2008	80% of 1100 = 880	1100
2009	400	120% of 250 = 300
2010	200	120% of 100 = 120
2011	700	120% of 500 = 600
2012	600	120% of 600 = 720
2013	800	120% of 400 = 480
2014	900	120% of 600 = 720

In a year when total cost is more than new revenue then in that year there is loss.
These years are 2004, 2005, 2006, 2007 and 2008
So, there are total 5 yr.

57. (b) Total decrease in revenues
$= 20\%$ of $(700+400+1200+900+1100)$
$= 1060$
Total increase in cost
$= 20\%$ of $(250+100+500+600+400+600)$
$= 490$
Decrease in cumulative profit
= Total decrease in revenues + Total increase in cost
$= 1060 + 490 = ₹1550$

GP_3386

Solutions (58-62):

	Total number of Mobiles Sold	Total Number of Mobiles Sold of Company A	Total Number of Mobiles Sold of Company B
July	7650	4080	3570
August	9900	4400	5500
September	11250	6750	4500
October	3600	2100	1500
November	5400	2520	2880
December	7200	3150	4050

58. (c) Number of mobiles sold of company B in July = 3570
Number of mobiles sold of company B in December = 4050
Required Ratio = 3570 : 4050 = 119 : 135

59. (c) Total mobiles sold by company A during November = 2520
Total mobiles sold by this company at discount = 35% of 2520 = 882
Total mobiles sold by company A without discount = 2520 – 882 = 1638

60. (d) Mobile phones sold of company B during October = 1500
Total profit earned on the mobile phones
= ₹(433 × 1500) = ₹6,49,500

61. (d) Number of mobile phones sold of company A during July = 4080
Number of mobile phones sold by company A during December = 3150
Required percentage

$$= \frac{4080}{3150} \times 100 = 129.5 \approx 130\%$$

62. (a) Mobile phones sold of company B during August = 5500
Mobile phones sold of company B during September = 4500
Total number of mobile phones
= 5500 + 4500 = 10,000

Solution (63-67):

63. (b) Required number of appeared candidates who qualified from state A in 2013
= 252/7 × (11+7) = 648
So Total number of appeared candidate from state A in 2013
= (648/60 ×100) = 1080

64. (c) Let the number of appeared candidate from state B in 2011 = 100
So number of appeared candidate in 2012 from state B in 2012 = 200
So Required number of appeared candidate from B in 2011
= 816/(30+90) ×100 = 680

65. (a) Required difference
= 900 × 60/100 – 1200 × 43/100
= 540 – 516 = 24

66. (d) Required number of qualified candidate from state B in 2015
= (3 × 420) – (560×60/100 + 1100×50/100)
= 1260 – (336 + 550)
= 1260 – 886 = 374

67. (c) Number of qualified candidate from state P in 2014 = 960 × 70/100 = 672
So Required number of qualified candidate from state A in 2015 = 672/14 × 9 = 432

Solution (68-71):

68. (a) No. of students from college R who attended at most 3 seminars

$$= 114 \times \frac{74}{100} = 84.36$$

So Required students :

$$= \frac{84.36}{87} \times 25 = 37$$

69. (a) Let total no. of students from college $T = 100$
Total no. of students from college

$$P = \frac{20}{35} \times 100 = 57\frac{1}{7}$$

$$\therefore \quad \text{Required \%} = \frac{57\frac{1}{7}}{100} \times 100$$

$$= 57\frac{1}{7}\%$$

70. (a) Total no. of students from college

$$P = \frac{48}{15} \times 100 = 320$$

Total – no.– of – students – from – college

$$S = \frac{(64+14)}{12} \times 100 = 650$$

Required %

$$= \frac{481-208}{481} \times 100 = 56.75 \approx 57\%$$

71. (c) Total no. of students from college

$$T = \frac{22}{8} \times 100 = 275$$

Total no. of students from college

$$R = \frac{60}{100} \times 275 = 165$$

Required No. of students $= 16.5 \times 2 = 33$

Solution (72-73):

72. (d) M.P of product P

$$= \frac{250}{100} \times 120 \times \frac{100}{75} = ₹400$$

So Required Difference

$= 1100 - 400 = ₹\ 700$

73. (b) Let CP of T = ₹ 'x'

$\therefore$ profit $= \left(\frac{27}{x} \times 100\right)$

$\Rightarrow 10 = \left(\frac{27}{x} \times 100\right)$

$\Rightarrow x = ₹270$

$\therefore$ SP $= 270 + 27 = ₹\ 297$

Discount%

$$= \left(\frac{1100-297}{1100} \times 100\right)\% = 73\%$$

Let CP of Q = ₹ x

$\therefore$ P % $= \left(\frac{22}{x} \times 100\right)$

$\Rightarrow 10 = \left(\frac{22}{x} \times 100\right)$

$\Rightarrow x = ₹\ 220$

$\therefore$ SP $= 220 + 22 = ₹\ 242$

Discount % $= \left(\frac{1100-242}{1100} \times 100\right)\%$

$= 78\%$

Discount % for R $= \left(\frac{300-(180+27)}{300} \times 100\right)\%$

$= 31\%$

$\therefore$ Required difference

$= (78+31) - 73$

$= 109 - 73 = 36\%$

74. (d) SP of product S $= 320 + 16$

$= ₹336$

$\therefore$ M.P $= \left(\frac{336}{27} \times 100\right) = ₹1244.44$

SP of product Q = ₹242

$\therefore$ Required total

$= 1244.44 + 242$

$= ₹\ 1286.44$

75. (b) S.P of product P $= 250 \times \frac{120}{100} = ₹300$

S.P of product T = ₹297

$\therefore$ Total SP $= 300 + 297$

$= ₹597$

MP of product P $= \frac{300}{75} \times 100$

Total MP $= 400 + 1100 = ₹1500$

Total Discount $= 1500 - 597 = ₹903$

$\therefore$ Discount% $= \left(\frac{903 \times 100}{1500}\right)\%$

$= 60.20\%$

GP_3386

76. (d) SP of product O $= 600 \times \frac{69}{100}$

$= ₹414$

$\therefore$ Profit % $\left(\frac{414-360}{360} \times 100\right)\% = 15\%$

Solution (77-81):

77. (b) No. of Mobile sold in February
$= 2392 \times 5 - (2200 + 2350 + 2540 + 2650)$
$= 11960 - 9740 = 2220$
No. of Mobile sold in April
$= 2200 \times 5 - (2250 + 2220 + 1950 + 2230)$
$= 11000 - 8650 = 2350$
No. of Computer sold in April
$= 2350 \times 5 - (2300 + 2500 + 2350 + 2450)$
$= 11750 - 9600 = 2150$
No. of Computer sold in January
$= 2310 \times 5 - (2650 + 2300 + 2150 + 2400)$
$= 11550 - 9500 = 2050$
So Required average

$$= \frac{2220 + 2300 + 2050}{3}$$

$$= \frac{6570}{3} = 2190$$

78. (d) No. of Fridge sold in January
$= 2150 \times 5 - (2050 + 2250 + 2100 + 2050)$
$= 10750 - 8450 = 2300$
No. of Laptop sold in March
$= 2242 \times 5 - (2100 + 2540 + 2450 + 2070)$
$= 11210 - 9160 = 2050$
No. of Fridge sold in January and April together
$= 2300 + 2500 = 4800$
No. of Laptop sold in March and April together
$= 2050 + 2450 = 4500$
So required %

$$= \left(\frac{(4800 - 4500)}{4500} \times 100\right)\%$$

$= 6.66\% \approx 7\%$

79. (c) No. of Fridge sold in May
$= 2404 \times 5 - (2300 + 2350 + 2250 + 2500)$
$= 12020 - 9400 = 2620$
No. of TV sold in June

$$= \frac{2620 + 2400}{2} = \frac{5020}{2} = 2510$$

No. of TV sold in March
$= 2190 \times 5 - (2300 + 2050 + 1950 + 2250)$
$= 10950 - 8550 = 2400$
No. of TV sold in May
$= 2374 \times 5 - (2400 + 2070 + 2230 + 2620)$
$= 11870 - 9320 = 2550$
So Required average

$$= \frac{2050 + 2200 + 2400 + 2300 + 2550 + 2510}{6}$$

$$= \frac{14010}{6} = 2335$$

80. (a) Highest sales of fridge in May = 2620

81. (d) No. of Laptop sold in March = 2050
No. of TV sold in February and May together
$= 2200 + 2550 = 4750$

$$\text{Required\%} = \left(\frac{2050}{4750} \times 100\right)\%$$

$= 43.15\% \approx 43\%$

Solution (82-86):

82. (d) Runs scored by D $= \frac{80 \times 4}{100} = \frac{32}{10} = 3.2$

Runs scored by F $= \frac{70 \times 5}{10} = \frac{35}{10} = 3.5$

$$\text{So Required\%} = \left(\frac{3.5 - 3.2}{3.2} \times 100\right)\%$$

$= 9.375\%$

83. (d) Total runs scored by E in 10 matches
$= 70 \times 10 = 700$
Total runs scored by E in 7 matches if last three matches are not considered) $= 7 \times 55 = 385$
So Total runs scored by E in last three matches $= 700 - 385 = 315$

Minimum run in 10th match, it means maximum run in 8th and 9th are below 100 and no two scores are equal.
So, run scores in 8th and 9th match are 98 and 99
So Run scored in 10^{th} match
$= 315 - 197 = 118$

84. (c) Let average of A = x
So total runs scored = 20x
And no. of ball faced = 20x – 600
According to question,

$$\text{Strike rate} = \frac{20x}{20x-600} \times 100$$

$$\Rightarrow 160 = \frac{20x}{20x-600} \times 100$$

$$\Rightarrow 8 = \frac{x}{20x-600} \times 100$$

$$\Rightarrow 160x - 4800 = 100x$$

$$\Rightarrow 60x = 4800$$

$$\Rightarrow x = \frac{4800}{60} = 80$$

85. (c) Let B played x no. of ball in first eight and last eight matches
So Run scored by B in first eight matches

$$= \frac{80 \times x}{100} = \frac{4x}{5}$$

and runs scored by B in last eight matches

$$= \frac{96 \times x}{100} = \frac{24x}{25}$$

According to question,

$$\frac{4x}{5} + \frac{24x}{25} = 16 \times 55$$

$$\Rightarrow \frac{20x + 24x}{25} = 880$$

$$\Rightarrow x = \frac{880 \times 25}{44} = 500$$

$\therefore$ Total no. of ball faced by B in the tournament = 500 + 500 = 1000

86. (c) Let the no. of matches played by C in tournament = x
According to question,

$$\text{Strike rate} = \left(\frac{60x}{400} \times 100\right)$$

$$\Rightarrow 120 = \frac{60x}{400} \times 100$$

$$\Rightarrow x = \frac{400 \times 120}{100 \times 60} = 8$$

87. (b) Total marks obtained by all the students in English

$$= 60 \times 5 \times \frac{70}{100} = 210$$

Marks obtained by M in English

$$= 210 - \left[\frac{60}{100} \times (75 + 80 + 70 + 60)\right]$$

$$= 210 - \frac{60}{100} \times 285$$

$$= 210 - 171 = 39$$

$$\text{Missing data} = \left(\frac{39}{60} \times 100\right)\%$$

$$= 65\%$$

88. (d) Marks obtained by K in History

$$= 50 \times \frac{60}{100} = 30$$

$\therefore$ Marks obtained by J in History

$$= 30 \times \frac{80}{100} = 24$$

89. (b) Marks obtained by K in Hindi

$$= 80 \times \frac{70}{100} = 56$$

Marks obtained by N in Hindi

$$= 56 \times \frac{90}{100} = 50.4$$

So, Average marks obtained by N in all the subjects together

$$= \frac{1}{5} \times \left(50.4 + 60 \times \frac{60}{100} + 50 \times \frac{90}{100} + 100 \times \frac{80}{100} + 150 \times \frac{70}{100}\right)$$

$$= \frac{50.4 + 36 + 45 + 80 + 105}{5}$$

$$= \frac{316.4}{5} = 63.28 \approx 63$$

90. (c) Marks obtained by M in English

$$= 60 \times \frac{75}{100} \times \frac{80}{100} = 36$$

Total marks obtained by M in all the subjects together = 338

$\therefore$ Marks obtained by M in Science

$$= 338 - \left[80 \times \frac{55}{100} + 36 + 50 \times \frac{80}{100} + 150 \times \frac{90}{100}\right]$$

$$= 338 - [44 + 36 + 40 + 135]$$

$$= 338 - 255 = 83$$

91. (a) Marks obtained by K in Maths

$$= 100 \times \frac{75}{100} \times \frac{80}{100} = 60$$

$\therefore$ Total marks obtained by all the students in Maths

$$= \left[\frac{150}{100} \times (80 + 85 + 90 + 70)\right] + 60$$

$$= \frac{150}{100} \times 325 + 60$$

$$= 487.5 + 60 = 547.5$$

CHAPTER

28

Data Sufficiency

INTRODUCTION

Data sufficiency is one of the most important parts of Quantitative Aptitude Section of various competitive examinations like IBPS-PO, IBPS-SO, SBI-PO, RBI, LIC, RBI, CAT, MAT etc.

Data sufficiency questions test your knowledge of basic mathematical facts and skills coupled with number system, time, speed and distance, profit and loss, percentage, simple interest and compound interest, algebra, geometry, mensuration, etc. Each data sufficiency question presents you with a question where you need to decide whether or not the information presented along with the question would be sufficent to answer the question.

Problem Format

The problem(s) below consist of a question/questions with two or three associated statements that provide information that might be useful in answering the question. You will need to use either statement alone is sufficient to answer the question whether two of them are needed to answer the question; or whether there is not enough information given to answer the question or whether there is all information needed to answer the question.

There are five possible answer choice for every data sufficiency question:

1. Statement (I) alone is sufficient, but statement (II) alone is not sufficient / Statement (I) and either statement (II) or statement (III) are sufficient
2. Statement (II) alone is sufficient, but statement (I) alone is not sufficient / Either statement (I) or statement (II) and statement (III) are sufficient.
3. Either statement (I) or statement (II) is sufficient to answer / Either statement (I) or statement (II) is sufficient but statement (III) is not sufficient
4. Statements (I) and (II) together are not sufficient to answer / Statement (I), (II) and (III) together are not sufficient
5. Both/all statements together are sufficient to answer.

GP_3386

LEVEL - 1

Directions (1–5):
(a) If statement 1 alone is sufficient to answer the question, but statement 2 alone is not sufficient
(b) If statement 2 alone is sufficient to answer the question, but statement 1 alone is not sufficient
(c) If both statements together are needed to answer the question, but neither statement alone is sufficient
(d) If either statement by itself is sufficient to answer the question
(e) If not enough facts are given to answer the question

1. Is three digit number N divisible by 9?
(i) All the digits in N is distinct and sum and product of digits is 6
(ii) N is divisible by only 1st 4 prime numbers.

2. Is number P a prime number if its square root is $K.abcd\ldots\ldots$
(i) P is not divisible by any prime number less than K
(ii) P is not divisible by any number more than K

3. Is product of 5 consecutive two digit natural numbers divisible by 25?
(i) Sum of those 5 natural numbers is more than 239
(ii) Sum of those 5 natural numbers is less than 260

4. Is number N has odd number of factors?
(i) N is divisible by only one prime number.
(ii) $61 < N < 67$

5. Is number N^2 has more than 35 factors?
(i) N has 12 factors
(ii) In 4 ways we can be write N as product of two of its co-prime factors.

Directions (6-17):
(a) If statement 1 alone is sufficient to answer the question, but statement 2 alone is not sufficient
(b) If statement 2 alone is sufficient to answer the question, but statement 1 alone is not sufficient
(c) If both statements together are needed to answer the question, but neither statement alone is sufficient
(d) If either statement by itself is sufficient to answer the question
(e) If not enough facts are given to answer the question

6. Is $A > B$ if their LCM is 30.
(i) HCF of A and B is 1 and when (A + B) divided by 9 remainder is 4.
(ii) A^B is odd.

7. What is HCF of $(A + B)$ and $(A - B)$ given that A is not equal to B?
(i) HCF of A and B is 4
(ii) LCM of A and B is 36

8. Find the unit digit of xy
(i) x and y are prime digits
(ii) unit digit of yx is 0

9. What is the unit digit of 7abcd
(i) Four digit number cbad is divisible by 4.
(ii) When 4 digit number 'abcd' divided by 72 it leaves remainder 1.

10. What is the remainder of $\left[\frac{3^x}{7}\right]$
(i) x is in the form of (4k + 2)
(ii) x is in the form of (3k + 1)

11. Find the remainder of $\left[\frac{5x}{9}\right]$
(i) x is a 2 digit prime no.
(ii) x when divided by 12 gives remainder 9.

12. Find the unit digit of a! + b! + c!
(i) Each of a, b and c is a two digit number.
(ii) Each of a, b and c is a prime number in the form of $6k \pm 1$.

13. Find the remainder of $\left[\frac{a!}{b!}\right]$
(i) Given that a = b
(ii) Given that a < b

14. Find the highest power of 7 in N!.
(i) Quotient of $\left[\frac{N}{9}\right]$ is 7
(ii) Quotient of $\left[\frac{N}{11}\right]$ is 5

15. Find the last two digits of a! + (a + 1)! + (a + 2)!
(i) a is a prime number.
(ii) a is a two digit number.

16. Is product of three numbers a, b and c a positive even number?
(i) At least 2 of the three is an even positive integer.
(ii) At most 1 of the three is an even negative integer.

17. What is minimum value of A/B + B/A?
(i) The product AB > 0
(ii) The sum A + B > 0

Directions (18-24):
Each of the following questions is followed by two statements. Mark
(a) If statement I alone is sufficient to answer the question.
(b) If statement II alone is sufficient to answer the question.
(c) If both statement I and II together are necessary to answer the question.
(d) If both statements I and II together are not sufficient to answer the question.

18. Is 'b' positive?
(I) a + b is positive.
(II) a – b is positive.

19. In a general body election, 3 candidates, p, q and r were contesting for a membership of the board. How many votes did each receive?
(I) p received 17 votes more than q and 103 votes more than r.
(II) Total votes cast were 1703.

20. If C1 and C2 are the circumferences of the outer and inner circles respectively. What is C1 : C2?
(I) The two circles are concentric.
(II) The area of the ring is 2/3 the area of greater circle.

21. What is the middle numbers of 7 consecutive whole numbers?
(I) Product of numbers is 702800.
(II) Sum of the numbers is 105.

22. Total marks obtained by P, Q, R and S in Mathematics is 360. How many marks did P secure in Mathematics?
(I) P secured one-third marks of the total of Q, R and S.
(II) Average marks obtained by Q and R are 20 more than that secured by S.

23. How many ice cubes can be accommodated in a container?
(I) The length and breadth of the container is 20 cm and 15 cm respectively.
(II) The edge of the ice cube is 2 cm.

24. Ram got ₹ 1500 as dividend from a company. What is the rate of interest given by the company?
(I) The dividend paid last year was 10%.
(II) Ram has 350 shares of ₹ 10 denomination.

Directions (25-27):
Each question is followed by two statements, A and B. Answer each question using the following instructions:
(a) if the question can be answered by using one of the statements alone, but cannot be answered using the other statement alone.
(b) if the question can be answered by using either statement alone.
(c) if the question can be answered by using both statements together, but cannot be answered using either statement alone.
(d) if the question cannot be answered even by using both statements together.

25. Consider three real numbers, X, Y and Z. Is Z the smallest of these numbers?
A. X is greater than at least one of Y and Z.
B. Y is greater than at least one of X and Z.

26. Let X be a real number. Is the modulus of X necessarily less than 3?
A. $X(X+3)<0$
B. $X(X-3)>0$

27. What are the ages of two individuals, X and Y?
A. The age difference between them is 6 years.
B. The product of their ages is divisible by 6.

Directions (28-32):
Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the questions. Read both the statements and give answer:
(a) if the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
(b) if the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
(c) if the data either in statement I alone or in statement II alone in sufficient to answer the question.
(d) if the data even in both the statements I and II together are not sufficient to answer the question.
(e) if the data in both the statements I and II together are necessary to answer the question.

28. What is the height of a triangle?
I. It is a right-angled triangle.
II. The area of the triangle is 5 times its base.

29. How much time will Dinesh take to walk a km distance?
I. The ratio of the speeds at which Dinesh and Ranjay walk is 5 : 6.
II. The average walking speed of Dinesh and Ranjay is known.

30. Is a two-digit integer 'x' divisible by 12?
I. When 'x' is divided by 5, the remainder is 2.
II. When 'x' is divided by 3, the remainder is 1.

31. Is the average of a, b and c equal to b?
I. $b - a = c - b$
II. a, b and c are positive integers.

32. What is the monthly income of Rahim?
I. Total monthly income of Rahim and Suresh is ₹ 27000, which is 150% of their total monthly expenditure.
II. The ratio of their monthly expenditures is 5 : 4.

Directions (33-36):
Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and give answer:

(a) If the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question
(b) If the data in Statement II alone are sufficient to answer the question, while the data in Statement 1 alone are not sufficient to answer the question
(c) If the data in Statement I alone or in Statement II alone are sufficient to answer the question
(d) If the data in both the Statements I and II are not sufficient to answer the question
(e) If the data in both the Statements I and II together are necessary to answer the question

33. In how many days will five women complete a work?
I. Two women, five boys and three men together complete the work in eight days.
II. Six women complete the same work in sixteen days.

34. What is Sonu's monthly income ?
I. Rita's monthly income is ₹ 5000 more than Sonu's monthly income.
II. Sonu's monthly income is ₹ 8000 more than Sohan's monthly income.

35. What will be Pravin's age after 4 years?
I. The ratio between Pravin's and Shekhar's present age is 2 : 3 respectively.
II. Shekhar is six years older than Pravin.

36. What is the area of a circle?
I. The radius of the circle is equal to length of a rectangle.
II. The breadth of the rectangle is equal to 22 cms.

LEVEL - 2

Directions (1-2):
are followed by two statements labeled as I and II. You have to decide if these statements are sufficient to conclusively answer the question. Choose the appropriate answer from options given below:

(a) If statement I alone is sufficient to answer the question.
(b) If statement II alone is sufficient to answer the question.
(c) If statement I and statement II together are sufficient but neither of the two alone is sufficient to answer the question.
(d) If either statement I or statement II alone is sufficient to answer the question.
(e) Both statement I and statement II are insufficient to answer the question.

1. The base of a triangle is 60 cms, and one of the base angles is 60°. What is length of the shortest sideof the triangle?
(I) The sum of lengths of other two sides is 80 cms.
(II) The other base angle is 45°.

2. A, B, C, D, E and F are six integers such that $E < F$, $B > A$, $A < D < B$. C is the greatest integer. Is A the smallest integer?
(I) $E + B < A + D$
(II) $D < F$

Directions (3-9):
Each question is followed by two statements, A and B. Answer each question using the following instructions:

(a) if the question can be answered by using one of the statements alone, but cannot be answered using the other statement alone.
(b) if the question can be answered by using either statement alone.
(c) if the question can be answered by using both

statements together, but cannot be answered using either statement alone.

(d) if the question cannot be answered even by using both statements together.

3. How many people are watching TV programme P?

A. Number of people watching TV programme Q is 1000 and number of people watching both the programmes, P and Q, is 100.

B. Number of people watching either P or Q or both is 1500.

4. Triangle PQR has angle PRQ equal to 90°, what is the value of PR + RQ?

A. Diameter of the inscribed circle of the triangle PQR is equal to 10 cm.

B. Diameter of the circumscribed circle of the triangle PQR is equal to 18 cm.

5. Harshad bought shares of a company on a certain day, and sold them the next day. While buying and selling he had to pay to the broker one percent of the transaction value of the shares as brokerage. What was the profit earned by him per rupee spent on buying the shares?

A. The sales price per share was 1.05 times that of its purchase price.

B. The number of shares purchased was 100.

6. For any two real numbers:

a + b = 1 if both a and b are positive or both a and b are negative.

= –1 if one of the two numbers a and b is positive and the other negative.

What is (2 + 0) Å (–5 + –6)?

A. a + b is zero if a is zero

B. a + b = b + a

7. There are two straight lines in the x-y plane with equations:

ax + by = c

dx + ey = f

Do the two straight lines intersect?

A. a, b, c, d, e and f are distinct real numbers.

B. c and f are non-zero

8. O is the centre of 2 concentric circle. ae is a chord of the outer circle and it intersects the inner circle at points b and d. C is a point on the chord in between b and d. What is the value of ac/ce?

A. bc/cd = 1

B. A third circle intersects the inner circle at b and d and the point c is on the line joining the centres of the third circle and the inner circle.

9. Ghosh Babu has decided to take a non-stop flight from Mumbai to No-man's-land in South America. He is scheduled to leave Mumbai at 5 am, Indian Standard Time on December 10, 2000. What is the local time at No-man's-land when he reaches there?

A. The average speed of the plane is 700 kilometres per hour.

B. The flight distance is 10,500 kilometres.

Directions (10-14):

In each of the following questions, a question is followed by information given in three Statements I, II and III. You have to study the question along with the statements and decide the information given in which of the statement(s) is necessary to answer the question.

10. In how many days 10 women can finish the work?

I. 10 men finish the work in 6 days.

II. 10 women and 10 men finish the work in $3\frac{3}{7}$ days.

III. If 10 men work 3 days and after that 10 women are deployed to work for men, the rest work is finished in 4 days.

(a) I and II (b) Any two of three

(c) I and III (d) II and III

(e) None of these

11. What is the present age of Sabir?

I. The present age of Sabir is half of his father's age.

II. After five years the ratio of ages of Sabir and his father is 6 : 11.

III. Sabir is younger to his brother by five years.

(a) I and II (b) I and III

(c) II and III (d) All of these

(e) Cannot be determined

12. What is two digit number?

I. The difference between the number and the number formed by interchanging the digit is 27.

II. The difference between two digits is 3.

III. The digit at unit's place is less than that at tenplace by 3.

(a) I and II (b) I and either II or III

(c) I and III (d) All of these

(e) None of these

13. What is the rate of interest Percent per annum?

I. An amount doubles itself in 5 yrs on simple interest;

II. Difference between the compound interest and the simple interest earned on a certain amount in two years is ₹ 400.

III. Simple interest earned per annum is ₹ 2000.

(a) Only I (b) II and III

(c) Any two of three (d) I or II and III

(e) Only I or II and III

14. What is the cost of flooring the rectangular hall?

I. Length and the breadth of the hall are in the ratio of 3 : 2

II. Length of the hall is 48 m and cost of flooring is ₹ 850 per sq m.

III. Perimeter of the hall is 160 m and cost of flooring is ₹ 850 per sq m.

(a) I and II (b) I and III

(c) Only III (d) I and either II or III

(e) Any two of these three

Directions (15-19):

In each of these questions, one question is given followed by data in three statements I, II and III. You have to study the question and the data in statements and decide the question can be answered with data in which of the statements and mark your answer accordingly.

15. What is the rate of interest pcpa?

Statements:

i. The difference between the compound interest and simple interest earned in two years on the amount invested is ₹100.

ii. The amount becomes ₹19,500 in three years on simple interest.

iii. The simple interest accrued in two years on the same amount at the same rate of interest is ₹3,000.

(a) Only I and II (b) Only I and III

(c) Only II and III

(d) Only I and either II or III

(e) None of these

16. What is the speed of the train in kmph?

Statements :

i. The train crosses an 'x' metre-long platform in 'n' seconds.

ii. The length of the train is 'y' metres.

iii. The train crosses a signal pole in 'm' seconds.

(a) Any two of the three

(b) Only II and III

(c) Only I and III (d) All I, II and III

(e) Either I and II or II and III

17. How many students passed in first class?

Statements :

i. 85% of the students who appeared in examination have passed either in first class or in second class or in pass class.

ii. 750 students have passed in second class.

iii. The number of students who passed in pass class is 28% of those passed in second class.

(a) All I, II and III (b) Only I and III

(c) Only II and III

(d) Question cannot be answered even with information in all three statements.

(e) None of these

18. What is the amount invested in Scheme 'B'?

Statements :

i. The amounts invested in Schemes 'A' and 'B' are in the ratio of 2 : 3.

ii. The amount invested in Scheme 'A' is 40% of the total amount invested.

iii. The amount invested in Scheme 'A' is ₹45,000.

(a) Only I and II (b) Only I and III
(c) Only II and III (d) All I, II and III
(e) Only III and either I or II

19. What is the cost of flooring a rectangular hall?

Statements :

i. The length of the rectangle is 6 metres.

ii. The breadth of the rectangle is two-thirds of its length.

iii. The cost of flooring the area of 100 cm^2 is ₹45.

(a) Only I and III (b) Only II and III
(c) All I, II and III
(d) Question cannot be answered even with data in all three statements.
(e) None of these

Directions (20-24):

Each question below is followed by two statements I and II. You are to determine whether the data given in the statement is sufficient to answer the question. You should use the data and your knowledge of Mathematics to choose between the possible answers.

Give answer—

(a) If the statement I alone is sufficient to answer the question, but the statement II alone is not sufficient.

(b) If the statement II alone is sufficient to answer the question, but the statement I alone is not sufficient.

(c) If either the statement I alone or the statement II alone is sufficient to answer the question.

(d) If you cannot get the answer from the statements I and II together, but need more data.

(e) If both statements I and II together are needed to answer the question.

20. What is the value of A + B?

I. A = 50% of B

II. 2A + 2B = 80

21. How fast can Ramesh row a boat in still water?

I. He takes 1 minutes to row upstream for a distance of 120 m.

II. He takes 20 seconds to row downstream for a distance of 120 m.

22. Find the rate of simple interest.

I. The principal triples in 9 years.

II. The principal is Rs 1000.

23. What is the area of rectangle ABCD?

I. Length of its diagonal is 26 m.

II. Perimeter is more than 40 m.

24. What is the value of 2A + 3B?

I. A + 3B = 60

II. A is twice of B.

Directions (25-29):

Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read the question and both the statements and give answer

(a) If the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) If the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) If the data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) If the data even in both the statements I and II together are not sufficient to answer the question.

(e) If the data in both the statements I and II together are necessary to answer the question.

25. A train running at a certain speed crosses a platform in 30 secs. What is the speed of the train?

I. Length of the train is 240 m.

II. The train crosses a man who is on the platform in 12 seconds.

26. The weights of A, B and C are in the ratio of 8 : 7 : 5. A's weight is 60% more than C's. Find the weight of B.

I. Total weight of A, B and C is 300 kg.

II. Difference between A's and C's weight is 45 kg.

27. A shopkeeper sells some articles, taking profit of 20% on the cost price. Find the amount of profit.

I. Cost price of the article is Rs. 2400.

II. Selling price of the article is Rs. 480 more than the cost price.

28. A 540m-long plot of rectangular land is to be fenced. Find the cost of fencing per square metre.

I. Breadth of the rectangular plot is 60 m.

II. Length of the rectangular plot is 120 m.

29. The difference between the simple interest and the compound interest on a sum of money at the end of four years is ₹ 1282. Find the sum.

I. Amount of simple interest accrued after four years is ₹ 4000.

II. Rate of interest is 10% per annum

Directions (30-34) :

In each of the following questions, a question is followed by information given in three Statements I, II and III. You have to study the question along with the statements and decide the information given in which of the statement(s) is necessary to answer the question.

30. What is the area of a circle?

I. Radius of circle is 70 cm.

II. Circumference of circle is 440 cm.

III. Perimeter of rectangle = 40 cm

(a) I and II statements alone

(b) Any two of three

(c) I and III

(d) II and III

(e) None of these

31. What is the present age of Sanjay?

I. The present age of Sanjay is half of his father's age.

II. After five years, the ratio of ages of Sanjay and his father is 6 :11.

III. Sanjay is younger to his brother by five years.

(a) I and II (b) I and III

(c) II and III (d) All of these

(e) Cannot be determined

32. What is the two digit number?

I. The difference between the number and the number formed by interchanging the digit is 27.

II. The difference between two digits is 3.

III. The digit at unit's place is less than that at ten's place by 3.

(a) I and II (b) I and either II or III

(c) I and III (d) All of these

(e) None of these

33. What is the rate of interest per cent per annum?

i. An amount doubles itself in 5 yr on simple interest.

ii. Difference between the compound interest and the simple interest earned on a certain amount in two years is ₹ 800.

iii. Simple interest earned per annum is ₹ 4000.

(a) Only I (b) II and III

(c) Any two of three (d) I and III

(e) Either I alone or II and III

34. What is the cost of flooring the rectangular hall?

i. Length and the breadth of the hall are in the ratio of 3 : 2.

ii. Length of the hall is 96 m and cost of flooring is ₹ 1700 per sq m.

iii. Perimeter of the hall is 320 m and cost of flooring is ₹ 1700 per sq m.

(a) I and II (b) I and III

(c) Only III (d) I and either II or III

(e) Any two of the three

Directions (Qs. 35-39):

Each question below is followed by two statements (I) and (II). You are to determine whether the data given in the statement in sufficient for answering the question. You should use the data and your knowledge of Mathematics to choose between the possible answers. Give answer:

(a) If the statement I alone is sufficient to answer the question, but the statement II alone is not sufficient.

(b) If the statement II alone is sufficient to answer the question, but the statement I alone is not sufficient.

(c) If both statement I and II together are needed to answer the question.

(d) If either the statement I alone or statement II alone is sufficient to answer the question.

(e) If you cannot get the answer from the statement I and II together, but need even more data.

35. What is the three-digit number?

I. Three-fifths of that number is less by 90 of that number.

II. One-fourth of that number is 25% of that number.

36. In how many days 14 men can complete a piece of work?

I. If 18 women can complete the same piece of work in 24 days.

II. If 28 children can complete the same piece of work in 56 days.

37. What is the salary of A, in a group of A, B, C, D and E whose average salary is ₹ 65780?

I. Total of the salary of B and C is ₹ 88545.

II. Total of the salary of D and E is ₹ 59020.

38. What is the profit earned by selling a Watch for ₹ 15675?

I. The cost price of 5 such Watches is equal to selling price of 4 such Watches.

II. 25% profit is earned by selling each Watch.

39. How many boys and girls are there in the family of 7 children?

I. Each boy in the family has as many sisters as brothers.

II. Each girl in the family has as many brothers as sisters.

Directions (40-44):

Each of the questions below consists of a question and two statements numbered I and II are given below it. You have to decide whether the data provided in the statements are sufficient to answer question.

Read both the statements and given answers.

(a) If the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question.

(b) If the data in statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question.

(c) If the data in Statement I alone or in Statement II alone sufficient to answer the question.

(d) If the data in both the Statements I and II are not sufficient to answer the question.

(e) If the data in both the Statements I and II together are necessary to answer the question.

40. What is Naina's share in the profit of ₹ 50,000 earned in the business run by her in partnership with Aparna?

I. Naina invested an amount 150% of the amount invested by Aprna.

II. Amount invested by Aprna is two-third of the amount invested by Naina.

41. What is the two digit number ?

I. Sum of the digits is 6.

II. Digit in the ten's place is double the digit in the unit's place.

42. What is the rate of interest p.c.p.a. ?

I. The amount doubles itself in 10 years.

II. The simple interest accrued in 5 years is ₹ 5,000.

43. In how many days can Q alone complete the work?

I. Q and R together can complete the work in 8 days.

II. P and Q together can complete the work in 12 days.

44. What is the mother's present age ?

I. Present ages of mother and child are in the ratio of 5 : 1 respectively.

II. Four years hence, the ratio of the ages of mother and child will be 17 : 5 respectively.

Directions (45-49):

Each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read the question and both the statements and give answer.

(a) if the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.

(b) if the data in statement II alone are sufficient to

answer the question, while the data in statement I alone are not sufficient to answer the question.

(c) if the data either in statement I alone or in statement II alone are sufficient to answer the question.

(d) if the data even in both the statements I and II together are not sufficient to answer the question.

(e) if the data in both the statements I and II together are necessary to answer the question.

45. Train 'L' running at a certain speed crosses another train 'M' running at a certain speed in the opposite direction in 12 seconds. What is the length of train 'M' ?

I. The length of both the trains together is 450 metres.

II. Train 'L' is slower than train 'M'.

46. Area of a rectangle is equal to the area of a right angled triangle. What is the length of the rectangle?

I. The base of the triangle is 40 cms.

II. The height of the triangle is 50 cms.

47. What will be the total compound interest on a sum after three years ?

I. The interest after one year was ₹100 and the sum will be ₹ 1,000.

II. The difference between simple interest and compound interest on a sum of ₹ 1,000 at the end of two years was ₹ 10.

48. What is the two digit number, where the digit at the unit place is smaller ?

I. The difference between the two digits is 5.

II. The sum of the two digits is 7.

49. What is the speed of the boat in still water?

I. It takes 2 hours to cover distance between points J and K downstream.

II. It takes 4 hours to cover distance between points J and K upstream.

Directions (50-54) : The following questions are accompanied by three statements A, B and C. You have to determine which statement(s) is/are necessary/ sufficient to answer the question.

50. P, Q and R together invested an amount of ₹ 42000 in the ratio of 4 : 3 : 7 for different periods of time. What was the amount of profit earned by them individually at the end of one year?

a. They invested for periods in the ratios of 1 : 2 : 1.

b. R's profit is ₹ 5500 less than Q's investment.

c. Total amount of profit at the end of one year is ₹8800.

(a) Only A and B together

(b) Only A and C together

(c) A and either B or C

(d) All statements are required

(e) Question can't be answered even after using all the informations

51. What was the sum of the ages of the mother and the son ten years earlier?

a. Ratio of the mother's present age to the son's age after 10 years is 5 : 3.

b. The difference between thrice the present age of the mother and five times the present age of the son is 50 years.

c. The ratio of the age of the mother after 10 years to the age of the son after 16 years is 5 : 3.

(a) Any two of them

(b) Any one of them

(c) B and either A or C

(d) All statements are required

(e) Question can't be answered even after using all the informations

52. What will be the cost of fencing a rectangular plot?

a. Cost of fencing a circular plot whose area is 616m^2 is ₹ 968.

b. Perimeter of the rectangular plot is 200 m.

c. Perimeter of the square whose length is equal to the breadth of the rectangular plot is 20 m.

(a) Only C (b) A and C together

(c) A and B together (d) A and either B or C

(e) Question can't be answered even after using all the informations

53. What is the cost price of an article?

a. After allowing a discount of 10% on marked price the shopkeeper charges ₹ 810.

b. If the shopkeeper does not give the discount the shopkeeper gets a profit of 50%.

c. If the shopkeeper gives only 5% discount on marked price, he will have 42.50% profit.

(a) Any two of them
(b) Only A and B together
(c) Only B and C together
(d) A and either B or C
(e) Only A

54. What is the rate of interest at which Binod has invested money?

a. The compound interest at this rate of ₹ 2500 in 2 yrs is equal to the simple interest in 3 yrs of ₹ $1716\frac{2}{3}$ at the same rate.

b. The total simple interest on an investment of ₹ 12000 for 3 yrs and ₹10000 for 5 yrs at this rate is ₹5160.

c. In 3 yrs ₹1500 at the same rate becomes ₹1725 by simple interest.

(a) Any of them (b) Either B or C
(c) Only C
(d) Only A and either B or C
(e) Any two of them

55. The following question is followed by information given in three Statements I, II and III. You have to study the question along with the statements and decide the information given in which of the statement(s) is necessary to answer the question.

What is the rate of interest Percent per annum?

I. An amount doubles itself in 5 yrs on simple interest;

II. Difference between the compound interest and the simple interest earned on a certain amount in two years is ₹ 400.

III. Simple interest earned per annum is ₹ 2000.

(a) Only I (b) II and III
(c) Any two of three (d) I or II and III
(e) Only I or II and III

GP_3386

Hints and Solutions

Level-1

1. (a) From statement (i) alone digits are 1, 2 and 3. Since sum of digit is 6 hence it can't be divisible by 9 hence this statement alone is sufficient enough.

2. (d) Here in this case either statement is sufficient enough to answer the question.

From statement (i) and (ii) alone we are sure that the number is prime

3. (c) From statement (1) alone: – since $46 + 47 + 48 + 49 + 50 = 240$ hence 1st of these natural numbers must be more than 45.

Similarly from statement (ii) alone: – since $50 + 51 + 52 + 53 + 54 = 260$ hence smallest of these numbers must be less than 50.

From both the statements together we are sure that one of the number must be 50 hence product must be divisible by 25.

4. (c) Odd number of factors means number should be perfect square

From statement (i) alone we can't sure about the number is perfect square or not.

From statement (ii) alone number n must be one of 62, 63, 64, 65, or 66, out of these only 64 is a perfect square.

If we combine both the statements then only 64 is a number that is divisible by only one prime number and it is a perfect square hence by using both the statements we can answer the question.

5. (c) From statement (i) alone: If a number has 12 factors then possible ways of writing n is a^{11}, a^1b^5, a^2b^3, $a^1b^1c^2$

Then N^2 must be one of a^{22}, a^2b^{10}, a^4b^6, $a^2b^2c^4$ and corresponding number of factors of N^2 is 23, 33, 35, 45, hence statement (i) alone is not sufficient enough.

From statement (ii) alone number n has three prime factors but from this we can not say anything about N.

If we combine both the statements then number must be in the form of $a^1b^1c^2$ or $N^2 = a^2b^2c^4$ has $3 \times 3 \times 5$ has 45 factors.

6. (e) From statement (i) alone the pairs are

(1, 30), (2, 15) (3, 10) and (5, 6) now from 2nd condition we may short the pairs as (1, 30) and (3, 10) but from this we can not conclude a definite relation between A and B, hence this statement alone is not sufficient enough.

If LCM of A and B is 30 then possible pair is (1, 30), (2, 15) (3, 10) (5, 6) (2, 30)

(3, 30) (5,30) (6, 30) (10, 30) and (15, 30)

Now if A^B is odd then A must be odd, this statement alone not sufficient enough as in (1, 30) here $A = 1$ and $B = 30$ while in

(2, 15) A = 15 and $B = 2$.

Even by using both the statement we can not find relationship between A and B.

7. (c) From statement (i) alone, two numbers

$A = 4x$ and $B = 4y$, $a + b = 4(x + y)$ and $A - b = 4(x - y)$ in this case since we don't know the HCF of $(x + y)$ and $(x - y)$ so we can not find HCF of A and B.

From statement (ii) alone, we can not conclude any thing.

If we combine both the statements then LCM = $4xy = 36$ or $xy = 9$ so possible pair of $(x, y) = (1, 9)$ another pair (3, 3) is ruled out as A cant be equal to B so possible value of $A + B$ is 40 and corresponding value of $A - B$ is 32 and their HCF is 8.

8. (e) From statement (i) alone we have following cases: –

$(x, y) = (2, 3)$

$x^y -> 2^3 -> 8$ or $3^2 -> 9$

We will not get unique value of unit digit hence statement (i) alone is not sufficient enough.

Statement (ii) alone y is a multiple of 10.

If we use both the statement then also we will not get a unique value of unit digit.

9. (b) We know that cyclicity of unit digit of 7 is four hence we will divide the given power by 4 if remainder is 1, 2, 3 and 0 then unit digit is same as that of 7^1, 7^2, 7^3, or 7^4.

So in this question we have to find the remainder when '*abcd*' is divided by 4 i.e when two digit number '*cd*' divided by 4.

From statement (i) alone two digit number 'ad' is divisible by 4. Or we can say that d must be an even number. This statement alone is not sufficient enough.

From statement (ii) alone since '*abcd*' divided by 72 it leaves remainder 1 hence

when it is divided by 4 it leaves remainder 1 and so required unit digit is 7. Statement (ii) alone is sufficient enough.

10. (d) Since cyclicity of remainder of $[3^x/7]$ is 6, hence we have to divide x by 6 and find the remainder. From statement (i) alone when x is divided by 6, we won't get a unique remainder.

Similarly from statement (ii) alone we cannot get A unique remainder. If we use both the statement together, then from L.C.M. type 2 x must be in the form of p (L.C.M. of 324) – 2 i.e. x = 12 p – 2. When any number in this format divided by 6 will give us remainder – 2 (or 4). Final remainder is same as that of 4th point of cycle or that of $[3^4/7]$ which is 4. Hence both the statements are required to answer the question.

11. (b) Cyclicity of remainder of $[5^x/9]$ is 6. From statement (i) alone 2 digit prime no is in the form of $6k \pm 1$ hence we cannot find remainder when x divided by 6, so this statement alone is not sufficient enough.

From statement (ii) alone number is in the form of $12k + 9$; it is when divided by 6 will give remainder 3. Hence this statement alone is sufficient enough.

12. (d) From statement (i) alone since a, b and c is a two digit number hence unit digit of their factorial must be 0, and hence required unit digit is 0, so this statement alone is sufficient enough.

From statement (ii) alone a, b and c is a prime number except 2 and 3 hence unit digit of their factorial must be 0 and hence required unit digit is zero. Hence statement (ii) alone is sufficient enough.

13. (a) From statement (i) alone since a b hence a! must be divisible by b! and hence remainder is zero, this statement is sufficient enough.

From statement (i) alone since a b if a = b then remainder of [a!/b!] is zero and when a < b then remainder depends on the value of a and b, hence this statement alone is not sufficient enough.

14. (c) From statement (i) alone $63 < N < 72$, if $63 < N < 70$, then highest power of 7 is 10 and if $70 < N < 72$ then highest power of 7 in N! is 11, hence this statement alone is not sufficient enough.

From statement (ii) alone $55 \le N < 65$, if N = 55 then highest power of 7 is 8, if $56 < N < 63$ then highest power of 7 is 9 and if

$63 < N < 65$ then highest power of 7 in N! is 10, hence this statement alone is not sufficient enough.

If we combine both the statements then we will get $63 < N < 65$ and in any case highest power of 7 in N! is 10, hence we can answer the question by using both the statements.

15. (b) From statement (i) alone it is not known that a is a two digit number or not (as factorial of a two digit number will have last two digits as 00) hence this statement alone is not sufficient enough.

Statement (ii) alone is sufficient enough.

16. (e) From statement (i) and (ii) alone no information is given about third number it may be integer or it may be fraction.

If we combine both the statements then we have 2 even integers and we may have 1 or none even negative integers, hence even by using both the statements we cannot answer the question.

17. (a) Here we have to find the minimum value of A/B + B/A let us assume that A/B = t, then A/B + B/A = t + 1/t, we know that its minimum value is 2 if t is positive i.e. A/B is positive i.e. either both of them are positive or both of them are negative.

From statement (i) alone since AB > 0 hence either both of them are positive or both of them are negative hence this statement alone is sufficient enough to find the answer.

From statement (ii) alone at least one of them is positive from this we cannot be sure about the parity of A/B.

18. (d) If we look at Statement I then we will get

If a = 3 and b = 2, a + b > 0. Here b > 0

If a = 3 and b = –2, a + b > 0. Here b < 0

Hence I alone is not sufficient.

Now if we look at Statement II only then we will get

If a = 3 and b = 2, a – b > 0. Here b > 0

If a = 3 and b = –2, a – b > 0. Here b < 0

Hence II alone is not sufficient.

Now by using statements and together

If a = 3 and b = 2, a – b > 0 and a + b > 0. Here b > 0

If a = 3 and b = –2, a – b > 0. and a + b > 0. Here b < 0.

Hence I and II together are also insufficient.

19. (c) If we look at Statement I

i = p – 17 and r = p – 103

Hence, we cannot find how many each received so this statement is not sufficient enough.

GP_3386

Now by considering Statement II alone.

$p + i + r = 170$

Hence, we cannot find how many each received. so this statement is not sufficient enough

Using I and II together, we get

$p + (p - 17) + (p - 103) = 170.$

Solving the above equation we get the value of p and the values of q and r.

20. (b) If we look at Statement I

It is given that the circles are concentric. But nothing is given about their dimensions.Hence alone is not sufficient.

In statement II ratio of area is given hence we can find the required ratio.

21. (b) Let the 7 consecutive whole numbers be $(n \pm 3), (n \pm 2) (n \pm 1), n$.

Now if we consider Statement I alone

Product of these 7 integers = 702800

Since $702800 = 24\ 52\ (251)(7)$, it cannot be the product of 7 consecutive whole numbers.Hence I alone is insufficient.

Now if we consider Statement II alone

Given that their sum = $105 = 7n$ or $n = 15$ and then 7 consecutive integers are 12, 13, 14, 15, 16, 17, 18

So, II alone is sufficient.

22. (a) Since sum is 360 hence $P + Q + R + S = 360$

From statement I alone we will get

$P = (Q + R + S)/3$ from this we can find the value of P hence statement I alone is sufficient enough.

From statement II alone we can not find the value of P.

23. (d) Statement is not sufficient as the size of the ice cube and height of the container is not known hence statement I is not sufficient alone.

Statement is also not sufficient as the dimension of the container is not known.

We cannot answer the question even by combining both the statements as the height of the container is not known.

24. (b) It is given that Ram got a dividend of ₹ 1500.

Statement I

Knowing the dividend paid last year, we cannot find the dividend paid this year.

Statement

Given that Ram bought 350 shares of face value ₹ 10, and so, their total face value is ₹ 3500. So here we know the investment and the return hence we can find out the rate of interest.

25. (c) A alone : X may be greater than Z. But doesn't say anything about Z being smallest.

B alone : Y may be greater than Z. But says nothing about Z being smallest

Now we will use both **statement A** and **B**

If $X > Y$, then $Y > Z$ and $X > Y > Z$

But if $X > Z$, then $Y > X$ and $Y > X > Z$

In either of these two cases, Z is the least and hence the question can be answered.

26. (a) From A alone: $x(x + 3) < 0$ hence $0 > x > -3$

Hence, $|x|$ is necessarily less than 3 in this range.

From B alone : $x(x - 3)$ hence $x > 3$ or $x < 0$

In this condition, any given $|x|$ value need not necessarily be less than 3. So B is not sufficient enough.

27. (d) From statement A alone it is given that $x - y = 6$ or $y - x = 6$, from this we cannot find out a unique value of x or y, hence this statement is not sufficient enough.

From statement B $xy = 6k$, again we cannot find out a unique value of x or y, hence this statement is also not sufficient enough.

Now if we use both the statement together then, let $x - y = 6$ and $xy = 6k$, or $x = 6k/y$ or $6k/y - y\ 6$ or $y2 + 6y = 6k$, and again we will not get a unique value of x and/or y.

28. (e) From statements I and II, since it is a right-angled triangle, area

$$\Rightarrow \frac{1}{2} \times b \times h = 5b \qquad \therefore h = 10$$

29. (e) Combining I & II, we get speed of Dinesh = 10 km/hr

$$\therefore \text{ to walk 1 km he takes } \frac{60}{10} = 6 \text{ minutes.}$$

30. (b) From statement II, we find the negative answer since the number is not divisible by 3 then it can't be divisible by 12.

31. (a)

32. (d) Using both the statements, we can find separate expenditures of Rahim and Suresh but can't find their separate incomes.

33. **(b)** From statement I, no equivalence relation among a woman, a boy and a man can be established.

From statement II

$M_1D_1 = M_2D_2$

$\Rightarrow 6 \times 16 = 5 \times D_2$

$\Rightarrow D_2 = \frac{6 \times 16}{5} = \frac{96}{5} = 19\frac{1}{5}$ days

34. **(d)** From both statements,

Rita = Sonu+5000

Sonu = Sonhan+8000

Two equation and three unknowns.

Hence, answer can't be determined.

35. **(e)** From statement I, and II,

Pravin's present age = 2x years

Shekhar's present age = 3x years

$3x = 2x + 6 \Rightarrow x = 6$

Thus, Pravin age after 4 years

$= 2x + 4 = 2 \times 6 + 4 = 16$ years

36. **(d)** From both the statements, answer can't be determined.

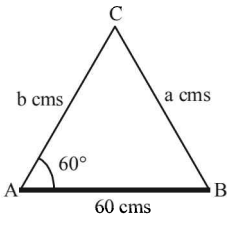
Radius = Length of rectangle

Breadth of rectangle = 22 cms

We get no information about the length of rectangle.

Level-1

1. **(d)**



Let, a cm and b cm are the the two unknown sides as shown in the fig.

From statement I, a + b = 80 cm, hence

b = (80 – a) cm

Now using cosine rule.

$\text{Cos } 60° = (AB^2 + AC^2 - CB^2)/2AB$

$\therefore \frac{1}{2} = [60^2 + b^2 - (80 - b)^2]/120$

By solving this we get, b = 28 cm. Hence, statement I is sufficient to answer.

From statement II,

Since $\angle B = 45°$ hence $\angle C = 75°$

According to sine rule: we know that a/sin a = b/sin b = c/sin c = a/sin45° = b/sin60°= 60/ sin75°

From this we can find the value of the sides.

Hence statement II is sufficient to answer the question.

2. **(a)** **From statement I,**

E + B < A + D, we easily say that E is less than A, because B > D and as the statement suggest E + B < A + D.E < A.

∴ A is not the smallest integer.

Statement I is sufficient to answer.

From statement II, D < F

This statement is not sufficient to find the relation between A and E.

3. **(c)** Consider A alone : from the formula $(P \cup Q) = n(P) + n(Q) - n(P\ Q)$

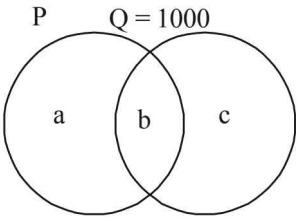
We get $n(P \cup Q) = n(P) + 1000 - 100$. From A alone, we get 2 unknowns and 1 equation. Hence A alone is not insufficient enough.

Consider B alone: $n(P \cup Q) = 1500$ given, but from Balone, n(P) cannot be determined.

Consider both together : 1500 = n(P) + 1000 – 100 AND n(P) = 1500 – 900 = 600

Hence both the statement required to solve the question.

Alternately- From statement A alone



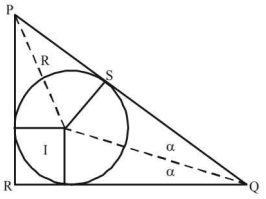
It is given that b + c = 1000 and b = 100 hence c = 900 but we cannot calculate a, so this statement is not sufficient enough.

From statement B alone a + b + c = 1500, and this statement is also not sufficient enough to find the value of a + b

GP_3386

If we use both the statement then we can calculate the value of a + b

4. (c)



Statement A: Gives us the radius of inscribed circle = 5, but with this information we cannot calculate PR & QR.

Statement B : Gives hypotenuse PQ = 18 cm (dia of circumcircle).

Now, $\angle P + \angle Q = 180° - 90° = 90°$

So $\angle P/2 + \angle Q/2 = 45°$ but this is not sufficient enough.

Now, If IS = 5cm (in radius = 5), then

Using both the information we can calculate

5. (a) Consider A alone : Given S.P / C.P = 1.05 and brokerage of 1%

Let CP =100x brokerage = x so total CP = 101x, and SP = 105x

A is sufficient.

Consider B alone : No. of shares = 100, since no price is given, profit cannot be calculated hence this statement is not sufficient enough.

6. (c) From the main information a@b is not define when a or b is zero.

Consider A alone : from this information we see that

a @ b = 0 only if a is zero.

However in the expression (2 + 0) : a 0. A is not sufficient.

B alone is not sufficient enough as it does not provide important information.

Consider A and B together a @ b = b @ a hence 2 @ 0 = 0 @ 2

So (2 @ 0) = 0 and (–5 @ –6) = 1

Hence A and B together is sufficient enough.

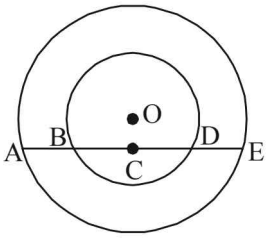
7. (d) Two straight line intersect if they are not parallel hence if a/d ≠ b/e

Now consider statement A alone, it does not say specifically that a/d ≠ b/e

From statement B alone, if c and f both are zero then also we can't say that a/d ≠ b/e

Even with both the statement together we cant determine whether the two straight line intersect or not.

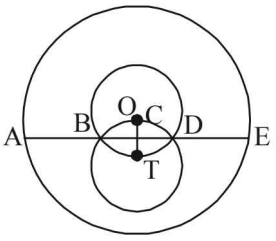
8. (b) **From statement A**



We see that BC = CD or C is at the center of B and D

So C will also be the centre of A and E, Hence (A) is sufficient to answer.

From Statement (B)



Let T be the center of the third circle. Then since C is the line joining O and T implies C is midpoint of B and D or bisects B and D.

Similar to last case now we can say that this statement is also sufficient enough.

9. (d) Consider statement A alone : Avg. speed of plane says nothing about local time in no-mans land. Statement B alone : Distance also says nothing about the local time.

Now if we use both the statement

together : Time required for flight can be obtained but still we cannot find the local time.

10. (b) From I and II, 10 women can finish the work in 1 day

$$= \frac{7}{24} - \frac{1}{6} = \frac{7-4}{24} = \frac{1}{8}$$

$\therefore$ 10 women can finish the work in 8 days.

From II and III,

Let 10 men can finish the work in x days and 10 women can finish the same work in y days.

Hence, $\frac{1}{x} + \frac{1}{y} = \frac{7}{24}$ ... (i)

and from III – II, $\frac{3}{x} + \frac{4}{y} = 1$ ... (ii)

from (i) & (ii)

y = 8 days

Again from I and III $\frac{3}{6} + \frac{4}{y} = 1 \Rightarrow y = 8$ days

11. (a) From I, Let present age of Sabir be x yr and age of his father be 2x yrs.

From I and II, $\frac{x+5}{2x+5} = \frac{6}{11}$

$\Rightarrow 12x + 30 = 11x + 55.$

x = 25 yrs

From I and II, age of Sabir = 25 yrs.

Hence, only from I & II, age of Sabir and his father can be obtained.

12. (e) Let two digit number be 10x + y.

From I, either $x - y = \frac{27}{9} = 3$

$\Rightarrow y - x = \frac{27}{9} = 3$

From II, $x - y = 3 \Rightarrow y - x = 3$

From III, $x - y = 3$

Hence, Even by (I) + (II) + (III) we cannot obtain the number.

13. (e) From I, If P = 100

A = 200 and SI = 200 – 100 = 100

$$\text{Rate} = \frac{SI \times 100}{P \times T} = \frac{100 \times 100}{100 \times 5} = 20\%$$

From II and III, $\text{Rate} = \frac{400 \times 100}{2000 \times 1} = 20\%$

Hence, either I alone or II + III will be sufficient.

14. (e) From I and II.

Length = 3x = 48 m

$\therefore$ x = 16

Breadth = 2x = 32 m

Hence, Area of floor = 48 × 32

Cost of flooring = 48 × 32 × 850 = ₹ 1305600

From I and III, 2(l + b) = 160

$\Rightarrow 2(3x + 2x) = 160 \Rightarrow 10x = 160$

$\therefore$ x = 16

$\therefore$ Length = 3 × 16 = 48 m

Breadth = 2 × 16 = 32 m

Cost of flooring = (48 × 32) × 850 = ₹ 1305600

Similarly, from II and III, we can find l = 48 m and b = 32 m

and Total cost of flooring = ₹ 1305600

15. (b) **From statement I,**

If the investment amount be ₹ P and rate of interest be R p.c.p.a. then

$$\text{Differenece} = \frac{PR^2}{10000} = 100 \quad \text{...(i)}$$

From statement II,

$$\frac{PR_1 \times 3}{100} = 19500 - P \quad \text{...(ii)}$$

From Statement III,

$$\frac{PR \times 2}{100} = 3000 \quad \text{...(iii)}$$

Dividing equation (i) by (iii),

$$\frac{PR^2}{PR} = \frac{1000000}{150000} = \frac{20}{3}$$

$\Rightarrow R = \frac{20}{3}$ p.c.p.a

GP_3386

16. (e) **From Statement I and II**

$$\text{Speed of train} = \frac{x+y}{n}\ \text{m/s}$$

$$= \left(\frac{x+y}{n} \times \frac{18}{5}\right)\text{km/h}$$

From statement II and III

$$\text{Speed of train} = \frac{y}{m}\text{m/s} = \left(\frac{y}{m} \times \frac{18}{5}\right)\text{km/h}$$

17. (d) Using statement II and III, we can find the number of students in second class and pass class only. As there is no link given between the first class and the other classes, we cannot find the number of students in first class.

18. (e) **Using Statement I :**

$$\frac{A}{B} = \frac{2}{3}$$

Using Statement II :

A is 40% of total. So B is 60% of total amount invested.

$$\frac{A}{B} = \frac{40}{60} = \frac{2}{3}$$

Using statement III:

A = 45000

Putting the value of statement III in any of the statements I or II, we can find the amount invested in scheme B.

19. (c) Using statement I and II we can find the area of the rectangle and using statement III we can find the cost.

20. (b) 2(A + B) = 80

A + B = 40

21. (e) Using I:

Speed upstream = 120/60 = 2 m/s

Using II:

Speed downstream = 120/20 = 6 m/s

Using both:

Speed of boat in still water = 2 m/s

22. (a) Using I:

Let Principal be ₹ x.

It become ₹ 3x in 9 years, so interest gained in 9 years is ₹ 2x. So,

S.I = PRT/100

$\Rightarrow (x \times r \times 9)/100 = 2x$

$\Rightarrow r = 200/9\ \%$

23. (d) Data are insufficient.

24. (e) Both statements are needed to find the value of 2A + 3B.

25. (e) Combining data of both the statements the speed of the train can be determined.

26. (c) From -(I)

Weight of B = (300×7)/20 = 105

From -(II)

Weight of B = (45×7)/3 = 105

27. (c) From -(I)

Amount of profit = 20% of 2400 = 480

From -(II)

SP – CP = 480 = Amount of profit

28. (d) Cost of fencing per square meter is not given. So, even combined both the statements together, total cost of fencing can't be determined.

29. (b) From II alone,

Let the sum be P

$$\Rightarrow \left[p\left(1+\frac{10}{100}\right)^4 - p\right] - \frac{p \times 10 \times 4}{100} = 1282$$

$\Rightarrow p = 2000$

30. (b) Answer can be derived by using I and II of the statements alone.

31. (a) From I and II

Sanjay age can be obtained.

32. (e) All the statements are giving same information, even combining all three statements, we cannot obtain the number.

33. (e) From I and II

R = (S.I×100)/(P×T) = (100×100)/(100×5)

= 20%

From II and III

$R = (800 \times 100)/(4000 \times 1) = 20\%$

34. (e) Any two of the three statements are sufficient to answer.

35. (a) From I,

$3/5 \times x = x - 90$

Or, $2x/5 = 90$

Hence, Number, $x = 5 \times 45 = 225$

36. (e) We cannot get the answer from the statement I and II together,

But need even more data.

37. (c) From I and II together, salary of A

$= 5 \times 65780 - (88545 + 59020)$

$= 328900 - 147565 = ₹\ 181335$

38. (d) From I, S.P. of 1 watch=15675 and C.P. of 1 watch

$= 15675 \times 4/5 = 12540$

So, Profit $= 15675 - 12540 = 3135$

From - II

Let CP = 'x'

So, 125% of CP = 15675

CP, (x) = $15675 \times 100/125 = 12540$

Hence, profit $= 15675 - 12540 = ₹\ 3135$

Hence, either statement I alone or statement II alone is sufficient.

39. (d) Either the statement I alone or statement II alone is sufficient to answer the question.

40. (c) The data in statement I alone or in statement II alone are sufficient. We can find the ratio of profit sharing as ratio of amounts of investment can be determined

41. (e) Let the number be $10x + y$.

From statement I,

$x + y = 6$

Many such combinations are possible.

From statement II

$x = 2y$

Taking both the statement together,

$3y = 6 \Rightarrow y = 2$ and hence, $x = 4$

So, number = 42

42. (a) From statement I alone,

Principal = x, R = ?, S.I = x

T = 10 years

$$\therefore \quad R = \frac{\text{S.I.} \times 100}{\text{Principal} \times \text{Time}} = \frac{x \times 100}{x \times 10} = 10\%$$

43. (d) Statement II gives no conclusion.

From statement II,

(P + Q)'s 1 day's work = 1/12

P's 1 day's work is not known.

44. (e) From both the statements,

$$= \frac{5x \times 4}{x \times 4} = \frac{17}{5}$$

$\Rightarrow 25x + 20 = 17x + 68 \Rightarrow 8x = 48 \Rightarrow x = 6$

$\therefore$ Mother's present age $= 5 \times 8 = 40$ years

45. (d) Time taken in crossing each other

= Total length of trains/ Relative speed

The information given in both statements is not sufficient length of train L and individual speed of each train are required.

46. (d) Area of rectangle = Area of triangle.

From the information given in both the statements, we can find area of triangle or area of rectangle. For finding length breadth is required, which is not known.

47. (c) From the statement I,

$$r = \frac{100 \times 100}{1000} = 10\%$$

Thus we have,; P = ₹ 1000, $r = 10\%$

t =3 years

Hence, C.I can be determined

From the statement II,

$$\text{S.I} = \frac{100 \times r \times 2}{100} = 20r$$

$$\text{CI} = 1000\left[\left(1 + \frac{r}{1000}\right)^2 - 1\right]$$

GP_3386

$\therefore$ C.I. – S.I.

$$=1000\left[\frac{200r+r^2}{10000}\right]-20r$$

$\Rightarrow 2000r + r^2 - 200r = 100 \Rightarrow r = 10\%$

Hence, C.I. can be determined

48. (e) Let the unit's digit be x and ten's digit be y and $x < y$.

$\therefore$ Number $= 10y + x$

From statement I,

$y - x = 5$ ...(i)

From statement II,

$y + x = 7$ ...(ii)

From (i) and (ii), x, y can be calculated and two digit number can be found.

49. (d) Let the distance between J and K be z km.

Again, let speed of boat in still water be x kmph and that a stream be y kmph.

$\therefore$ Rate downstream $= (x + y)$ kmph

Rate upstream $= (x - y)$ kmph

From statement I,

$$=\frac{z}{x+y}=2$$

From statement II,

$$=\frac{z}{x-y}=4$$

50. (c) The given question gives the amounts of investment of P, Q and R, which is P = ₹12000, Q = ₹9000 and S = ₹21000. Statement (A) combined with the question's information will give us the ratio of their profits, i.e. 4 : 6 : 7. Now, combine this either with B or C, and profit earned by them can be determined. Therefore, A and either B or C is sufficient.

51. (e) Let the present ages of mother and son be M and S yrs respectively.

a. $\frac{M}{S+10}=\frac{5}{3}$ or, $3M - 5S = 50$

b. $3M - 5S = 50$

c. $\frac{M+10}{S+16}=\frac{5}{3}$ or, $3M - 5S = 50$

Here, all the three equations are the same. Therefore option (e) is the correct choice.

52. (c) Statement (A) gives the cost of fencing one metre of the plot. Combining this with statement (B), total cost of fencing can be determined.

53. (d) A. Market price of the article

$$=\frac{100}{90}\times 810 = ₹900$$

B. CP of the article (with the help of A)

$$=\frac{900\times 100}{150}=₹600$$

Now, combining (A) with (C),

SP $= 900 \times 0.95 = ₹855$

$$CP = 855\times\frac{100}{142.50}=₹600$$

Hence, A and either B or C are sufficient.

54. (a) Let the rate of interest be r%.

A. $$2500\left[\left(1+\frac{r}{100}\right)^2-1\right]=\frac{5150\times r\times 3}{3\times 100}$$

B. $$\frac{12000\times r\times 3}{100}+\frac{10000\times r\times 5}{100}=5160$$

C. $$r=\frac{1725-1500}{3\times 150}\times 100 = 5\%$$

Hence, any one of them is sufficient.

55. (e) From I, If P = 100

A = 200 and SI = 200 – 100 = 100

$$\text{Rate}=\frac{SI\times 100}{P\times T}=\frac{100\times 100}{100\times 5}=20\%$$

From II and III, $\text{Rate}=\frac{400\times 100}{2000\times 1}=20\%$

Hence, either I alone or II + III will be sufficient.